Systems and control theory

Katholieke Universiteit Leuven

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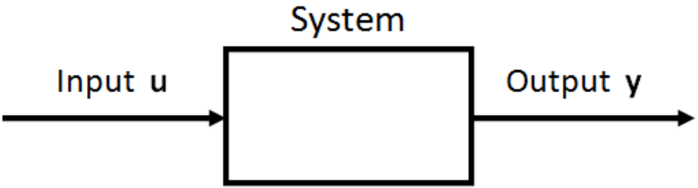
Overview

1. System theory
   - Discrete systems
   - Continuous systems
2. Control theory
3. Examples

Dynamical system

A dynamical system is a constantly changing system that connects outputs (denoted by $\mathbf{y}$) and inputs (denoted by $\mathbf{u}$).
The word dynamical refers to the fact that the system relates time-changing signals.

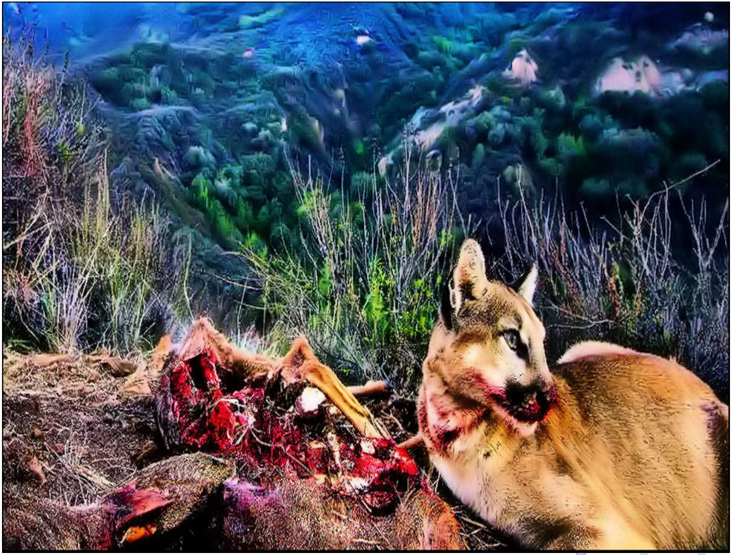


Everything is a dynamical system.

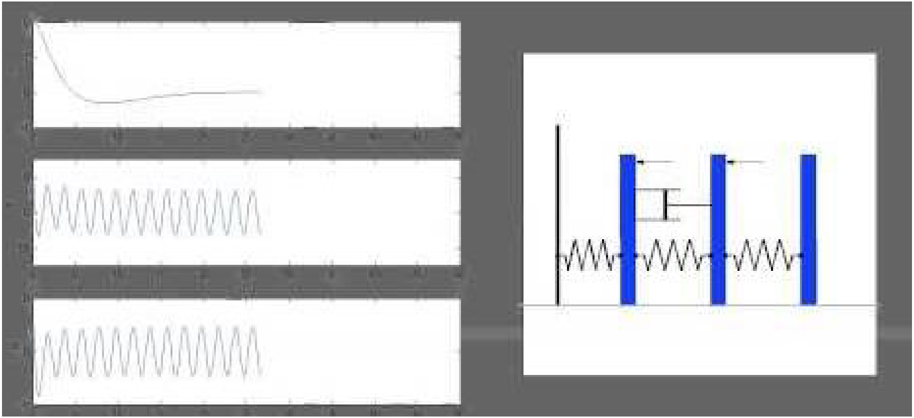
Antique racecar

Everything is a dynamical system

Predator-prey system



Mechanical system



System theory

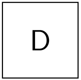
System theory occupies itself with the mathematical description and study of systems. Models describe the connections between input, output and states.

Next to inputs and outputs, states (denoted by $\mathbf{x}$) are a third type of variable used to describe a system. They represent the internal state of the system at a given time. The next state is a function of the previous states. The order of a system is the number of state-variables (i.e. the size of the vector $\mathbf{x}$).

