Stochastic Dynamic Model of Infectious Disease Spread

- A simplified approach

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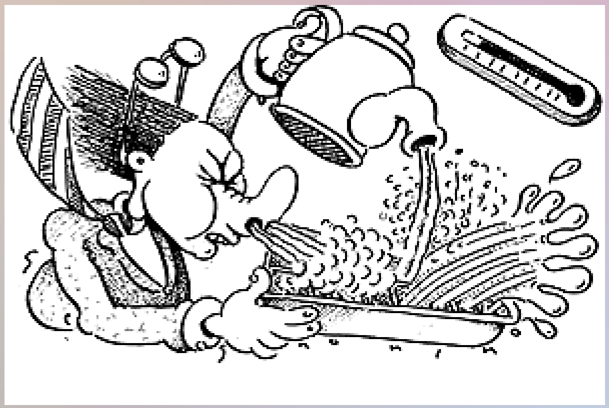


Overview

1. A few words about Epidemic Models, Dynamic Behavior and Stochasticity.

2. Theoretical basis and assumptions of the model :
   - SEIR model and multi-compartmental formulation (age structure)
   - Seasonality – time-dependent transmission rate
   - Stochasticity – event-driven approach
   - Other assumptions

Overview (cont.)

3. Mathematical formulation.
4. Modeling approach – tau-leap method.
5. Parameters initialization.
6. Results.
7. Extensions for the model – future work.

1. Epidemic Models, Dynamic Behavior and Stochasticity

- SIR model (Kermack and McKendrick, 1927) :
  - **S**usceptible : previously unexposed to the pathogen
  - **I**nfected : currently colonized by the pathogen
  - **R**ecovered : have successfully cleared the infection

$$S \rightarrow I \rightarrow R$$

Generalized SIR :

(demography included,

lifelong immunity)

$$\frac{dS}{dt} = \mu - \beta SI - \mu S$$

$$\frac{dI}{dt} = \beta SI - \gamma I - \mu I$$

$$\frac{dR}{dt} = \gamma I - \mu R$$

μ=natural mortality rate

β=transmission rate

γ=removal (recovery) rate

1. Epidemic Models, Dynamic Behavior and Stochasticity (cont.)

- Other Epidemic models :
  - SI (infection-induced mortality)
  - SIS (without immunity)
  - SIRS (waning immunity)
  - SEIR (latent period)

- Dynamic behavior : $S(t)$, $I(t)$, $R(t)$, $\beta(t)$

1. Epidemic Models, Dynamic Behavior and Stochasticity (cont.)

- Stochasticity: to include randomness in the model in the form of probabilities.

  Events of disease transmission, recovery, birth, death (natural) do not happen at a constant rate, but probabilities rule their appearance.

  Role of chance most important when the population size and the number of infectious individuals are relatively small.

2. Theoretical basis and assumptions

- SEIR model and multi-compartmental formulation (age structure).
  - **S**usceptibles (X)
  - **E**xposed (W) S→E→I→R
  - **I**nfectious (Y)
  - **R**ecovereds (Z)
  - Multi-compartmental model : age structure is introduced to the model to depict the higher risk some age groups may experience.
    - 0-5 , 6-9 , 10-19, 20+ years old

2. Theoretical basis and assumptions

- Seasonality – time-dependent transmission rate.

  Assuming the disease occurs more often during specific times of the year (ex. December, January, February) :

$$\beta(t) = \beta o * (1 + b_1 * term(t)),$$
$$where\; term(t) = +1 \;\; for\; t \in (Dec, Jan, Feb),$$
$$term(t) = -1 \;\; for\; t \notin (Dec, Jan, Feb)$$

2. Theoretical basis and assumptions

- Stochasticity – event-driven approach

  "Demographic stochasticity is defined as fluctuations in population processes that arise form the random number of events at the level of the individual"(M.Keeling, P.Rohani , 2008).

- Event driven methods require explicit consideration of events. In the presented model, there are 8 events.

  We therefore calculate the probability of :

- Birth
- Exposure
- Infection
- Recovery
- Death of recovered
- Death of exposed
- Death of infected
- Death of susceptibles

2. Theoretical basis and assumptions

- Other assumptions
  - Lifetime immunity
  - The Exposed group is not infectious
  - No vaccination
  - Only those at 20+ age group give births
  - Only those at 20+ age group suffer (natural) deaths
  - 6-9 age group has the highest value of transmission coefficient (assortative mixing at school combined with probable lack of previous exposure)

3. Mathematical formulation

- SEIR model with four (4) age classes :

We are now dealing with (integer) numbers of S,E,I,R .

