Tensor Estimation Diffusion MRI

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Abstract

Diffusion tensor imaging (DTI) is a magnetic resonance imaging technology to probe the architectures of biological samples through measuring water diffusion and to produce neural fiber tracts. DTI is widely used to study neuro-degenerative diseases and segmentation or connectivity of brain.
The most commonly used DTI model is the single tensor model where a tensor is used to represent water diffusion at each voxel. This work focuses on developing and comparing two tensor estimators: one based on linear regression, another based on nonlinear regression with positive definite constraint. The leading eigenvector at each voxel is then extracted and used as input for fiber tracking algorithms. We compare different estimators using simulation experiments. We find that tensor estimation based on nonlinear methods are superior to the commonly used linear regression estimator when studying anisotropic tensors. If isotropic tensors are included, nonlinear method performs similar to linear method or even worse in some cases.

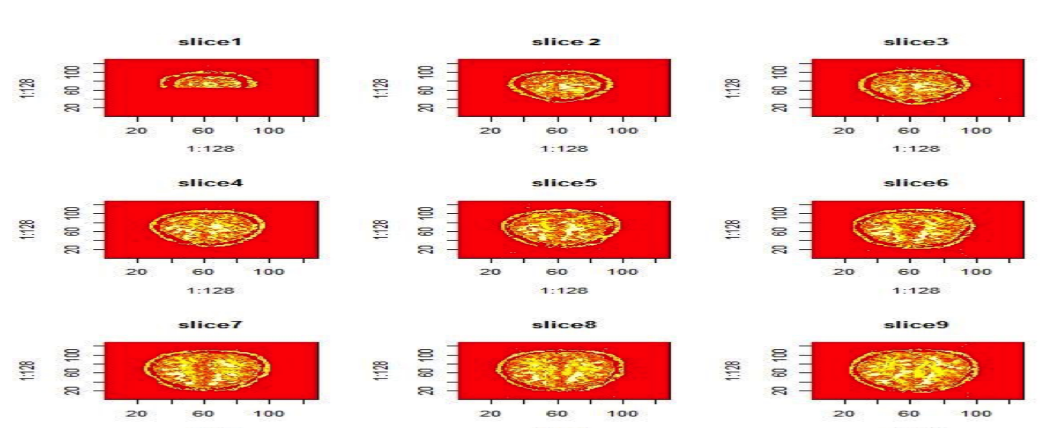

Figure: FA Map of Brain

Two Tensor Estimators

Linear Methods:

$$\hat{D} = \arg\min_{D} \sum_{q \in Q} (log\hat{S}_q - logS_0 + bq^T Dq))^2 \quad (1)$$

Nonlinear Methods:

$$\hat{D} = \arg\min_{D} \sum_{q \in Q} (\hat{S}_q - S_0 \exp(-bq^T Dq))^2$$

$$D = L(L^T)$$

The **Cholesky Factorization** is a method to decompose a positive-definite matrix into the product of a lower triangular matrix and its conjugate transpose.

Simulation Procedure

DTI Model:

$$S_q = S_0 \exp(-bq^T Dq)$$

- q: Gradient direction (3 by 1 vector)
- S_q: Signal intensity
- S_0: Baseline intensity
- D: Diffusion tensor (positive definite matrix)
- b: Experimental constant

33 Gradient directions(roughly equiangular)
Eigenvalues of true tensor:
(1,1,1)–isotropic tensor
(4,0.25,0.25)–anisotropic tensor
Procedure:

- Simulate data from the intensity model plus Rician noise with scale parameter σ (SNR=$\frac{S_0}{\sigma}$)
- Use two methods to estimate diffusion tensor
- Evaluate the estimated tensor by comparing with true tensors

Fractional Anisotropy:

$$FA = \sqrt{\frac{1}{2}} \frac{\sqrt{(\lambda_1 - \lambda_2)^2 + (\lambda_2 - \lambda_3)^2 + (\lambda_3 - \lambda_1)^2}}{\sqrt{(\lambda_1)^2 + (\lambda_2)^2 + (\lambda_3)^2}}$$

Note:The FA value reflecs the anisotrpy of water diffusion at corresponding voxel.

