7-1 INTRODUCTION

The dc–dc converters are widely used in regulated switch-mode dc power supplies and in dc motor drive applications. As shown in Fig. 7-1, often the input to these converters is an unregulated dc voltage, which is obtained by rectifying the line voltage, and therefore it will fluctuate due to changes in the line-voltage magnitude. Switch-mode dc-to-dc converters are used to convert the unregulated dc input into a controlled dc output at a desired voltage level.

Looking ahead to the application of these converters, we find that these converters are very often used with an electrical isolation transformer in the switch-mode dc power supplies and almost always without an isolation transformer in case of dc motor drives. Therefore, to discuss these circuits in a generic manner, only the nonisolated converters are considered in this chapter, since the electrical isolation is an added modification.

The following dc–dc converters are discussed in this chapter:

1. Step-down (buck) converter
2. Step-up (boost) converter
3. Step-down/step-up (buck–boost) converter
4. Cúk converter
5. Full-bridge converter

Of these five converters, only the step-down and the step-up are the basic converter topologies. Both the buck–boost and the Cúk converters are combinations of the two basic topologies. The full-bridge converter is derived from the step-down converter.

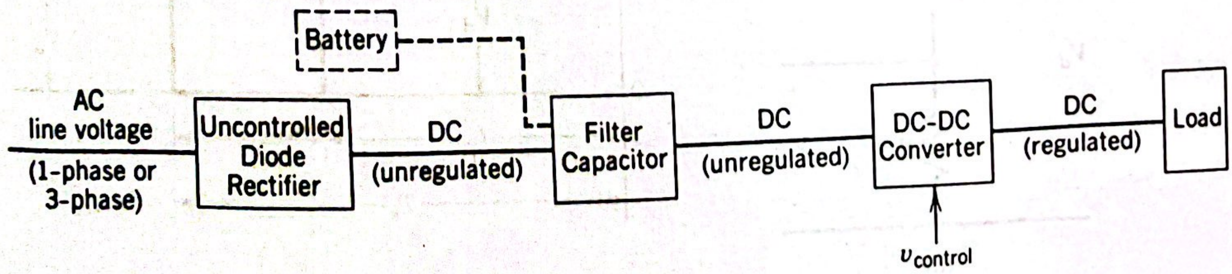

Figure 7-1 A dc–dc converter system.

The converters listed are discussed in detail in this chapter. Their variations, as they apply to specific applications, are described in the chapters dealing with switch-mode dc power supplies and dc motor drives.

In this chapter, the converters are analyzed in steady state. The switches are treated as being ideal, and the losses in the inductive and the capacitive elements are neglected. Such losses can limit the operational capacity of some of these converters and are discussed separately.

The dc input voltage to the converters is assumed to have zero internal impedance. It could be a battery source; however, in most cases, the input is a diode rectified ac line voltage (as is discussed in Chapter 5) with a large filter capacitance, as shown in Fig. 7-1 to provide a low internal impedance and a low-ripple dc voltage source.

In the output stage of the converter, a small filter is treated as an integral part of the dc-to-dc converter. The output is assumed to supply a load that can be represented by an equivalent resistance, as is usually the case in switch-mode dc power supplies. A dc motor load (the other application of these converters) can be represented by a dc voltage in series with the motor winding resistance and inductance.

7-2 CONTROL OF dc–dc CONVERTERS

In dc–dc converters, the average dc output voltage must be controlled to equal a desired level, though the input voltage and the output load may fluctuate. Switch-mode dc–dc converters utilize one or more switches to transform dc from one level to another. In a dc–dc converter with a given input voltage, the average output voltage is controlled by controlling the switch on and off durations (t_{on} and t_{off}). To illustrate the switch-mode conversion concept, consider a basic dc–dc converter shown in Fig. 7-2*a*. The average value V_o of the output voltage v_o in Fig. 7-2*b* depends on t_{on} and t_{off}. One of the methods for controlling the output voltage employs switching at a constant frequency (hence, a constant switching time period $T_s = t_{on} + t_{off}$) and adjusting the on duration of the switch to control the average output voltage. In this method, called *pulse-width modulation* (PWM) switching, the switch duty ratio D, which is defined as the ratio of the on duration to the switching time period, is varied.

The other control method is more general, where both the switching frequency (and hence the time period) and the on duration of the switch are varied. This method is used only in dc–dc converters utilizing force-commutated thyristors and therefore will not be discussed in this book. Variation in the switching frequency makes it difficult to filter the ripple components in the input and the output waveforms of the converter.

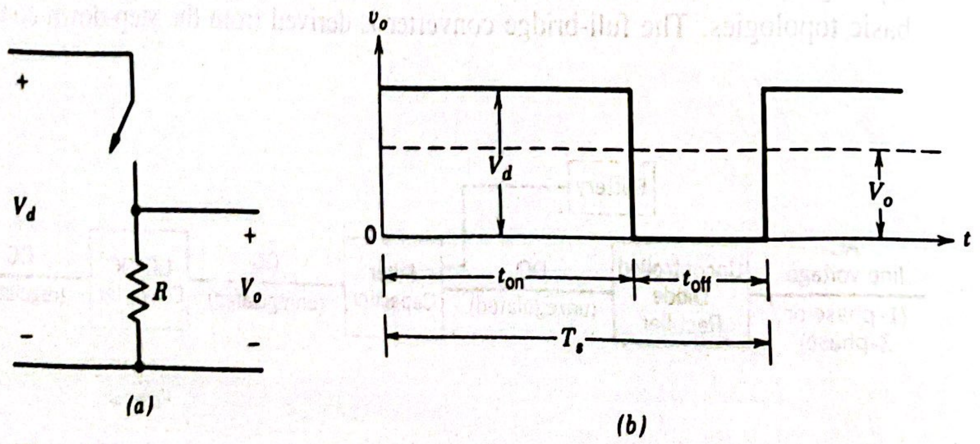

Figure 7-2 Switch-mode dc–dc conversion.

