Introduction to Control

July 22, 2015

Outline

1. Basics
   - Classical control loop
   - Demo: Inverted Pendulum
   - Elements
2. Control Goals
   - stability
   - disturbance rejection
   - Reference tracking
   - Exercise
3. Closed-loop system
   - Sensitivity
   - Robustness
   - Types of systems and Steady State Error
   - Noise- and disturbance rejection

An introduction to control

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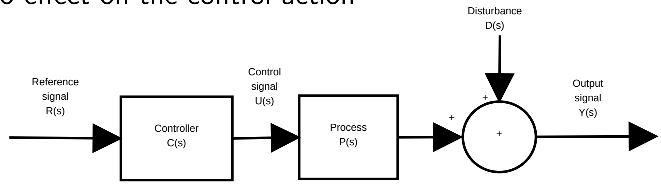
What is control?

- The goal is to find an input (control signal U(s)) such that the process $P(s)$ produces the desired output
- Open loop control system: the actual output signal $Y(s)$ has no effect on the control action

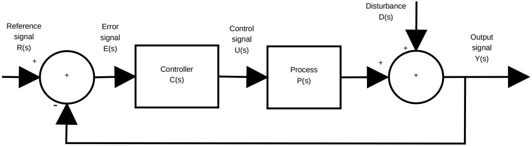


$$Y(s) = P(s)U(s) = P(s)C(s)R(s)$$

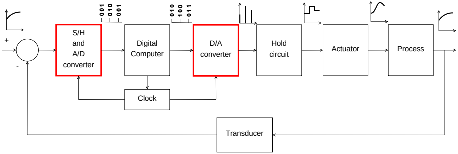
Remember the example of pouring a glass of water without looking at the glass

A general set-up of a closed loop system

- We will focus on closed loop control systems



- Example: Inverted pendulum
- Digital Control Loop:



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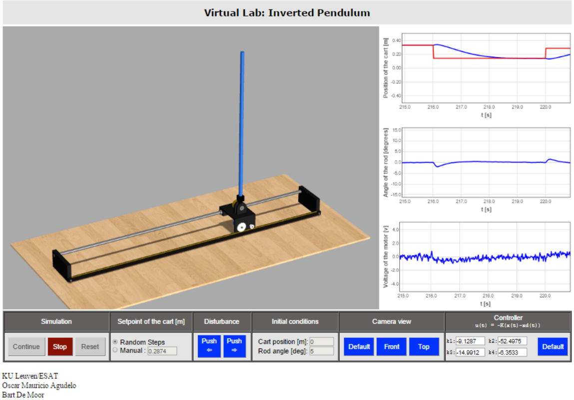

Inverted Pendulum

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Concrete Controllers

- On-off controller
  - Thermostate at home
- **PID controllers, Lead and lag compensators (this course)**
  - Cruise-control in your car
- More advanced controllers
  - STATE-space feedback controllers
  - Model Predictive Controller (MPC)
  - Fuzzy Control
  - Neuro-fuzzy Control
  - ...

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What is good control?

- Before we will start to design control systems we will first focus on the question. What is good control?
- It depends on the application
  - Stability
  - Disturbance rejection
  - Reference tracking (speed)
  - Sensitivity to errors on model
  - Etc...

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Examples: stability

Space shuttles are like inverted pendulums. How do you make sure they don't flip over.

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Examples: Disturbance rejection

- Your body will try to keep the temperature in your body as constant as possible. No matter what the outside temperature is. Two people will have almost the same body temperature.

Flickr.com, tent86, Marathon Des Sables 046

Jack Zalium, Enduring, https://creativecommons.org/licenses/by-nd/2.0/

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Examples: Reference tracking

Audi has a system for automatic driving in traffic jams. The audi will follow the car in front of him at an appropriate distance.
https://www.youtube.com/watch?v=Qa_ZSRj0WM0

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Exercise: name the correct property

