Chapter 13 - PID Controllers

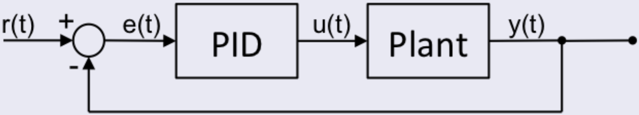
August 7, 2015

Outline

What is a PID controller?

Definition

A **P**roportional-**I**ntegral-**D**eriviative (PID) controller is a control-loop feedback mechanism (controller) widely used in process industry.



Continuous-time text book equation:

$$u(t) = \underbrace{K_p e(t)}_{\text{Proportional Action}} + \underbrace{K_i \int_0^t e(\tau) d\tau}_{\text{Integral Action}} + \underbrace{K_d \frac{de(t)}{dt}}_{\text{Derivative Action}}$$

Note: 90% (or more) of control-loops in process industry are PID.

What is a PID controller?

- **P**roportional action $u_p(t) = K_p e(t)$: it depends on the instaneous value of the error.
  - \+ Reduces rise time
  - \+ Reduces but **does not eliminate the steady-state error**: Only when $K \to \infty$, error $\to 0$ (unless the plant has pole(s) at $s = 0$)
- **I**ntegral action $u_i(t) = K_i \int_0^t e(\tau)d\tau$: it is proportional to the accumulated error.
  - \+ **Eliminates the steady-state error** in some cases
  - – Makes transient response slower
- **D**erivative action $u_d(t) = K_d \frac{de(t)}{dt}$: it is proportional to the rate of change of the error.
  - \+ Increases the stability of the system, reduces overshoot, improves the transient response
  - – Amplifies the noise present in the error signal

Outline

Proportional Control

The continuous-time and discrete-time implementation are identical.

For the continuous-time case we have:

$$u_p(t) = K_p e(t) \quad \rightarrow \quad \frac{U_p(s)}{E(s)} = K_p$$

and for the discrete-time case:

$$u_p[k] = K_p e[k] \quad \rightarrow \quad \frac{U_p(z)}{E(z)} = K_p$$

where e(t) or e[k] is the error signal.

Derivative Control

In continuous-time it is given by:

$$u_d(t) = K_d \frac{de(t)}{dt} \quad \rightarrow \quad \frac{U_d(s)}{E(s)} = K_d s$$

and in discrete-time by (using backward Euler):

$$u_d[k] = K_d \frac{e[k] - e[k-1]}{T_s} \quad \rightarrow \quad \frac{U_d(z)}{E(z)} = K_d \frac{z-1}{T_s z}$$

with T_s the sampling time.

Integral Control

In continuous-time it is given by:

$$u_i(t) = K_i \int_0^t e(\tau) d\tau \quad \rightarrow \quad \dot{u}_i(t) = K_i e(t) \quad \rightarrow \quad \frac{U_i(s)}{E(s)} = \frac{K_i}{s}$$

and in discrete-time by (using backward Euler)

$$u_i[k] = u_i[k-1] + K_i T_s e[k] \quad \rightarrow \quad \frac{U_i(z)}{E(z)} = K_i \frac{zT_s}{z-1}$$

with T_s the sampling time.

Digital formulation (conventional version)

Digital PID controller (conventional version)

$$u[k] = K_p e[k] + \frac{K_d}{T_s}(e[k] - e[k-1]) + u_i[k]$$
$$\text{with } u_i[k] = u_i[k-1] + K_i T_s e[k]$$

In the $\mathcal{Z}$-domain:

$$\frac{U(z)}{E(z)} = K_p + K_i T_s \frac{z}{z-1} + \frac{K_d}{T_s} \frac{z-1}{z}$$

where $\frac{K_d}{T_s}$ and $K_i T_s$ are the new derivative and gains.

