Examples: Robustness

It is not possible to drive a formula 1 car using your knowledge of regular cars. However, you can drive a wide variety of road cars.

Outline

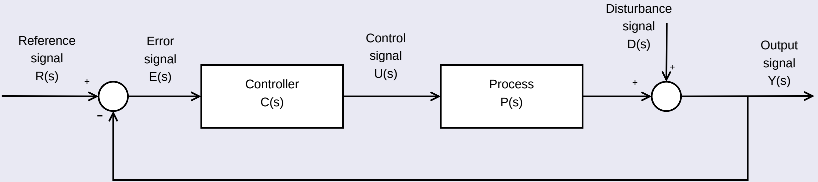
1. Basics
   - Control Theory
   - Demo: Inverted Pendulum
2. Control Goals
   - Examples
   - Exercise
3. Closed-loop systems
   - Sensitivity - Robustness
   - Types of systems and Steady State Error
   - Noise and disturbance rejection

Exercise: Could you name the correct property?

Outline

Transfer function of a closed-loop system

Classical control loop



$$Y(s) - D(s) = P(s)U(s) \quad \text{with } U(s) = C(s)E(s)$$
$$Y(s) - D(s) = P(s)C(s)E(s) \quad \text{with } E(s) = R(s) - Y(s)$$

$$Y(s) - D(s) = P(s)C(s)(R(s) - Y(s))$$
$$Y(s) - D(s) = P(s)C(s)R(s) - P(s)C(s)Y(s)$$

$$Y(s) = \frac{P(s)C(s)}{1 + P(s)C(s)}R(s) + \frac{1}{1 + P(s)C(s)}D(s)$$

Transfer function from $R(s)$ to $Y(s)$

Definition

We define $H(s)$ as the transfer function from $R(s)$ to $Y(s)$

$$H(s) \triangleq \frac{Y(s)}{R(s)} = \frac{P(s)C(s)}{1+P(s)C(s)} \quad \text{when } D(s) = 0.$$

- This transfer function $H(s)$ will help us to evaluate tracking
- Almost perfect tracking: the output $Y(s)$ will follow $R(s)$ very closely $\Rightarrow H(s) \approx 1$

Transfer function from $D(s)$ to $Y(s)$

Definition

We define $M(s)$ as the transfer function from $D(s)$ to $Y(s)$

$$M(s) \triangleq \frac{Y(s)}{D(s)} = \frac{1}{1 + P(s)C(s)} \quad \text{when } R(s) = 0.$$

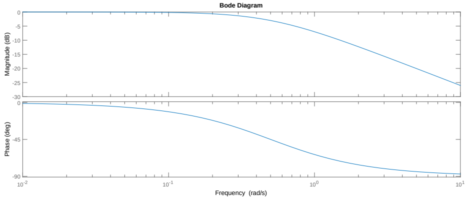
If the disturbance rejection of the control system is very good, the disturbances will have almost no effect on the output $\Rightarrow M(s) \approx 0$

if $\begin{cases} |H(j\omega)| \cong 1 \\ |M(j\omega)| \cong 0 \end{cases}$ then $|P(j\omega)C(j\omega)|$ **(open loop gain)** is very large.

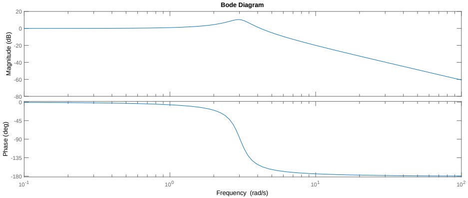
A large open loop amplification might lead to instabilities!

Exercise: Which controller do you prefer?

