Classifications of systems

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Overview

1. Number of inputs and outputs
2. Continuous vs. Discrete time
3. Linear vs. Nonlinear
4. Causal vs. Non-causal
5. Time-invariant vs. Time-varying
6. Lumped vs. Distributed

Based on the number of inputs and outputs

1. **SISO**: Single Input Single Output
2. **SIMO**: Single Input Multiple Output
3. **MISO**: Multiple Input Single Output
4. **MIMO**: Multiple Input Multiple Output
5. **Autonomous**: No inputs and one or more outputs

Continuous vs. Discrete time

We will discuss both types simultaneously in order to emphasize the similarities (and differences).

Continuous system

1. It has continuous input and output signals
2. We denote continuous time by $t \in \Re$
3. We denote functions of continuous time with round brackets, e.g.: $x(t)$

Discrete system

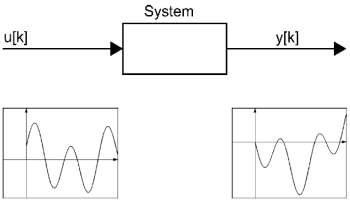
1. It has discrete input and output signals
2. We denote discrete time by $k \in Z$
3. We denote functions of continuous time with square brackets, e.g.: $x[k]$

Continuous vs. Discrete time

Continuous

For every moments $t \in \Re$, the system has:

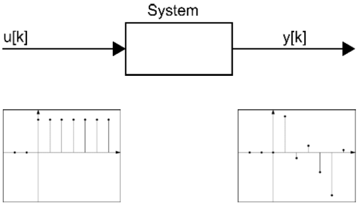
1. A vector of inputs $\mathbf{u}(t)$
2. A vector of outputs $\mathbf{y}(t)$
3. A vector of states $\mathbf{x}(t)$



Discrete

For every moments $k \in Z$, the system has:

1. A vector of inputs $\mathbf{u}[k]$
2. A vector of outputs $\mathbf{y}[k]$
3. A vector of states $\mathbf{x}[k]$



Linear vs. Nonlinear: a linear system

Definition

A system is linear if $u_1(t) \rightarrow y_1(t)$ (*input* $u_1(t)$ *results in output* $y_1(t)$) and $u_2(t) \rightarrow y_2(t)$ imply that

$$\alpha u_1(t) + \beta u_2(t) \rightarrow \alpha y_1(t) + \beta y_2(t)$$

Properties of a linear system (contained in the definition):

- Superposition
- Homogeneity

Linear vs. Nonlinear: a linear system

Properties of a linear system (contained in the definition):

- Superposition

  $u_a(t) \to y_a(t),\ u_b(t) \to y_b(t) \Leftrightarrow u_a(t) + u_b(t) \to y_a(t) + y_b(t)$

  This means the output of a system can be found by splitting up the input and solving it separately (analogous to the homogeneous part of an ordinary differential equation).

- Homogeneity

  $\alpha u(t) \to \alpha y(t)$

How to recognize a linear system:

- Linear in all of the variables
- No constant factors

Examples

- $$\begin{cases} \dot{x} = u \\ \dot{y} = x + 2u \end{cases}$$

  Linearity of this system is easily verified, based on the linearity of the derivative:

  $$\begin{cases} \alpha\dot{x}_a(t) + \beta\dot{x}_b(t) = \alpha u_a(t) + \beta u_b(t) \\ \alpha\dot{y}_a(t) + \beta\dot{y}_b(t) = \alpha x_a(t) + \beta x_b(t) + 2\alpha u_a(t) + 2\beta u_b(t) \end{cases}$$

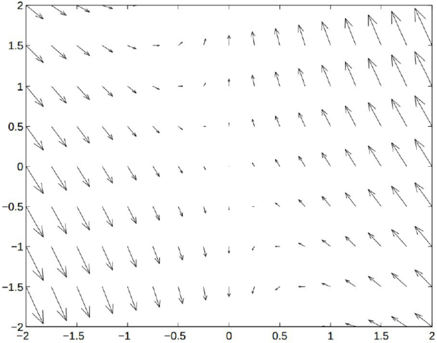
- $$\begin{cases} \dot{x}_1 = u \\ \dot{x}_2 = \frac{3}{2}x_1 + u \\ \dot{y} = ax_1 - x_2 + 2u \end{cases}$$