Digital PI controller

$$\frac{U(z)}{E(z)} = K_p + K_i T_s \frac{z}{z-1}$$

Digital PD controller

$$\frac{U(z)}{E(z)} = K_p + \frac{K_d}{T_s} \frac{z-1}{z}$$

Alternative Digital PID controller

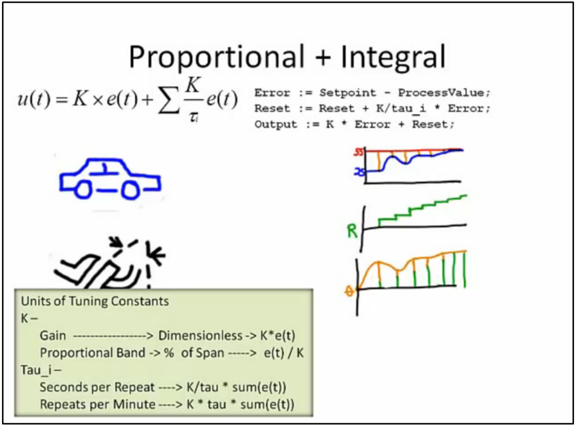
If we discretize the continuous-time (analog) PID controller using the bilinear transformation,

$$\frac{U(z)}{E(z)} = K_p + \frac{K_i}{s} + K_d s \bigg|_{s=\frac{2}{T_s}\left(\frac{z-1}{z+1}\right)}$$

we obtain an alternative form for a digital PID controller

$$\begin{aligned}\frac{U(z)}{E(z)} &= K_p + \frac{K_i T_s(z+1)}{2(z-1)} + \frac{2K_d(z-1)}{T_s(z+1)} \\ &= \frac{\alpha_2 z^2 + \alpha_1 z + \alpha_0}{(z-1)(z+1)}\end{aligned}$$

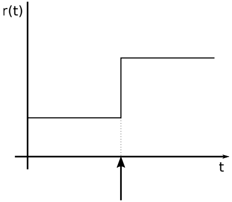
where $\alpha_2, \alpha_1, \alpha_0$ are design parameters.



https://www.youtube.com/watch?v=JEpWlTl95Tw

Alterantive Derivative Action (Continous-time)

Imagine a step change in reference signal $r(t)$. This results in a theoretically infinite, practically very large response of the derivative term.



$\Rightarrow$ Add a low-pass filter to the derivative term:

$$\frac{U_d(s)}{E(s)} = \frac{K_d s}{1 + s\tau}$$

With $s = j\omega$, breakpoint at $\omega = 1/\tau$. This prevents amplification of high frequencies.

Alterantive Derivative Action (Continous-time)

$$\frac{U_d(s)}{E(s)} = \frac{K_d s}{1 + s\tau}$$

Further $e(t)$ is replaced by $c \cdot r(t) - y(t)$ with c the set point weighting, which is often set to zero to further reduce immediate influence of a sudden set-point jump.

In the time-domain:

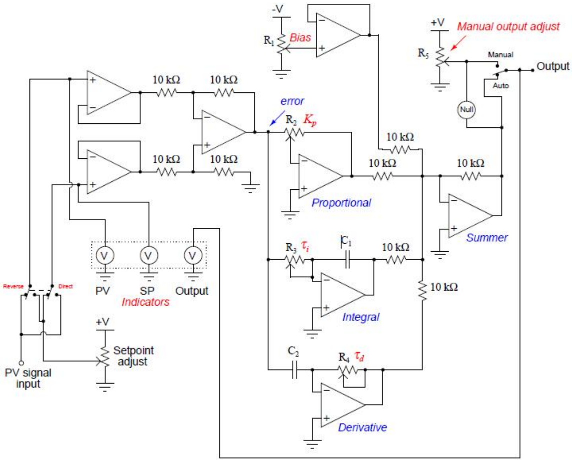
$$u_d(t) = -\tau \frac{du_d}{dt} + K_d \frac{d}{dt}(c \cdot r(t) - y(t))$$

Outline

1. Introduction
2. Analog and Digital formulations
3. Implementations
4. PID Tuning

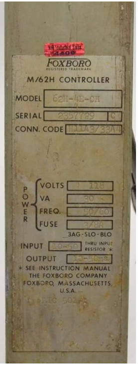
Analog Implementation

The key building block is the operational amplifier (op-amp).



