

# Outline

- 1 Introduction
- 2 Analog and Digital formulations
- 3 Implementations
  - Analog Implementation
  - Digital Implementation
- 4 PID Tuning
  - Manual Tuning
  - Heuristic Methods
  - Numerical Optimization Methods

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# Manual Tuning

The effects of each of the controller parameters  $K_p$ ,  $K_i$  and  $K_d$  on the closed-loop system are summarized in the table below.

PID gains	Closed-loop response			
	Rise Time	Overshoot	Settling time	Steady-State error
$K_p \uparrow$	Decrease	Increase	Small Change	Decrease
$K_i \uparrow$	Decrease	Increase	Increase	Eliminate
$K_d \uparrow$	Small change	Decrease	Decrease	No change

## Important note

Changing one parameter can influence the effect of the other two. Therefore, use this table only as a reference.

# Manual Tuning

One possible way is as follows (the controller is connected to the plant):

- 1 Set  $K_i$  and  $K_d$  equal to 0.
- 2 Increase  $K_p$  until you observe that the step response is fast enough and the steady-state error is small.
- 3 Start adding some integral action in order to get rid of the steady-state error. Keep in mind that too much  $K_i$  can cause instability!
- 4 Add some derivative action in order to quickly react to disturbances and/or dampen the response.

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# Heuristic Methods

When the **mathematical model of a plant is unknown** or it is too complicated to obtain, an analytical approach to design a PID controller is not possible. Therefore in these cases we have to use an experimental approach to tune the PID controller. Among the existing experimental approaches we can mention:

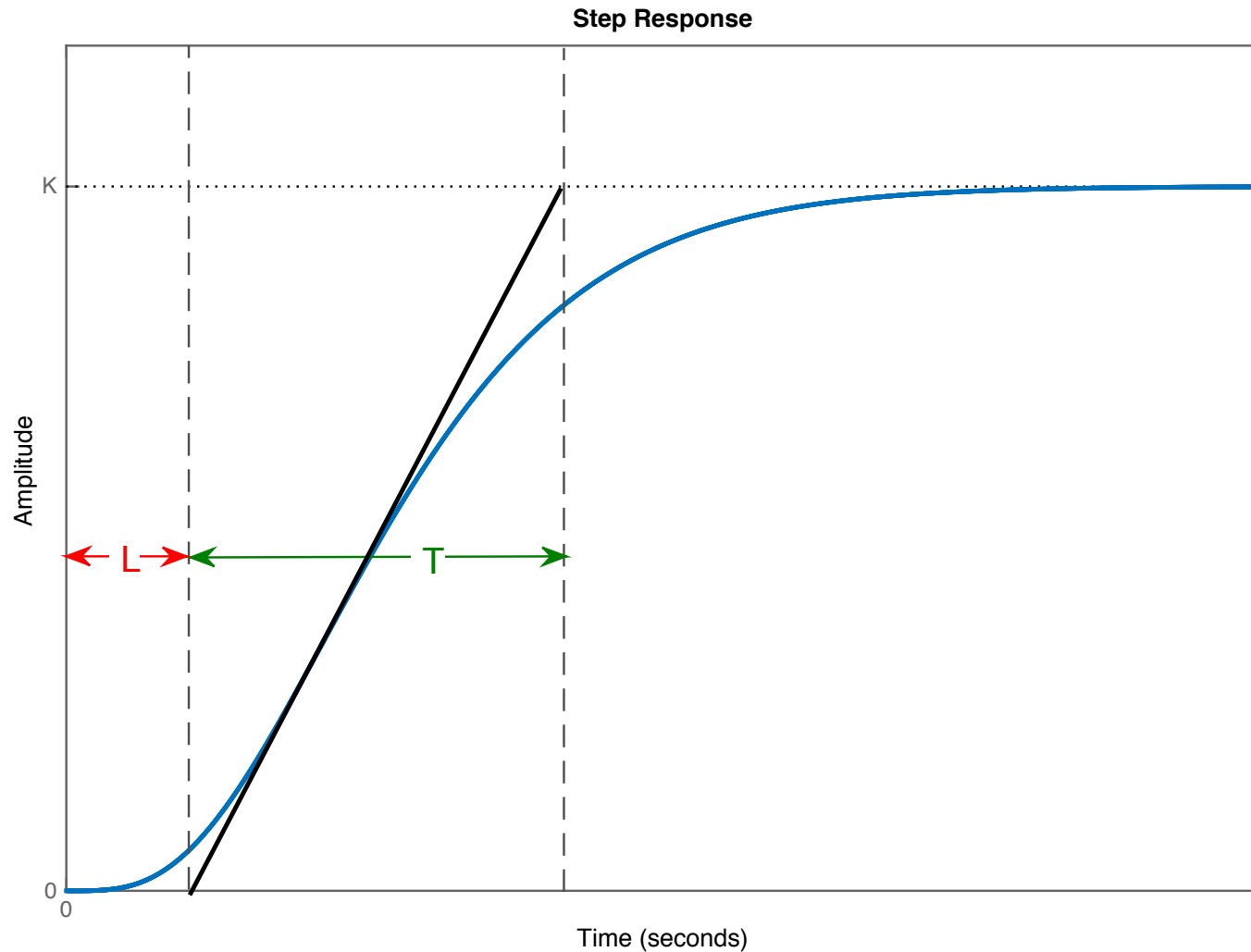
- Ziegler-Nichols tuning rule based on step response (First method)
- Ziegler-Nichols tuning rule based on critical gain and critical period (Second method)

