Introduction to Control

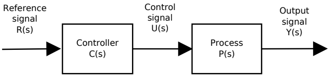
July 20, 2015

Outline

An introduction to control

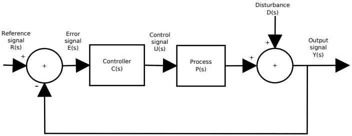
What is control?

- The goal is to find an input (control signal U(s)) such that the process produces the desired output
- Open loop control system: the actual output signal has no effect on the control action



A general set-up of a closed loop system

- We will focus on closed loop control systems



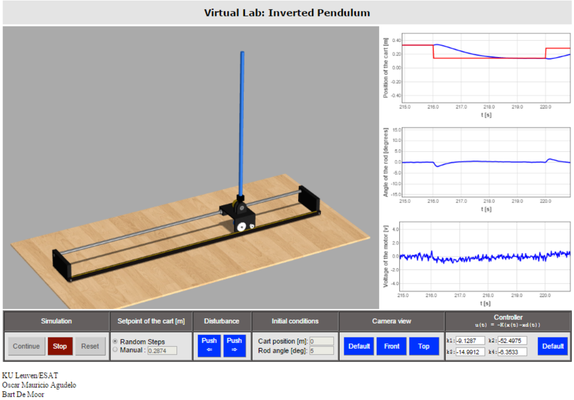


Figure: Inverted Pendulum

Concrete Control

- On-off controller
  - Thermostate at home
- **PID controllers, Lead and lag compensators (this course)**
  - Cruise-control in your car
- More advanced controllers
  - STATE-space feedback controllers
  - Model Predictive Controller (MPC)
  - Fuzzy Control
  - Neuro-fuzzy Control
  - ...

Outline

What is good control?

- Before we will start to design control systems we will first focus on the question. What is good control?
- It depends on the application
  - Stability
  - Disturbance rejection
  - Reference tracking (speed)
  - Sensitivity to errors on model
  - Etc...

Examples: stability

Figure: Space shuttles are like inverted pendulums. How do you make sure they don't flip over.

Examples: Disturbance rejection

- Your body will try to keep the temperature in your body as constant as possible. No matter what the outside temperature is. Two people will have almost the same body temperature.

Figure: Flickr.com, tent86, Marathon Des Sables 046

Figure: Jack Zalium, Enduring, https://creativecommons.org/licenses/by-nd/2.0/

Examples: Reference tracking

Figure: Audi has a system for automatic driving in traffic jams. The audi will follow the car in front of him at an appropriate distance. youtube

Exercise: name the correct property

Outline

Transfer function of a closed-loop system

$$
\begin{aligned}
Y(s) - D(s) &= P(s)U(s) \\
&\quad \text{with } U(s) = C(s)E(s) \\
Y(s) - D(s) &= P(s)C(s)E(s) \\
&\quad \text{with} E(s) = R(s) - Y(s) \\
Y(s) - D(s) &= P(s)C(s)(R(s) - Y(s)) \\
Y(s) - D(s) &= P(s)C(s)R(s) - P(s)C(s)Y(s) \\
&\Rightarrow Y(s) = \frac{P(s)C(s)}{1 + P(s)C(s)}R(s) + \frac{1}{1 + P(s)C(s)}D(s)
\end{aligned}
$$

Transfer function from $R(s)$ to $Y(s)$

We define S as the transfer functin from $R(s)$ to $Y(s)$

$$S(s) \triangleq \frac{P(s)C(s)}{1 + P(s)C(s)}$$

This transfer function $S(s)$ will help us to evaluate tracking
Almost perfect tracking: the output $Y(s)$ will follow $R(s)$ very closely $\Rightarrow S(s) \approx 1$

Transfer function from $D(s)$ to $Y(s)$

We define $T(s)$ as the transfer function from $D(s)$ to $Y(s)$.

$$T(s) \triangleq \frac{1}{1 + P(s)C(s)}$$

If the disturbance rejection is very good the disturbances will have almost no effect on the output $\Rightarrow T \approx 0$

$$\begin{cases} |S(j\omega)| \cong 1 \\ |T(j\omega)| \cong 0 \end{cases}$$

$\Rightarrow |P(j\omega)C(j\omega)|$ **(open loop gain)** is very large.
! a large open loop amplification might lead to an unstable system

Model errors

- In practice, transfer function of $P(s)$ might be unknown. It is important to know what the effect of the model errors will be. Sensitivity and robustness are key-concepts to evaluate these effects.
- Sensitivity
  - Quantifies the effect of a small perturbation on the model will be on the output.
- Robustness
  - This concept refers to bigger changes on the model. A controller is robust if it works properly over a given set of parameters.