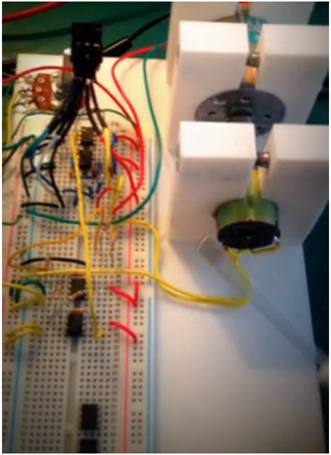
- PV - Process Variable $y(t)$
- SP - Set Point $r(t)$
- Output - Control action $u(t)$

Analog Implementation



Analog PID controller: FOXBORO 62H-4E-OH M/62H

Analog PI Motor Speed Control



https://youtu.be/6W3PLiVIcmE

Digital Implementation

The difference equations are typically implemented in a microcontroller in an FPGA (field-programmable gate array device):

$$u[k] = K_p e[k] + \frac{K_d}{T_s}(e[k] - e[k-1]) + u_i[k]$$

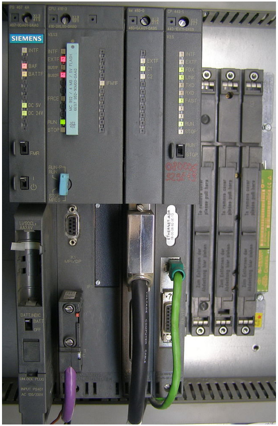
$$\text{with } u_i[k] = u_i[k-1] + K_i T_s e[k]$$

Pseudocode

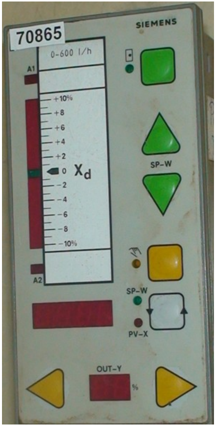
```
previous_error = 0
integral = 0
Start:
error = setpoint - measured_value
proportional = KP * error
integral = integral + Ki*sampling_time*error
derivative = Kd*(error-previous_error)/sampling_time
output = proportional + integral + derivative
previous_error = error
wait (sampling_time)
goto Start
```

Digital Implementation

PLC with a digital PID module:



Digital PID:



PLC

A Programmable **L**ogic **C**ontroller (PLC) is a digital computer used for automation in process industry.

What is a PLC? Basics of PLCs

https://youtu.be/iWgHqqunsyE

Outline

Manual Tuning

The effects of each of the controller parameters K_p, K_i and K_d on the closed-loop system are summarized in the table below.

	Closed-loop response			
PID gains	Rise Time	Overshoot	Setlling time	Steady-State error
$K_p \uparrow$	Decrease	Increase	Small Change	Decrease
$K_i \uparrow$	Decrease	Increase	Increase	Eliminate
$K_d \uparrow$	Small change	Decrease	Decrease	No change

Important note

Changing one parameter can influence the effect of the other two. Therefor, use this table only as a reference.

Manual Tuning

One possible way is as follows (the controller is connected to the plant):

1. Set K_i and K_d equal to 0.
2. Increase K_p until you observe that the step response is fast enough and the steady-state error in small.
3. Start adding some integral action in order to get rid of the steady-state error. Keep in mind that too much K_i can cause instability!
4. Add some derivative action in order to quickly react to disturbances and/or dampen the response.

