System Modeling - Part 1

July 6, 2015

Outline

Introduction

We can derive the mathematical model of a system in two ways mainly:

- Physical Modeling:
  Applying the laws of physics, chemistry, thermodynamics,...
  Also called modeling from *First Principles*

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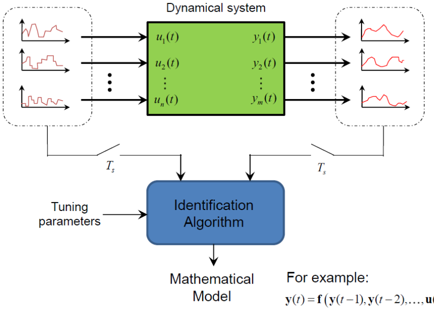
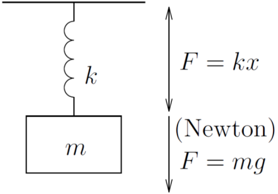
- Physical Modeling:
  Applying the laws of physics, chemistry, thermodynamics,...
  Also called modeling from *First Principles*
- System identification or *Empirical Modeling*:
  Developing models from observed or collected data



For example:
$\mathbf{y}(t) = \mathbf{f}\left(\mathbf{y}(t-1), \mathbf{y}(t-2), \ldots, \mathbf{u}(t), \mathbf{u}(t-1), \ldots\right)$

Outline

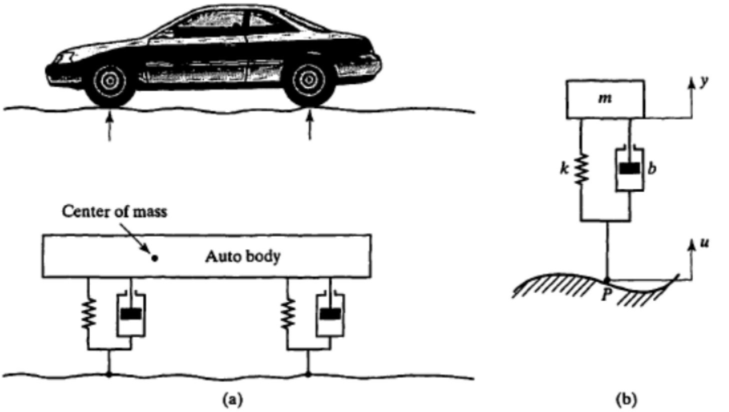
Example 1: Mass-Spring System



If spring is at rest at $x = 0$:

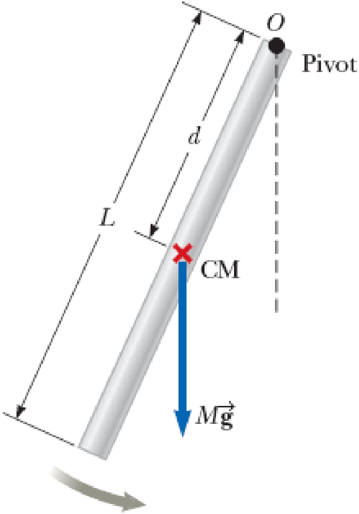
$$m \cdot \frac{d^2x}{dt^2} + k \cdot x = m \cdot g$$

Example 2: Mass-Spring Damped



Animation

Example 3: Pendulum



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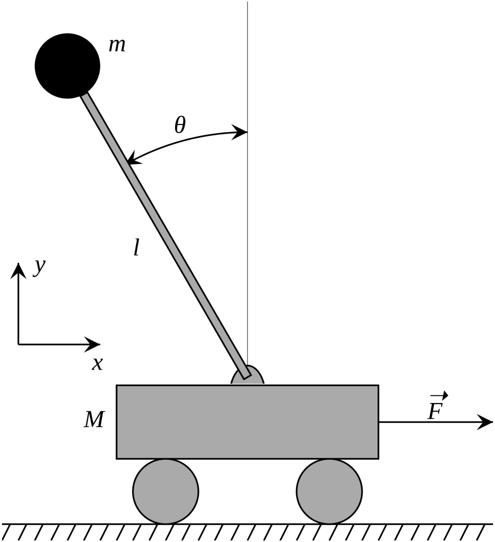
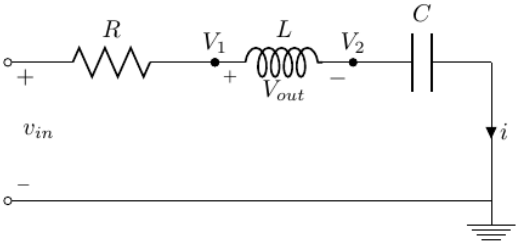
Example 4: Inverted Pendulum



Example 5: RLC Circuit



Besides input v_{in}, two internal variables needed to determine output $\Rightarrow$ Second-order System

Inputs	Ouputs	Choosen States
v_{in}	v_{out}	V_2
		i

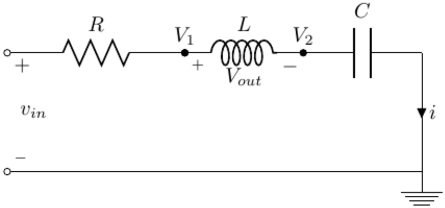
Example 6: RLC Circuit

Equations for each component:

$$i = \frac{V_{in} - V_1}{R}$$

$$V_1 - V_2 = L \cdot \frac{di}{dt}$$

$$i = C \cdot \frac{dV_2}{dt}$$



Example 7: RLC Circuit

- Writing derivatives of state variables in function of state variables and inputs:
- Writing output in function of state variables and inputs:

State Space Representation

This yields the State Space Representation of the dynamic system. In Matrix form:

$$\begin{bmatrix} a & b \end{bmatrix}$$