The design of lead and lag compensators

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Outline

Definitions

The main objective of this chapter: design and compensation of single-input-sigle-output linear time-invariant control systems.

- Compensation: the modification of the system dynamics to satisfy the given specifications.
- Specifications (transient response and steady-state requirements): given before the design.
- Design by root-locus method: making a new root locus by adding poles and zeros to the system's open-loop transfer function.
- Compensator: an other system inserted in parallel or in cascade with the system for the purpose of satisfying the specifications of the original system (e.g. lead, lag, lag-lead compensator's or PID controllers).

Compensators

A sinusoidal input is applied to the input of a network. We got a:

- lead network: if the steady-state output has a phase lead.
- lag network: if the steady-state output has a phase lag.
- lag-lead network: if we have phase lag and phase lead in the output but in different frequentcy regions (lag when the input has low frequency and lead in high frequentcy).

The amount of lag/lead is a function of the frequency.

A compensator with characteristic of a lead network, lag network, or lag-lead network is called a lead compensator, lag compensator, or lag-lead compensator.

Remark: with trail and error we find the optimal compensator

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2. General design
3. Lead copensations
4. Lag compensation

Controllers

Scetching the problem:

1. We got a plant that not achieve all the specifications (and we cannot change the parameters of the original system).
2. We have to change other parameters such that the system will achieve all the specifications.
3. These other parameters can be changed by changing the root loci of the closed-loop system.
4. So we search the root loci of a compensator such that the overall system achieve the specifications.
5. We let the original system interact (cascade or parallel) with the compensator.

Remark: we discuss only continu time systems.

Root locus approach

The method:
Given: the root loci of the open-loop system (this are the parameters).
Now, the method consist of graphical determining the root loci of the closed-loop system.

Example: We want a certain gain of a system:

- The system is not stable at that gain.
- We have to change the root loci (and make the system stable) by designing a compensator
- Now, we got different root loci and the system is stable at the certain gain. So, the specifications are achieved.

In **general**: we want to have the root loci of the system on the good locations.

Addition of poles/zeros

Addition of poles

The addition of a pole to the open-loop transfer function has the effect of pulling the root locus to the right, tending to lower the system's relative stability and to slow down the settling of the response.

Addition of zeros

The addition of a zero to the open-loop trans- fer function has the effect of pulling the root locus to the left, tending to make the system more stable and to speed up the settling of the response. It also increase:

- the degree of anticipation into the system
- the speed of the transient response

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Lead compensators

Steps for creating a compensator:

1. Determine the desired location for the dominant closed-loop poles (from specifications).
2. If the adjustment of the gain connot yield the desired closed loop poles, then calculate the angle deficiency ϕ. This deficiency has to by contributed by the lead compensator.
3. We take a compensator of this form: $C(s) = K\frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}$ with $0 < \alpha < 1$.
4. α and T must be created such that there isn't a angle deficiency. If this already be done, then take α as graet as possible. K can be found by the requirements of the open loop gain.

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Lag compensators

When the system unsatisfactory the steady state (but can satisfy the transient response)

We go increase the open loop gain, while we try to untouch the transient response (concreet: don't change the root locus in the neigbourhood of the dominant closed-loops, but increase the open loop gain)

How to fix a small changing of the root loci -¿ the angle contribution of the lag compensator should be not larger than 5 (how? we place the pole and zero of the lag network close togheter and close to the origin of the s-plane)

$C(s) = K\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}$ with $\beta > 1$ beta can e large if the poles and zeros are close to the origin, T has to be taken large.