Heuristic Methods

Sometimes the **mathematical model** of the plant is **not known**. In these cases we will uses the **heuristic methods**:

- Ziegler-Nichols tuning rule based on step response (First method)
- Ziegler-Nichols tuning rule based on critical gain and critical period (Second method)

Heuristic Methods: Ziegler-Nichols tuning rule

This method relies on empirically determining two parameters of the system:

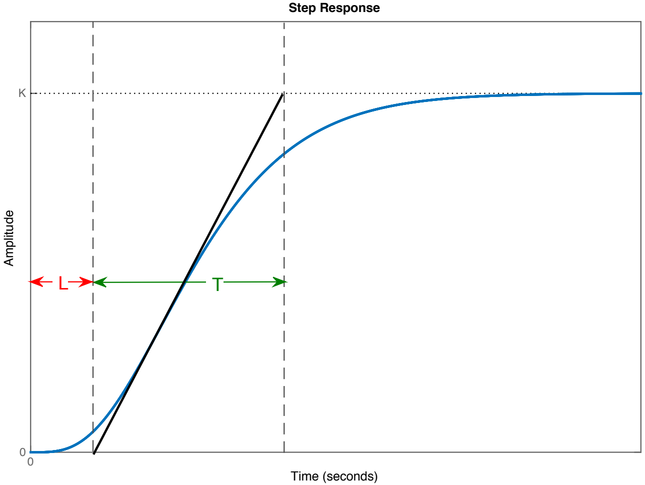
1. Set the integral and derivative gains to 0
2. Increase the proportional gain K_p until the output of the control loop starts oscillating with at constant amplitude. The value of K_p at this point is referred to as ultimate gain $K_u \triangleq K_p$
3. Measure the period of the oscillations T_u at the output
4. Adjust the controller parameters according the table on the next slide.

First Method

In this method, the response of the plant to a unit-step input is obtained experimentally (because we do not know the mathematical model). If the plant involves neither integrators nor dominant complex-conjugate poles, then such a unit-step response curve may look S-shaped as shown in the figure on the next slide. This S-shaped curve may be characterized by two constants: delay time L and time constant T. The delay time and the time constant are determined by drawing a tangent line at the inflection point of the S-shaped curve and determining the intersections of the tangent line with the time axis and line Amplitude = K.

If the response to a step input does not exhibit an S-shaped curve, the first method does not apply.

First Method



First Method

Control Type	K_P	K_I	K_D
P	$\frac{T}{L}$	0	0
PI	$0.9\frac{T}{L}$	$K_p\frac{0.3}{L}$	0
PID	$1.2\frac{T}{L}$	$\frac{K_p}{2L}$	$0.5LK_P$

The PID controller tuned by the first method of Ziegler-Nichols rules gives:

$$
\begin{aligned}
\frac{U(s)}{E(s)} &= K_P + \frac{K_I}{s} + K_D s \\
&= 1.2\frac{T}{L}\left(1 + \frac{1}{2Ls} + 0.5Ls\right) \\
&= 0.6T\frac{\left(s + \frac{1}{L}\right)^2}{s}
\end{aligned}
$$

with a pole at the origin and a double zero at $s = -\frac{1}{L}$.

Second Method

With K_u and T_u defined as before, a starting point for the parameters can be determined:

Control Type	K_p	K_i	K_d
P	$0.5K_u$	-	-
PI	$0.45K_u$	$1.2K_p/T_u$	-
PD	$0.8K_u$	-	$K_pT_u/8$
PID	$0.6K_u$	$2K_p/T_u$	$K_pT_u/8$
Pessen Integral Rule	$0.7K_u$	$2.5K_p/T_u$	$3K_pT_u/20$
Some overshoot	$0.33K_u$	$2K_p/T_u$	$K_pT_u/3$
No overshoot	$0.2K_u$	$2K_p/T_u$	$K_pT_u/3$

If the output does not exhibit sustained oscillations for whatever value K_P may take, then the second method does not apply.