Linear vs. Nonlinear: autonomous linear systems

Continuous-time autonomous linear dynamical systems are described by:

$$\dot{x}(t) = Ax(t)$$

Example: $\dot{x}(t) = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix} x(t)$



Linear vs. Nonlinear: violating homogeneity

All nonhomogeneous systems are strictly speaking nonlinear, e.g.:

$$\begin{cases} \dot{x}(t) = x(t) + u(t)^2 \\ \dot{y}(t) = x(t) \end{cases} \qquad \Rightarrow \text{nonhomogeneous}$$

This is nonlinear, because the term $u(t)^2$ violates homogeneity.
It can be turned into a linear system with inputs $z(t) = u(t)^2$.

$$\begin{cases} \dot{x}(t) = x(t) + z(t) \\ \dot{y}(t) = x(t) \end{cases} \qquad \Rightarrow \text{linear}$$

$\rightarrow$ nonhomogeneous systems that are linear apart from some function of inputs are often treated as linear systems.

Linear vs. Nonlinear: nonlinear systems

Some examples of nonlinear systems:

$$\begin{cases} \dot{x}_1(t) = x_1(t) + u(t) \\ \dot{x}_2(t) = x_1(t)x_2(t) \\ y(t) = x_1(t) + x_2(t) \end{cases}$$

$$\begin{cases} \dot{x}(t) = sin(x(t)) + u(t) \\ y(t) = x(t) \end{cases}$$

$$\begin{cases} \dot{x}(t) = 2u(t) + 1 \\ y(t) = cos(x(t)) \end{cases}$$

Linear vs. Nonlinear

Predominantly linear

Simple electrical systems

- Circuits with ideal resistors, capacitors and inductors

Simple mechanical systems

- Systems with ideal springs

Inherently nonlinear

Chemical systems
Biological systems
Economical systems
More involved electrical or mechanical systems
...

Linear vs. Nonlinear

- Reality is nonlinear
- However, this course will only deal with linear systems
- Why we prefer linear systems:

  The previously mentioned properties will allow for a thorough study of the system
- Why we are allowed to use linear systems, even in a nonlinear setting:

  You can linearize around an equilibrium point (we will do this in the next lecture)

Causal vs. Non-causal

- A causal system only depends on the present and the past, not on the future
- A non-causal system (also) depends on the future
- (Almost) all physical systems are causal
  - A telephone:
    - It will not ring for future calls
  - Any human:
    - Is a system that will only react on inputs it has already received
    - If we react because we expect something to happen in the future, then that expectation arose from past or present inputs

source:
http://www.deekshith.in/2013/03/causal-and-non-causal-systems-better-explained.html

Causal vs. Non-causal

How do non-causal systems arise?

A possibility is by greatly reducing the complexity of a system, in which some causes of events are taken out of the equations.
Example:

- A model of the economical consumption (**output**)
- A lot of influencing factors, but the only **input** is the employment numbers
- Current and past employment numbers determine consumption, but when someone gets fired, they will continue to work for several weeks in most instances, but their consumption will drop immediately
  $\rightarrow$ A correcct model for this relation would have to be **non-causal**
- The non-causal model for this input-output relation is not useful if you want to determine the level of consumption
- You could use the relation to see a drop in employment, before it is visible in the employment numbers

Causal vs. Non-causal

Examples of non-causal systems: **expectations**

Modelling housing prices

- People are willing to offer more for houses if they expect rising prices
- It is hard to measure the expectations or housing prices
- Sometimes economists use their own predictions of housing prices to replace the expectations

Causal vs. Non-causal

Examples of non-causal systems: **image processing**

- The input to our system (the image processor) is a two dimensional series of values ($u(k,l)$): the color values at the different pixels of the original image
- The output is a processed image ($y(k,l)$)
- There is now no reason to want causality; the input depends on position and not on time
- $y(k,l)$ can rely on 'later' values like $u(k+1,l+1)$, without that being a problem

Original image

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