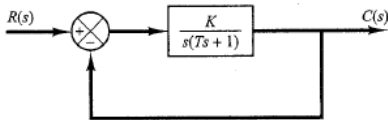


# Brainteasers

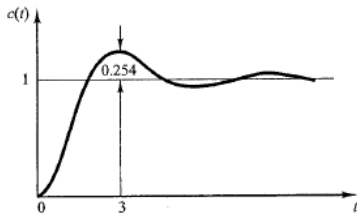
July 10, 2015

# Brainteaser 1

When the system shown in Figure (a) is subjected to a unit-step input, the system output responds as shown in Figure (b). Determine the values of  $K$  and  $T$  from the response curve.



(a)



(b)

# Solution 1

The maximum overshoot of 0.254 corresponds to  $\zeta = 0.4$ .

From the response curve we have  $t_p = 3$ ,

$$t_p = \pi\omega_d = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 3$$

It follows that  $\omega_n = 1.14$

From the block diagram we have  $\frac{C(s)}{R(s)} = \frac{K}{Ts^2 + s + K}$

from which  $\omega_n = \sqrt{\frac{K}{T}}$ ,  $2\zeta\omega_n = \frac{1}{T}$

Therefore, the values of T and K are determined as

$$T = \frac{1}{2\zeta\omega_n} = \frac{1}{2 \times 0.4 \times 1.14} = 1.09$$

$$K = \omega_n^2 T = 1.14^2 \times 1.09 = 1.42$$

Source: Ogata, Modern Control Engineering 4th edition, p.298

## Brainteaser 2

Two linear, time invariant systems are connected in series.  
For a periodic input  $u(t) = \sin(\alpha t)$ ,  
is  $y_{ss} = |H_1(j\alpha)||H_2(j\alpha)|\sin(\alpha t + \angle H_1(j\alpha) + \angle H_2(j\alpha))$  the steady state output ?

Old exam question, answer should be around

# Brainteaser 3

We have found the step-response of a physical system with 1 input and 1 output. This response converges to a constant value. Can we conclude that the system is internal stable?

Old exam question, answer should be around.

# Brainteaser 4

Find the laplace transformation of 
$$\begin{cases} f(t) = 0 & t < 0 \\ f(t) = \sin(\omega t + \theta) & t \geq 0 \end{cases}$$



# Solution 4