Ziegler-Nichols tuning rule(example)

Example

Consider a plant with a given model:

$$P(s) = \frac{1}{(s+1)^3}$$

- We compute the critical gain K_c. This is the value of K_p for which $\angle(K_p P(s)) = -180^\circ$. On the Nyquist plot this the value of K_p for which $K_p P(s)$ passes through $(-1, 0)$.

$$K_c P(j\omega_c) = -1$$

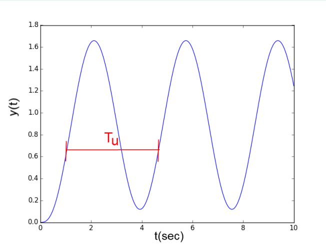
$$\Leftrightarrow K_c = -(j\omega_c + 1)^3$$

$$= (3\omega_c^2 - 1) + j(\omega_c^3 - 3\omega_c)$$

$$\omega_c^3 - 3\omega_c = 0 \Rightarrow \omega_c = \sqrt{3}$$

$$K_c = 8,\ T_u = \frac{2\pi}{\omega} = 3.628$$

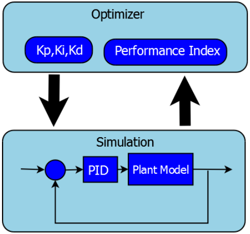
$$K_p = 4.8,\ K_I = 0.551 K_p,\ K_d = 0.45 K_p$$



Numerical Optimization Methods

The tuning of a PID controller is posed as a constrained optimization problem.

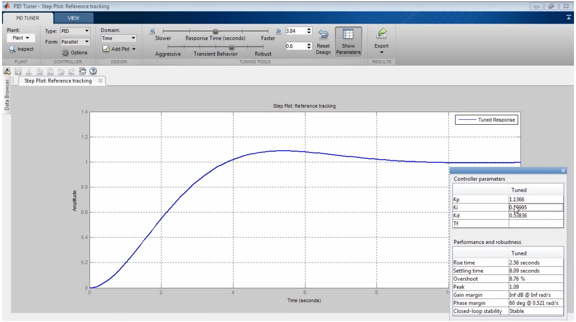
- For a given set of parameters K_p, K_i and K_d run a simulation of the closed-loop system, and compute some performance parameters (e.g. setting time, rise time, etc.) and a performance index.
- Optimize the performance index over the three PID gains.



Some Software Tools

Software Tool	Brief Description
pidtool / pidTuner	It is a Matlab tool to interactively design a SISO PID controller in the feed-forward path of single-loop, unity-feedback control configuration
Pidpy	It is a modular PID control library for python that supports PID auto tuning. `https://pypi.python.org/pypi/pypid/`
INCA PID Tuner	It is a commercial tuning tool developed by IPCOS. It has a vast library of PID structures for DCS and PLC Systems including Siemens, ABB, Honeywell, Emerson, etc. `http://www.ipcos.com/advancedprocesscontrol/advanced-process-control/pid-tuning-software/inca-pid-tuning/`

pidtool /pidTuner - Demo



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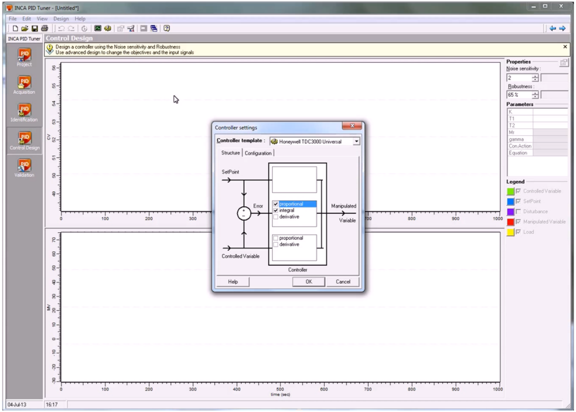
INCA PID Tuner Demo



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