Outline

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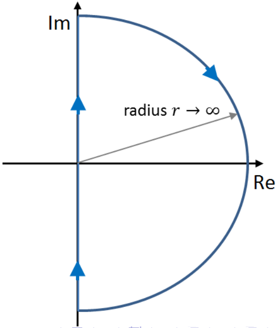
Nyquist plot

We know how to deduce stability of the closed loop system from the Nyquist plot.
Now we will discuss how these plots can be found.

First some simplifications:

1. For physically realizable systems (i.e. relevant systems to engineers) the circle bow will be mapped on a point.
2. The image of the positive imaginary axis is the mirror image of the negative imaginary axis



Nyquist plot: physically realizable systems

1. **Every physically realizable system is causal**
   This is logical: you cannot build a system that knows the future
2. **Every causal system has a transfer function with a degree of the denominator that is larger than or equal to the degree of the numerator**
   Take for example the following transfer function:

$$H(z) = \frac{b_2 z^2 + b_1 z + b_0}{a_0 z + a_1}$$

The corresponding difference equation is:

$$a_0 y_{n-1} + a_1 y_n = b_0 u_{n-1} + b_1 u_n + b_2 u_{n+2}$$

It is non-causal, because the output depends on future input.

Nyquist plot: physically realizable systems

3. **From the transfer function, we can easily show that the circle bow maps onto a point.**
    - If the degree of the denominator is strictly higher than the degree of the numerator:
      If $s \to \infty$, then $P(s)C(s) \to 0$
    - If the degree of the denominator is equal to the degree of the numerator:
      If $s \to \infty$, then $P(s)C(s) \to c$, a real number

Nyquist plot: symmetry

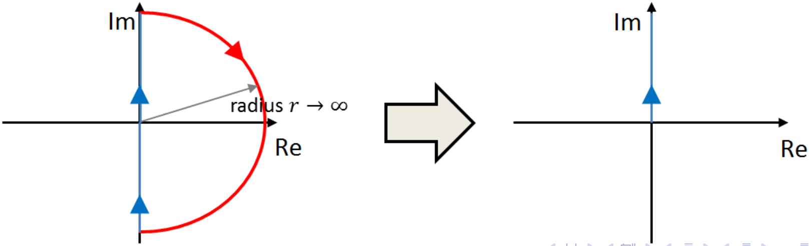
The symmetry follows directly from the way $f(c)$ can be evaluated.

Since the position of $f(c)$ only depends on the location of the poles and zeros, and those poles and zeros only occur symmetrically round the real axis, the Nyquist plot will be symmetrical round the real axis.

Nyquist plot

So we will only have to study the positive (or the negative) imaginary axis!

- The circular part maps onto one point, which is the same as where $j\omega$ maps onto
- The other half of the imaginary axis will give the mirror image of the studied half



Nyquist plot

Extracting the number of times $(-1, 0)$ is encircled isn't that difficult anymore:

- You search for the image of $j0^+$
- You search for the image of $j\infty$
- You search for the positive real y for which $f(jy)$'s imaginary part changes sign

This information allows you to determine if you encircle $(-1, 0)$
Let's visualize this with a simple example, but of course you can use software to do this (e.g. nyquist in Matlab)

Nyquist plot: a simple example

Let's take the following open loop system:

$$P(s)C(s) = \frac{1}{s^2 - 2s + 2}$$

Substitute s with $j\omega$

$$\frac{1}{-\omega^2 - 2j\omega + 2} = \frac{1}{-\omega^2 + 2 - 2j\omega} \frac{-\omega^2 + 2 + 2j\omega}{-\omega^2 + 2 + 2j\omega} = \frac{-\omega^2 + 2 + 2j\omega}{(-\omega^2 + 2)^2 + 4\omega^2}$$

- $f(j0^+) = \frac{1}{2}$
- $f(j\infty) = 0$
- Imaginary part: $\frac{2\omega}{(-\omega^2+2)^2+4\omega^2} = 0 \Rightarrow \omega = 0$

The real axis gets crossed 2 times: first at $\frac{1}{2}(\omega = 0^+)$ and then at $0(\omega = \infty) \rightarrow (-1, 0)$ is not encircled

Nyquist plot: a simple example

Remember our open loop system:

$$P(s)C(s) = \frac{1}{s^2 - 2s + 2}.$$

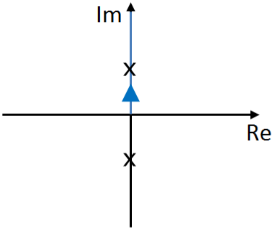
Z and P are respectively the number of zeros and poles of $1 + P(s)C(s)$.
The poles of $P(s)C(s)$ and $1 + P(s)C(s)$ are the same ($P = 2$ in our example).
Since $(-1, 0)$ is not encircled: $Z - P = 0$, hence there are 2 zeros in the right half plane.
Remember that the zeros of $1 + P(s)C(s)$ are the poles of the closed loop system and thus, the unity feedback controller is unstable.

Outline

Nyquist plot: poles on the imaginary axis

Poles on the imaginary axis: why are they a problem?

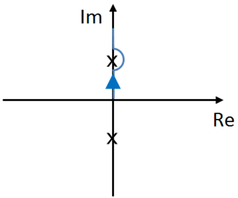
- Take for instance the case with one pair of imaginary poles at jc and $-jc$
- When coming close to jc, the argument will remain 0 and the gain will increase to infinity
- At jc itself, the gain will be infinite, but the argument is undetermined, hence we cannot map this point



Nyquist plot: poles on the imaginary axis

How do we solve this?

- Instead of going through the poles, we will evade them by an infinitesimally small amount (see figure)
- That way we do not have the problem of an undetermined mapping at the pole
- Since we avoid them by an infinitesimally small amount, we also know we will not wrongly avoid a pole that lies in the right half plane



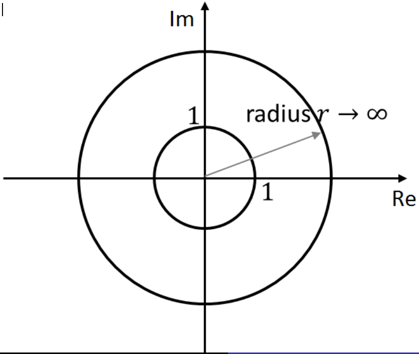
Now the Nyquist plot will go to infinity as the pole is approached, then the argument will change from 0 to π as the semi-circle is traversed and then the Nyquist plot will return from infinity.

Outline

Discrete-time case

The Cauchy argument principle still applies for discrete-time systems, since $P(z)C(z)$ also has the shape of a rational polynomial.

The contour will have to encircle the entire complex plane except for the unity ci



Discrete-time case

However, this is not a contour, but it can be solved with the following trick:
the two horizontal pieces are both infinitely close to the real axis, that way they are identical but with opposite signs. They will cancel each other out