Sensitivity

- Sensitivity is a measure for the effect of a (small) disturbance on the model (e.g. variations in the process parameters)

$$Y(s) + \Delta Y(s) = \frac{(P(s) + \Delta P(s))C(s)}{1 + (P(s) + \Delta P(s))C(s)} R(s) + \frac{1}{1 + (P(s) + \Delta P(s))C(s)} D(s)$$

- Look at the effect on the system without disturbances ($D(s)$=0)

$$\begin{aligned}
\Delta Y &= \frac{(P + \Delta P)C}{1 + (P + \Delta P)C} R - \frac{PC}{1 + PC} R \\
&= \frac{(P + \Delta P)C(1 + PC) - PC - PC(P + \Delta P)C}{(1 + (P + \Delta P)C)(1 + PC)} R \\
&= \frac{\Delta PC}{(1 + (P + \Delta P)C)(1 + PC)} R \\
&= Y \frac{\Delta P}{P} \frac{1}{1 + (P + \Delta P)C}
\end{aligned}$$

Sensitivity

- Now take the relative change due to this disturbance of the model and take the limit for $\partial x \to 0$; this gives the following (measure of the) sensitivity:

$$S_P^Y(s) = \frac{\frac{\partial Y}{Y}(s)}{\frac{\partial P}{P}(s)} = \frac{1}{1 + P(s)C(s)}$$

- Again, a very large $|P(s)C(s)|$ looks like a good choice, but again there is a risk for instability!
- Note that the sensitivity can be determined for any parameter

Example 1: robustness

- Suppose we want to stabilize the system $P(s) = 1/(s-a)$. We only know that $a \in [0.20; 0.80]$. We propose to use a proportional controller $C(s) = K$. The closed loop transfer function becomes

$$S(s) = \frac{P(s)C(s)}{1 + P(s)C(s)} = \frac{\frac{1}{s-a}K}{1 + \frac{1}{s-a}K} = \frac{K}{s - a + K}$$

The controller stabilizes the system if $a + K > 0$. If we choose K very large the system will be robust against changes in a.

Example 2: robustness

Suppose we want to stabilize the system $P(s) = \frac{1}{(s-a)}$ with $a \in [0.20; 0.80]$. If we choose the control law $C(s) = \frac{K}{s}$ this results in the same transfer function.

$$S(s) = \frac{P(s)C(s)}{1 + P(s)C(s)} = \frac{\frac{s}{s-a}\frac{K}{s}}{1 + \frac{s}{s-a}\frac{K}{s}} = \frac{K}{s - a + K}$$

Again we choose $K > a$ to ensure stability. But how does the controller perform on the slightly perturbed system $\tilde{P}(s) = \frac{(s+\epsilon)}{(s-a)}$? The transfer function from $R(s)$ to $Y(s)$ becomes

$$S(s) = \frac{\frac{s+\epsilon}{s-a}\frac{K}{S}}{1 + \frac{s+\epsilon}{s-a}\frac{K}{s}} = \frac{(s+\epsilon)K}{s^2 + (K-a)s + \epsilon K}$$

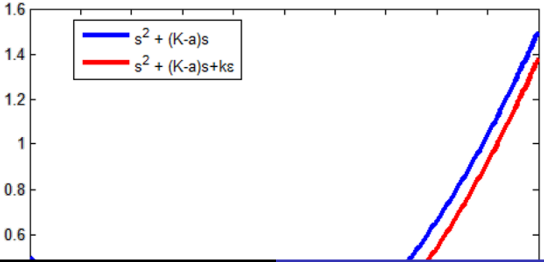
Example2: robustness

$$\tilde{S}(s) = \frac{(s+\epsilon)K}{s^2 + (K-a)s + \epsilon K}$$

The figure on the right plots

$$s^2 + (K-a)s$$

$$s^2 + (K-a)s + \epsilon K$$



Example 3: Robustness of steady-state error

- The steady state error is defined as follows:

$$\begin{aligned}\lim_{t\to\infty} e(t) &= \lim_{t\to\infty} (r(t) - y(t)) \\ &= \lim_{s\to 0} s(R(s) - Y(s)) \quad \text{(final value theorem)}\end{aligned}$$

- A very small steady state error (preferably zero) indicates that the controller tracks the reference very well
- For the open loop system with a step reference

$$e_{ol}(\infty) = \lim_{s\to 0} s(1 - C(s)P(s))R(s) = 1 - C(0)P(0)$$

  - So the open loop controller can be free of a steady state error (for a step reference $\epsilon(t)$) by calibrating the controller such that $C(0)P(0) = 1$
  - $\Rightarrow$ a precise calibration of the DC gain

Example 3: Steady state error

- Closed loop system:

$$\begin{aligned}
\epsilon_{cl} &= \lim_{s\to 0} s\left(1-\frac{C(s)P(s)}{1+C(s)P(s)}\right)R(s) \\
&= \lim_{s\to 0}\frac{s}{1+C(s)P(s)}R(s) = \frac{1}{1+C(0)P(0)}
\end{aligned}$$

  - The steady state error is small if $C(0)P(0)$ is very large
  - Again, calibrating the DC gain
  - However, the difference is that here, we only need a large value for it, which is far less demanding than having to make it equal to 1

An open loop controller is not robust

- We can now show how this results in a great advantage of the closed loop strategy over the open loop strategy:
  - If P changes slightly (for instance due to a factor that has not been taken up into the model) to $P + \Delta P$
  - Then making $e_{ol}(\infty)$ small would require to calibrate anew
  - Whereas $e_{cl}(\infty)$ would remain small, as long as $(P(0) + \Delta P(0))C(0)$ remains large
- Hence an open loop controller cannot control the output **robustly** against changes in P