SEIR $\rightarrow$(X,W,Y,Z)

(for simplicity, probability symbols are excluded from the equations)

$$\frac{dX_i}{dt} = \nu_i N_4 - \sum_{j=1}^{c_a} \frac{\beta_{ij}(t) Y_j X_i}{N} - \mu_i X_i$$

$$\frac{dW_i}{dt} = \sum_{j=1}^{c_a} \frac{\beta_{ij}(t) Y_j X_i}{N} - \mu_i W_i - \sigma W_i$$

$$\frac{dY_i}{dt} = \sigma W_i - \mu_i Y_i - \gamma Y_i$$

$$\frac{dZ_i}{dt} = \gamma Y_i - \mu_i Z_i$$

$$where\ \nu_1 = \mu_4 \neq 0$$

3. Mathematical formulation

- SEIR model with four (4) age classes (cont.) :

A fraction of each class moves to the age class above:

$$
\begin{aligned}
Q_1 &= Q_1 - Q_1/6 \\
Q_2 &= Q_2 + Q_1/6 - Q_2/4 \\
Q_3 &= Q_3 + Q_2/4 - Q_3/10 \\
Q_4 &= Q_4 + Q_3/10
\end{aligned}
$$

$$where\ Q_i \in (X_i, W_i, Y_i, Z_i)$$

since age groups are 0-5, 6-9, 10-19, 20+ years

3. Mathematical formulation

- SEIR model with four (4) age classes (cont.) :

  Additionally , the transmission coefficient is now a transmission matrix, where b_{ij} is the transmission coefficient for Xi's meeting Y_i's.

$$\beta_o = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix}$$

- Reminding

$$\beta(t) = \begin{cases} \beta o * (1 + b_1), t \in (Dec, Jan, Feb) \\ \beta o * (1 - b_1), t \notin (Dec, Jan, Feb) \end{cases}$$

4. Modeling approach – tau-leap method

- Gillespie's "τ-leap method" (Gillespie, 2001)
  - Discrete-time simulation.
  - 1. Time increment δt small and fixed
  - 2. Mi number of (ex.) transmission events by time t
  - 3. Define $\delta M_i = Mi(t+\delta t) - Mi(t)$ (i = events of the model, here transmission events), then

$$P(\delta M_T = 1 | X, Y) = \frac{\beta XY}{N} \delta t, \qquad N = total\ population$$

defines the transition probability for transmission events occurring in the time interval δt.

4. Modeling approach – tau-leap method (cont.)

- Gillespie's "τ-leap method" (Gillespie, 2001) (cont.)
  - 4. For δt small, δM_T is approximately Poisson :

$$\delta M_T \approx Poisson\left(\frac{\beta XY}{N}\delta t\right)$$

  - 5. Variables can be updated :

$$X(t+\delta t) = X(t) - \delta M_T \qquad Y(t+\delta t) = Y(t) + \delta M_T$$

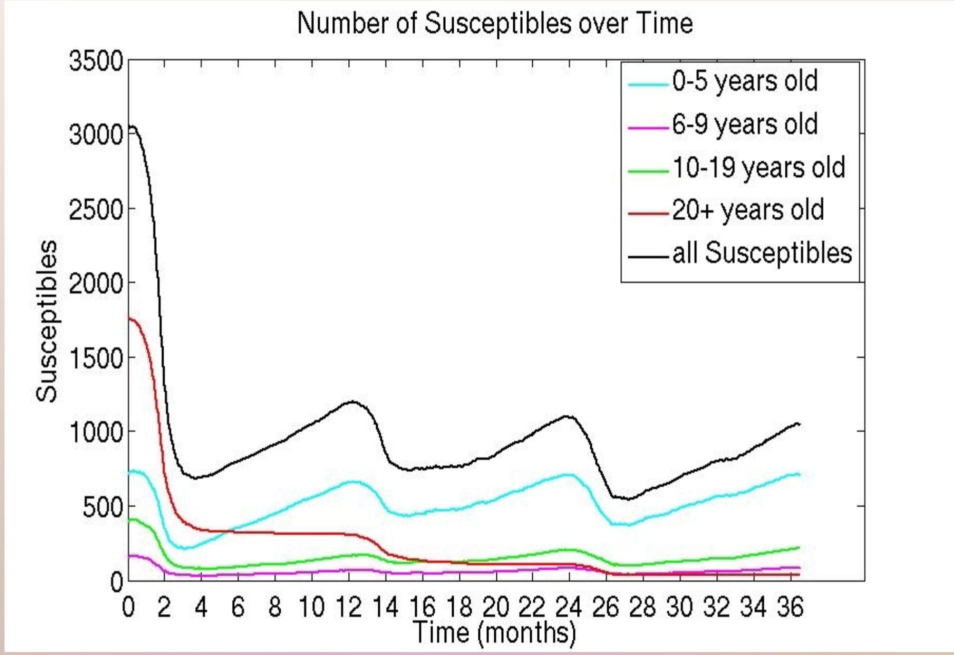
  - 6. Time is updated , $t = t + \delta t$. Return to 4.

