

# Complex Numbers

July 23, 2015

# Outline

## 1 Complex numbers

# Complex numbers

## Definition

$$j = \sqrt{-1}$$

## Cartesian form

$$x = a + b j$$

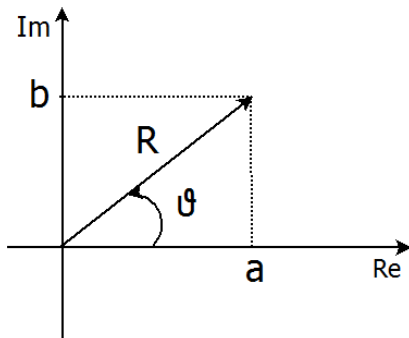
$$\Re(x) = a \quad \Im(x) = b$$

## Polar form

$$x = R e^{j\theta}$$

$$R = \sqrt{a^2 + b^2}$$

$$\theta = \arctan\left(\frac{b}{a}\right)$$



# Complex numbers

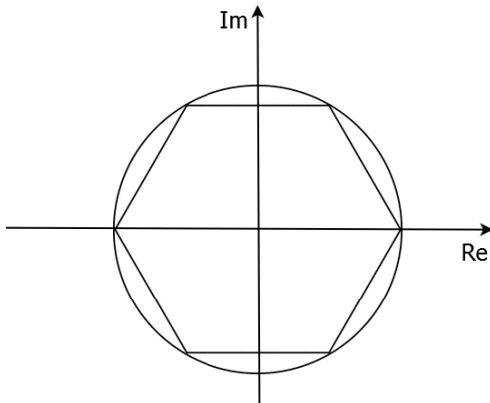
How many zero's has  
 $x^n - 1$ ?

$$x^6 = 1$$

$$x^6 = e^{2k\pi j}$$

$$x = e^{\frac{k\pi}{3}j}$$

$$x = \{1, e^{\frac{\pi}{3}j}, e^{\frac{2\pi}{3}j}, -1, e^{\frac{4\pi}{3}j}, e^{\frac{5\pi}{3}j}\}$$



# Complex numbers

How to calculate power of complex number?

$$\begin{aligned}(a + bj)^n &= R(\cos(\theta) + \sin(\theta)j) \\ &= (Re^{\theta j})^n \\ &= R^n e^{n\theta j} \\ &= R^n (\cos(n\theta) + \sin(n\theta)j)\end{aligned}$$

# Complex conjugate

## Definition

The conjugate of a complex number is the complex number with same real part and an opposite imaginary part.  $\overline{a + bj} = a - bj$

Product of 2 complex conjugate numbers results in a complex number with only a real part.

$$(a + bj)(a - bj) = a^2 - abj + abj + b^2 = a^2 + b^2$$

## Dividing complex numbers

$$\frac{a+bj}{c+dj} = \frac{(a+bj)(c-dj)}{(c^2+d^2)}$$

Alternative:

$$\frac{a+bj}{c+dj} = \frac{R_1 e^{j\theta_1}}{R_2 e^{j\theta_2}} = \left(\frac{R_1}{R_2}\right) e^{j(\theta_1 - \theta_2)}$$