The closed-loop transfer functions for two different controllers for high precision surgery are:



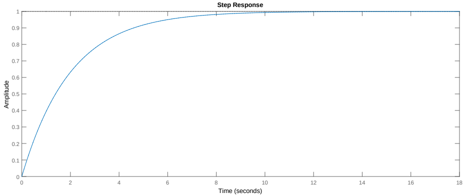
$$\frac{Y(s)}{R(s)} = \frac{0.5}{s+0.5}$$



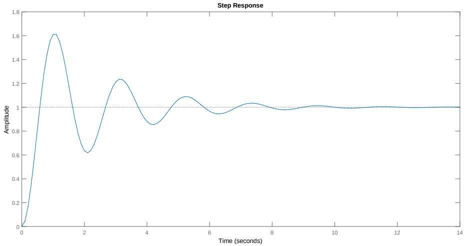
$$\frac{Y(s)}{R(s)} = \frac{10}{1.09s^2 + s + 10}$$

Exercise: Which controller do you prefer?

The output $Y(s)$ of the closed-loop system when $R(s)$ is a step function is as follows:



$$\frac{Y(s)}{R(s)} = \frac{0.5}{s + 0.5}$$

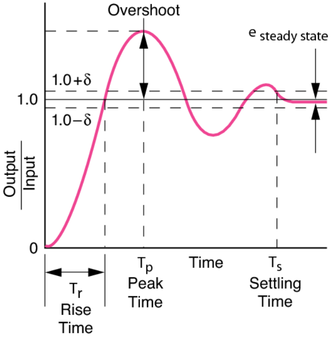


$$\frac{Y(s)}{R(s)} = \frac{10}{1.09s^2 + s + 10}$$

Quality of reference tracking

Look at the step response of the transfer function from $R(s)$ to $Y(s)$. The quality can be determined using these criteria:

- Rise-time
- Settling time
- Steady-state error
- Overshoot
- ...



Model errors

- The plant characteristics may be variable or time-varying. System modeling techniques identify the plant within a certain model class and with a certain amount of inaccuracy. So there always exists a plant uncertainty, which cannot be described exactly by the mathematical models.
- Control systems need to be made robust against this plant variability and uncertainty.

Outline

Sensitivity

Sensitivity is a measure of the relative change in the system due to a relative change in a chosen parameter. Here we look at $\Delta Y(s)$ due to $\Delta P(s)$.

$$Y(s) + \Delta Y(s) = \frac{(P(s) + \Delta P(s))C(s)}{1 + (P(s) + \Delta P(s))C(s)} R(s) + \frac{1}{1 + (P(s) + \Delta P(s))C(s)} D(s)$$

Look at the effect on the system without disturbances ($D(s){=}0$)

$$\Delta Y(s) = \frac{(P(s) + \Delta P(s))C(s)}{1 + (P(s) + \Delta P(s))C(s)} R(s) - \frac{P(s)C(s)}{1 + P(s)C(s)} R(s)$$

Sensitivity

$$
\begin{aligned}
&= \frac{(P(s)+\Delta P(s))C(s)(1+P(s)C(s)) - P(s)C(s) - P(s)C(s)(P(s)+\Delta P(s))C(s)}{(1+(P(s)+\Delta P(s))C(s)(1+P(s)C(s))} R(s) \\
&= \frac{\Delta P(s)C(s)}{(1+(P(s)+\Delta P(s))C(s))(1+P(s)C(s))} R(s) \\
&= Y(s)\frac{\Delta P(s)}{P(s)} \frac{1}{1+(P(s)+\Delta P(s))C(s)}
\end{aligned}
$$

Sensitivity

Now take the relative change due to this model uncertainty and take the limit for $\Delta x \rightarrow 0$; this gives the following (measure of the) sensitivity:

$$S_P^Y(s) = \frac{\frac{\partial Y}{Y}(s)}{\frac{\partial P}{P}(s)} = \frac{\partial Y}{\partial P}(s) \cdot \frac{P(s)}{Y(s)} = \frac{1}{1 + P(s)C(s)}$$

- Again, a very large $|P(s)C(s)|$ looks like a good choice, but again there is a risk for instability!
- Note that the sensitivity can be determined for any parameter