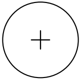
Dynamical systems can be devided into two types: **discrete** and **continuous** systems.

Discrete systems

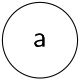
Delay element



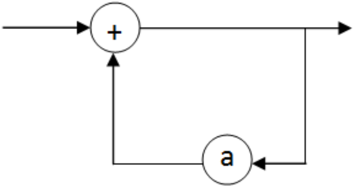
Adder



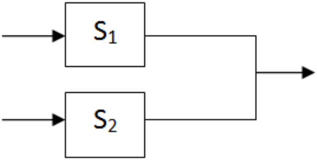
Constant multiplier



Restrictions:

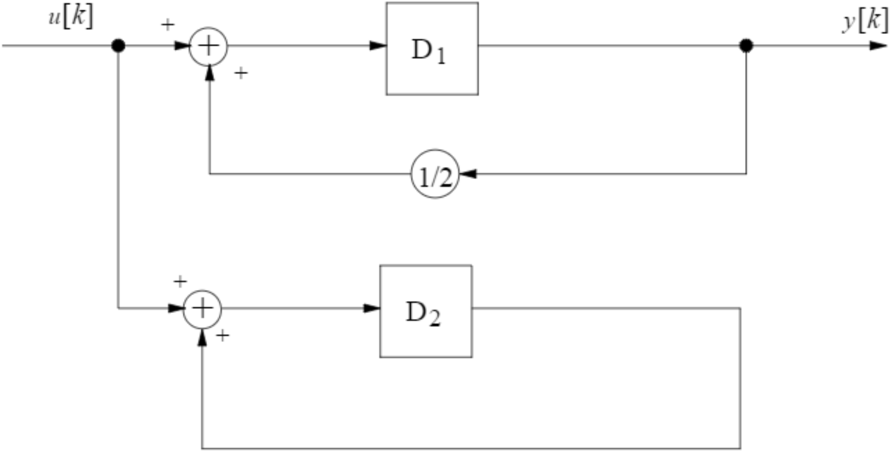


No loops without delay element



No connecting outputs without adder

Example



Methods

State space representation

$$x[k+1] = A\,x[k] + B\,u[k]$$
$$y[k] \quad = C\,x[k] + C\,u[k]$$

Difference equations

$$\sum_{i=0}^{n} a_i y[k+i] = \sum_{i=0}^{n} b_i u[k+i]$$

Impulse response

$$h[k] = y[k]\ when\ u[k] = \delta[k]$$

Z-transform

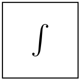
$$X(z) = \sum_{k=-\infty}^{+\infty} x[k] z^{-k}$$

Transfer function

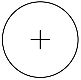
$$H(z) = \frac{Y(z)}{U(z)} \ \Rightarrow\ Y(z) = H(z)U(z)$$

Continuous systems

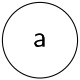
Integrator



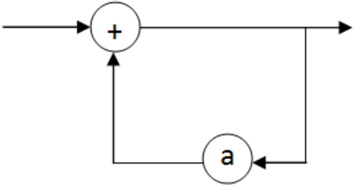
Adder



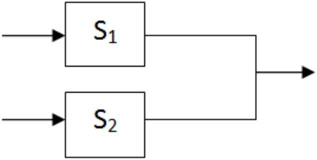
Constant multiplier



Restrictions:

