

Outline

- 1 Introduction
- 2 Main Approaches
 - Numerical Integration
 - Impulse Invariant Method
 - Zero-pole Equivalent
 - Hold Equivalent
- 3 Sampling Time
- 4 Discretization and MATLAB
 - Commands
 - Exercises

Sampling time T_s based on the time response

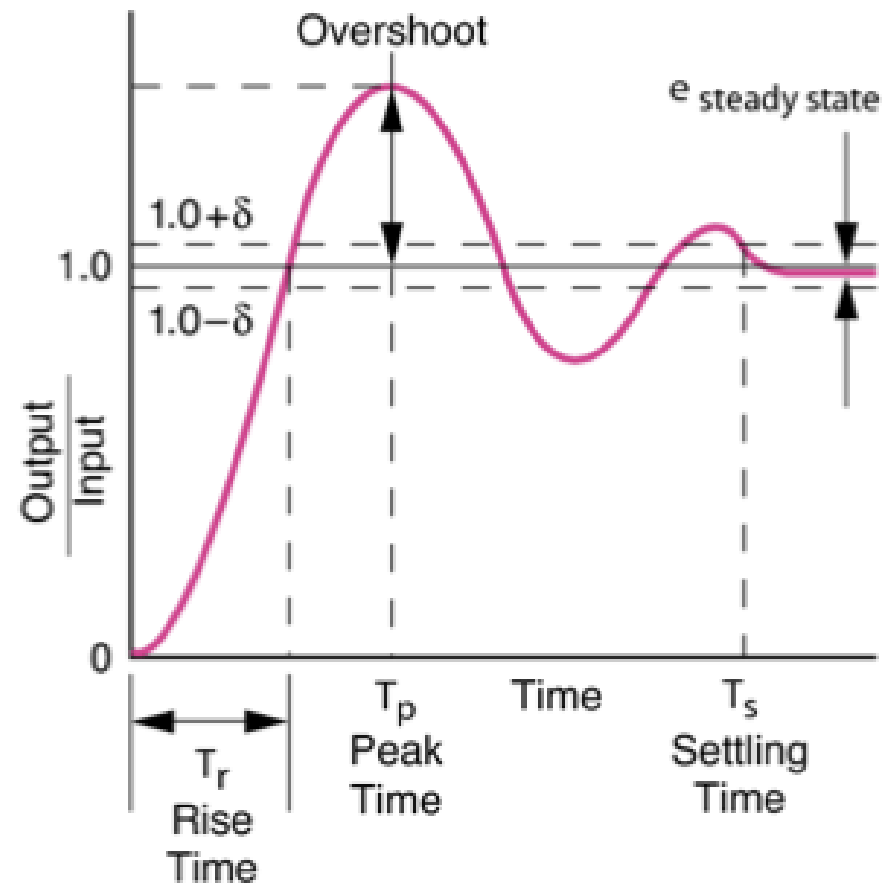
Rise time

Time needed to reach the steady state for the first time.

Practical rule

A good rule of thumb is

$$T_s = \frac{T_{rise}}{10}.$$



Sampling time T_s based on the frequency response

Practical rule

A good rule of thumb is $T_s = \frac{1}{2 \cdot 2 \cdot \text{Bandwidth}}$.

Signal vs System

It is very important to understand that although the previous rule of thumb for a system seems the same as the rule of thumb for a signal, they differ! The rule of thumb for a signal uses the bandwidth of the signal and the rule of thumb for a system uses the bandwidth of the system, which can be found in the bodeplot or using MATLAB.

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MATLAB

MATLAB Commands for calculations

MATLABs Control System Toolbox offers extensive support for discretization and re-sampling of linear systems:

- "c2d(system,sampling time,method)" used for discretization
- "d2c(system,sampling time,method)" used for transforming a discrete-time system to a continuous-time system (not used in the examples).

These commands can also have extra options. It is necessary to specify the options in an additional command:

"c2dOptions('OptionName',OptionValue)".

