Chapter 13 - PID Controllers

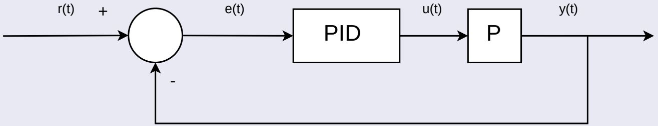
August 6, 2015

Outline

What is a PID controller?

Definition

A **P**roportional **I**ntegral **D**eriviative controller is a control loop feedback mechanism (controller) widely used in process industry.



Continuous-time text book equation:

$$u(t) = \underbrace{K_p e(t)}_{\text{Proportional Action}} + \underbrace{K_i \int_0^t e(\tau) d\tau}_{\text{Integral Action}} + \underbrace{K_d \frac{de(t)}{dt}}_{\text{Derivative Action}}$$

Note: More than 90% of all closed loop controllers are PID.

What is a PID controller?

- **P**roportional action $u_p(t) = K_p e(t)$: Action depends on the instaneous value of the control error.
  - \+ Reduces rise time
  - − Reduces but **does not eliminate steady-state error**: Only when $K \to \infty$, error $\to 0$ (unless plant has pole(s) at $s = 0$)
- **I**ntegral action $u_i(t) = K_i \int_0^t e(\tau) d\tau$: Gives a controller output that is proportional to the accumulated error. Reacts on constant errors
  - \+ **Can eliminate steady state error** in some cases
  - − Makes transient response slower
- **D**erivative action $u_d(t) = K_d \frac{de(t)}{dt}$: Acts on the rate of change of the control error.
  - \+ Damping effect: reduces overshoot, improves transient response
  - − Sensitive for noise, amplifies it if present

Outline

Proportional Control

The continuous-time and discrete implementation are identical

Continuous:

$$u_p(t) = K_p e(t) \quad \rightarrow \quad \frac{U_p(s)}{E(s)} = K_p$$

Discrete:

$$u_p[k] = K_p e[k] \quad \rightarrow \quad \frac{U_p(z)}{E(z)} = K_p$$

where e(t) or e[k] is the error signal.

Derrivative Control

Continuous:

$$u_d(t) = K_d \frac{de(t)}{dt} \quad \rightarrow \quad \frac{U_d(s)}{E(s)} = K_d s$$

Discrete (using **backward Euler**):

$$u_d[k] = K_d \frac{e[k] - e[k-1]}{T_s} \quad \rightarrow \quad \frac{U_d(z)}{E(z)} = K_d \frac{z-1}{T_s z}$$

with T_s the sampling time.

Integral Control

The continuous equation is:

$$u_i(t) = K_i \int_0^t e(\tau)d\tau \quad \rightarrow \quad \frac{U_i(s)}{E(s)} = \frac{K_i}{s}$$

Differentiating this gives :

$$\dot{u}_i = K_i e(t)$$

Then applying backward Euler:

$$u_i[k] = u[k-1] + K_i Te[k] \quad \rightarrow \quad \frac{U_i(z)}{E(z)} = K_i \frac{zT_s}{z-1}$$

with T_s the sampling time.

Digital formulation (conventional version)

Digital PID controller (conventional version)

$$u[k] = K_p e[k] + \frac{K_d}{T_s}(e[k] - e[k-1]) + u_i[k]$$
$$\text{with } u_i[k] = u_i[k-1] + K_i T_s e[k]$$

In z-domain:

$$\frac{U(z)}{E(z)} = K_p + \frac{K_d}{T_s}\frac{z-1}{z} + K_i T_s \frac{z}{z-1}$$

where $\frac{K_d}{T_s}$ and $K_i T_s$ are the new derivative and gains.

Digital PI controller

$$\frac{U(z)}{E(z)} = K_p + K_i T \frac{z}{z-1}$$

Digital PD controller

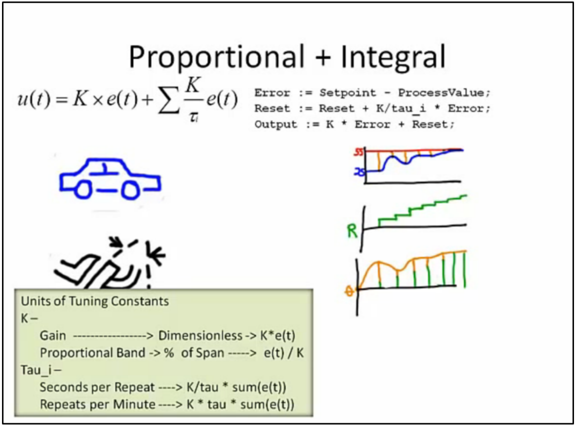
$$\frac{U(z)}{E(z)} = K_p + \frac{K_d}{T}\frac{z-1}{z}$$

