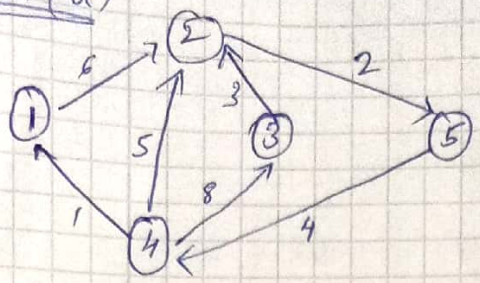
Task 1(a)



$$A^1 = \begin{bmatrix} 0 & 6 & \infty & \infty & \infty \\ \infty & 0 & \infty & \infty & 2 \\ \infty & 3 & 0 & \infty & \infty \\ 1 & 5 & 8 & 0 & \infty \\ \infty & \infty & \infty & 4 & 0 \end{bmatrix}$$

$C_{11} = \min(A_{11}+A_{11}, A_{12}+A_{21}, A_{13}+A_{31}, A_{14}+A_{41}, A_{15}+A_{51})$
$= \min(0, \infty, \infty, \infty)$
$= 0$

$C_{12} = \min(A_{12}+A_{12}, A_{12}+A_{22}, A_{13}+A_{32}, A_{14}+A_{42}, A_{15}+A_{52})$
$= \min(0+6, 6+0, \infty, \infty, \infty)$
$= 6$

$C_{13} = \infty$
$C_{14} = \infty$
$C_{15} = 8$
$C_{21} = \infty$
$C_{22} = 0$
$C_{23} = \infty$
$C_{24} = 6$
$C_{25} = 2$

$C_{31} = \infty$
$C_{32} = 3$
$C_{33} = 0$
$C_{34} = \infty$
$C_{35} = 5$
$C_{41} = 1$
$C_{42} = 5$
$C_{43} = 8$

$C_{44} = 0$
$C_{45} = 7$
$C_{51} = 5$
$C_{52} = 9$
$C_{53} = 12$
$C_{54} = 4$
$C_{55} = 0$

$$A^2 = \begin{bmatrix} 0 & 6 & \infty & \infty & 8 \\ \infty & 0 & \infty & 6 & 2 \\ \infty & 3 & 0 & \infty & 5 \\ 1 & 5 & 8 & 0 & 7 \\ 5 & 9 & 12 & 4 & 0 \end{bmatrix}$$

$C_{11} = 0$

$C_{12} = 6$

$C_{13} = 8+12 = 20$

$C_{14} = 12$

$C_{15} = 8$

$C_{21} = 7$

$C_{22} = 0$

$C_{23} = 14$

$C_{24} = 6$

$C_{25} = 2$

$C_{31} = 10$

$C_{32} = 3$

$C_{33} = 0$

$C_{34} = 9$

$C_{35} = 5$

$C_{41} = 1$

$C_{42} = 5$

$C_{43} = 8$

$C_{44} = 0$

$C_{45} = 7$

$C_{51} = 5$

$C_{52} = 9$

$C_{53} = 12$

$C_{54} = 4$

$C_{55} = 0$

$$A^3 = \begin{bmatrix} 0 & 6 & 20 & 12 & 8 \\ 7 & 0 & 14 & 6 & 2 \\ 10 & 3 & 0 & 9 & 5 \\ 1 & 5 & 8 & 0 & 7 \\ 5 & 9 & 12 & 4 & 0 \end{bmatrix}$$

Task 1(b)

$$A' = \begin{bmatrix} 0 & 6 & \infty & \infty & \infty \\ \infty & 0 & \infty & \infty & 2 \\ \infty & 3 & 0 & \infty & \infty \\ 1 & 5 & 8 & 0 & \infty \\ \infty & \infty & \infty & 4 & 0 \end{bmatrix}$$

$C_{11} = \min(A_{11}, \max(A_{12}, A_{21}), \max(A_{13}, A_{31}), \max(A_{14}, A_{41}), \max(A_{15}, A_{51}))$

$= \min(0, \infty, \infty, \infty, \infty)$

$= 0$

$C_{12} = \min(A_{12}, \max(A_{11}, A_{12}), \max(A_{13}, A_{32}), \max(A_{14}, A_{42}), \max(A_{15}, A_{52}))$