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Transfer function of a closed-loop system

$$Y(s) - D(s) = P(s)U(s)$$

$$\text{with } U(s) = C(s)E(s)$$

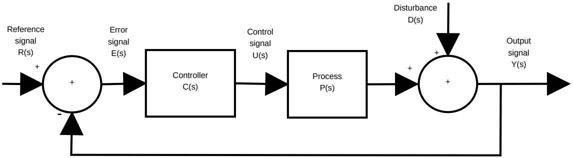
$$Y(s) - D(s) = P(s)C(s)E(s)$$

$$\text{with } E(s) = R(s) - Y(s)$$

$$Y(s) - D(s) = P(s)C(s)(R(s) - Y(s))$$

$$Y(s) - D(s) = P(s)C(s)R(s) - P(s)C(s)Y(s)$$

$$\Rightarrow Y(s) = \frac{P(s)C(s)}{1 + P(s)C(s)} R(s) + \frac{1}{1 + P(s)C(s)} D(s)$$



Transfer function from $R(s)$ to $Y(s)$

We define S as the transfer function from $R(s)$ to $Y(s)$

$$S(s) \triangleq \frac{P(s)C(s)}{1+P(s)C(s)}$$

This transfer function $S(s)$ will help us to evaluate tracking
Almost perfect tracking: the output $Y(s)$ will follow $R(s)$ very closely $\Rightarrow S(s) \approx 1$

Transfer function from $D(s)$ to $Y(s)$

We define $T(s)$ as the transfer function from $D(s)$ to $Y(s)$.

$$T(s) \triangleq \frac{1}{1 + P(s)C(s)}$$

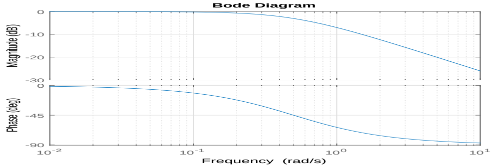
If the disturbance rejection is very good the disturbances will have almost no effect on the output $\Rightarrow T \approx 0$

$$\begin{cases} |S(j\omega)| \cong 1 \\ |T(j\omega)| \cong 0 \end{cases}$$

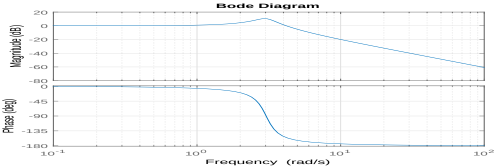
$\Rightarrow |P(j\omega)C(j\omega)|$ **(open loop gain)** is very large.
! a large open loop amplification might lead to an unstable system

Example: Which controller do you prefer

The transfer function from $R(s)$ to $Y(s)$ for two different controllers for high precision surgery



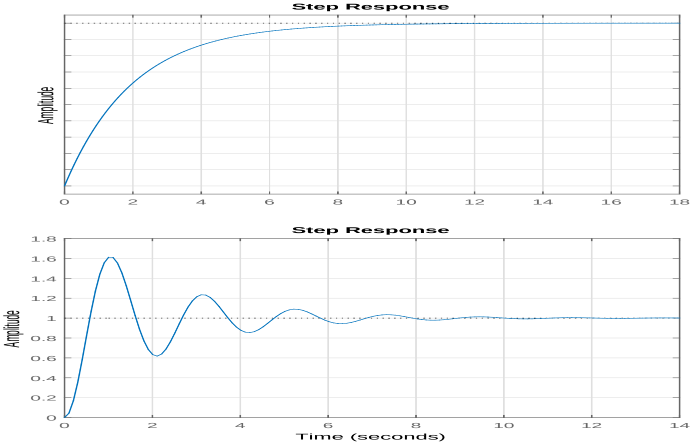
$$\frac{0.5}{s+0.5}$$



$$\frac{10}{1.09s^2+s+10}$$

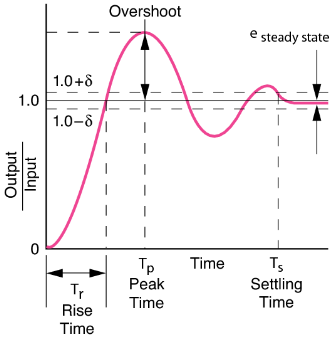
Example: step response of both controllers

The ouput $Y(s)$ when $R(s)$ is a step function.



