Chapter 13 - PID Controllers

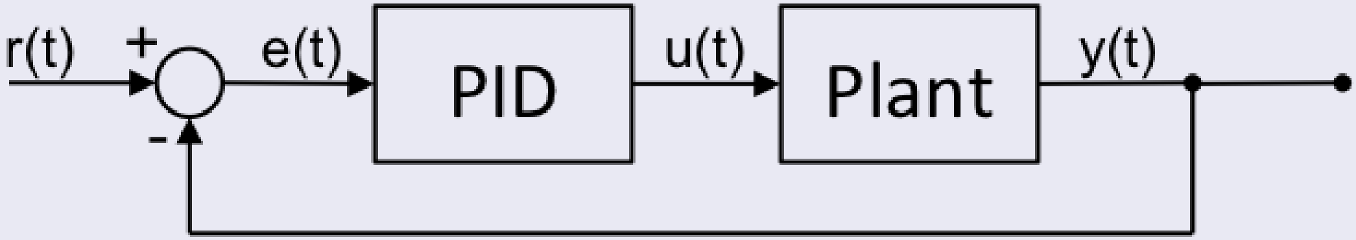
August 7, 2015

Outline

What is a PID controller?

Definition

A **P**roportional-**I**ntegral-**D**eriviative (PID) controller is a control-loop feedback mechanism (controller) widely used in process industry.



Continuous-time text book equation:

$$u(t) = \underbrace{K_p e(t)}_{\text{Proportional Action}} + \underbrace{K_i \int_0^t e(\tau) d\tau}_{\text{Integral Action}} + \underbrace{K_d \frac{de(t)}{dt}}_{\text{Derivative Action}}$$

Note: 90% (or more) of control-loops in process industry are PID.

What is a PID controller?

- **P**roportional action $u_p(t) = K_p e(t)$: it depends on the instaneous value of the error.
  - $+$ Reduces rise time
  - $+$ Reduces but **does not eliminate the steady-state error**: Only when $K \to \infty$, error $\to 0$ (unless the plant has pole(s) at $s = 0$)
- **I**ntegral action $u_i(t) = K_i \int_0^t e(\tau)d\tau$: it is proportional to the accumulated error.
  - $+$ **Eliminates the steady-state error** in some cases
  - $-$ Makes transient response slower
- **D**erivative action $u_d(t) = K_d \frac{de(t)}{dt}$: it is proportional to the rate of change of the error.
  - $+$ Increases the stability of the system, reduces overshoot, improves the transient response
  - $-$ Amplifies the noise present in the error signal

Outline

Proportional Control

The continuous-time and discrete-time implementations are identical.

For the continuous-time case we have:

$$u_p(t) = K_p e(t) \quad \rightarrow \quad \frac{U_p(s)}{E(s)} = K_p$$

and for the discrete-time case:

$$u_p[k] = K_p e[k] \quad \rightarrow \quad \frac{U_p(z)}{E(z)} = K_p$$

where e(t) or e[k] is the error signal.

Derivative Control

In continuous-time it is given by:

$$u_d(t) = K_d \frac{de(t)}{dt} \quad \rightarrow \quad \frac{U_d(s)}{E(s)} = K_d s$$

and in discrete-time by (using backward Euler):

$$u_d[k] = K_d \frac{e[k] - e[k-1]}{T_s} \quad \rightarrow \quad \frac{U_d(z)}{E(z)} = K_d \frac{z-1}{T_s z}$$

with T_s the sampling time.

Integral Control

In continuous-time it is given by:

$$u_i(t) = K_i \int_0^t e(\tau)d\tau \quad \rightarrow \quad \dot{u}_i(t) = K_i e(t) \quad \rightarrow \quad \frac{U_i(s)}{E(s)} = \frac{K_i}{s}$$

and in discrete-time by (using backward Euler)

$$u_i[k] = u_i[k-1] + K_i T_s e[k] \quad \rightarrow \quad \frac{U_i(z)}{E(z)} = K_i \frac{zT_s}{z-1}$$

with T_s the sampling time.

Digital formulation (conventional version)

Digital PID controller (conventional version)

$$u[k] = K_p e[k] + \frac{K_d}{T_s}(e[k] - e[k-1]) + u_i[k]$$

$$\text{with } u_i[k] = u_i[k-1] + K_i T_s e[k]$$

In the $\mathcal{Z}$-domain:

$$\frac{U(z)}{E(z)} = K_p + K_i T_s \frac{z}{z-1} + \frac{K_d}{T_s}\frac{z-1}{z}$$

where $K_i T_s$ and $\frac{K_d}{T_s}$ are the new derivative and gains.

Digital PI controller

$$\frac{U(z)}{E(z)} = K_p + K_i T_s \frac{z}{z-1}$$

Digital PD controller

$$\frac{U(z)}{E(z)} = K_p + \frac{K_d}{T_s}\frac{z-1}{z}$$

Alternative Digital PID controller

If we discretize the continuous-time (analog) PID controller using the bilinear transformation,

$$\frac{U(z)}{E(z)} = K_p + \frac{K_i}{s} + K_d s \bigg|_{s=\frac{2}{T_s}\left(\frac{z-1}{z+1}\right)}$$

we obtain an alternative form for a digital PID controller

$$\begin{aligned}\frac{U(z)}{E(z)} &= K_p + \frac{K_i T_s(z+1)}{2(z-1)} + \frac{2K_d(z-1)}{T_s(z+1)} \\ &= \frac{\alpha_2 z^2 + \alpha_1 z + \alpha_0}{(z-1)(z+1)}\end{aligned}$$

where $\alpha_2, \alpha_1, \alpha_0$ are design parameters.