- For all 8 events in the presented model we assume Poisson distributions with appropriate parameters.

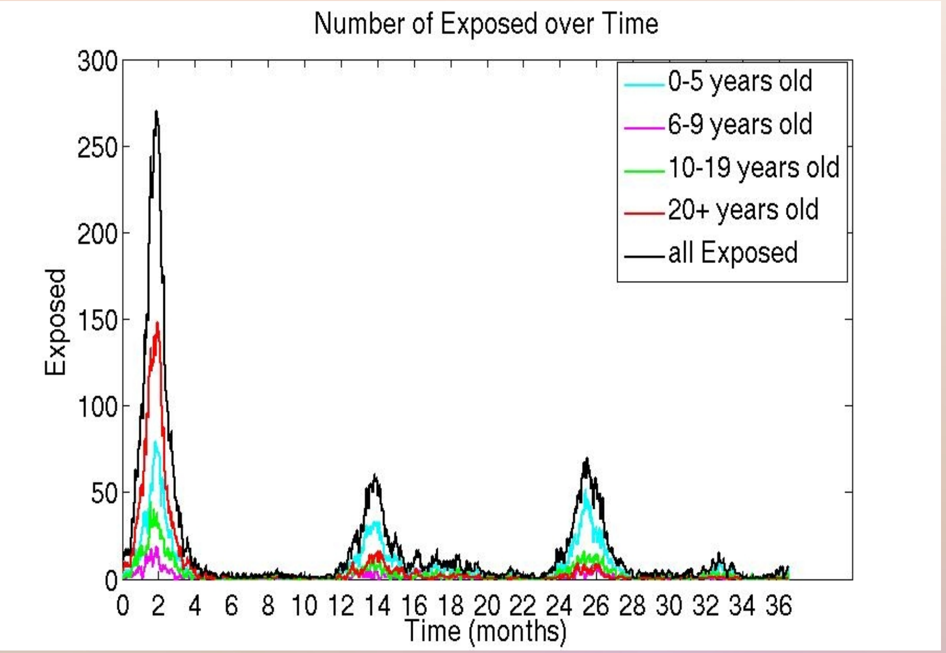
5. Parameters initialization

- $\sigma = 1/4$, meaning 4 days average latent (incubation) period
- $\gamma = 1/10$, meaning 10 days average infectious period
- $\mu_4 = \nu_1 = 0.02$ average birth and death rates per year
- $b_0 \simeq 0.5 \pm 0.25$ average values in the transmission matrix, ± seasonality (b1)
- N=50000 total population
- $\delta t = \tau = 1$ day
- time simulated = 3 years
- Starting point 5 infectious and 5 exposed in every age group
- Life expectancy 75 years