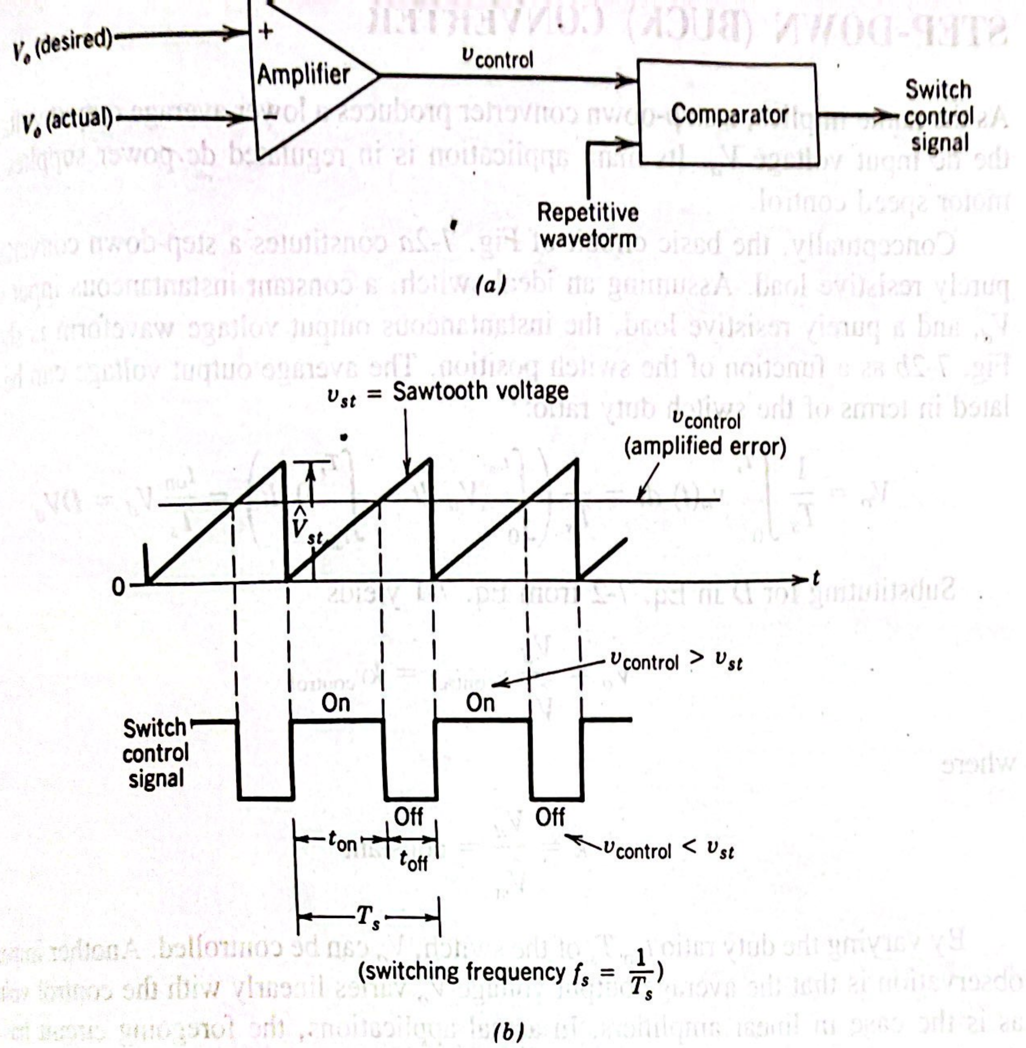

Figure 7-3 Pulse-width modulator: (*a*) block diagram; (*b*) comparator signals.

In the PWM switching at a constant switching frequency, the switch control signal, which controls the state (on or off) of the switch, is generated by comparing a signal-level control voltage $v_{control}$ with a repetitive waveform as shown in Figs. 7-3*a* and 7-3*b*. The control voltage signal generally is obtained by amplifying the error, or the difference between the actual output voltage and its desired value. The frequency of the repetitive waveform with a constant peak, which is shown to be a sawtooth, establishes the switching frequency. This frequency is kept constant in a PWM control and is chosen to be in a few kilohertz to a few hundred kilohertz range. When the amplified error signal, which varies very slowly with time relative to the switching frequency, is greater than the sawtooth waveform, the switch control signal becomes high, causing the switch to turn on. Otherwise, the switch is off. In terms of $v_{control}$ and the peak of the sawtooth waveform $\hat{V}_{st}$ in Fig. 7-3, the switch duty ratio can be expressed as

$$D = \frac{t_{on}}{T_s} = \frac{v_{control}}{\hat{V}_{st}} \tag{7-1}$$

The dc–dc converters can have two distinct modes of operation: (1) continuous current conduction and (2) discontinuous current conduction. In practice, a converter may operate in both modes, which have significantly different characteristics. Therefore, a converter and its control should be designed based on both modes of operation.

7-3 STEP-DOWN (BUCK) CONVERTER

As the name implies, a step-down converter produces a lower average output voltage than the dc input voltage V_d. Its main application is in regulated dc power supplies and dc motor speed control.

Conceptually, the basic circuit of Fig. 7-2*a* constitutes a step-down converter for a purely resistive load. Assuming an ideal switch, a constant instantaneous input voltage V_d, and a purely resistive load, the instantaneous output voltage waveform is shown in Fig. 7-2*b* as a function of the switch position. The average output voltage can be calculated in terms of the switch duty ratio:

$$V_o = \frac{1}{T_s}\int_0^{T_s} v_o(t)\,dt = \frac{1}{T_s}\left(\int_0^{t_{on}} V_d\,dt + \int_{t_{on}}^{T_s} 0\,dt\right) = \frac{t_{on}}{T_s}V_d = DV_d \qquad (7\text{-}2)$$

Substituting for D in Eq. 7-2 from Eq. 7-1 yields

$$V_o = \frac{V_d}{\hat{V}_{st}} v_{\text{control}} = k v_{\text{control}}$$

where

$$k = \frac{V_d}{\hat{V}_{st}} = \text{constant}$$

By varying the duty ratio t_{on}/T_s of the switch, V_o can be controlled. Another important observation is that the average output voltage V_o varies linearly with the control voltage, as is the case in linear amplifiers. In actual applications, the foregoing circuit has two drawbacks: (1) In practice the load would be inductive. Even with a resistive load, there would always be certain associated stray inductance. This means that the switch would have to absorb (or dissipate) the inductive energy and therefore it may be destroyed. (2) The output voltage fluctuates between zero and V_d, which is not acceptable in most applications. The problem of stored inductive energy is overcome by using a diode as shown in Fig. 7-4*a*. The output voltage fluctuations are very much diminished by using a low-pass filter, consisting of an inductor and a capacitor. Figure 7-4*b* shows the waveform of the input v_{oi} to the low-pass filter (same as the output voltage in Fig. 7-2*b* without a low-pass filter), which consists of a dc component V_o, and the harmonics at the switching frequency f_s and its multiples, as shown in Fig. 7-4*b*. The low-pass filter characteristic with the damping provided by the load resistor R is shown in Fig. 7-4*c*. The corner frequency f_c of this low-pass filter is selected to be much lower than the switching frequency, thus essentially eliminating the switching frequency ripple in the output voltage.

During the interval when the switch is on, the diode in Fig. 7-4*a* becomes reverse biased and the input provides energy to the load as well as to the inductor. During the interval when the switch is off, the inductor current flows through the diode, transferring some of its stored energy to the load.

In the steady-state analysis presented here, the filter capacitor at the output is assumed to be very large, as is normally the case in applications requiring a nearly constant instantaneous output voltage $v_o(t) \simeq V_o$. The ripple in the capacitor voltage (output voltage) is calculated later.

From Fig. 7-4*a* we observe that in a step-down converter, the average inductor current is equal to the average output current I_o, since the average capacitor current in steady state is zero (as discussed in Chapter 3, Section 3-2-5-1).