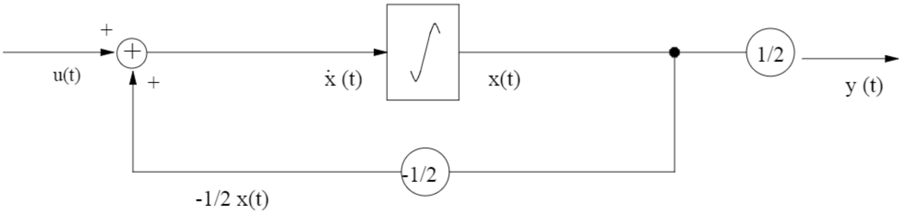


No loops without integrator



No connecting outputs without adder

Example



Methods

State space representation

$$\dot{x}(t) = A\,x(t) + B\,u(t)$$
$$y(t) = C\,x(t) + C\,u(t)$$

Differential equations

$$\sum_{i=0}^{n} a_i y^{(i)} = \sum_{i=0}^{n} b_i u^{(i)}$$

Impulse response

$$h(t) = y(t)\ when\ u(t) = \delta(t)$$

Laplace transform

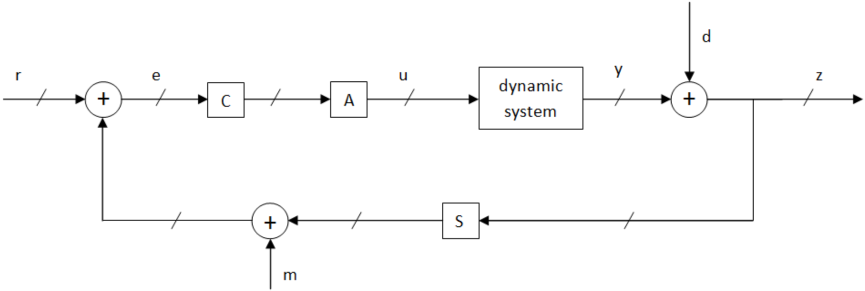
$$X(s) = \int_0^\infty e^{-st} f(t)\,\mathrm{d}t$$

Transfer function

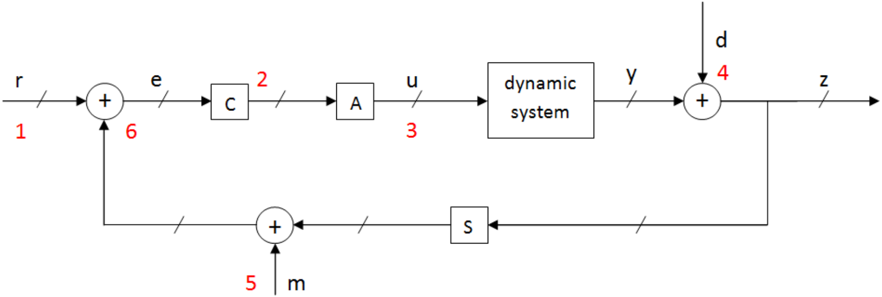
$$H(s) = \frac{Y(s)}{U(s)} \Rightarrow Y(s) = H(s)U(s)$$

Control theory

Control theory deals with the behavior of dynamical systems and how their behavior is modified by feedback.
The output is compared to the desired output and the error is used to adjust the system.



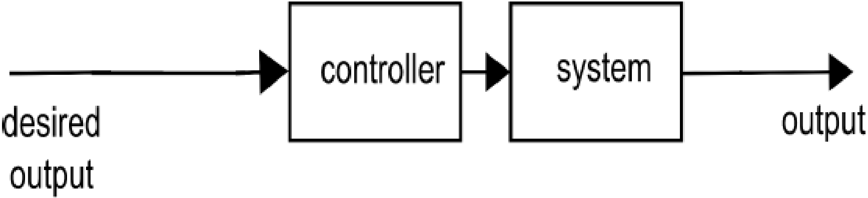
Control theory



1. reference signals: the desired output signals
2. controller with model (controller = dynamic)
3. actuators that control the system
4. Noise
5. Measurement noise
6. Negative feedback $\rightarrow$ system = stable

Open loop

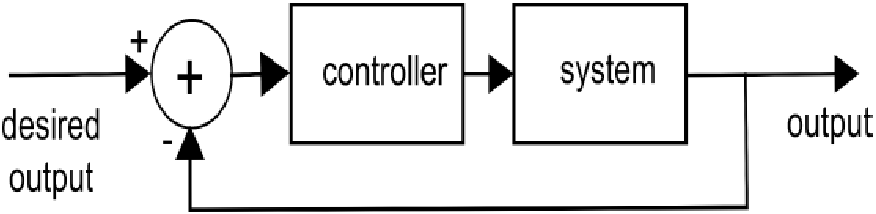
In an open loop system, the output is not fed back into the controller.
Therefore, the controller cannot *'see'* the effect of its actions.
This way it is hard to get the desired output.



