Outline

Solving LDEs with the Laplace transform 1/3

The Laplace transform can be used to solve LDEs with given initial conditions (the previous approach gave us the basis functions). This is done by using the following property (differentiation):

$$\mathcal{L}\{f^{(1)}\} = sF(s) - f(0),$$
$$\mathcal{L}\{f^{(2)}\} = s^2F(s) - sf(0) - f^{(1)}(0).$$

Via induction, the Laplace transform of the *n*th order derivative:

$$\mathcal{L}\{f^{(n)}\} = s^nF(s) - \sum_{i=1}^{n} s^{n-i} f^{(n-i)}(0)$$

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$$\mathcal{L}\{f^{(n)}\} = s^n F(s) - \sum_{i=1}^{n} s^{n-i} f^{(n-i)}(0)$$

We want to solve the following LDE:

$$\sum_{i=0}^{n} A_i y^{(n-i)}(t) = f(t),$$
$$y^{(i)}(0) = c_i \quad \forall i = 0 \ldots n.$$

Via the linearity of the Laplace transform:

$$\sum_{i=0}^{n} A_i \mathcal{L}\{y^{(n-i)}(t)\} = \mathcal{L}\{f(t)\}$$

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$$\sum_{i=0}^{n} A_i \mathcal{L}\{y^{(n-i)}(t)\} = \mathcal{L}\{f(t)\} \tag{1}$$

$$\mathcal{L}\{f^{(n)}\} = s^n F(s) - \sum_{i=1}^{n} s^{n-i} f^{(n-i)}(0) \tag{2}$$

Expanding Eq. (2) into (1) yields:

$$Y(s)\sum_{i=0}^{n} A_i s^i - \sum_{i=1}^{n}\sum_{j=1}^{i} A_i s^{i-j} y^{j-1}(0) = F(s)$$

The solution in the time domain is obtained via the inverse Laplace transform: $y(t) = \mathcal{L}^{-1}\{Y(s)\}$.