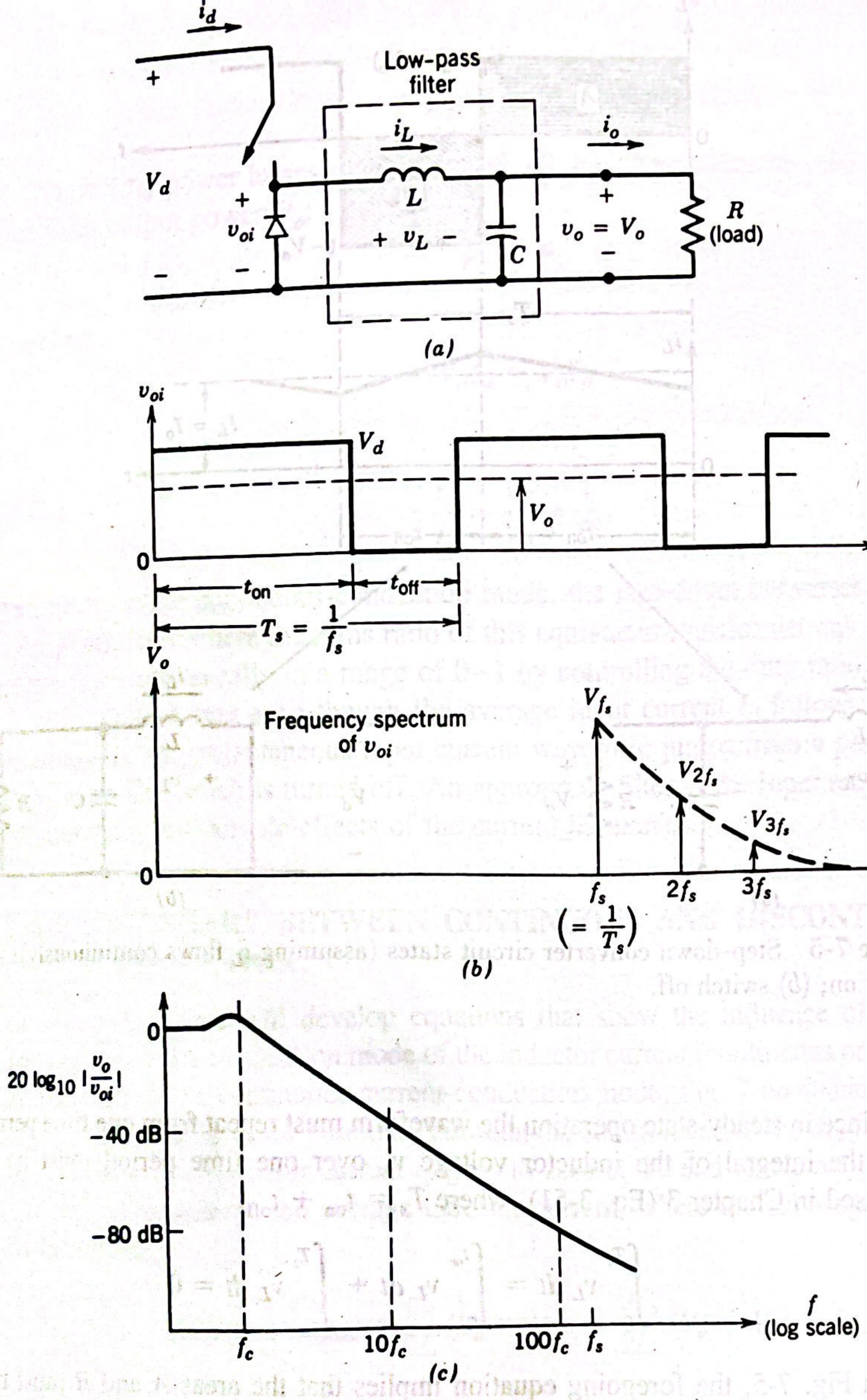

Figure 7-4 Step-down dc–dc converter.

7-3-1 CONTINUOUS-CONDUCTION MODE

Figure 7-5 shows the waveforms for the continuous-conduction mode of operation where the inductor current flows continuously $[i_L(t) > 0]$. When the switch is on for a time duration t_{on}, the switch conducts the inductor current and the diode becomes reverse biased. This results in a positive voltage $v_L = V_d - V_o$ across the inductor in Fig. 7-5*a*. This voltage causes a linear increase in the inductor current i_L. When the switch is turned off, because of the inductive energy storage, i_L continues to flow. This current now flows through the diode, and $v_L = -V_o$ in Fig. 7-5*b*.

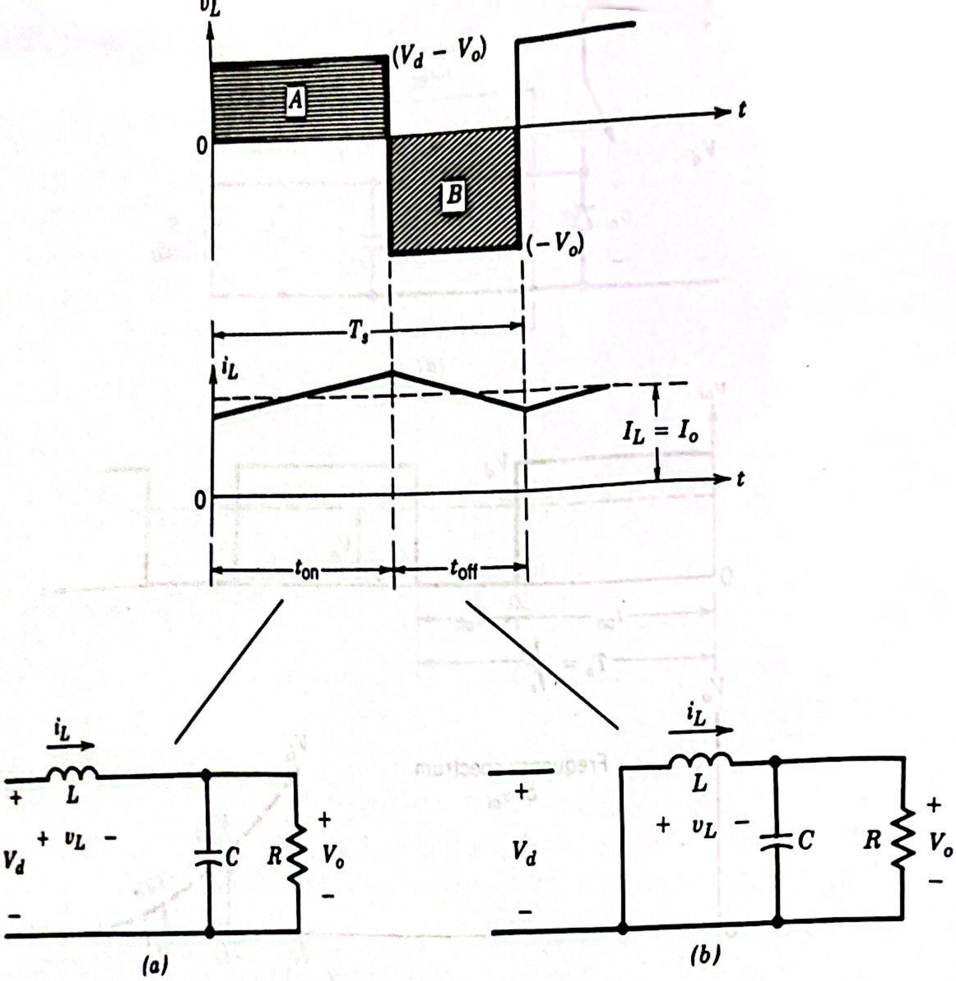

Figure 7-5 Step-down converter circuit states (assuming i_L flows continuously): (*a*) switch on; (*b*) switch off.

Since in steady-state operation the waveform must repeat from one time period to the next, the integral of the inductor voltage v_L over one time period must be zero, as discussed in Chapter 3 (Eq. 3-51), where $T_s = t_{on} + t_{off}$:

$$\int_0^{T_s} v_L \, dt = \int_0^{t_{on}} v_L \, dt + \int_{t_{on}}^{T_s} v_L \, dt = 0$$

In Fig. 7-5, the foregoing equation implies that the areas A and B must be equal. Therefore,

$$(V_d - V_o)t_{on} = V_o(T_s - t_{on})$$

or

$$\frac{V_o}{V_d} = \frac{t_{on}}{T_s} = D \quad \text{(duty ratio)} \tag{7-3}$$

Therefore, in this mode, the voltage output varies linearly with the duty ratio of the switch for a given input voltage. It does not depend on any other circuit parameter. The foregoing equation can also be derived by simply averaging the voltage v_{oi} in Fig. 7-4*b* and recognizing that the average voltage across the inductor in steady-state operation is zero:

$$\frac{V_d t_{on} + 0 \cdot t_{off}}{T_s} = V_o$$

$$\frac{V_o}{V_d} = \frac{t_{on}}{T_s} = D$$

Neglecting power losses associated with all the circuit elements, the input power P_d equals the output power P_o:

$$P_d = P_o$$

Therefore,

$$V_d I_d = V_o I_o$$

and

$$\frac{I_o}{I_d} = \frac{V_d}{V_o} = \frac{1}{D} \tag{7-4}$$

Therefore, in the continuous-conduction mode, the step-down converter is equivalent to a dc transformer where the turns ratio of this equivalent transformer can be continuously controlled electronically in a range of 0–1 by controlling the duty ratio of the switch.

We observe that even though the average input current I_d follows the transformer relationship, the instantaneous input current waveform jumps from a peak value to zero every time the switch is turned off. An appropriate filter at the input may be required to eliminate the undesirable effects of the current harmonics.

7-3-2 BOUNDARY BETWEEN CONTINUOUS AND DISCONTINUOUS CONDUCTION

In this section, we will develop equations that show the influence of various circuit parameters on the conduction mode of the inductor current (continuous or discontinuous). At the edge of the continuous-current-conduction mode, Fig. 7-6*a* shows the waveforms for v_L and i_L. Being at the boundary between the continuous and the discontinuous mode, by definition, the inductor current i_L goes to zero at the end of the off period.