For example: try pouring water in a glass without looking.

Feedback

In a feedback system, the output is fed back to the controller. This way, the input to the controller becomes the error on the output.



There are two types of feedback systems. The output can either be added to the desired output (positive feedback) or substracted from it (negative feedback).

Positive vs. Negative Feedback

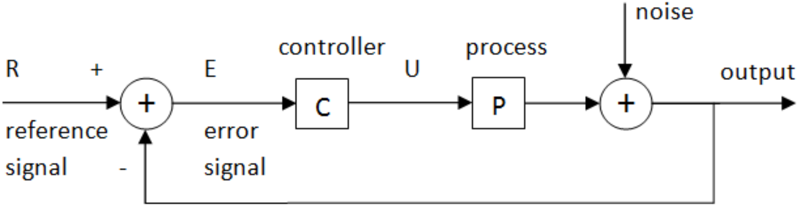
Negative Feedback

1. Continuous behavior
2. Analog technology
3. Output primarily reflects the input
4. Loops enhance of amplify the changes between input and output

Positive Feedback

1. On-Off behavior
2. Digital Technology
3. Output primarily reflects memory of the past
4. Loops tend to dampen or buffer the changes between input and output

Types of controllers



Proportional controller

$C(s) = K$

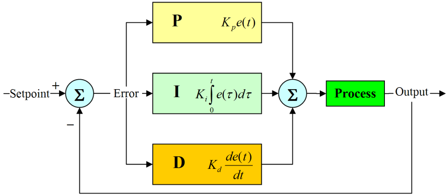
Phase lead controller

$C(s) = K \frac{b}{a} \frac{s+a}{s+b} \qquad a < b$

Phase lag controller

$C(s) = K \frac{b}{a} \frac{s+a}{s+b} \qquad a > b$

PID controller

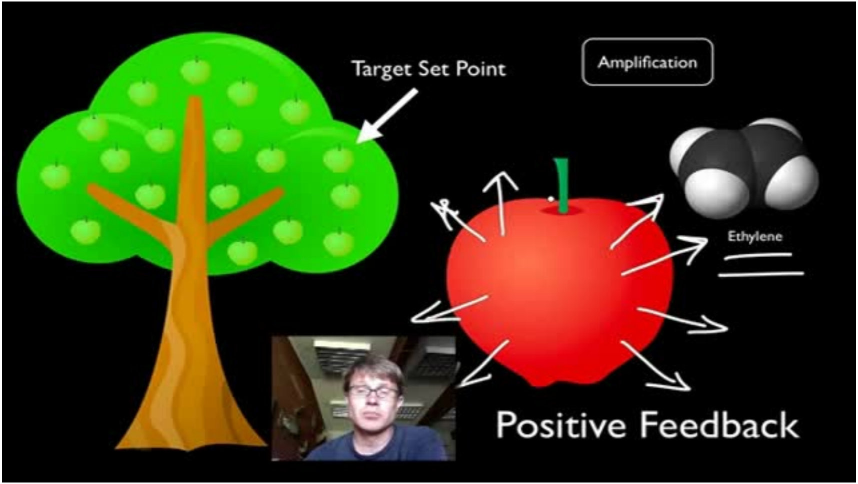


Millenium Bridge London

Resonance on the Millenium Bridge in London due to the rithm of walking people.

