2. To prove $\hat{\beta} = X^T\hat{\alpha}$,

We know that $\hat{\beta} = (X^TX + \tau\mathbb{I}_D)^{-1}X^Ty$, $\hat{\alpha} = (XX^T + \tau\mathbb{I}_N)^{-1}y$.

we only need to prove $(X^TX + \tau\mathbb{I}_D)^{-1}X^T = X^T(XX^T + \tau\mathbb{I}_N)^{-1}$ --(1)

Then we decomposite X using SVD:

$X_{N\times D} = U\Sigma V^T$ | U is $N\times N$ orthogonal matrix; Σ is $N\times D$ diagonal matrix; V is $D\times D$ orthogonal matrix

$\therefore X^T = V\Sigma^TU^T$

$X^TX = V\Sigma^TU^T \cdot U\Sigma V^T = V\Sigma^T\Sigma V^T$

$XX^T = U\Sigma V^T \cdot V\Sigma^TU^T = U\Sigma\Sigma^TU^T$

| $UU^T = VV^T = \mathbb{I}$; $\Sigma^T\Sigma$ $(D\times D)$; $\Sigma\Sigma^T$ $(N\times N)$

So Left of (1)
$$
\begin{aligned}
&= (V\Sigma^T\Sigma V^T + \tau\mathbb{I}_D)^{-1}V\Sigma^TU^T \\
&= (V\Sigma^T\Sigma V^T + \tau VV^T)^{-1}V\Sigma^TU^T \\
&= [V(\Sigma^T\Sigma + \tau\mathbb{I}_D)V^T]^{-1}V\Sigma^TU^T \\
&= V(\Sigma^T\Sigma + \tau\mathbb{I}_D)^{-1}V^TV\Sigma^TU^T \\
&= V(\Sigma^T\Sigma + \tau\mathbb{I}_D)^{-1}\Sigma^TU^T \\
&= V[\Sigma^T\Sigma + \tau\Sigma^T(\Sigma^T)^{-1}]^{-1}\Sigma^TU^T \\
&= V[\Sigma^T(\Sigma + \tau(\Sigma^T)^{-1})]^{-1}\Sigma^TU^T \\
&= V[\Sigma + \tau(\Sigma^T)^{-1}]^{-1}(\Sigma^T)^{-1}\Sigma^TU^T \\
&= V\Sigma^T\cdot(\Sigma^T)^{-1}[\Sigma + \tau(\Sigma^T)^{-1}]^{-1}U^T \\
&= V\Sigma^T[(\Sigma + \tau(\Sigma^T)^{-1})\Sigma^T]^{-1}U^T \\
&= V\Sigma^T(\Sigma\Sigma^T + \tau\mathbb{I}_N)^{-1}U^T
\end{aligned}
$$

Right of (1)
$$
\begin{aligned}
&= V\Sigma^TU^T(U\Sigma\Sigma^TU^T + \tau\mathbb{I}_N)^{-1} \\
&= V\Sigma^TU^T(U\Sigma\Sigma^TU^T + \tau UU^T)^{-1} \\
&= V\Sigma^TU^T[U(\Sigma\Sigma^T + \tau\mathbb{I}_N)U^T]^{-1} \\
&= V\Sigma^TU^TU(\Sigma\Sigma^T + \tau\mathbb{I}_N)^{-1}U^T \\
&= V\Sigma^T(\Sigma\Sigma^T + \tau\mathbb{I}_N)^{-1}U^T
\end{aligned}
$$

QED.