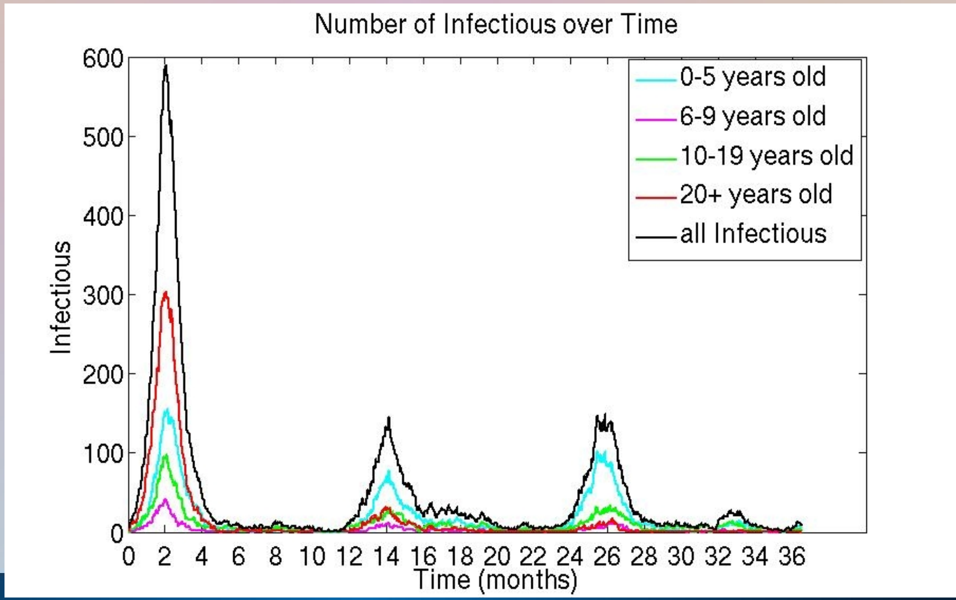
6. Results



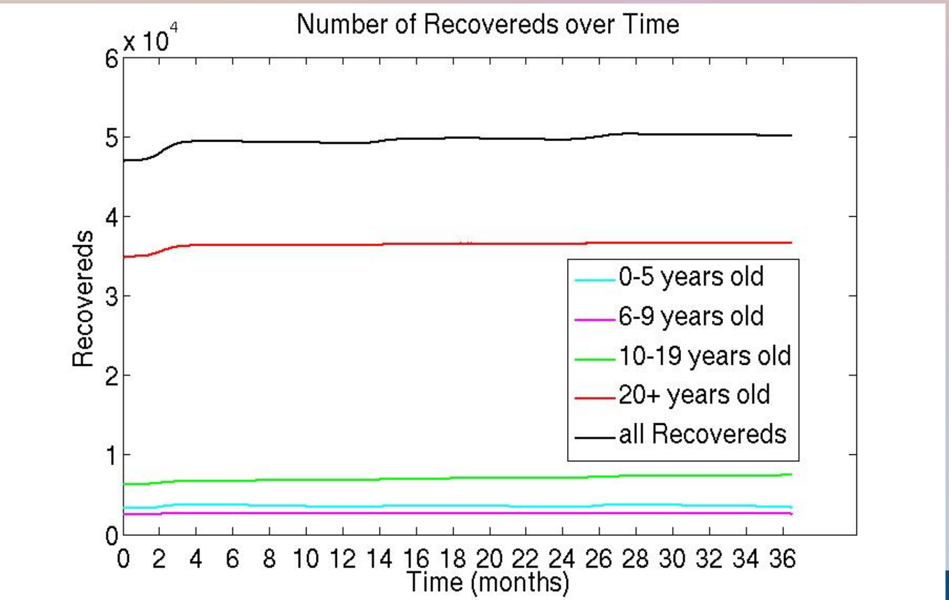
Number of Susceptibles over Time
Susceptibles
Time (months)
0-5 years old
6-9 years old
10-19 years old
20+ years old
all Susceptibles



Number of Exposed over Time
Exposed
Time (months)
0-5 years old
6-9 years old
10-19 years old
20+ years old
all Exposed

