

Outline

- 1 Linear differential equations
- 2 Laplace transform
- 3 Solving LDEs with the Laplace transform**
- 4 Properties of state-space representation
- 5 Transfer functions
 - Impulse response and time constant
 - Relationship between state space and transfer functions
- 6 Transient response analysis of first order and second order systems
 - First order systems
 - Second order systems

Solving LDEs with the Laplace transform 1/3

The Laplace transform can be used to solve LDEs with given initial conditions (the previous approach gave us the basis functions). This is done by using the following property (differentiation):

$$\mathcal{L}\{f^{(1)}\} = sF(s) - f(0),$$

$$\mathcal{L}\{f^{(2)}\} = s^2F(s) - sf(0) - f^{(1)}(0).$$

Via induction, the Laplace transform of the n th order derivative:

$$\mathcal{L}\{f^{(n)}\} = s^n F(s) - \sum_{i=1}^n s^{n-i} f^{(n-i)}(0)$$

Solving LDEs with the Laplace transform 2/3

$$\mathcal{L}\{f^{(n)}\} = s^n F(s) - \sum_{i=1}^n s^{n-i} f^{(n-i)}(0)$$

We want to solve the following LDE:

$$\sum_{i=0}^n A_i y^{(n-i)}(t) = f(t),$$

$$y^{(i)}(0) = c_i \quad \forall i = 0 \dots n.$$

Via the linearity of the Laplace transform:

$$\sum_{i=0}^n A_i \mathcal{L}\{y^{(n-i)}(t)\} = \mathcal{L}\{f(t)\}$$

Solving LDEs with the Laplace transform 3/3

$$\sum_{i=0}^n A_i \mathcal{L}\{y^{(n-i)}(t)\} = \mathcal{L}\{f(t)\} \quad (1)$$

$$\mathcal{L}\{f^{(n)}\} = s^n F(s) - \sum_{i=1}^n s^{n-i} f^{(n-i)}(0) \quad (2)$$

Expanding Eq. (2) into (1) yields:

$$Y(s) \sum_{i=0}^n A_i s^i - \sum_{i=1}^n \sum_{j=1}^i A_i s^{i-j} y^{j-1}(0) = F(s)$$

The solution in the time domain is obtained via the inverse Laplace transform: $y(t) = \mathcal{L}^{-1}\{Y(s)\}$.