## Ziegler-Nichols tuning rule: First Method

In this method, the response of the plant to a unit-step input is obtained experimentally. If the plant involves neither integrators nor dominant complex-conjugate poles, then such a unit-step response curve may look S-shaped as shown in the figure on the next slide. This S-shaped curve may be characterized by two constants: delay time  $L$  and time constant  $T$ . The delay time and the time constant are determined by drawing a tangent line at the inflection point of the S-shaped curve and determining the intersections of the tangent line with the time axis and a horizontal line crossing the y-axis at point  $(0,K)$ .

If the response to a step input does not exhibit an S-shaped curve, the first method does not apply.

# Ziegler-Nichols tuning rule: First Method





# Ziegler-Nichols tuning rule: First Method

Ziegler and Nichols suggested to set the values of  $K_p$ ,  $K_i$ , and  $K_d$  according to the formulas shown in the following table:

Control Type	$K_p$	$K_i$	$K_d$
P	$T/L$	0	0
PI	$0.9 \frac{T}{L}$	$K_p \frac{0.3}{L}$	0
PID	$1.2 \frac{T}{L}$	$\frac{K_p}{2L}$	$0.5 L K_p$

# Ziegler-Nichols tuning rule: First Method

The PID controller tuned by the first method gives:

$$\begin{aligned}\frac{U(s)}{E(s)} &= K_p + \frac{K_i}{s} + K_d s \\ &= 1.2 \frac{T}{L} \left( 1 + \frac{1}{2Ls} + 0.5Ls \right) \\ &= 0.6 T \frac{\left( s + \frac{1}{L} \right)^2}{s}\end{aligned}$$

with a pole at the origin and a double zero at  $s = -\frac{1}{L}$ .

## Ziegler-Nichols tuning rule: Second Method

This method is based on the value of  $K_p$  that results in marginal stability when only proportional control action is used.

- 1 Set the integral and derivative gains to zero ( $K_p = K_d = 0$ );
- 2 Increase the proportional gain  $K_p$  until the output of the control loop starts oscillating with a constant amplitude. The value of  $K_p$  at this point is referred to as ultimate gain ( $K_u \triangleq K_p$ );
- 3 Measure the period of the oscillations  $T_u$  at the output of the closed-loop system;
- 4 Use  $K_u$  and  $T_u$  to determine the gains of the PID controller according to the tuning rule table shown in the next slide.

# Ziegler-Nichols tuning rule: Second Method

Control Type	$K_p$	$K_i$	$K_d$
P	$0.5K_u$	-	-
PI	$0.45K_u$	$1.2K_p/T_u$	-
PD	$0.8K_u$	-	$K_p T_u/8$
PID	$0.6K_u$	$2K_p/T_u$	$K_p T_u/8$
Pessen Integral Rule	$0.7K_u$	$2.5K_p/T_u$	$3K_p T_u/20$
Some overshoot	$0.33K_u$	$2K_p/T_u$	$K_p T_u/3$
No overshoot	$0.2K_u$	$2K_p/T_u$	$K_p T_u/3$

If the output does not exhibit sustained oscillations for whatever value  $K_p$  may take, then the second method does not apply.

Keep in mind that we are working with heuristic tuning rules, and therefore some additional fine tuning might be necessary.

# Ziegler-Nichols tuning rule: Second Method (example)

## Example

Consider a plant with a given model:

$$P(s) = \frac{1}{(s+1)^3}$$

- We compute the critical gain  $K_u$ . This is the value of  $K_p$  for which  $\angle(K_p P(s)) = -180^\circ$ . On the Nyquist plot this is the value of  $K_p$  for which  $K_p P(s)$  passes through  $(-1, 0)$ .