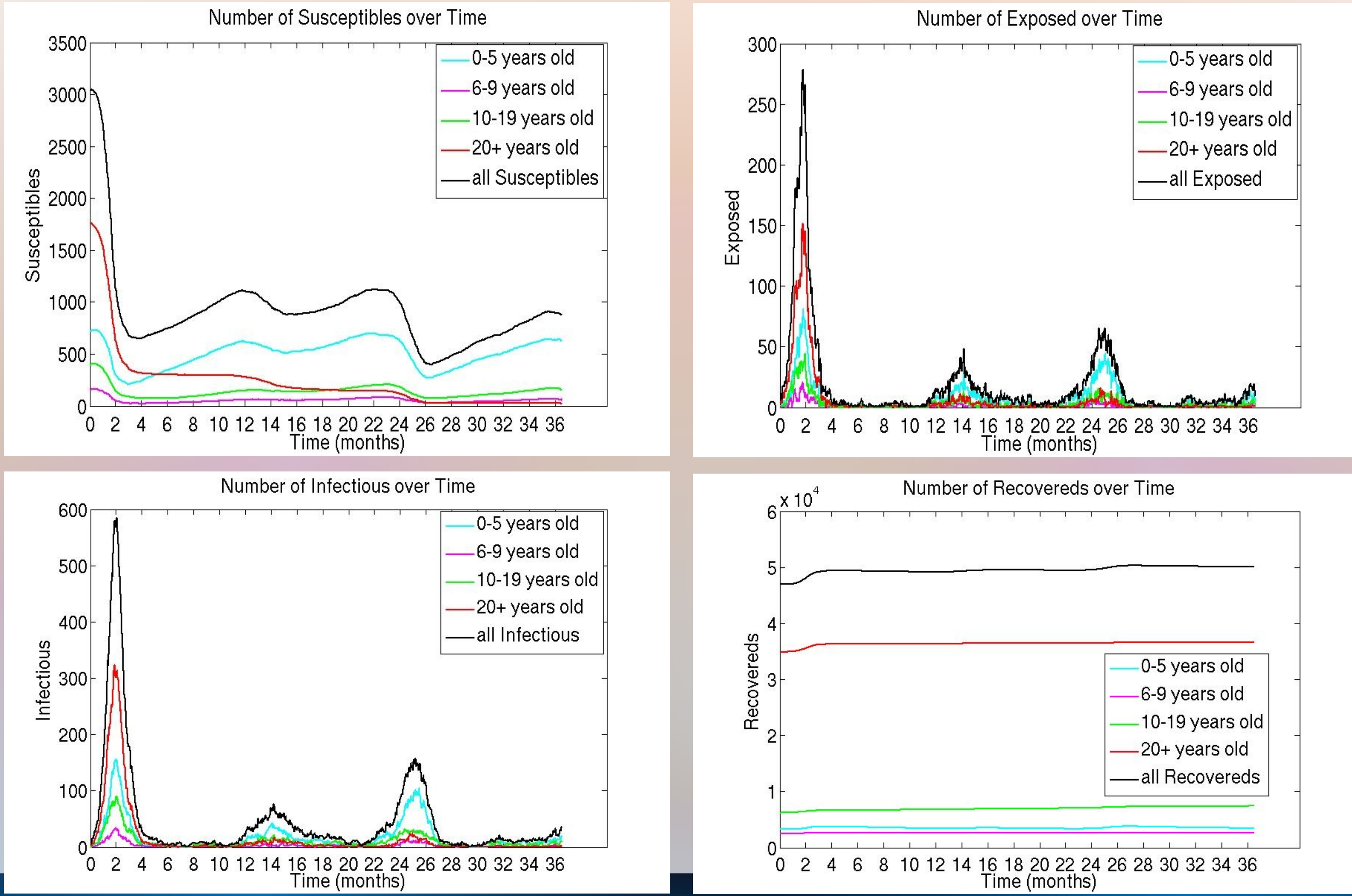


Number of Infectious over Time
Infectious
Time (months)
0-5 years old
6-9 years old
10-19 years old
20+ years old
all Infectious

