Theory of Computation

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References: Lectures by Stephen Cranefield in COSC341 at the University of Otago, New Zealand; Lecture Slides mostly by Michael Albert; tikz tutorial by Satyaki Sikdar

Contents

1 Deterministic Finite State Automaton (DFAs)

1.1 Introduction to DFAs

DEF: A *deterministic finite state automaton (DFA)*, **A**, consists of the following:

- A finite set Σ called its alphabet,
- A finite set $\mathcal{S}$ called its states,
- A function $T : \mathcal{S} \times \Sigma \to \mathcal{S}$ called its transition function,
- A single element $s \in \mathcal{S}$ called its start state,
- A subset $A \subseteq \mathcal{S}$ called its final states or accepting states.

Note: changes to the state of any model always occur sequentially, not in parallel.
We begin with an example. Consider a light with two switches. Flipping either switch changes the state of the light.

Ex:

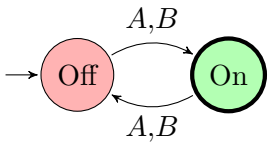


Figure 1: Two buttons, one light

- The light starts in the **Off** state.
- In either state, making an input of either A or B switches to the other state.
- We consider the **On** state to be *accepting* – any sequence of inputs that leads to this state is considered successful
- The successful inputs are all strings consisting of characters from A, B that have an odd number of characters.

Ex:

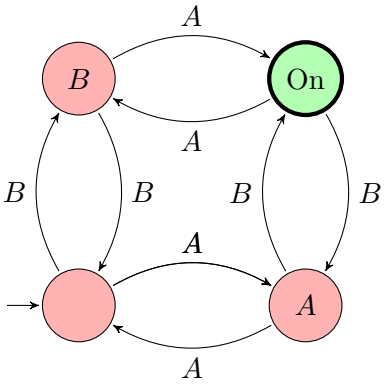
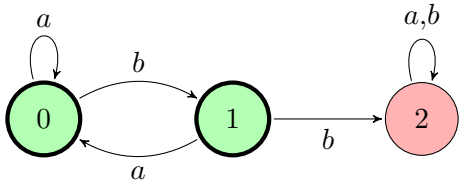


Figure 2: Two different buttons, one light

- The light starts in the lower left state.

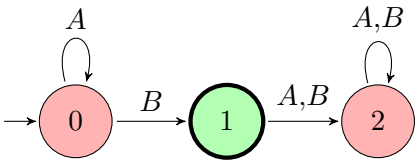
- The successful inputs are all strings consisting of characters from A, B that have an odd number of each letter.

Ex: What does this machine do?



- The machine starts in the state.
- There are two buttons to press - a and b. These define the alphabet of the machine.
- Each button press causes a transition according to the labelled arrows.
- The sequence of button presses leaving us in an accepting state (coloured green) are the language accepted by the machine.

Ex:



In this example, the accepted language is exactly "zero or more copies of A followed by exactly one B".

1.2 Automata and Grammars