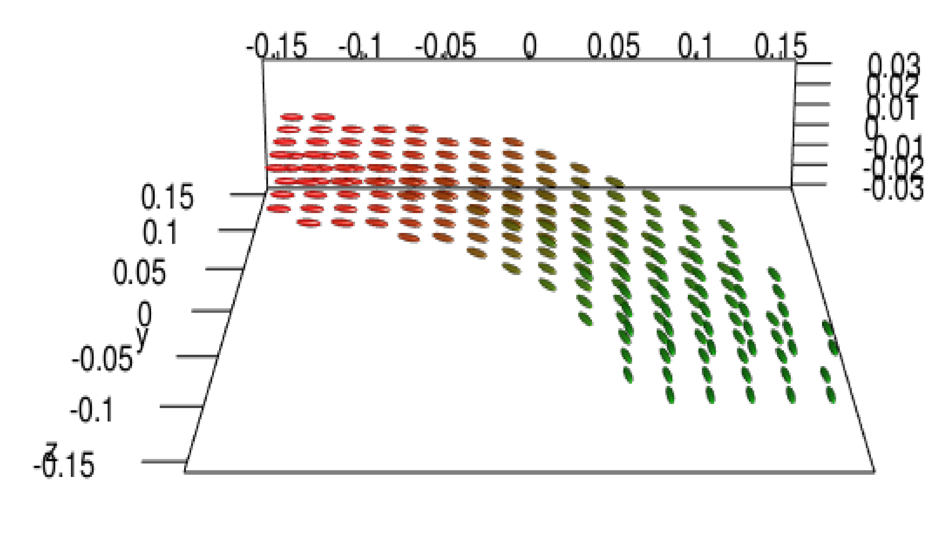

Figure: True Tensor Ellipsoids

Results

		1^{st} Qu.	Median	Mean	3^{rd} Qu.
Anisotropic	Cholesky 20	0.0182	0.0333	1.4210	3.1180
	Linear 20	0.0449	0.0923	1.4210	3.0800
	Cholesky 100	0.0036	0.0068	1.4200	3.1380
	Linear 100	0.0154	0.0296	1.4220	3.1260
Isotropic	Cholesky 20	0.8965	1.4490	1.4630	2.0430
	Linear 20	0.8891	1.4560	1.4620	2.0390
	Cholesky 100	0.9423	1.4800	1.4690	2.0010
	Linear 100	0.9136	1.4450	1.4550	1.9860

Table: Angle difference between leading eigenvectors in both methods and true vectors.

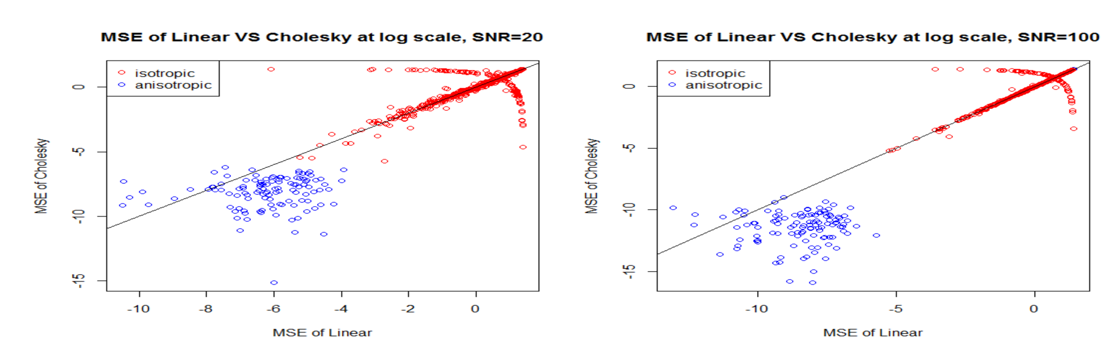

Figure: Scatter plot of Linear vs Cholesky at log scale, with SNR=20 (left) and SNR=100 (right).