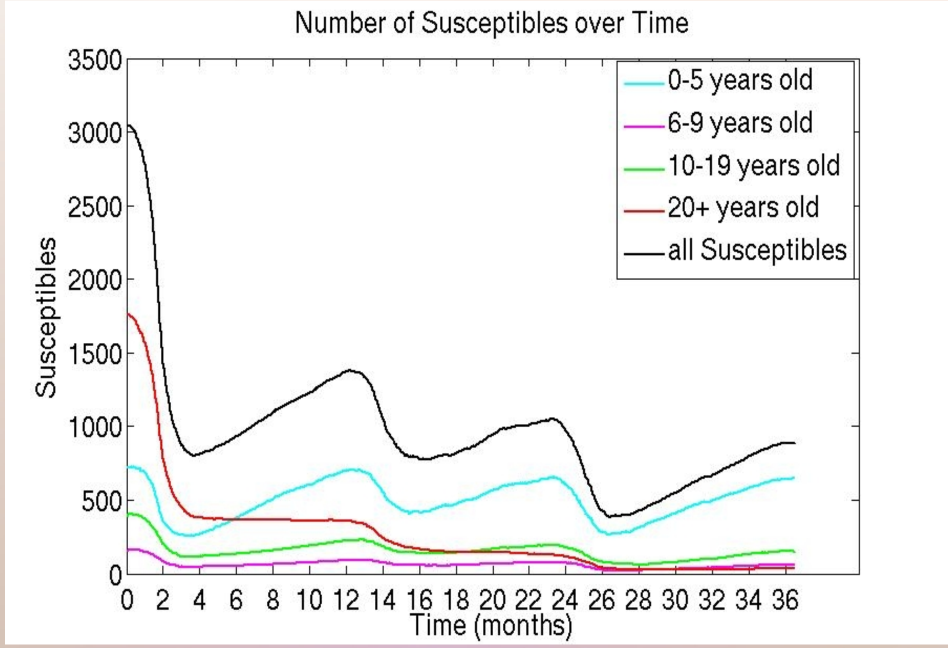


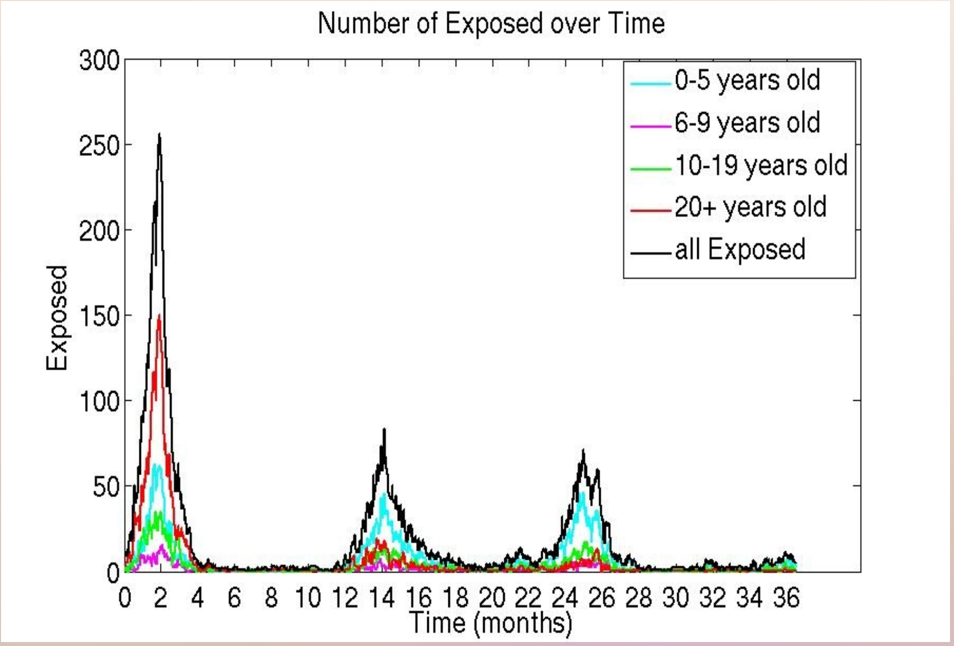
Number of Recovereds over Time
x 10^4
Recovereds
Time (months)
0-5 years old
6-9 years old
10-19 years old
20+ years old
all Recovereds

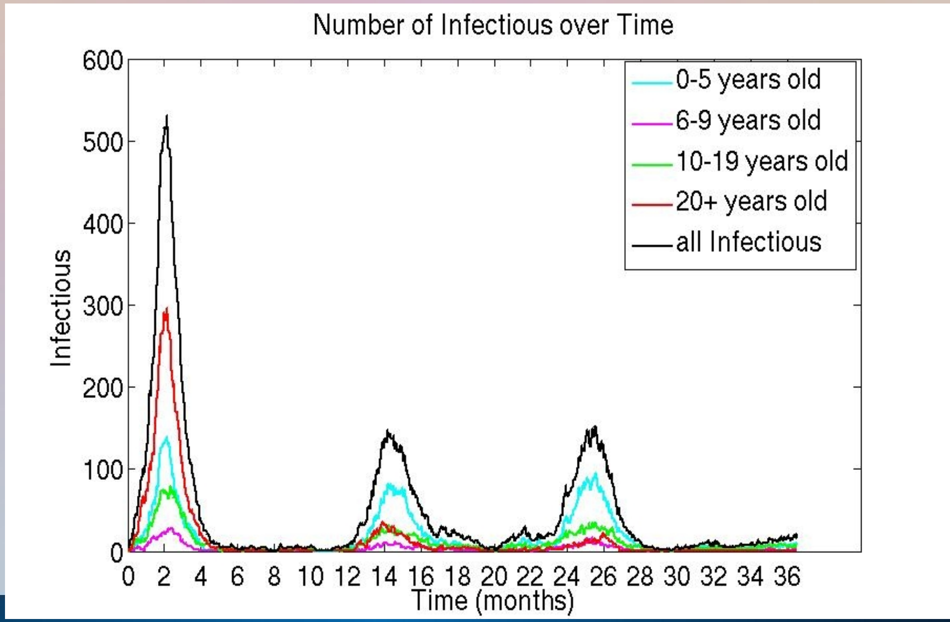
6. Results (cont.)