MATLAB

Available options

- 'Method'
- 'PrewarpFrequency'
- 'FractDelayApproxOrder'

Available methods

- zero-order hold equivalent: "zoh"
- first-order hold equivalent: "foh"
- impulse invariant rule: "impuls"
- zero-pole matching equivalent: "matched"
- bilinear: "tustin"

MATLAB

MATLAB Commands for visualisation

- MATLAB can draw the bode plots of the continuous system
→ command: `"bode(system)"`;
- MATLAB can carry out a graphic showing the impulse response of the continuous system and the discretized system
→ command: `"impulse(H,'b',Hd,'r')"` in which 'b' and 'r' stand for blue and red. By using colours, the step responses of both systems can easily be distinguished;
- MATLAB can carry out a graphic showing the step response of the continuous system and the discretized system
→ command: `"step(H,'b',Hd,'r')"`.

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Exercise 1 with MATLAB

Exercise 1

We will now discuss 4 methods applied on the same continuous system: $H(s) = \frac{s+1}{s^2+s+1}$

- The sampling time can be determined by the rule of thumb previously explained: $T_s = 2.2\text{Bandwidth}$;
- MATLAB can calculate the bandwidth of a given continuous system, using the command: "bandwidth(system)".

This results in a sampling time of 0.25033 seconds for the given system

Zero-order hold equivalent

Example

Given:

$$H(s) = \frac{s+1}{s^2+s+1}$$

Sampling time = 0.25033sec

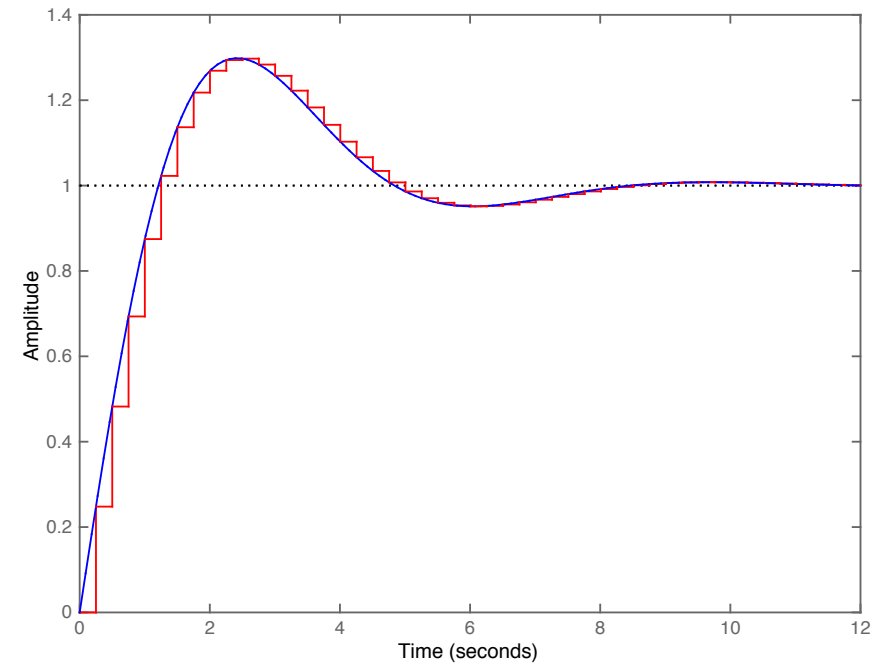
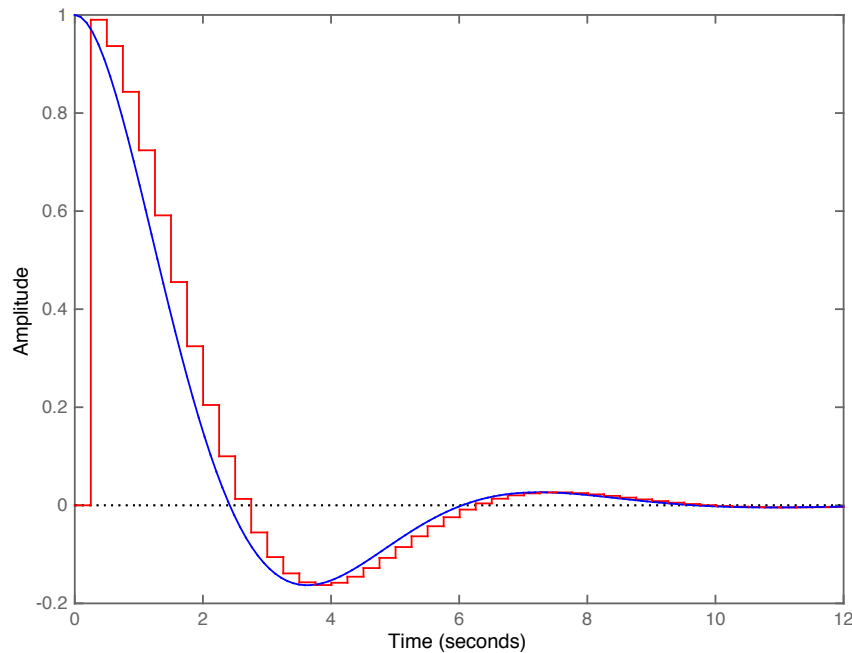
MATLAB commands:

$H = \text{tf}([1 \ 1],[1 \ 1 \ 1])$ we define the system H by its transfer function

$H_d = \text{c2d}(H,0.25033,'zoh')$ we calculate the discrete-time model

Result: $H_{zoh}(z) = \frac{0.2479z-0.1927}{z^2-1.723z+0.7785}$

Zero-order hold equivalent (impulse and step response)



First-order hold equivalent

Example

Given:

$$H(s) = \frac{s+1}{s^2+s+1}$$

Sampling time = 0.25033sec

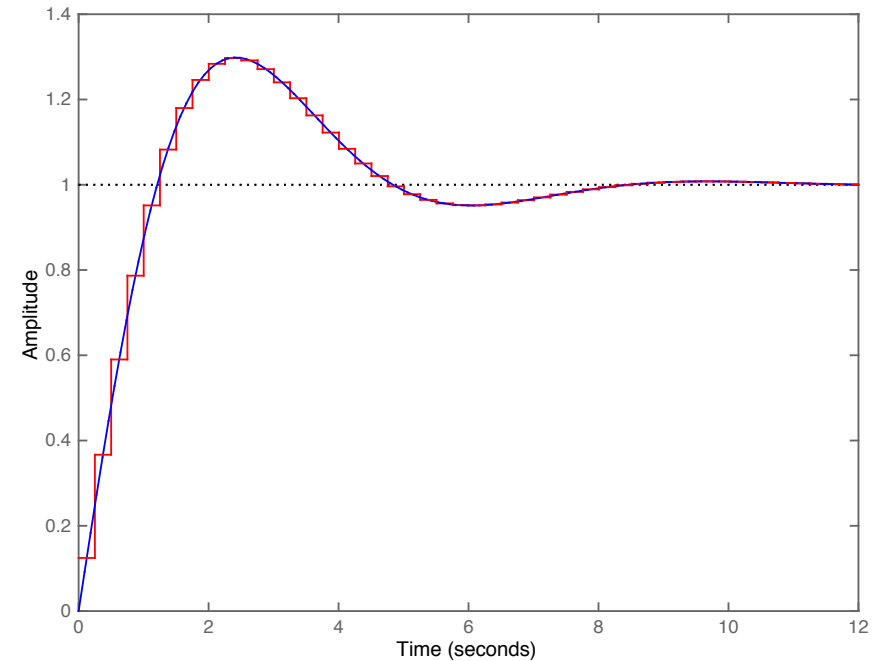
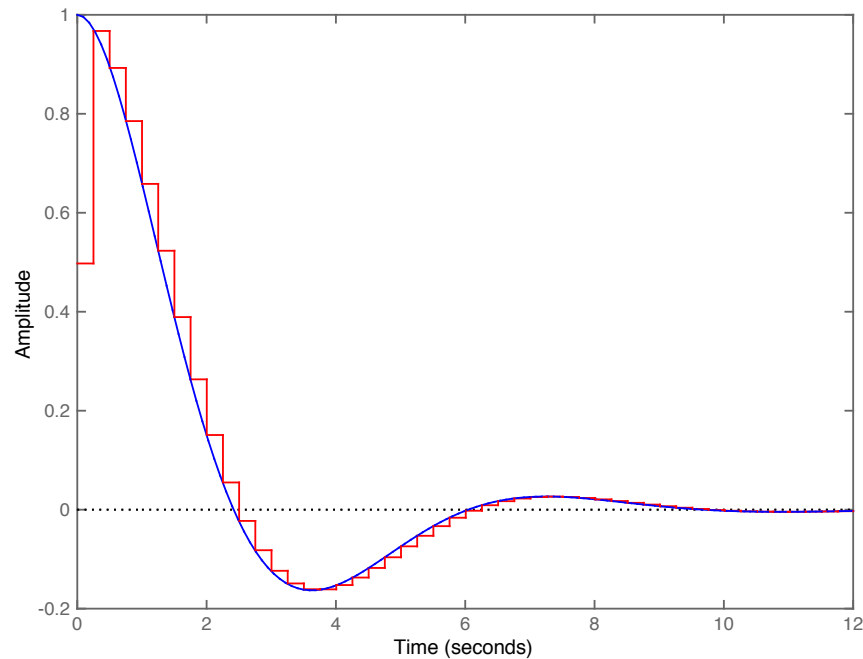
MATLAB commands:

$H = \text{tf}([1 \ 1],[1 \ 1 \ 1])$ we define the system H by its transfer function

$H_d = \text{c2d}(H,0.25033,'foh')$ we calculate the discrete-time model

$$\text{Result: } H_{foh}(z) = \frac{0.1245z^2+0.02752z-0.09691}{z^2-1.723z+0.7785}$$

First-order hold equivalent (impulse and step response)



Impulse invariant rule

Example

Given:

$$H(s) = \frac{s+1}{s^2+s+1}$$

Sampling time = 0.25033sec

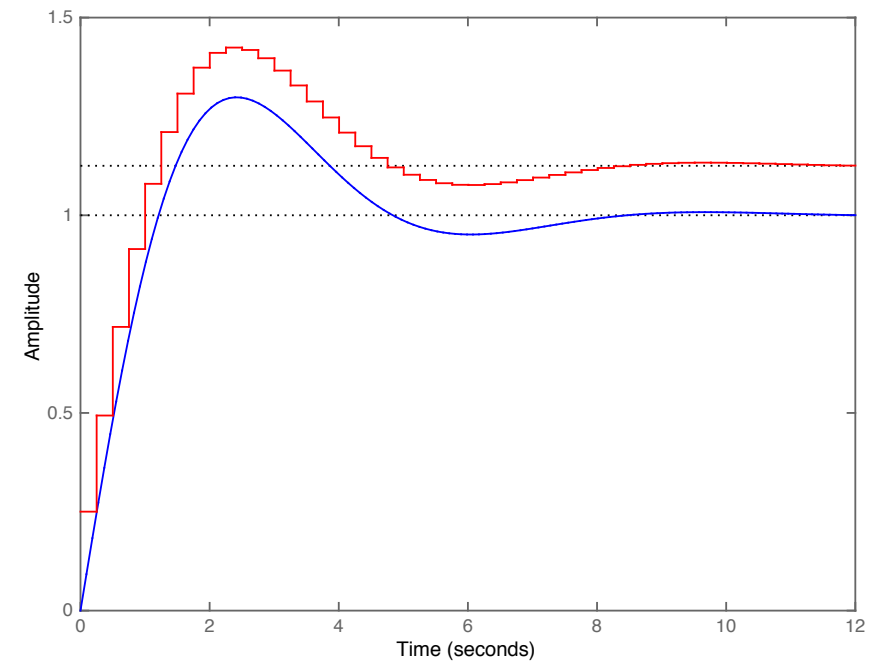
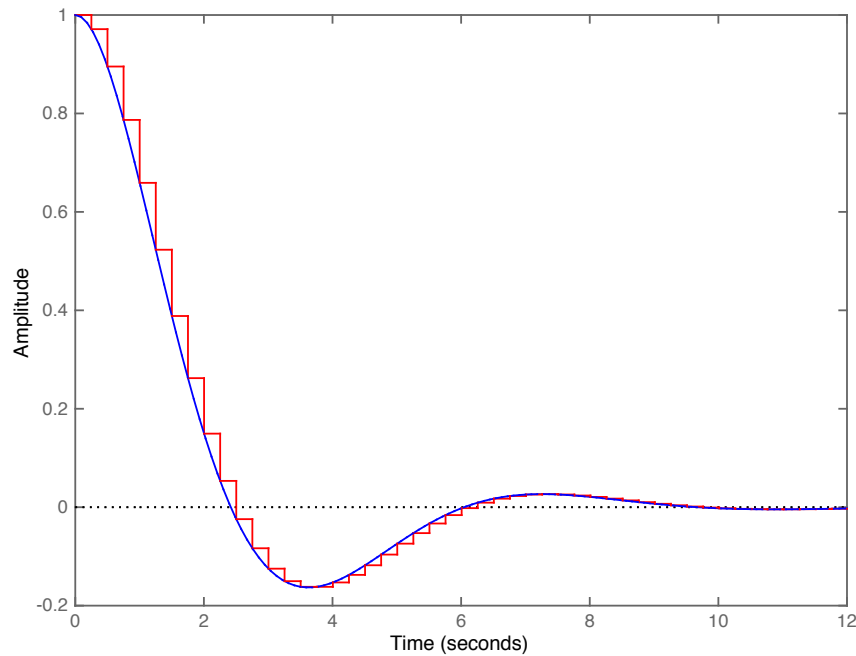
MATLAB commands:

$H = \text{tf}([1 \ 1],[1 \ 1 \ 1])$ we define the system H by its transfer function

$H_d = \text{c2d}(H,0.25033,'impuls')$ we calculate the discrete-time model

$$\text{Result: } H_{\text{impulse}}(z) = \frac{0.2503z^2 - 0.1883z}{z^2 - 1.723z + 0.7785}$$

Impuls invariant rule (impulse and step response)



Zero-pole equivalent

Example

Given:

$$H(s) = \frac{s+1}{s^2+s+1}$$

Sampling time = 0.25033sec

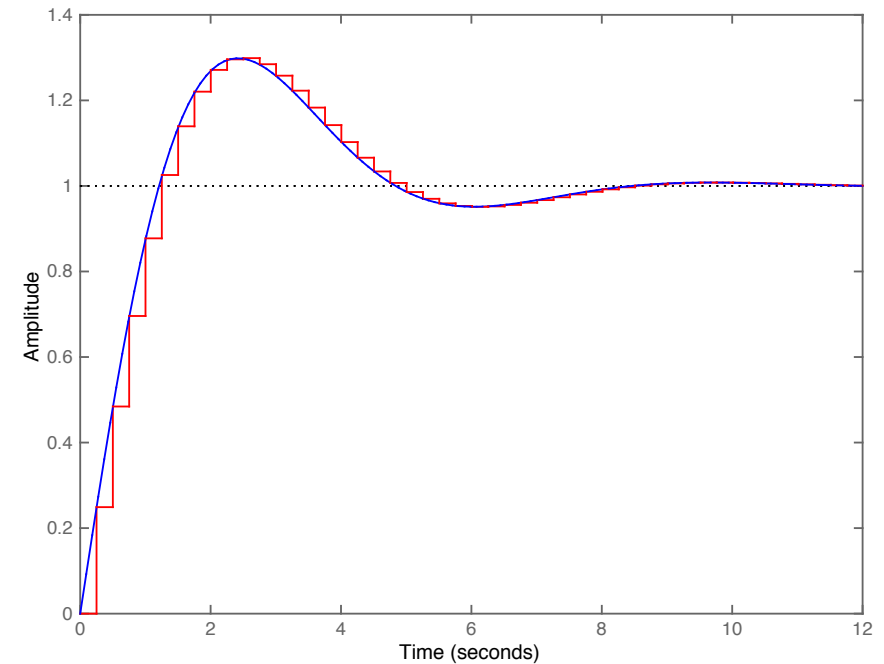
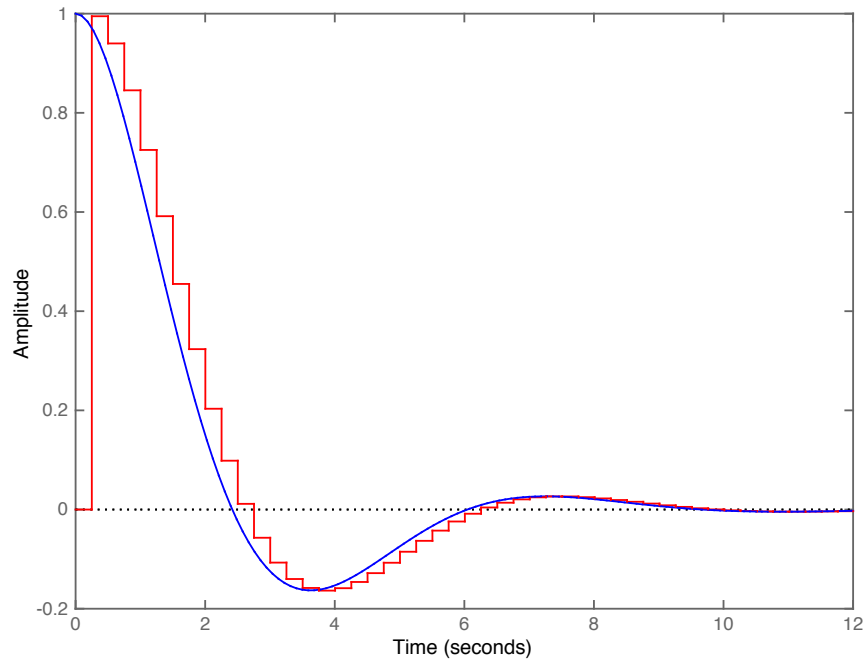
MATLAB commands:

`H = tf([1 1],[1 1 1])` we define the system H by its transfer function

`Hd = c2d(H,0.25033,'matched')` we calculate the discrete-time model

$$\text{Result: } H_{\text{matched}}(z) = \frac{0.249z-0.1939}{z^2-1.723z+0.7785}$$

Zero-pole equivalent (impulse and step response)



Exercise 2 with MATLAB

Exercise 2

We will now apply the bilinear rule with and without prewarping on the same continuous system: $H(s) = \frac{s^2+0.5s+9}{s^2+5s+9}$

Criteria:

- 1 The sampling time is chosen to be 0.5 seconds;
- 2 The discrete system must have the same behaviour as the continuous one at 3rad/s.

The last criteria applies to this specific exercise. It is possible that the parity of the magnitude, of the discrete and the continuous system, at a different frequency (e.g. the peak frequency) is required.

Tustin rule

Example

Given:

$$H(s) = \frac{s^2 + 0.5s + 9}{s^2 + 5s + 9}$$

Sampling time = 0.5sec

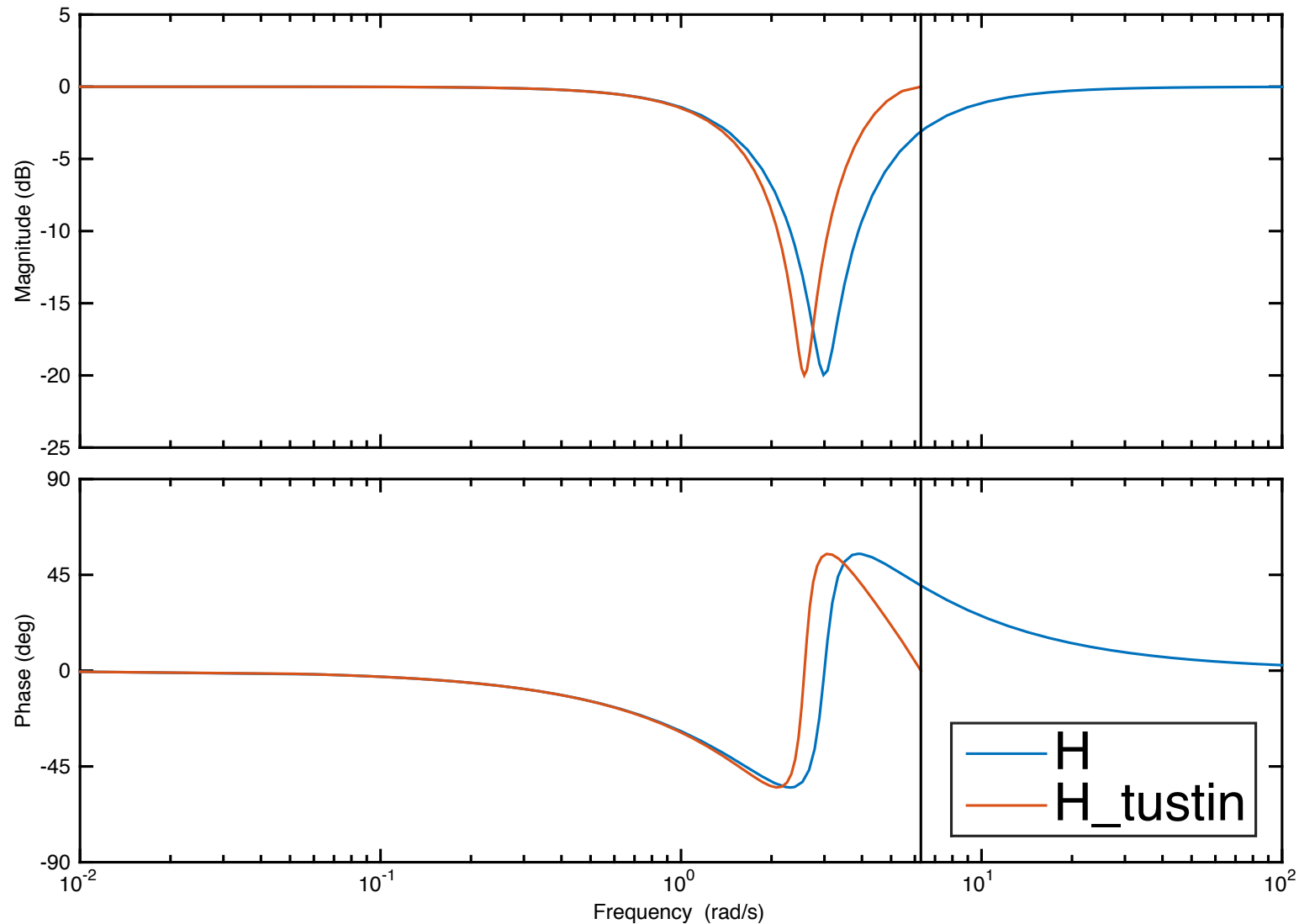
MATLAB commands:

`H = tf([1,0.5,9],[1,5,9])` definition of the system H

`Hdt = c2d(H,0.5,'tustin')` we calculate the discrete-time model

$$\text{Result: } H_{Tustin}(z) = \frac{0.6z^2 - 0.3111z + 0.5111}{z^2 - 0.3111z + 0.1111}$$

Tustin rule (bode plot)



Tustin rule with prewarping

Example

Given:

$$H(s) = \frac{s^2 + 0.5s + 9}{s^2 + 5s + 9}$$

Sampling time = 0.5sec

MATLAB commands:

`H = tf([1,0.5,9],[1,5,9])` definition of the system H

`damp(H)` results in 3.0Hz

`discopts = c2dOptions('Method','tustin','PrewarpFrequency',3.0)`

at the prewarpfrequency, the discrete-time model will have the same behavior as the continuous-time one

`Hdtp = c2d(H,0.5,discopts)` we calculate the discrete-time model

$$\text{Result: } H_{TustinPrewarp}(z) = \frac{0.5915z^2 - 0.07726z + 0.5007}{z^2 - 0.07726z + 0.09215}$$

Tustin rule with prewarping (bode plot)