Quality of reference tracking

- Look at the step response from $R(s)$ to $Y(s)$. All the known criteria can help you determine the quality.
  - Rise-time
  - Settling time
  - Steady state error
  - Overshoot
  - ...



http://www.newport.com/Control-Theory-Terminology/178319/1033/content.aspx

Model errors

- In practice, the transfer function $P(S)$ is often unknown. It is important to know how model errors affect the result. Sensitivity and robustness are key concepts to evaluate these effects.
- Sensitivity
  - Quantifies the effect of small model errors on the output.
- Robustness
  - Refers to bigger changes of the model. A controller is robust if it works properly over a given set of parameters.

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Sensitivity

- Sensitivity is a measure of the effect of a (small) disturbance on the model (e.g. variations in the process parameters)

$$Y(s) + \Delta Y(s) = \frac{(P(s) + \Delta P(s))C(s)}{1 + (P(s) + \Delta P(s))C(s)} R(s) + \frac{1}{1 + (P(s) + \Delta P(s))C(s)} D(s)$$

- Look at the effect on the system without disturbances ($D(s)$=0)

$$\begin{aligned}
\Delta Y &= \frac{(P + \Delta P)C}{1 + (P + \Delta P)C} R - \frac{PC}{1 + PC} R \\
&= \frac{(P + \Delta P)C(1 + PC) - PC - PC(P + \Delta P)C}{(1 + (P + \Delta P)C)(1 + PC)} R \\
&= \frac{\Delta PC}{(1 + (P + \Delta P)C)(1 + PC)} R \\
&= Y \frac{\Delta P}{P} \frac{1}{1 + (P + \Delta P)C}
\end{aligned}$$

Sensitivity

- Now take the relative change due to this disturbance of the model and take the limit for $\partial x \to 0$; this gives the following (measure of the) sensitivity:

$$S_P^Y(s) = \frac{\frac{\partial Y}{Y}(s)}{\frac{\partial P}{P}(s)} = \frac{1}{1 + P(s)C(s)}$$

- Again, a very large $|P(s)C(s)|$ looks like a good choice, but again there is a risk for instability!
- Note that the sensitivity can be determined for any parameter

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Example 1: robustness

- Suppose we want to stabilize the system $P(s) = \frac{1}{(s-a)}$. We only know that $a \in [0.20; 0.80]$. We propose to use a proportional controller $C(s) = K$. The closed loop transfer function becomes

$$S(s) = \frac{P(s)C(s)}{1 + P(s)C(s)} = \frac{\frac{1}{s-a}K}{1 + \frac{1}{s-a}K} = \frac{K}{s - a + K}$$

The controller stabilizes the system if $-a + K > 0$. If we choose K very large the system will be robust against changes in a.

Example 2: robustness

Suppose we want to stabilize the system $P(s) = \frac{s}{(s-a)}$ with $a \in [0.20; 0.80]$. If we choose the control law $C(s) = \frac{K}{s}$ this results in the same transfer function.

$$S(s) = \frac{P(s)C(s)}{1 + P(s)C(s)} = \frac{\frac{s}{s-a}\frac{K}{s}}{1 + \frac{s}{s-a}\frac{K}{s}} = \frac{K}{s - a + K}$$

Again we choose $K > a$ to ensure stability. But how does the controller perform on the slightly perturbed system $\tilde{P}(s) = \frac{(s+\epsilon)}{(s-a)}$? The transfer function from $R(s)$ to $Y(s)$ becomes

$$S(s) = \frac{\frac{s+\epsilon}{s-a}\frac{K}{S}}{1 + \frac{s+\epsilon}{s-a}\frac{K}{s}} = \frac{(s+\epsilon)K}{s^2 + (K-a)s + \epsilon K}$$

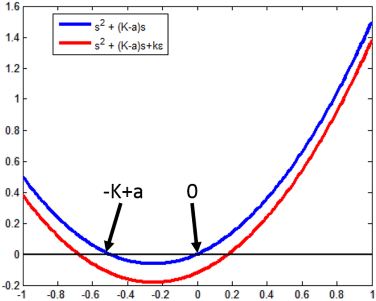
Example 2: robustness

$$\tilde{S}(s) = \frac{(s+\epsilon)K}{s^2 + (K-a)s + \epsilon K}$$

The figure on the right plots

$$s^2 + (K-a)s$$
$$s^2 + (K-a)s + \epsilon K$$

For ϵ negative the system becomes unstable. The control system is not robust and useless in practical situations. You shouldnt use control laws which rely on a pole-zero cancellation.