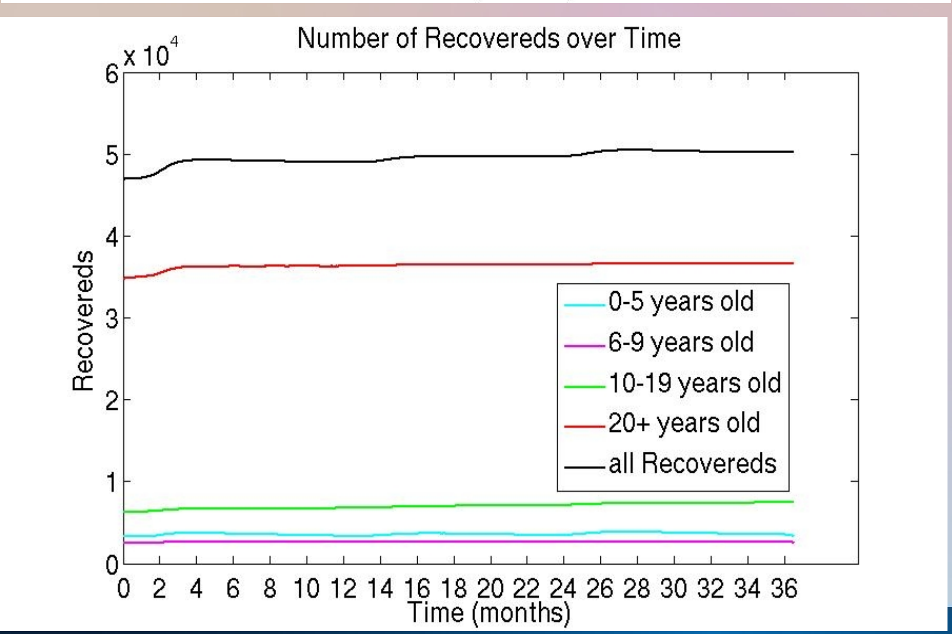


6. Results (cont.)

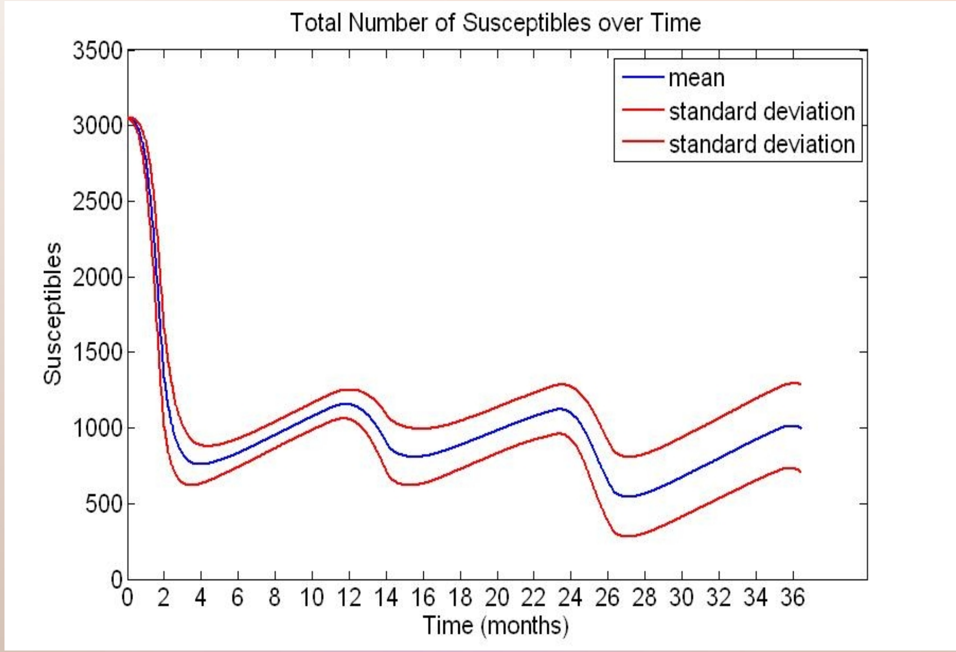