At this boundary, the average inductor current, where the subscript B refers to the boundary, is

$$I_{LB} = \frac{1}{2} i_{L,\text{peak}} = \frac{t_{on}}{2L}(V_d - V_o) = \frac{DT_s}{2L}(V_d - V_o) = I_{oB} \tag{7-5}$$

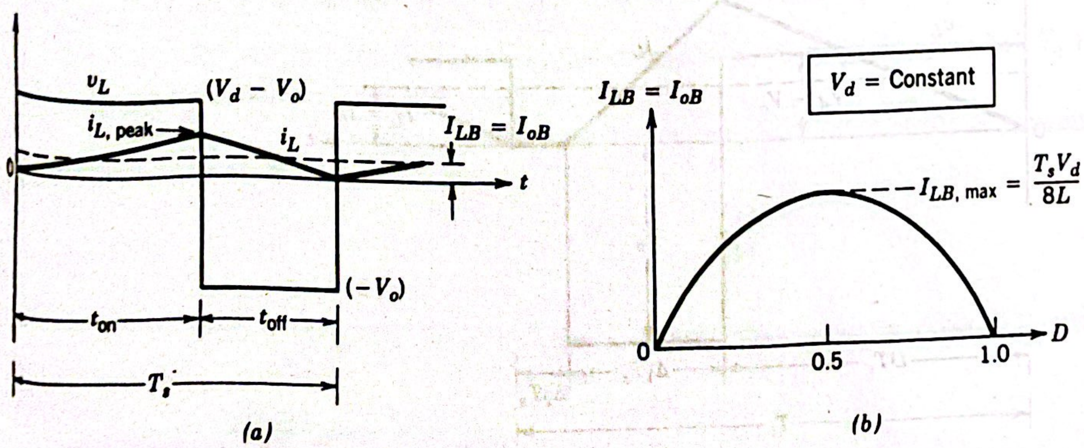

Figure 7-6 Current at the boundary of continuous–discontinuous conduction: (*a*) current waveform; (*b*) I_{LB} versus D keeping V_d constant.

Therefore, during an operating condition (with a given set of values for T_s, V_d, V_o, L, and D), if the average output current (and, hence, the average inductor current) becomes less than I_{LB} given by Eq. 7-5, then i_L will become discontinuous.

7-3-3 DISCONTINUOUS-CONDUCTION MODE

Depending on the application of these converters, either the input voltage V_d or the output voltage V_o remains constant during the converter operation. Both of these types of operation are discussed below.

7-3-3-1 Discontinuous-Conduction Mode with Constant V_d

In an application such as a dc motor speed control, V_d remains essentially constant and V_o is controlled by adjusting the converter duty ratio D.

Since $V_o = DV_d$, the average inductor current at the edge of the continuous-conduction mode from Eq. 7-5 is

$$I_{LB} = \frac{T_s V_d}{2L} D(1 - D) \tag{7-6}$$

Using this equation, we find that Fig. 7-6b shows the plot of I_{LB} as a function of the duty ratio D, keeping V_d and all other parameters constant. It shows that the output current required for a continuous-conduction mode is maximum at $D = 0.5$:

$$I_{LB,\max} = \frac{T_s V_d}{8L} \tag{7-7}$$

From Eqs. 7-6 and 7-7

$$I_{LB} = 4I_{LB,\max} D(1 - D) \tag{7-8}$$

Next the voltage ratio V_o/V_d will be calculated in the discontinuous mode. Let us assume that initially the converter is operating at the edge of continuous conduction, as in Fig. 7-6a, for given values of T, L, V_d, and D. If these parameters are kept constant and the output load power is decreased (i.e., the load resistance goes up), then the average inductor current will decrease. As is shown in Fig. 7-7, this dictates a higher value of V_o than before and results in a discontinuous inductor current.

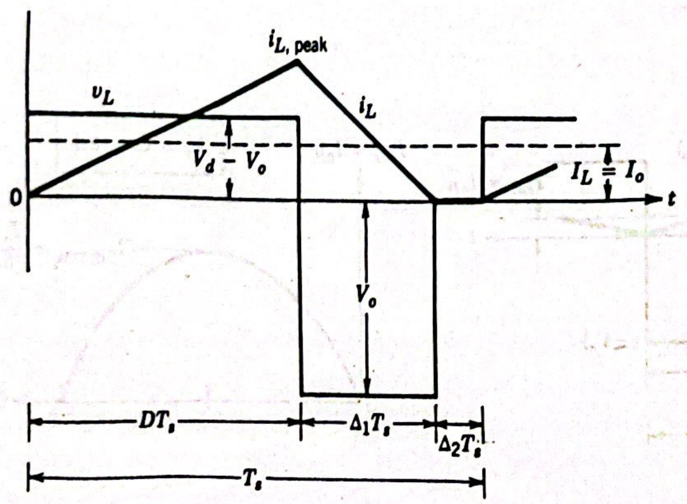

Figure 7-7 Discontinuous conduction in step-down converter.

During the interval $\Delta_2 T_s$ where the inductor current is zero, the power to the load resistance is supplied by the filter capacitor alone. The inductor voltage v_L during this interval is zero. Again, equating the integral of the inductor voltage over one time period to zero yields

$$(V_d - V_o)\, DT_s + (-V_o)\Delta_1 T_s = 0 \tag{7-9}$$

$$\therefore \frac{V_o}{V_d} = \frac{D}{D + \Delta_1} \tag{7-10}$$

where $D + \Delta_1 < 1.0$. From Fig. 7-7,

$$i_{L,\text{peak}} = \frac{V_o}{L}\Delta_1 T_s \tag{7-11}$$

Therefore,

$$I_o = i_{L,\text{peak}} \frac{D + \Delta_1}{2} \tag{7-12}$$

$$= \frac{V_o T_s}{2L}(D + \Delta_1)\, \Delta_1 \quad \text{(using Eq. 7-11)} \tag{7-13}$$

$$= \frac{V_d T_s}{2L} D\Delta_1 \quad \text{(using Eq. 7-10)} \tag{7-14}$$

$$= 4 I_{LB,\max} D\Delta_1 \quad \text{(using Eq. 7-7)} \tag{7-15}$$

$$\therefore \Delta_1 = \frac{I_o}{4 I_{LB,\max} D} \tag{7-16}$$

From Eqs. 7-10 and 7-16

$$\frac{V_o}{V_d} = \frac{D^2}{D^2 + \frac{1}{4}(I_o/I_{LB,\max})} \tag{7-17}$$

Figure 7-8 shows the step-down converter characteristic in both modes of operation for a constant V_d. The voltage ratio (V_o/V_d) is plotted as a function of $I_o/I_{LB,\max}$ for various values of duty ratio using Eqs. 7-3 and 7-17. The boundary between the continuous and the discontinuous mode, shown by the dashed curve, is established by Eq. 7-3 and 7-8.

7-3-3-2 Discontinuous-Conduction Mode with Constant V_o

In applications such as regulated dc power supplies, V_d may fluctuate but V_o is kept constant by adjusting the duty ratio D.