Alternative Digital PID controller

We can also discretize using the **bilinear transformation**:

$$\begin{aligned}
\frac{U(z)}{E(z)} &= K_p + \frac{K_i}{s} + K_d s \bigg|_{s=\frac{2}{T}\left(\frac{z-1}{z+1}\right)} \\
&= K_p + \frac{K_i T(z+1)}{2(z-1)} + \frac{2K_d(z-1)}{T(z+1)} \\
&= \frac{\alpha_2 z^2 + \alpha_1 z + \alpha_0}{(z-1)(z+1)}
\end{aligned}$$

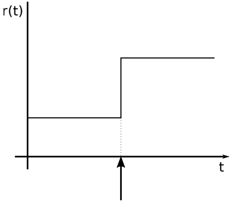
where $\alpha_2, \alpha_1, \alpha_0$ are design parameters.



https://www.youtube.com/watch?v=JEpWlTl95Tw

Alterantive Derivative Action(Continous time)

Imagine a jumping set point or rapidly changing signal. This results in a theoretically infinite, practically very large response of the derivative term.



$\Rightarrow$ Add a low-pass filter to the derivative term:

$$\frac{U_d(s)}{E(s)} = \frac{K_d s}{1 + s\tau}$$

With $s = j\omega$, breakpoint at $\omega = 1/\tau$. This prevents amplification of high frequencies.

Alterantive Derivative Action(Continous time)

$$\frac{U_d(s)}{E(s)} = \frac{K_d s}{1 + s\tau}$$

Further $e(t)$ is replaced by $c \cdot r(t) - y(t)$ with c the set point weighting, which is often set to zero to further reduce immediate influence of a sudden set point jump.
In time domain:

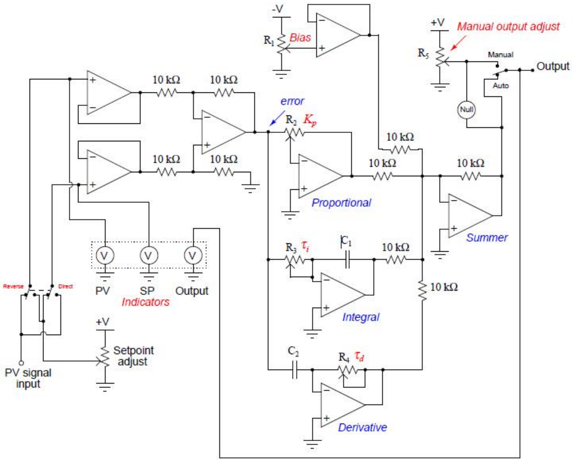
$$u_d(t) = -\tau \frac{du_d}{dt} + K_d(c \cdot r(t) - y(t))$$

This can also be discretized, but the bilinear method then introduces *ringing*, i.e. large oscillations in transient response.

Outline

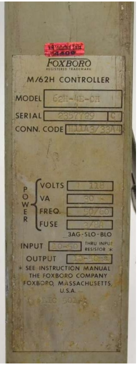
Analog Implementation

The key building block is the operational amplifier (op-amp).



- PV - Process Variable $y(t)$
- SP - Set Point $r(t)$
- Output - Control action $u(t)$

Analog Implementation



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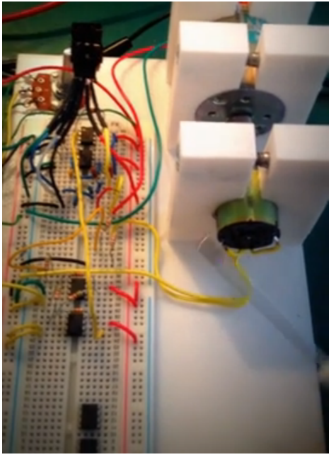
Analog PI Motor Speed Control



https://youtu.be/6W3PLiVIcmE

Digital Implementation

The difference equations are typically implemented in a micro controller or FPGA (field-programmable gate array):

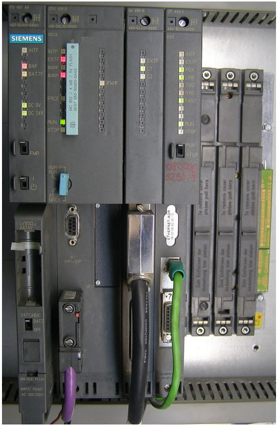
$$u[k] = K_p e[k] + \frac{K_d}{T}(e[k] - e[k-1]) + u_i[k]$$
$$\text{with } u_i[k] = u_i[k-1] + K_i T e[k]$$

Steps to be implemented:

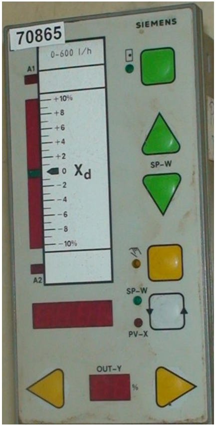
```
previous_error = 0
integral = 0
Start:
        error = setpoint - measured_value
        proportional = K_P * error
        integral = integral + K_i*sampling_time*error
        derivative = K_d*(error-previous_error)/sampling_time
        output = proportional + integral + derivative
        previous_error = error
        wait(sampling_time)
        goto Start
```

Digital Implementation Example

PLC with a digital PID module:



Digital PID's:



PLC

Programmable **L**ogic **C**ontroller is a digital computer used for automation in the process industry.

What is a PLC? Basics of PLCs

https://youtu.be/iWgHqqunsyE

Outline

Manual Tuning

The controller can be tuned while connected to the plant. Following routine can be used:

1. Set K_i and K_d equal to 0
2. Increase K_p until you observe that the step response is fast enough and the steady-state error in small
3. Start adding some integral action in order to get rid of the steady state error. Keep in mind that too much K_i can cause instability!
4. Add some derivative action in order to quickly react to disturbance and/or dampen the response

Manual Tuning

PID gains	Rise Time	Overshoot	Setlling time	Steady-State error
$K_p \uparrow$	Decrease	Increase	Small Change	Decrease
$K_i \uparrow$	Decrease	Increase	Increase	Eliminate
$K_d \uparrow$	Small change	Decrease	Decrease	No change

Important note

Changing one parameter can influence the effect of the other two. Use this table only as an indication.

Heuristic Methods

Manual tuning is used on a plant from which we know the mathematical model.

Sometimes the **mathematical model** of the plant is **not known**. In these cases we will uses the **heuristic methods**:

- Ziegler-Nichols tuning rule based on step response (First method)
- Ziegler-Nichols tuning rule based on critical gain and critical period (Second method)

Heuristic Methods: Ziegler-Nichols tuning rule

This method relies on empirically determining two parameters of the system:

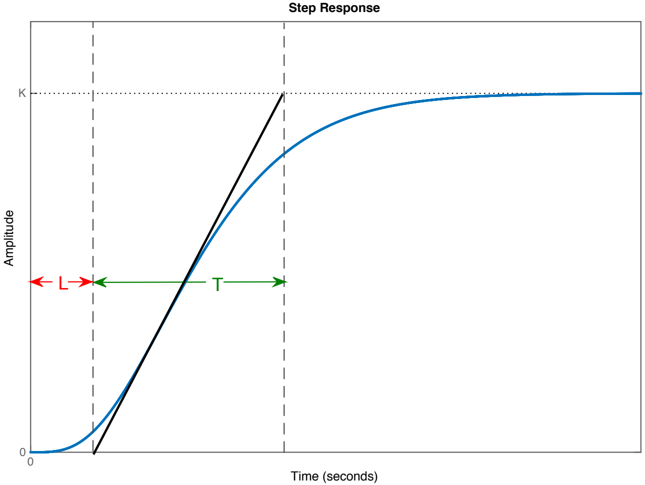
1. Set the integral and derivative gains to 0
2. Increase the proportional gain K_p until the output of the control loop starts oscillating with at constant amplitude. The value of K_p at this point is referred to as ultimate gain $K_u \triangleq K_p$
3. Measure the period of the oscillations T_u at the output
4. Adjust the controller parameters according the table on the next slide.

First Method

In this method, the response of the plant to a unit-step input is obtained experimentally (because we do not know the mathematical model). If the plant involves neither integrators nor dominant complex-conjugate poles, then such a unit-step response curve may look S-shaped as shown in the figure on the next slide. This S-shaped curve may be characterized by two constants: delay time L and time constant T. The delay time and the time constant are determined by drawing a tangent line at the inflection point of the S-shaped curve and determining the intersections of the tangent line with the time axis and line Amplitude = K.

If the response to a step input does not exhibit an S-shaped curve, the first method does not apply.

First Method



First Method

Control Type	K_P	K_I	K_D
P	$\frac{T}{L}$	0	0
PI	$0.9\frac{T}{L}$	$K_p\frac{0.3}{L}$	0
PID	$1.2\frac{T}{L}$	$\frac{K_p}{2L}$	$0.5LK_P$

The PID controller tuned by the first method of Ziegler-Nichols rules gives:

$$\begin{aligned}\frac{U(s)}{E(s)} &= K_P + \frac{K_I}{s} + K_D s \\ &= 1.2\frac{T}{L}\left(1 + \frac{1}{2Ls} + 0.5Ls\right) \\ &= 0.6T\frac{\left(s + \frac{1}{L}\right)^2}{s}\end{aligned}$$

with a pole at the origin and a double zero at $s = -\frac{1}{L}$.