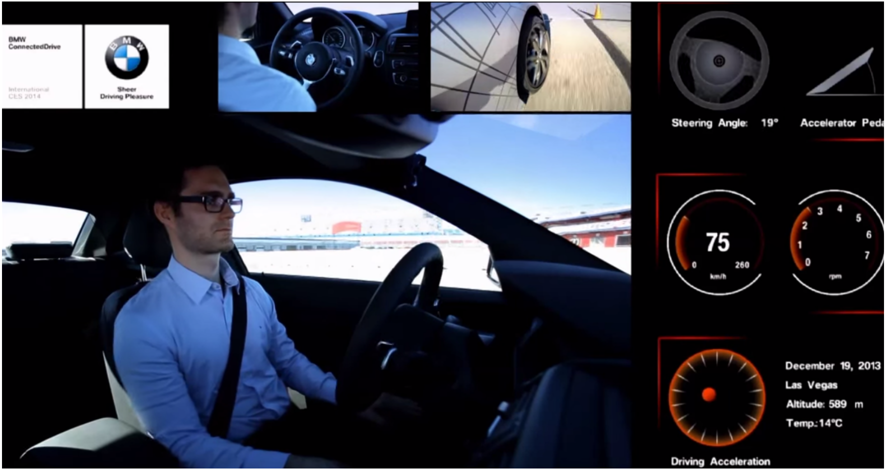
https://www.youtube.com/watch?v=eAXVa__XWZ8

Feedback in biology



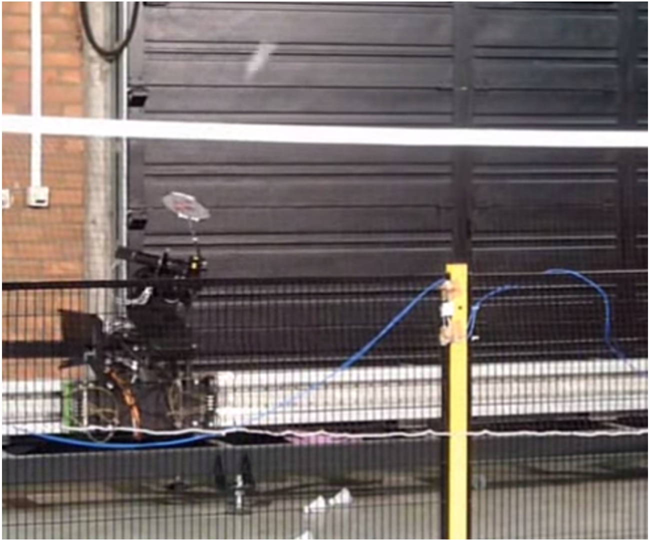
https://youtu.be/CLv3SkF_Eag

Autonomous car



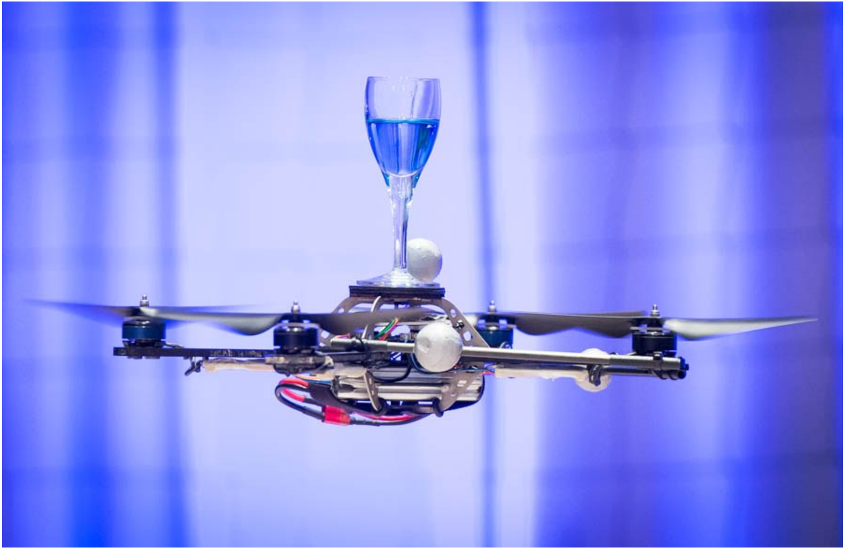
https://youtu.be/1FVglbJZ_tg

Badminton robot



https://youtu.be/LSax71cn6A4

Quadcopters



https://youtu.be/w2itwFJCgFQ