Since $V_d = V_o/D$, the average inductor current at the edge of the continuous-conduction mode from Eq. 7-5 is

$$I_{LB} = \frac{T_s V_o}{2L}(1 - D) \tag{7-18}$$

Equation 7-18 shows that if V_o is kept constant, the maximum value of I_{LB} occurs at $D = 0$:

$$I_{LB,\max} = \frac{T_s V_o}{2L} \tag{7-19}$$

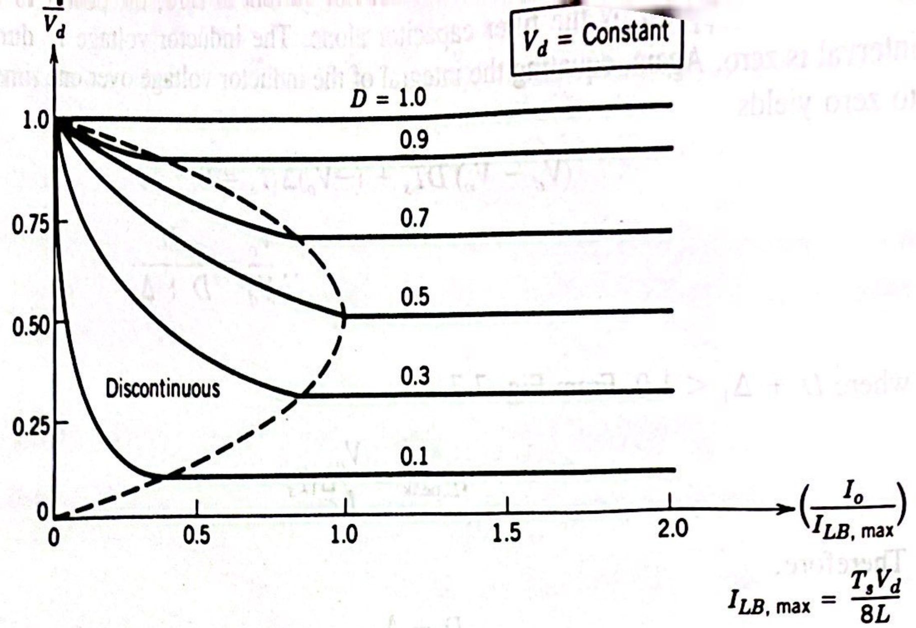

Figure 7-8 Step-down converter characteristics keeping V_d constant.

It should be noted that the operation corresponding to $D = 0$ and a finite V_o is, of course, hypothetical because it would require V_d to be infinite.

From Eqs. 7-18 and 7-19

$$I_{LB} = (1 - D)I_{LB,\max} \tag{7-20}$$

For the converter operation where V_o is kept constant, it will be useful to obtain the required duty ratio D as a function of $I_o/I_{LB,\max}$. Using Eqs. 7-10 and 7-13 (which are valid in the discontinuous-conduction mode whether V_o or V_d is kept constant) along with Eq. 7-19 for the case where V_o is kept constant yields

$$D = \frac{V_o}{V_d}\left(\frac{I_o/I_{LB,\max}}{1 - V_o/V_d}\right)^{1/2} \tag{7-21}$$

The duty ratio D as a function of $I_o/I_{LB,\max}$ is plotted in Fig. 7-9 for various values of V_d/V_o, keeping V_o constant. The boundary between the continuous and the discontinuous mode of operation is obtained by using Eq. 7-20.

7-3-4 OUTPUT VOLTAGE RIPPLE

In the previous analysis, the output capacitor is assumed to be so large as to yield $v_o(t) = V_o$. However, the ripple in the output voltage with a practical value of capacitance can be calculated by considering the waveforms shown in Fig. 7-10 for a continuous-conduction mode of operation. Assuming that all of the ripple component in i_L flows through the capacitor and its average component flows through the load resistor, the shaded area in Fig. 7-10 represents an additional charge ΔQ. Therefore, the peak-to-peak voltage ripple ΔV_o can be written as

$$\Delta V_o = \frac{\Delta Q}{C} = \frac{1}{C}\frac{1}{2}\frac{\Delta I_L}{2}\frac{T_s}{2}$$

From Fig. 7-5 during t_{off}

$$\Delta I_L = \frac{V_o}{L}(1 - D)T_s \tag{7-22}$$

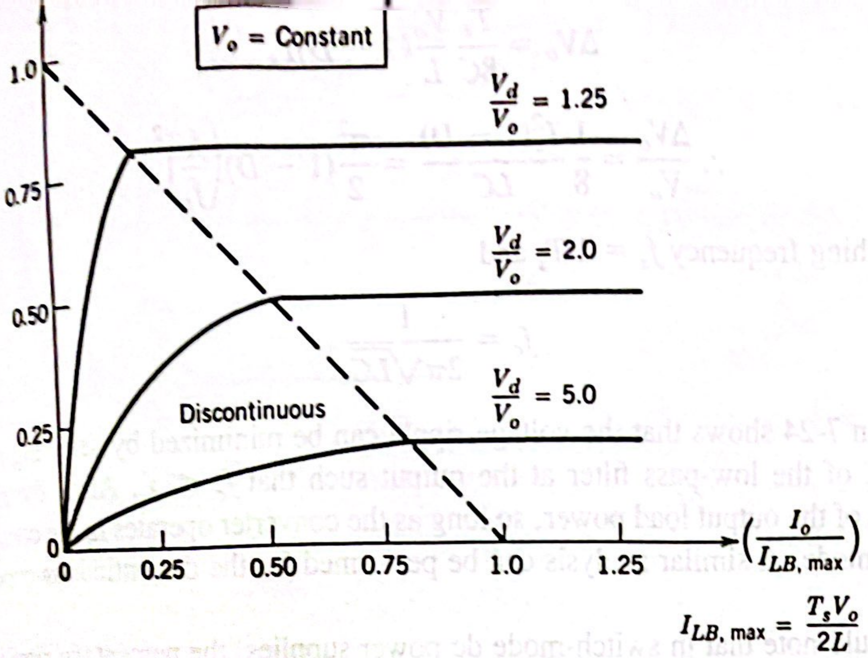

Figure 7-9 Step-down converter characteristics keeping V_o constant.

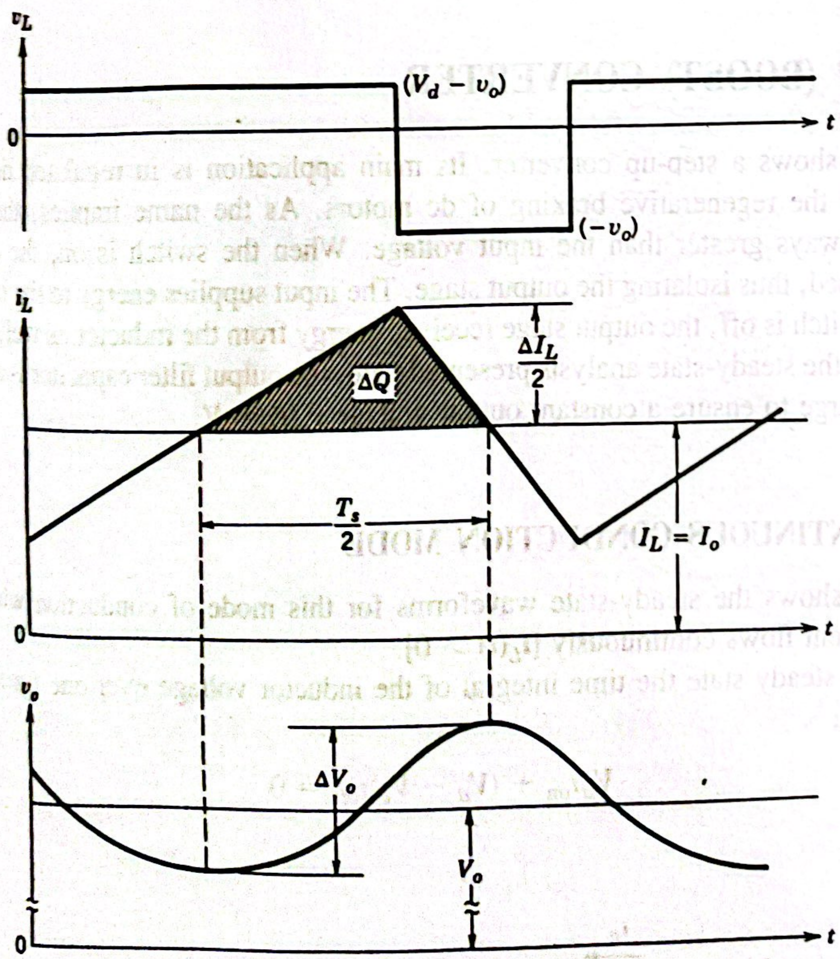

Figure 7-10 Output voltage ripple in a step-down converter.

