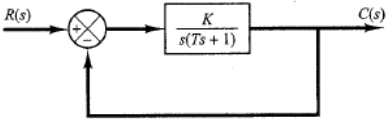
Brainteasers

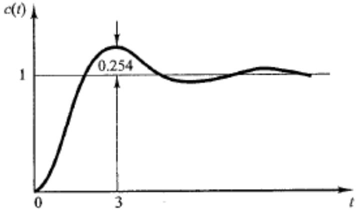
July 10, 2015

Brainteaser 1

When the system shown in Figure (a) is subjected to a unit-step input, the system output responds as shown in Figure (b) Determine the values of K and T from the response curve.



(a)



(b)

Solution 1

The maximum overshoot of 0.254 corresponds to $\zeta = 0.4$.
From the response curve we have $t_p = 3$,
$t_p = \pi\omega_d = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 3$
It follows that $\omega_n = 1.14$
From the block diagram we have $\frac{C(s)}{R(s)} = \frac{K}{Ts^2+s+K}$
from which $\omega_n = \sqrt{\frac{K}{T}}$, $2\zeta\omega_n = \frac{1}{T}$
Therefore, the values of T and K are determined as
$T = \frac{1}{2\zeta\omega_n} = \frac{1}{2x0.4x1.14} = 1.09$
$K = \omega_n^2 T = 1.14^2 x1.09 = 1.42$
Source: Ogata, Modern Control Engineering 4th edition, p.298

Brainteaser 2

Two linear, time invariant systems are connected in series.
For a periodic input $u(t) = sin(\alpha t)$,
is $y_{ss} = |H_1(j\alpha)||H_2(j\alpha)|sin(\alpha t + \angle H_1(j\alpha) + H_2(j\alpha))$ the steady state output ?

Solution 2

Old exam question, answer should be around

Brainteaser 3

We have found the step-response of a physical system with 1 input and 1 output. This response converges to a constant value. Can we conclude that the system is internal stable?

Old exam question, answer should be around.

Brainteaser 4

Find the laplace transformation of $\begin{cases} f(t) = 0 & t < 0 \\ f(t) = sin(\omega t + \theta) & t \geq 0 \end{cases}$

Solution 4

Noting that $sin(\omega t + \theta) = sin(\omega t)cos(\theta) + cos(\omega t)sin(\theta)$
We have $\mathcal{L}\left[sin(\omega t + \theta)\right] =$
$cos(\theta)\mathcal{L}\left[sin(\omega t)\right] + sin(\theta)\mathcal{L}\left[cos(\omega t)\right] = \frac{\omega cos(\theta) + sin(\theta)s}{s^2 + \omega^2}$