$$K_u P(j\omega_u) = -1$$

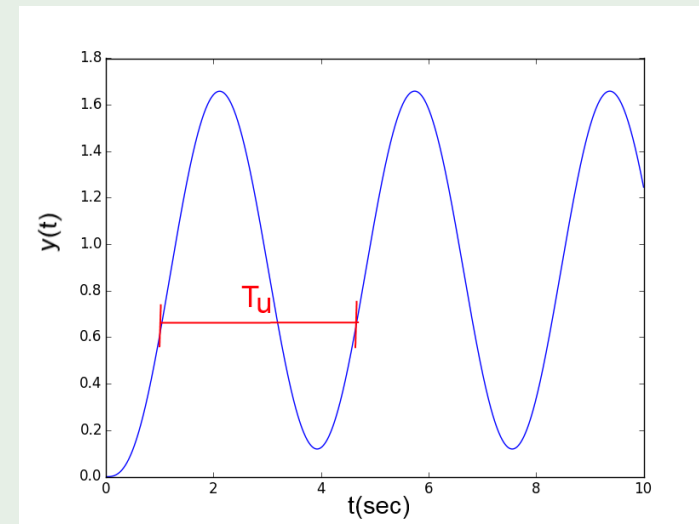
$$\Leftrightarrow K_u = -(j\omega_u + 1)^3$$

$$= (3\omega_u^2 - 1) + j(\omega_u^3 - 3\omega_u)$$

$$\omega_u^3 - 3\omega_u = 0 \Rightarrow \omega_u = \sqrt{3}$$

$$K_u = 8, T_u = \frac{2\pi}{\omega_u} = 3.628$$

$$K_p = 4.8, K_i = 2.6448, K_d = 2.16$$



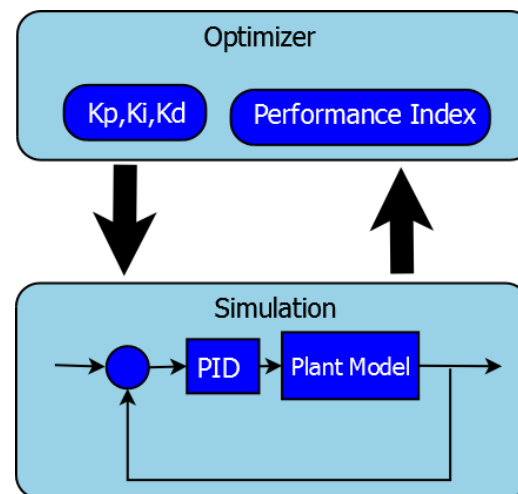
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# Numerical Optimization Methods

The tuning of a PID controller is posed as a constrained optimization problem.

- For a given set of parameters  $K_p$ ,  $K_i$  and  $K_d$  run a simulation of the closed-loop system, and compute some performance parameters (e.g. settling time, rise time, etc.) and a performance index.
- Optimize the performance index over the three PID gains.