Example 3: Robustness of steady-state error

- The steady state error is defined as follows:

$$\lim_{t\to\infty} e(t) = \lim_{t\to\infty} (r(t) - y(t))$$
$$= \lim_{s\to 0} s(R(s) - Y(s)) \quad \text{(final value theorem)}$$

- A very small steady state error (preferably zero) indicates that the controller tracks the reference very well
- For the open loop system with a step reference

$$e_{ol}(\infty) = \lim_{s\to 0} s(1 - C(s)P(s))R(s) = 1 - C(0)P(0)$$

  - So the open loop controller can be free of a steady state error (for a step reference $\epsilon(t)$) by calibrating the controller such that $C(0)P(0) = 1$
  - $\Rightarrow$ a precise calibration of the DC gain

Example 3: Steady state error

- Closed loop system:

$$e_{cl}(\infty) = \lim_{s \to 0} s \left(1 - \frac{C(s)P(s)}{1 + C(s)P(s)}\right) R(s)$$
$$= \lim_{s \to 0} \frac{s}{1 + C(s)P(s)} R(s) = \frac{1}{1 + C(0)P(0)}$$

  - The steady state error is small if $C(0)P(0)$ is very large
  - Again, calibrating the DC gain
  - The difference is that here we only need a large gain, which is far less demanding than having to make it equal to 1

An open loop controller is not robust

- We can now show how this results in a great advantage of the closed loop strategy over the open loop strategy:
  - If P changes slightly (for instance due to a factor that has not been taken up into the model) to $P + \Delta P$
  - Then making $e_{ol}(\infty)$ small would require to calibrate anew
  - Whereas $e_{cl}(\infty)$ would remain small, as long as $(P(0) + \Delta P(0))C(0)$ remains large
- Hence an open loop controller can't control the output **robustly** against changes in P

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Type of a system

- For a step reference $r(t) = A\epsilon(t) \Rightarrow R(s) = \frac{A}{s}$

$$e_{cl}(\infty) = \frac{A}{1 + C(0)P(0)}$$

So $e_{cl}(\infty) = 0$ if C(s)P(s) has a least one pole in zero $(C(0)P(0) \to \infty)$

- For a ramp reference function $r(t) = At \Rightarrow R(s) = \frac{A}{s^2}$, we have

$$e_{cl}(\infty) = \lim_{s \to 0} \frac{s}{1 + C(s)P(s)} \frac{A}{s^2}$$

So $e_{cl}(\infty) = 0$ if $C(s)P(s)$ has at least two poles in zero. $\left(\lim_{s \to 0} sC(s)P(s) = \infty\right)$

Type of a system

- The type of a system is determined by the number of poles $P(s)C(s)$ has in zero, and hence it is linked to what type of references it can track perfectly
- Write $P(s)C(s)$ as $\frac{K\prod_{k=1}^{m}(s-z_k)}{s^l\prod_{k=1}^{n-l}(s-p_k)}$ with l the amount of poles in zero
- We say this system is of type l
  - It will be able to track references of shape $At^0\epsilon(t)$ up to $At^{(l-1)}\epsilon(t)$ perfectly (for $t \to \infty$, $\lim_{t\to\infty} e_{cl}(t) = 0$)
  - A reference of shape $At^l\epsilon(t)$ will be missed by a finite factor (for $t \to \infty$, $\lim_{t\to\infty} e_{cl}(t) = C$)
  - A reference of shape $At^{l+\cdots}\epsilon(t)$ will be missed by infinity (for $t \to \infty$, $\lim_{t\to\infty} e_{cl}(t) = \infty$)