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Therefore, substituting ΔI_L from Eq. 7-22 into the previous equation gives

$$\Delta V_o = \frac{T_s}{8C}\frac{V_o}{L}(1 - D)T_s \tag{7-23}$$

$$\therefore \frac{\Delta V_o}{V_o} = \frac{1}{8}\frac{T_s^2(1 - D)}{LC} = \frac{\pi^2}{2}(1 - D)\left(\frac{f_c}{f_s}\right)^2 \tag{7-24}$$

where switching frequency $f_s = 1/T_s$ and

$$f_c = \frac{1}{2\pi\sqrt{LC}} \tag{7-25}$$

Equation 7-24 shows that the voltage ripple can be minimized by selecting a corner frequency f_c of the low-pass filter at the output such that $f_c \ll f_s$. Also, the ripple is independent of the output load power, so long as the converter operates in the continuous-conduction mode. A similar analysis can be performed for the discontinuous-conduction mode.

We should note that in switch-mode dc power supplies, the percentage ripple in the output voltage is usually specified to be less than, for instance, 1%. Therefore, the analysis in the previous sections assuming $v_o(t) = V_o$ is valid. It should be noted that the output ripple in Eq. 7-24 is consistent with the discussion of the low-pass filter characteristic in Fig. 7-4c.

7-4 STEP-UP (BOOST) CONVERTER

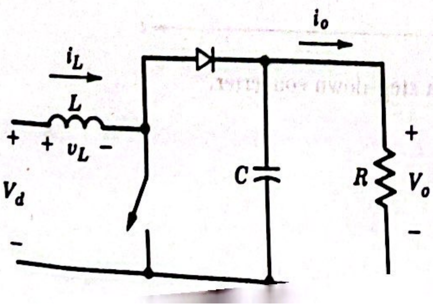
Figure 7-11 shows a step-up converter. Its main application is in regulated dc power supplies and the regenerative braking of dc motors. As the name implies, the output voltage is always greater than the input voltage. When the switch is on, the diode is reversed biased, thus isolating the output stage. The input supplies energy to the inductor. When the switch is off, the output stage receives energy from the inductor as well as from the input. In the steady-state analysis presented here, the output filter capacitor is assumed to be very large to ensure a constant output voltage $v_o(t) \simeq V_o$.

7-4-1 CONTINUOUS-CONDUCTION MODE

Figure 7-12 shows the steady-state waveforms for this mode of conduction where the inductor current flows continuously [$i_L(t) > 0$].

Since in steady state the time integral of the inductor voltage over one time period must be zero,

$$V_d t_{on} + (V_d - V_o)t_{off} = 0$$



dc converter.

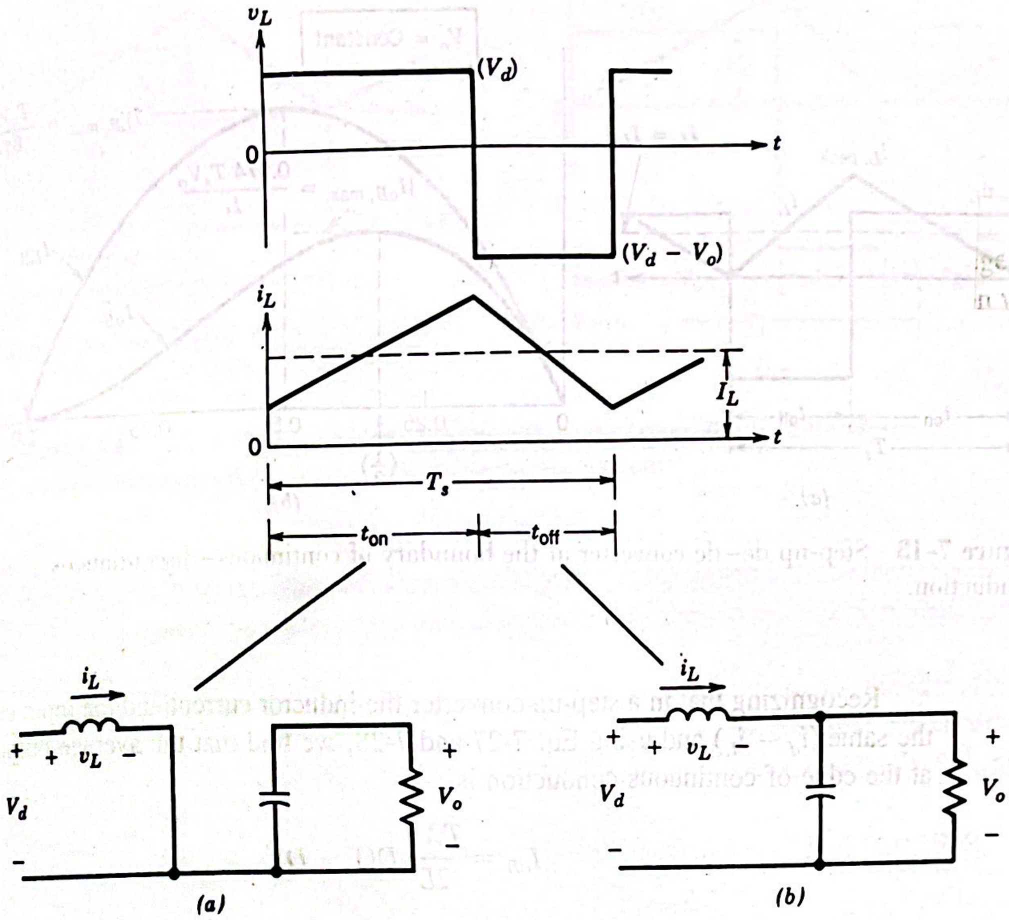


Figure 7-12 Continuous-conduction mode: (*a*) switch on; (*b*) switch off.

Dividing both sides by T_s and rearranging terms yield

$$\frac{V_o}{V_d} = \frac{T_s}{t_{off}} = \frac{1}{1 - D} \tag{7-26}$$

Assuming a lossless circuit, $P_d = P_o$,

$$\therefore V_d I_d = V_o I_o$$

and

$$\frac{I_o}{I_d} = (1 - D) \tag{7-27}$$

7-4-2 BOUNDARY BETWEEN CONTINUOUS AND DISCONTINUOUS CONDUCTION

Figure 7-13*a* shows the waveforms at the edge of continuous conduction. By definition, in this mode i_L goes to zero at the end of the off interval. The average value of the inductor current at this boundary is

$$I_{LB} = \frac{1}{2} i_{L,\text{peak}} \quad \text{(Fig. 7-13}a\text{)}$$

$$= \frac{1}{2}\frac{V_d}{L} t_{on}$$

$$= \frac{T_s V_o}{2L} D(1 - D) \quad \text{(using Eq. 7-26)} \tag{7-28}$$

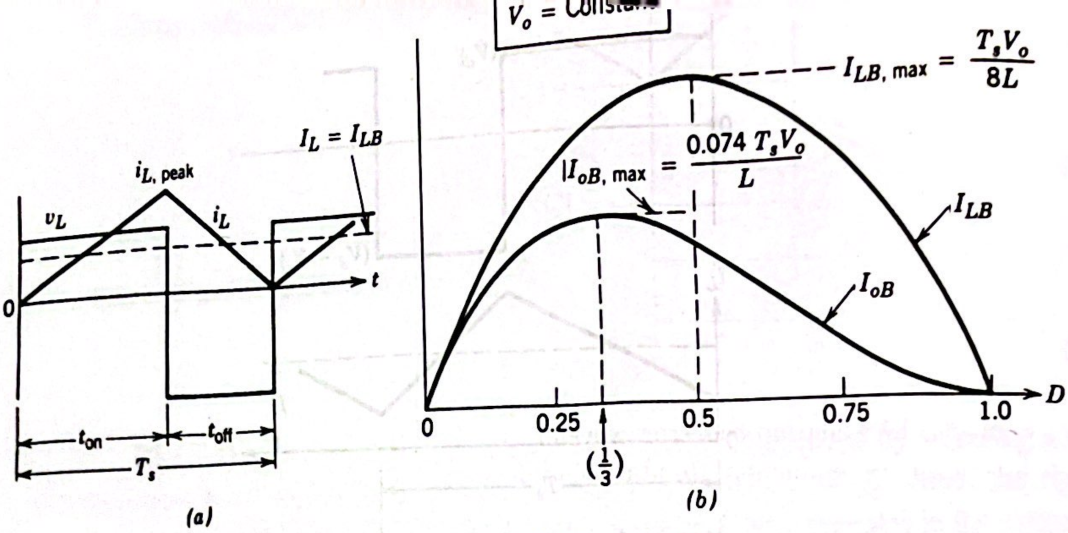


Figure 7-13 Step-up dc–dc converter at the boundary of continuous–discontinuous conduction.