Second Method

With K_u and T_u defined as before, a starting point for the parameters can be determined:

Control Type	K_p	K_i	K_d
P	$0.5K_u$	-	-
PI	$0.45K_u$	$1.2K_p/T_u$	-
PD	$0.8K_u$	-	$K_pT_u/8$
PID	$0.6K_u$	$2K_p/T_u$	$K_pT_u/8$
Pessen Integral Rule	$0.7K_u$	$2.5K_p/T_u$	$3K_pT_u/20$
Some overshoot	$0.33K_u$	$2K_p/T_u$	$K_pT_u/3$
No overshoot	$0.2K_u$	$2K_p/T_u$	$K_pT_u/3$

If the output does not exhibit sustained oscillations for whatever value K_P may take, then the second method does not apply.

Ziegler-Nichols tuning rule(example)

Example

Consider a plant with a given model:

$$P(s) = \frac{1}{(s+1)^3}$$

- We compute the critical gain K_c. This is the value of K_p for which $\angle(K_p P(s)) = -180^\circ$. On the Nyquist plot this the value of K_p for which $K_p P(s)$ passes through $(-1, 0)$.

$$K_c P(j\omega_c) = -1$$

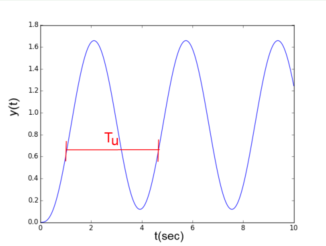
$$\Leftrightarrow K_c = -(j\omega_c + 1)^3$$

$$= (3\omega_c^2 - 1) + j(\omega_c^3 - 3\omega_c)$$

$$\omega_c^3 - 3\omega_c = 0 \Rightarrow \omega_c = \sqrt{3}$$

$$K_c = 8,\ T_u = \frac{2\pi}{\omega} = 3.628$$

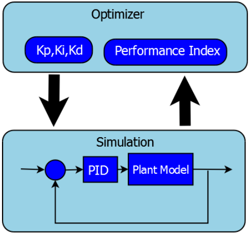
$$K_p = 4.8,\ K_I = 0.551 K_p,\ K_d = 0.45 K_p$$



Numerical Optimization Methods

The tuning of a PID controller is posed as a constrained optimization problem.

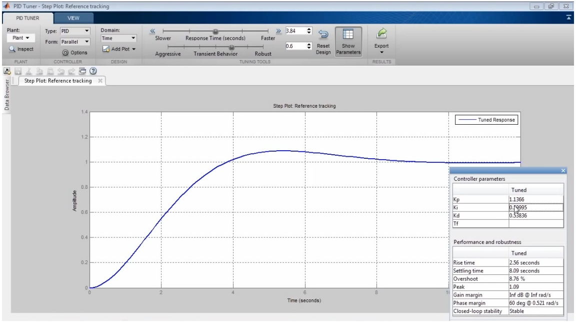
- For a given set of parameters K_p, K_i and K_d run a simulation of the closed-loop system, and compute some performance parameters (e.g. setting time, rise time, etc.) and a performance index.
- Optimize the performance index over the three PID gains.



Some Software Tools

Software Tool	Brief Description
pidtool / pidTuner	It is a Matlab tool to interactively design a SISO PID controller in the feed-forward path of single-loop, unity-feedback control configuration
Pidpy	It is a modular PID control library for python that supports PID auto tuning. `https://pypi.python.org/pypi/pypid/`
INCA PID Tuner	It is a commercial tuning tool developed by IPCOS. It has a vast library of PID structures for DCS and PLC Systems including Siemens, ABB, Honeywell, Emerson, etc. `http://www.ipcos.com/advancedprocesscontrol/advanced-process-control/pid-tuning-software/inca-pid-tuning/`

pidtool /pidTuner - Demo



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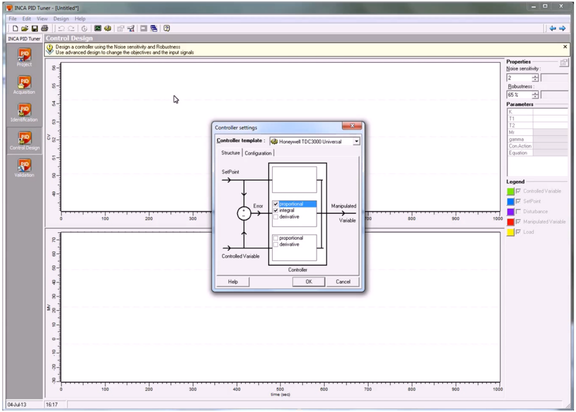
INCA PID Tuner Demo



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