Estimation Vector Field

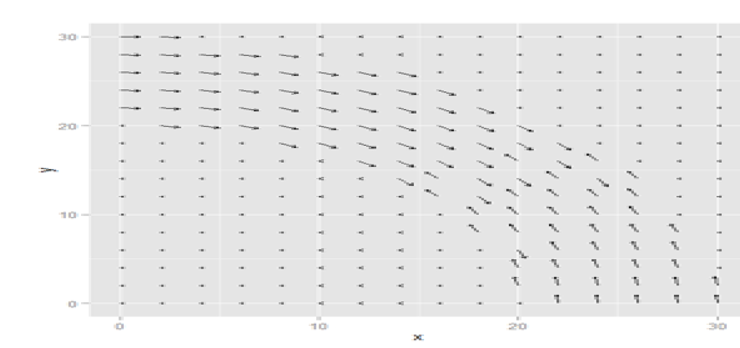

Figure: Vector Field of True Tensors

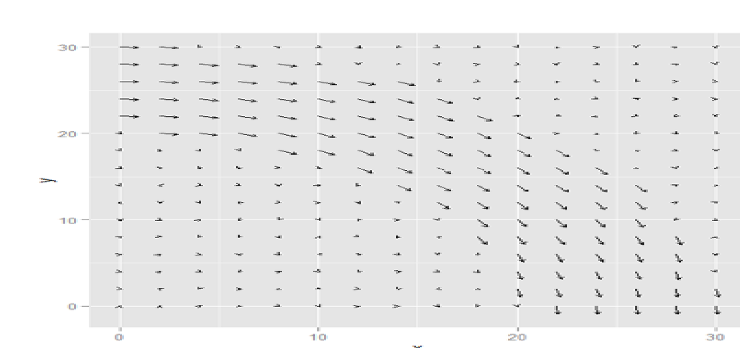

Figure: Vector Field of Tensors from Cholesky method (SNR=20)

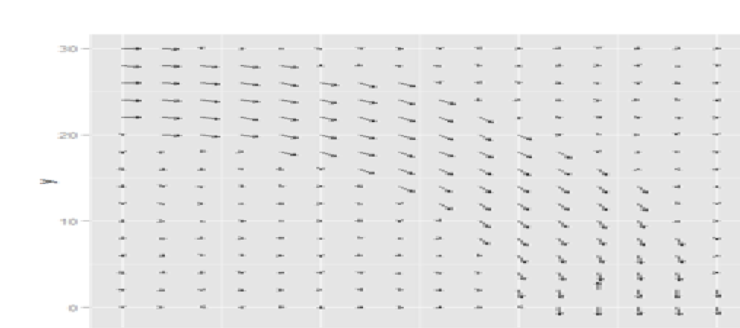

Figure: Vector Field of Tensors from Linear method (SNR=20)

Fiber Tractography

Fiber Tractography is a 3D modeling technique used to visually represent neural tracts and identify anatomical connections given a set of diffusion weighted images.
Input: Diffusion tensors

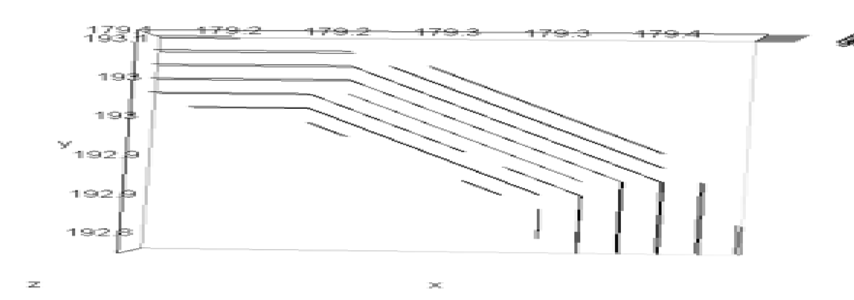

Figure: Fiber Tractography of True tensors

Results:

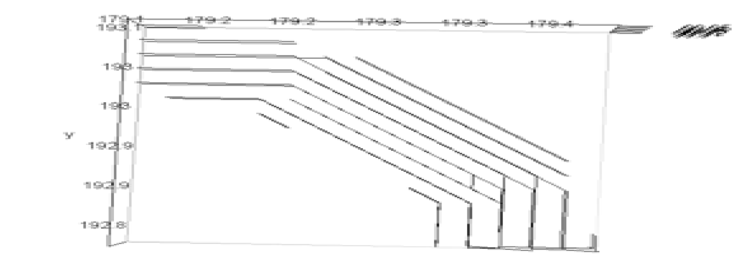

Figure: Fiber Tractography of Simulated Tensors from Linear Method(SNR=20)

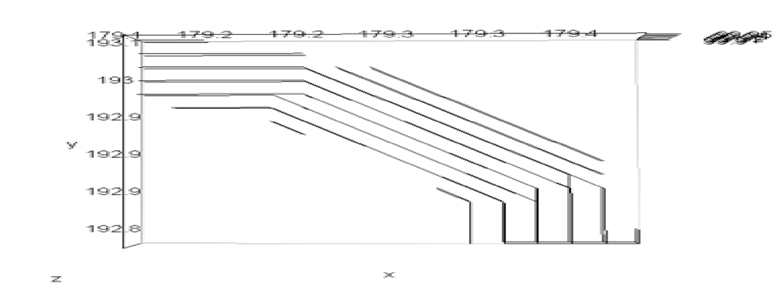

Figure: Fiber Tractography of Simulated Tensors from Cholesky method(SNR=20)

Conclusion

Tensor estimations based on nonlinear method are superior to the commonly used linear regression estimator when studying anisotropic tensors. If isotropic tensors are included, nonlinear method performs similar to linear method or even worse in some cases.

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