"In the third step, you actually do the prediction," Ott said. The reservoir, having learned the system's dynamics, can reveal how it will evolve. The network essentially asks itself what will happen. Outputs are fed back in as the new inputs, whose outputs are fed back in as inputs, and so on, making a projection of how the heights at the five positions on the flame front will evolve. Other reservoirs working in parallel predict the evolution of height elsewhere in the flame.

In a plot in their *PRL* paper, which appeared in January, the researchers show that their predicted flamelike solution to the Kuramoto-Sivashinsky equation exactly matches the true solution out to eight Lyapunov times before chaos finally wins, and the actual and predicted states of the system diverge.

The usual approach to predicting a chaotic system is to measure its conditions at one moment as accurately as possible, use these data to calibrate a physical model, and then evolve the model forward. As a ballpark estimate, you'd have to measure a typical system's initial conditions 100,000,000 times more accurately to predict its future evolution eight times further ahead.

That's why machine learning is "a very useful and powerful approach," said <u>Ulrich Parlitz</u> of the Max Planck Institute for Dynamics and Self-Organization in Göttingen, Germany, who, like Jaeger, also applied machine learning to low-dimensional chaotic systems in the early 2000s. "I think it's not only working in the example they present but is universal in some sense and can be applied to many processes and systems." In a paper soon to be published in *Chaos*, Parlitz and a collaborator applied reservoir computing to predict the dynamics of "excitable media," such as cardiac tissue. Parlitz suspects that deep learning, while being more complicated and computationally intensive than reservoir computing, will also work well for tackling chaos, as will other machine-learning algorithms. Recently, researchers at the Massachusetts Institute of Technology and ETH Zurich achieved similar results as the Maryland team using a "long short-term memory" neural network, which has recurrent loops that enable it to store temporary information for a long time.

Since the work in their *PRL* paper, Ott, Pathak, Girvan, Lu and other collaborators have come closer to a practical implementation of their prediction technique. In <u>new research accepted for publication in *Chaos*</u>, they showed that improved predictions of chaotic systems like the Kuramoto-Sivashinsky equation become possible by hybridizing the data-driven, machine-learning approach and traditional model-based prediction. Ott sees this as a more likely avenue for improving weather prediction and similar efforts, since we don't always have complete high-resolution data or perfect physical models. "What we should do is use the good knowledge that we have where we have it," he said, "and if we have ignorance we should use the machine learning to fill in the gaps where the ignorance resides." The reservoir's predictions can essentially calibrate the models; in the case of the Kuramoto-Sivashinsky equation, accurate predictions are extended out to 12 Lyapunov times.

The duration of a Lyapunov time varies for different systems, from milliseconds to millions of years. (It's a few days in the case of the weather.) The shorter it is, the touchier or more prone to the butterfly effect a system is, with similar states departing more rapidly for disparate futures. Chaotic systems are everywhere in nature, going haywire more or less quickly. Yet strangely, chaos itself is hard to pin down. "It's a term that most people in dynamical systems use, but they kind of hold their noses while using it," said Amie Wilkinson, a professor of mathematics at the University of Chicago. "You feel a bit cheesy for saying something is chaotic," she said, because it grabs people's attention while having no agreed-upon mathematical definition or necessary and sufficient conditions. "There is no easy concept," Kantz agreed. In some cases, tuning a single parameter of a system can make it go from chaotic to stable or vice versa.

Wilkinson and Kantz both define chaos in terms of stretching and folding, much like the repeated