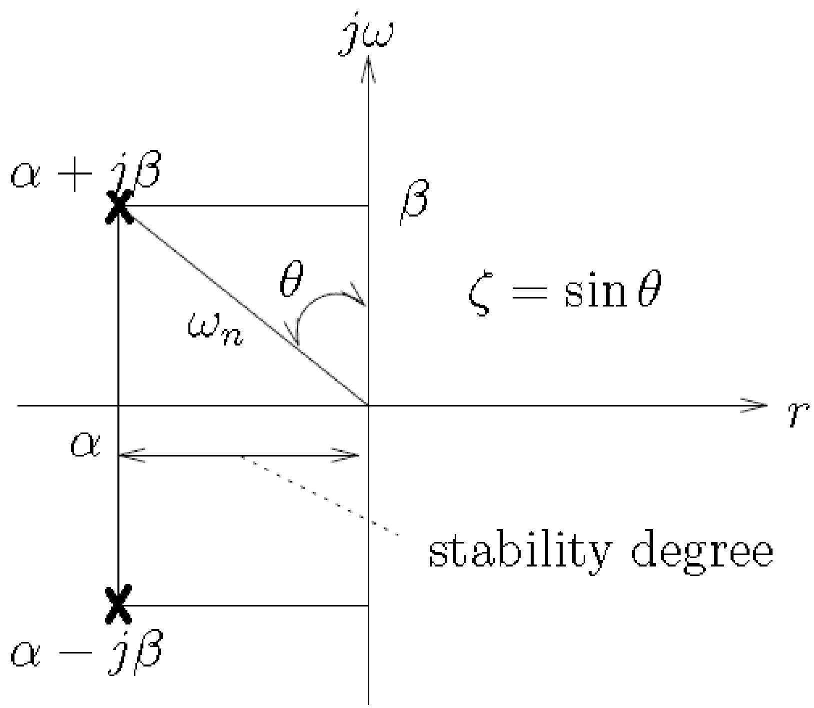
Outline

Second order system

Recall from chapter 5 that the dynamic behavior of the second-order system can be described in terms of only two parameters ζ and ω_n, where $\alpha = -\zeta\omega_n$ and $\beta = \omega_n\sqrt{1-\zeta^2}$.



Root locus method: design criteria

If you are able to express your design criteria in terms of the poles ($\alpha + j\beta$, with $\alpha < 0$ and $\beta > 0$), you can find out if there are some appropriate K-values and more importantly, which K-values.

Criteria

- The damping ratio: $\zeta = \frac{\beta}{|\alpha + j\beta|}$ $(0 \leq \zeta \leq 1)$;
- The natural frequency: $\omega_n = -\frac{\alpha}{\zeta} = \frac{\beta}{\sqrt{1-\zeta^2}}$;
- The rise time: $t_r \cong \frac{1.8}{\omega_n}$;
- The settling time: $t_s = \frac{4.6}{\zeta\omega_n}$;
- The peak time: $t_p = \frac{\pi}{\omega_d}$, with $\omega_d = \omega_n\sqrt{1-\zeta^2}$;
- The overshoot: $M_p = e^{-\pi\frac{\zeta}{\sqrt{1-\zeta^2}}}$.

Root locus method: design criteria

Example

Let's go back to the problem of the DC motor discussed in the first subsection, whose transfer function is:

$$G(s) = \frac{1}{s(s+1)}. \tag{10}$$

For this particular case, we are interested in finding a gain K for which the poles of the closed-loop system has a damping ratio equal to 0.5.

Root locus method: design criteria

We know that if $\zeta = 0.5$, then $\theta = 30^\circ$ and the magnitude of the imaginary part of the root is $\sqrt{3}$ times the magnitude of the real part. Previously, we already found the following equation and its roots:

$$s^2 + s + K = 0$$
$$r_1, r_2 = -\tfrac{1}{2} \pm \tfrac{\sqrt{1-4K}}{2}$$

The magnitude of the real part is $\frac{1}{2}$ and thus, we have:

$$\frac{\sqrt{4K-1}}{2} = \frac{\sqrt{3}}{2} \tag{11}$$

and therefore, $K = 1$.

Root locus method: design criteria

The possible design criteria for second order systems are very useful since they represent physically important measures in terms of the poles.

Second order approximation

For most of the time only one or two poles dominate the behavior of the system. These are called the dominant poles. Consequently many systems behave more or less as if they are of second order.

- A single pole at position $-a$ results in a e^{-at} term in the output. So after some time, the poles with the largest real part will dominate the behavior;
- A single pole at position $me^{j\omega}$ results in a $m^k e^{j\omega k}$ term in the output. So after some time the poles with the largest modulus will dominate.