Recognizing that in a step-up converter the inductor current and the input current are the same ($i_d = i_L$) and using Eq. 7-27 and 7-28, we find that the average output current at the edge of continuous conduction is

$$I_{oB} = \frac{T_s V_o}{2L} D(1 - D)^2 \tag{7-29}$$

Most applications in which a step-up converter is used require that V_o be kept constant. Therefore, with V_o constant, I_{oB} are plotted in Fig. 7-13*b* as a function of duty ratio D. Keeping V_o constant and varying the duty ratio imply that the input voltage is varying.

Figure 7-13*b* shows that I_{LB} reaches a maximum value at $D = 0.5$:

$$I_{LB,\max} = \frac{T_s V_o}{8L} \tag{7-30}$$

Also, I_{oB} has its maximum at $D = \frac{1}{3} = 0.333$:

$$I_{oB,\max} = \frac{2}{27} \frac{T_s V_o}{L} = 0.074 \frac{T_s V_o}{L} \tag{7-31}$$

In terms of their maximum values, I_{LB} and I_{oB} can be expressed as

$$I_{LB} = 4D(1 - D) I_{LB,\max} \tag{7-32}$$

and

$$I_{oB} = \frac{27}{4} D(1 - D)^2 I_{oB,\max} \tag{7-33}$$

Figure 7-13*b* shows that for a given D, with constant V_o, if the average load current drops below I_{oB} (and, hence, the average inductor current below I_{LB}), the current conduction will become discontinuous.

7-4-3 DISCONTINUOUS-CONDUCTION MODE

To understand the discontinuous-current-conduction mode, we would assume that as the output load power decreases, V_d and D remain constant (even though, in practice, D

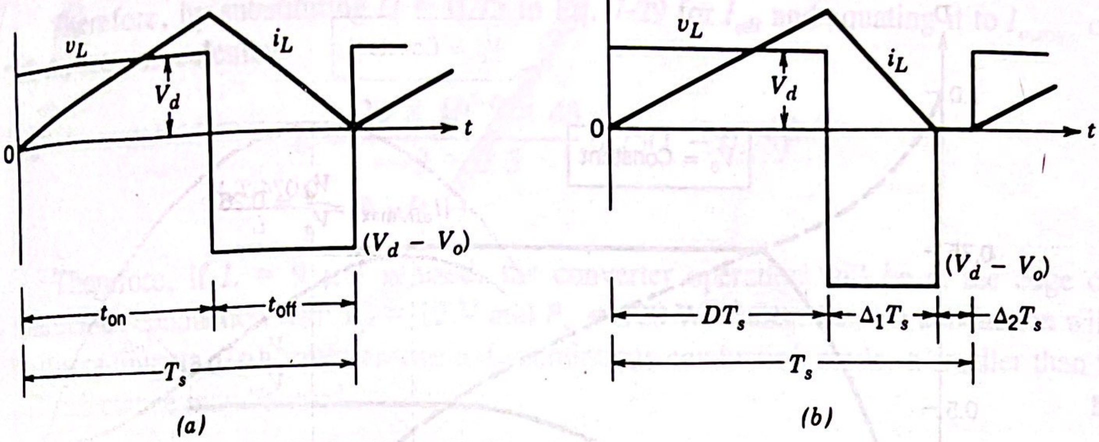


Figure 7-14 Step-up converter waveforms: (*a*) at the boundary of continuous–discontinuous conduction; (*b*) at discontinuous conduction.

would vary in order to keep V_o constant). Figure 7-14 compares the waveforms at the boundary of continuous conduction and discontinuous conduction, assuming that V_d and D are constant.

In Fig. 7-14*b*, the discontinuous current conduction occurs due to decreased P_o $(=P_d)$ and, hence, a lower I_L $(=I_d)$, since V_d is constant. Since $i_{L,\text{peak}}$ is the same in both modes in Fig. 7-14, a lower value of I_L (and, hence a discontinuous i_L) is possible only if V_o goes up in Fig. 7-14*b*.

If we equate the integral of the inductor voltage over one time period to zero,

$$V_d DT_s + (V_d - V_o)\Delta_1 T_s = 0$$

$$\therefore \frac{V_o}{V_d} = \frac{\Delta_1 + D}{\Delta_1} \tag{7-34}$$

and

$$\frac{I_o}{I_d} = \frac{\Delta_1}{\Delta_1 + D} \quad (\text{since } P_d = P_o) \tag{7-35}$$

From Fig. 7-14*b*, the average input current, which is also equal to the inductor current, is

$$I_d = \frac{V_d}{2L} DT_s(D + \Delta_1) \tag{7-36}$$

Using Eq. 7-35 in the foregoing equation yields

$$I_o = \left(\frac{T_s V_d}{2L}\right) D\Delta_1 \tag{7-37}$$

In practice, since V_o is held constant and D varies in response to the variation in V_d, it is more useful to obtain the required duty ratio D as a function of load current for various values of V_o/V_d. By using Eqs. 7-34, 7-37, and 7-31, we determine that

$$D = \left[\frac{4}{27}\frac{V_o}{V_d}\left(\frac{V_o}{V_d} - 1\right)\frac{I_o}{I_{oB,\max}}\right]^{1/2} \tag{7-38}$$

In Fig. 7-15, D is plotted as a function of $I_o/I_{oB,\max}$ for various values of V_d/V_o. The boundary between continuous and discontinuous conduction is shown by the dashed curve.

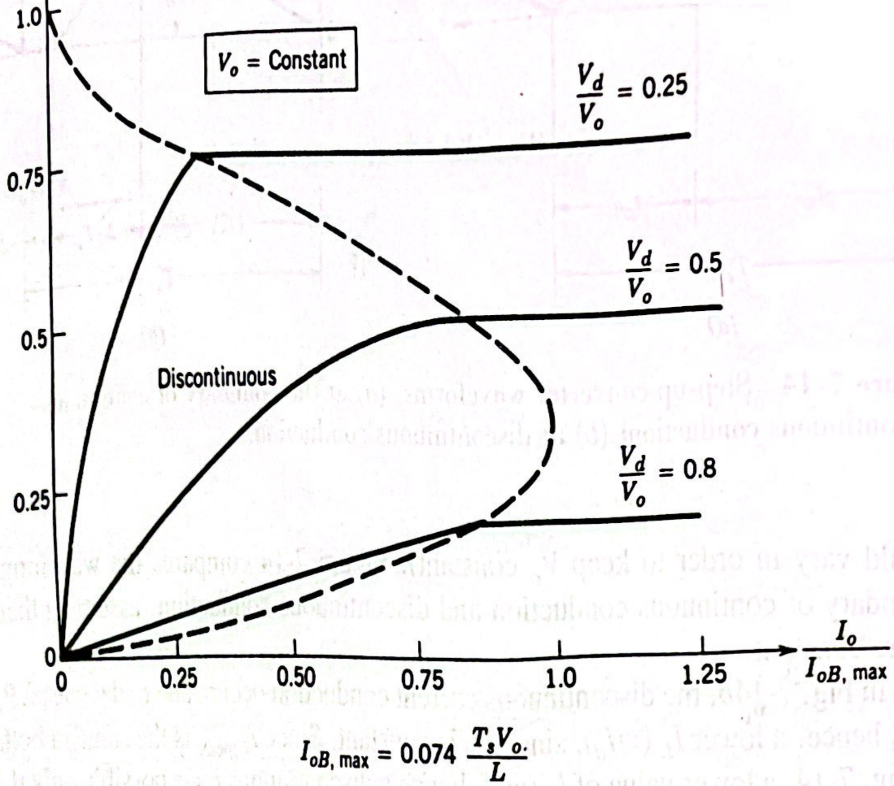


Figure 7-15 Step-up converter characteristics keeping V_o constant.