$= \min(6, 6, \infty, \infty, \infty)$

$= 6$

$C_{13} = \min(\infty, \infty, \infty, \infty, \infty)$

$= \infty$

$C_{14} = \min(\infty, \infty, \infty, \infty, \infty)$

$= \infty$

$C_{15} = \min(\infty, 6, \infty, \infty, \infty)$

$= 6$

$C_{21} = \min(\infty, \infty, \infty, \infty, \infty)$

$= \infty$

$C_{22} = 0$

$C_{23} = \min(\infty, \infty, \infty, \infty, \infty)$

$= \infty$

$C_{24} = \min(\infty, \infty, \infty, \infty, 4)$

$= 4$

$C_{25} = \min(2, \infty, \infty, \infty, \infty)$

$= 2$

$C_{31} = \min(\infty, \infty, \infty, \infty, \infty)$

$= \infty$

$C_{32} = 3$

$C_{33} = 0$

$C_{34} = \min(\infty, \infty, \infty, \infty, \infty)$

$= \infty$

$C_{35} = \min(\infty, \infty, 3, \infty, \infty)$

$= 3$

$C_{41} = 1$

$C_{42} = \min(5, 6, \infty, \infty, \infty)$

$= 5$

$C_{43} = \min(8, \infty, \infty, 8, \infty)$

$= \min 8$

$C_{44} = \min(0$

$C_{45} = \min(\infty, \infty, 5, \infty, \infty)$

$= 5$

$C_{51} = \min(\infty, \infty, \infty, 4, \infty)$
$= 4$

$C_{52} = \min(\infty, \infty, \infty, 5, \infty)$
$= 5$

$C_{53} = \min(\infty, \infty, \infty, \infty, 8, \infty)$
$= 8$

$C_{54} = \min(4, \infty, \infty, \infty, 4)$
$= 4$

$C_{55} = 0$

$$A^2 = \begin{bmatrix} 0 & 6 & \infty & \infty & 6 \\ \infty & 0 & \infty & 4 & 2 \\ \infty & 3 & 0 & \infty & 3 \\ 1 & 5 & 8 & 0 & 5 \\ 4 & 5 & 8 & 4 & 0 \end{bmatrix}$$

$C_{31} = \min(\infty, \infty, \infty, \infty, 4)$
$= 4$

$C_{32} = \min(\infty, 3, 3, \infty, 5)$
$= 3$

$C_{33} = 0$

$C_{11} = 0$

$C_{12} = \min(6, 6, \infty, \infty, 6)$
$= 6$

$C_{13} = \min(\infty, \infty, \infty, \infty, 8)$
$= 8$

$C_{14} = \min(\infty, 6, \infty, \infty, 6)$
$= 6$

$C_{15} = \min(6, 6, \infty, \infty, 6)$
$= 6$

$C_{21} = \min(\infty, \infty, \infty, 4, 4)$
$= 4$

$C_{22} = 0$

$C_{23} = \min(\infty, \infty, \infty, 8, 8)$
$= 8$

$C_{24} = \min(4, \infty, 4, \infty, 4)$
$= 4$

$C_{25} = \min(2, \infty, \infty, 5, 2)$
$= 2$

$C_{34} = \min(\infty, \infty, 4, \infty, 4)$
$= 4$

$C_{35} = \min(3, \infty, 3, \infty, 3) = 3$

$C_{41} = \min(1, \infty, \infty, 1, 5) = 1$

$C_{42} = \min(5, 6, 8, 5, 5) = 5$

$C_{43} = \min(8, \infty, \infty, 8, 8) = 8$

$C_{44} = 0$

$C_{45} = \min(5, 6, 5, 8, 5) = 5$

$C_{51} = \min(4, \infty, \infty, 4, 4) = 4$

$C_{52} = \min(5, 6, 8, 5, 5) = 5$

$C_{53} = \min(8, \infty, \infty, 8, 8) = 8$

$C_{54} = \min(4, \infty, 5, \infty, 4) = 4$

$C_{55} = 0$

$$A^3 = \begin{bmatrix} 0 & 6 & 8 & 6 & 6 \\ 4 & 0 & 8 & 4 & 2 \\ 4 & 3 & 0 & 4 & 3 \\ 1 & 5 & 8 & 0 & 5 \\ 4 & 5 & 8 & 4 & 0 \end{bmatrix}$$

Task 1(c)

In order to detect negative cycles, we run the standard all pairs shortest path algorithm without assigning 0 where $i == j$. Instead we let the value get modified like the rest of the values in the adjacency matrix.

If a negative cycle exists, we will find a case where $A[i][i] < 0$.