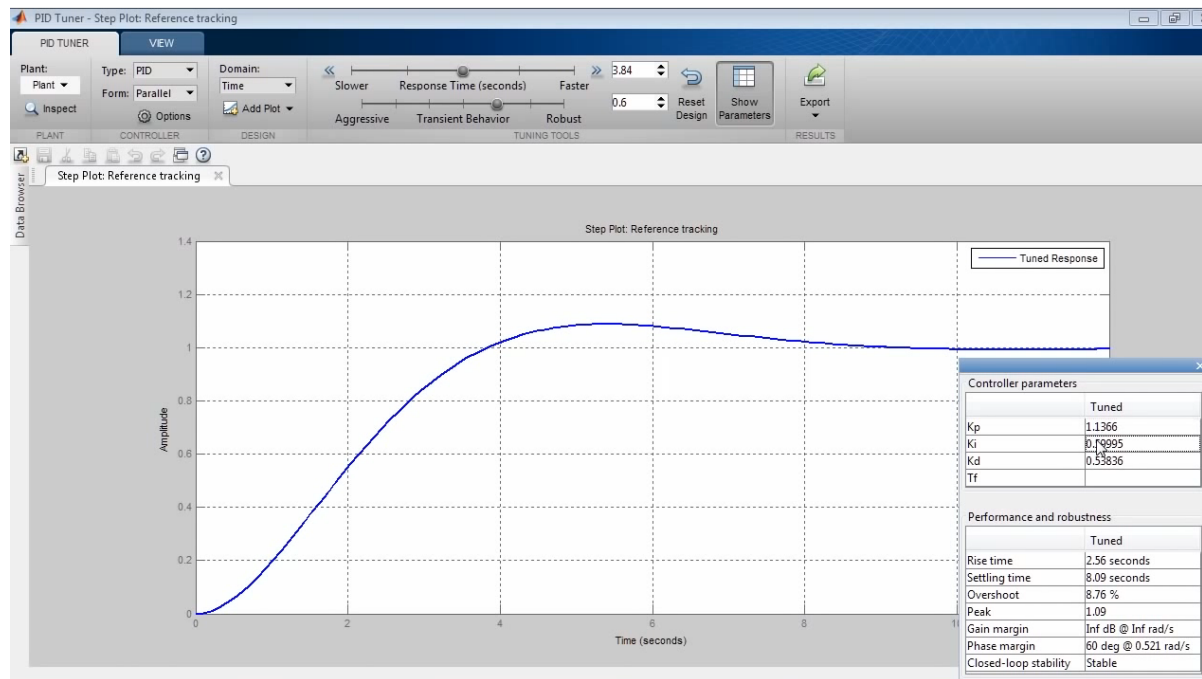


## Some Software Tools

Software Tool	Brief Description
pidtool/pidTuner	It is a Matlab tool to interactively design a SISO PID controller in the feed-forward path of single-loop, unity-feedback control configuration
Pidpy	It is a modular PID control library for python that supports PID auto tuning. <a href="https://pypi.python.org/pypi/pypid/">https://pypi.python.org/pypi/pypid/</a>
INCA PID Tuner	It is a commercial tuning tool developed by IPCOS. It has a vast library of PID structures for DCS and PLC Systems including Siemens, ABB, Honeywell, Emerson, etc. <a href="http://www.ipcos.com/advancedprocesscontrol/advanced-process-control/pid-tuning-software/inca-pid-tuning/">http://www.ipcos.com/advancedprocesscontrol/advanced-process-control/pid-tuning-software/inca-pid-tuning/</a>

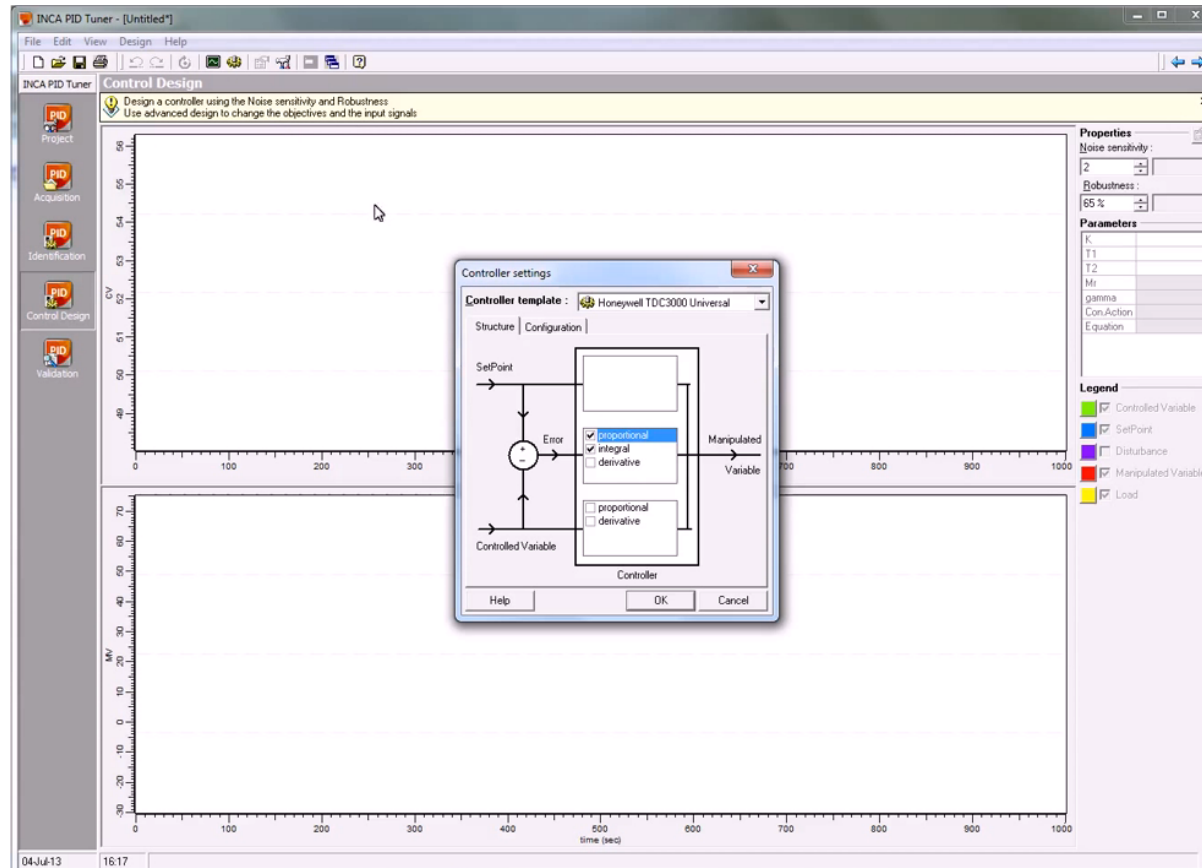


# pidtool/pidTuner - Demo



<https://www.youtube.com/watch?v=2tKe0caUv1I>

# INCA PID Tuner - Demo



<https://www.youtube.com/watch?v=XH2bkq1URSg>