Example of a type 0 system

- Step reference $r(t) = A\epsilon(t) \Rightarrow R(s) = \frac{A}{s}$

$$e_{cl}(\infty) = \lim_{s\to 0} \frac{s}{1 + C(s)P(s)} \frac{A}{s}$$
$$= \lim_{s\to 0} \frac{A}{1 + \frac{K\prod_{k=1}^{m}(-z_k)}{\prod_{k=1}^{n-l}(-p_k)}} = \frac{A}{1+K_p}$$

  with K_p the static position constant

- Ramp reference $r(t) = At\epsilon(t) \Rightarrow R(s) = \frac{A}{s^2}$

$$e_{cl}(\infty) = \lim_{s\to 0} \frac{s}{1 + C(s)P(s)} \frac{A}{s^2}$$
$$= \lim_{s\to 0} \frac{1}{1 + \frac{K\prod_{k=1}^{m}(-z_k)}{\prod_{k=1}^{n-l}(-p_k)}} = \infty$$

Steady state errors - type of a system

- With:

$$K_p = \lim_{s\to 0} P(s)C(s) \qquad K_p = \text{Static position constant}$$
$$K_v = \lim_{s\to 0} sP(s)C(s) \qquad K_v = \text{Static velocity constant}$$
$$K_a = \lim_{s\to 0} s^2P(s)C(s) \qquad K_a = \text{Static acceleration constant}$$

- And the respective steady state errors for different system types are:

Type I	**Step $A\epsilon(t)$**	**Ramp $At\epsilon(t)$**	**Parabola $\frac{At^2\epsilon(t)}{2}$**
0	$\frac{A}{1+K_p}$	∞	∞
1	0	$\frac{A}{K_v}$	∞
2	0	0	$\frac{A}{K_a}$

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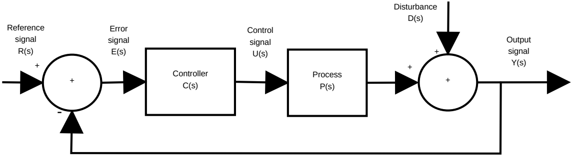
Noise rejection and disturbance rejection

- Both refer to rejection of the input D but are not the same
- Measurement noise is often modelled by white noise. (High frequency signal)
- Disturbances are actual changes to the state of the system



Noise versus Disturbances

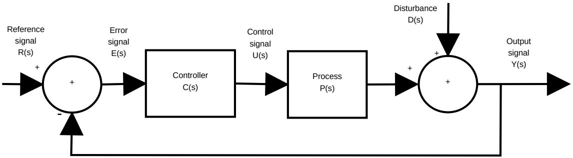
- Cruise control:
  Measurement errors on the speed are noise
  A change in slope is a disturbance

Noise versus Disturbances

- Remember from the start of the lecture

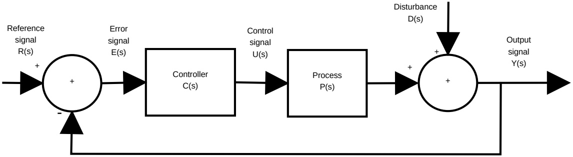
$$Y(s) = \frac{P(s)C(s)}{1+P(s)C(s)}R(s) + \frac{1}{1+P(s)C(s)}D(s)$$

- Choosing $T(s) = \frac{1}{1+P(s)C(s)}$ sufficiently small results in the rejection of D. This can be achieved by choosing $C(s)$ sufficiently high



Disturbance vs noise

- But what happens to measurement noise
  - The amplified measurement noise will be applied to the input of the plant.
  - In reality measurement noise requires no control actions. Good noise rejection means that the controller ignores this noise.



Disturbance versus noise rejection

- Good disturbance rejection requires fast action to bring the state back to the desired state
- Good noise rejection requires that measurement noise will be ignored
- Note that a controller can not see the difference between measurement noise and disturbances. Slow controllers will be less sensitive to measurement noise. Fast system will have better disturbance rejection

Classical trade-offs in control theory

Slow Controllers:		**Fast controllers:**
Not sensitive to noise Small control inputs	vs.	Good disturbance rejection Fast tracking
Robust controllers:		**Aggresive controllers:**
Model errors will not affect the behaviour of the system strongly	vs.	Exchanges robustness for better performance