# Introduction to Control

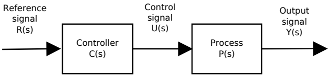
July 20, 2015

# Outline

# An introduction to control

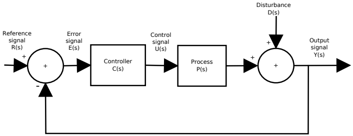
# What is control?

- The goal is to find an input (control signal U(s)) such that the process produces the desired output
- Open loop control system: the actual output signal has no effect on the control action



# A general set-up of a closed loop system

- We will focus on closed loop control systems



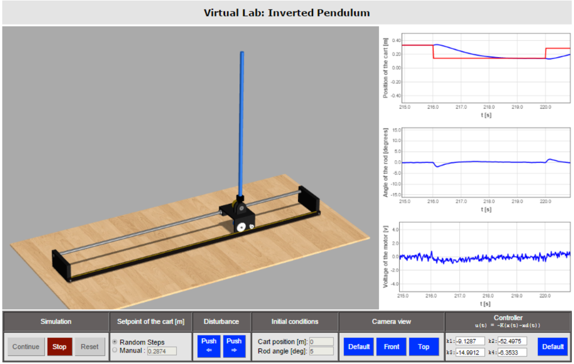



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Figure: Inverted Pendulum

# Concrete Control

- On-off controller
  - Thermostate at home
- **PID controllers, Lead and lag compensators (this course)**
  - Cruise-control in your car
- More advanced controllers
  - STATE-space feedback controllers
  - Model Predictive Controller (MPC)
  - Fuzzy Control
  - Neuro-fuzzy Control
  - ...

# Outline

# What is good control?

- Before we will start to design control systems we will first focus on the question. What is good control?
- It depends on the application
  - Stability
  - Disturbance rejection
  - Reference tracking (speed)
  - Sensitivity to errors on model
  - Etc...

# Examples: stability


Figure: Space shuttles are like inverted pendulums. How do you make sure they don't flip over.

# Examples: Disturbance rejection

- Your body will try to keep the temperature in your body as constant as possible. No matter what the outside temperature is. Two people will have almost the same body temperature.

Figure: Flickr.com, tent86, Marathon Des Sables 046

Figure: Jack Zalium, Enduring, https://creativecommons.org/licenses/by-nd/2.0/

# Examples: Reference tracking

Figure: Audi has a system for automatic driving in traffic jams. The audi will follow the car in front of him at an appropriate distance. youtube

# Exercise: name the correct property

# Outline

# Transfer function of a closed-loop system

$$
\begin{aligned}
Y(s) - D(s) &= P(s)U(s) \\
&\quad \text{with } U(s) = C(s)E(s) \\
Y(s) - D(s) &= P(s)C(s)E(s) \\
&\quad \text{with} E(s) = R(s) - Y(s) \\
Y(s) - D(s) &= P(s)C(s)(R(s) - Y(s)) \\
Y(s) - D(s) &= P(s)C(s)R(s) - P(s)C(s)Y(s) \\
&\Rightarrow Y(s) = \frac{P(s)C(s)}{1 + P(s)C(s)}R(s) + \frac{1}{1 + P(s)C(s)}D(s)
\end{aligned}
$$

# Transfer function from $R(s)$ to $Y(s)$

We define S as the transfer functin from $R(s)$ to $Y(s)$

$$S(s) \triangleq \frac{P(s)C(s)}{1 + P(s)C(s)}$$

This transfer function $S(s)$ will help us to evaluate tracking
Almost perfect tracking: the output $Y(s)$ will follow $R(s)$ very closely $\Rightarrow S(s) \approx 1$

# Transfer function from $D(s)$ to $Y(s)$

We define $T(s)$ as the transfer function from $D(s)$ to $Y(s)$.

$$T(s) \triangleq \frac{1}{1 + P(s)C(s)}$$

If the disturbance rejection is very good the disturbances will have almost no effect on the output $\Rightarrow T \approx 0$

$$\begin{cases} |S(j\omega)| \cong 1 \\ |T(j\omega)| \cong 0 \end{cases}$$

$\Rightarrow |P(j\omega)C(j\omega)|$ **(open loop gain)** is very large.

! a large open loop amplification might lead to an unstable system

# Model errors

- In practice, transfer function of $P(s)$ might be unknown. It is important to know what the effect of the model errors will be. Sensitivity and robustness are key-concepts to evaluate these effects.
- Sensitivity
  - Quantifies the effect of a small perturbation on the model will be on the output.
- Robustness
  - This concept refers to bigger changes on the model. A controller is robust if it works properly over a given set of parameters.

# Sensitivity

- Sensitivity is a measure for the effect of a (small) disturbance on the model (e.g. variations in the process parameters)

$$Y(s) + \Delta Y(s) = \frac{(P(s) + \Delta P(s))C(s)}{1 + (P(s) + \Delta P(s))C(s)} R(s) + \frac{1}{1 + (P(s) + \Delta P(s))C(s)} D(s)$$

- Look at the effect on the system without disturbances ($D(s)$=0)

$$\begin{aligned}
\Delta Y &= \frac{(P + \Delta P)C}{1 + (P + \Delta P)C} R - \frac{PC}{1 + PC} R \\
&= \frac{(P + \Delta P)C(1 + PC) - PC - PC(P + \Delta P)C}{(1 + (P + \Delta P)C)(1 + PC)} R \\
&= \frac{\Delta PC}{(1 + (P + \Delta P)C)(1 + PC)} R \\
&= Y \frac{\Delta P}{P} \frac{1}{1 + (P + \Delta P)C}
\end{aligned}$$

# Sensitivity

- Now take the relative change due to this disturbance of the model and take the limit for $\partial x \to 0$; this gives the following (measure of the) sensitivity:

$$S_P^Y(s) = \frac{\frac{\partial Y}{Y}(s)}{\frac{\partial P}{P}(s)} = \frac{1}{1 + P(s)C(s)}$$

- Again, a very large $|P(s)C(s)|$ looks like a good choice, but again there is a risk for instability!