In the discontinuous mode, if V_o is not controlled during each switching time period, at least

$$\frac{L}{2} i_{L,\text{peak}}^2 = \frac{(V_d D T_s)^2}{2L} \qquad \text{W-s}$$

are transferred from the input to the output capacitor and to the load. If the load is not able to absorb this energy, the capacitor voltage V_o would increase until an energy balance is established. If the load becomes very light, the increase in V_o may cause a capacitor breakdown or a dangerously high voltage to occur.

■ ***Example 7-1*** In a step-up converter, the duty ratio is adjusted to regulate the output voltage V_o at 48 V. The input voltage varies in a wide range from 12 to 36 V. The maximum power output is 120 W. For stability reasons, it is required that the converter always operate in a discontinuous-current-conduction mode. The switching frequency is 50 kHz.

Assuming ideal components and C as very large, calculate the maximum value of L that can be used.

Solution In this converter, $V_o = 48$ V, $T_s = 20\ \mu s$, and $I_{o,\max} = 120$ W/48 V = 2.5 A. To find the maximum value of L that keeps the current conduction discontinuous, we will assume that at the extreme operating condition, the inductor current is at the edge of continuous conduction.

For the given range of V_d (12–36 V), D is in a range of 0.75–0.25 (corresponding to the current conduction bordering on being continuous). For this range of D, from Fig. 7-13b, I_{oB} has the smallest value at $D = 0.75$.

Therefore, by substituting $D = 0.75$ in Eq. 7-29 for I_{oB} and equating it to $I_{o,\max}$ of 2.5 A, we can calculate

$$L = \frac{20 \times 10^{-6} \times 48}{2 \times 2.5} 0.75(1 - 0.75)^2$$
$$= 9\ \mu H$$

Therefore, if $L = 9\ \mu$H is used, the converter operation will be at the edge of continuous conduction with $V_d = 12$ V and $P_o = 120$ W. Otherwise, the conduction will be discontinuous. To further ensure a discontinuous-conduction mode, a smaller than 9 μH inductance may be used. ■

7-4-4 EFFECT OF PARASITIC ELEMENTS

The parasitic elements in a step-up converter are due to the losses associated with the inductor, the capacitor, the switch, and the diode. Figure 7-16 qualitatively shows the effect of these parasitics on the voltage transfer ratio. Unlike the ideal characteristic, in practice, V_o/V_d declines as the duty ratio approaches unity. Because of very poor switch utilization at high values of duty ratio (as discussed in Section 7-8), the curves in this range are shown as dashed. These parasitic elements have been ignored in the simplified analysis presented here; however, these can be incorporated into circuit simulation programs on computers for designing such converters.

7-4-5 OUTPUT VOLTAGE RIPPLE

The peak-to-peak ripple in the output voltage can be calculated by considering the waveforms shown in Fig. 7-17 for a continuous mode of operation. Assuming that all the ripple current component of the diode current i_D flows through the capacitor and its average value flows through the load resistor, the shaded area in Fig. 7-17 represents charge ΔQ. Therefore, the peak–peak voltage ripple is given by

$$\Delta V_o = \frac{\Delta Q}{C} = \frac{I_o D T_s}{C} \quad \text{(assuming a constant output current)}$$
$$= \frac{V_o}{R} \frac{D T_s}{C} \tag{7-39}$$

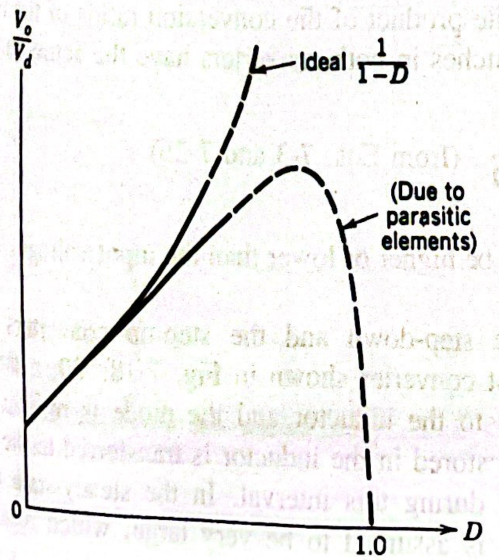


Figure 7-16 Effect of parasitic elements on voltage conversion ratio (step-up converter).

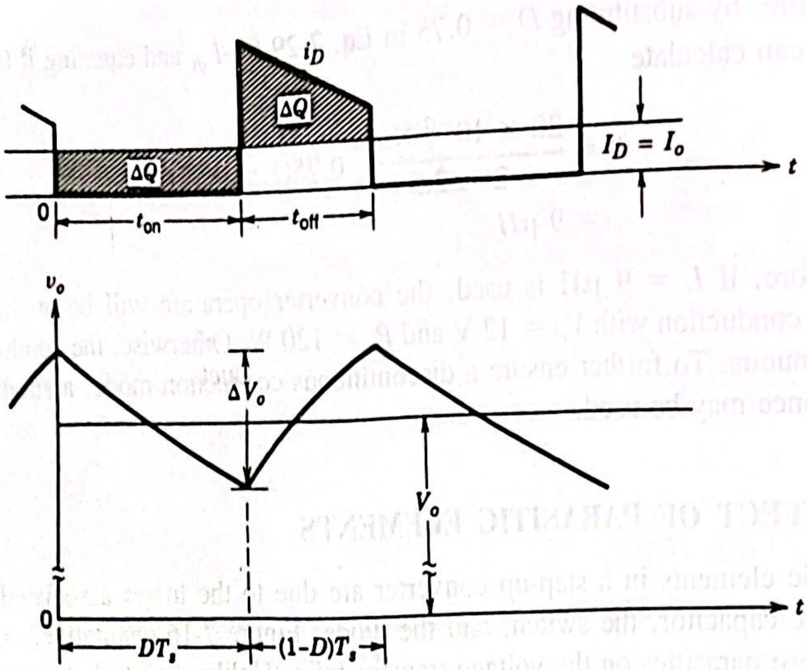


Figure 7-17 Step-up converter output voltage ripple.

$$\therefore \frac{\Delta V_o}{V_o} = \frac{DT_s}{RC}$$

$$= D\frac{T_s}{\tau} \quad \text{(where } \tau = RC \text{ time constant)} \tag{7-40}$$

A similar analysis can be performed for the discontinuous mode of conduction.

7-5 BUCK–BOOST CONVERTER

The main application of a step-down/step-up or buck–boost converter is in regulated dc power supplies, where a negative-polarity output may be desired with respect to the common terminal of the input voltage, and the output voltage can be either higher or lower than the input voltage.

A buck–boost converter can be obtained by the cascade connection of the two basic converters: the step-down converter and the step-up converter. In steady state, the output-to-input voltage conversion ratio is the product of the conversion ratios of the two converters in cascade (assuming that switches in both converters have the same duty ratio):

$$\frac{V_o}{V_d} = D\frac{1}{1-D} \quad \text{(from Eqs. 7-3 and 7-26)} \tag{7-41}$$

This allows the output voltage to be higher or lower than the input voltage, based on the duty ratio D.

The cascade connection of the step-down and the step-up converters can be combined into the single buck–boost converter shown in Fig. 7-18. When the switch is closed, the input provides energy to the inductor and the diode is reverse biased. When the switch is open, the energy stored in the inductor is transferred to the output. No energy is supplied by the input during this interval. In the steady-state analysis presented here, the output capacitor is assumed to be very large, which results in a constant output voltage $v_o(t) \simeq V_o$.