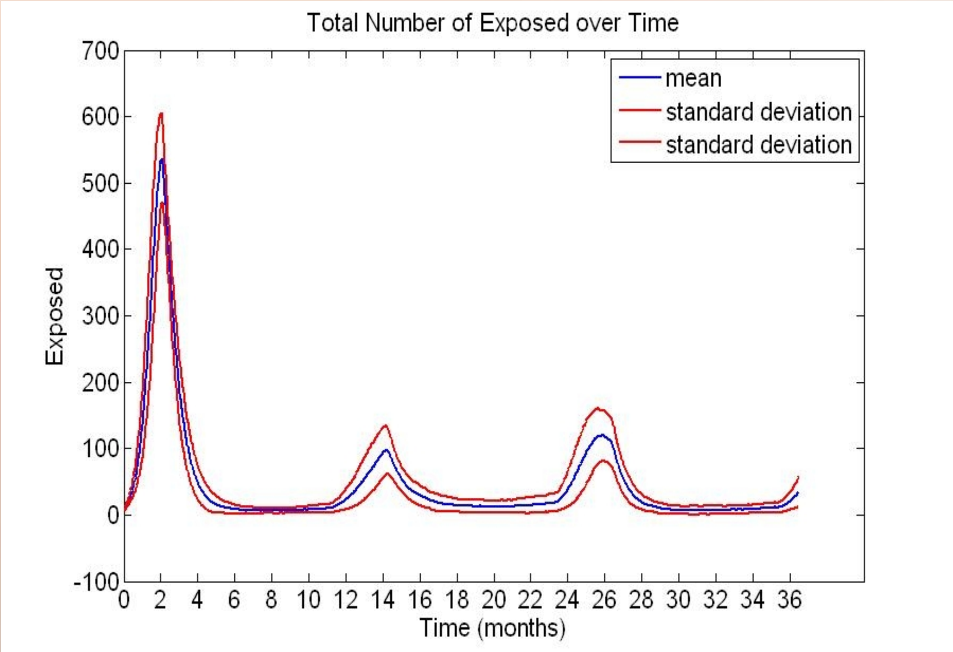


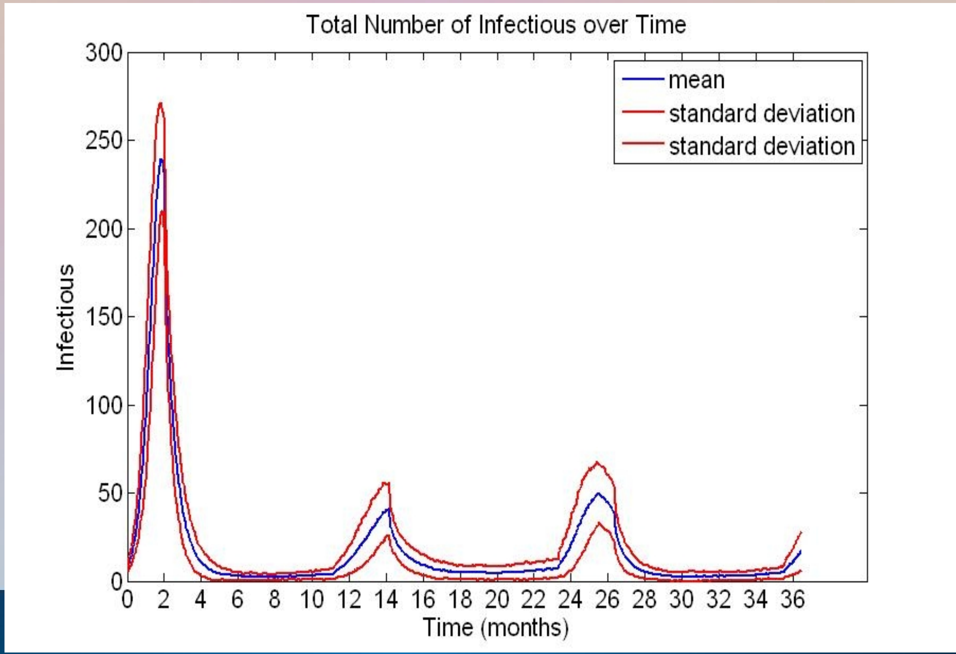


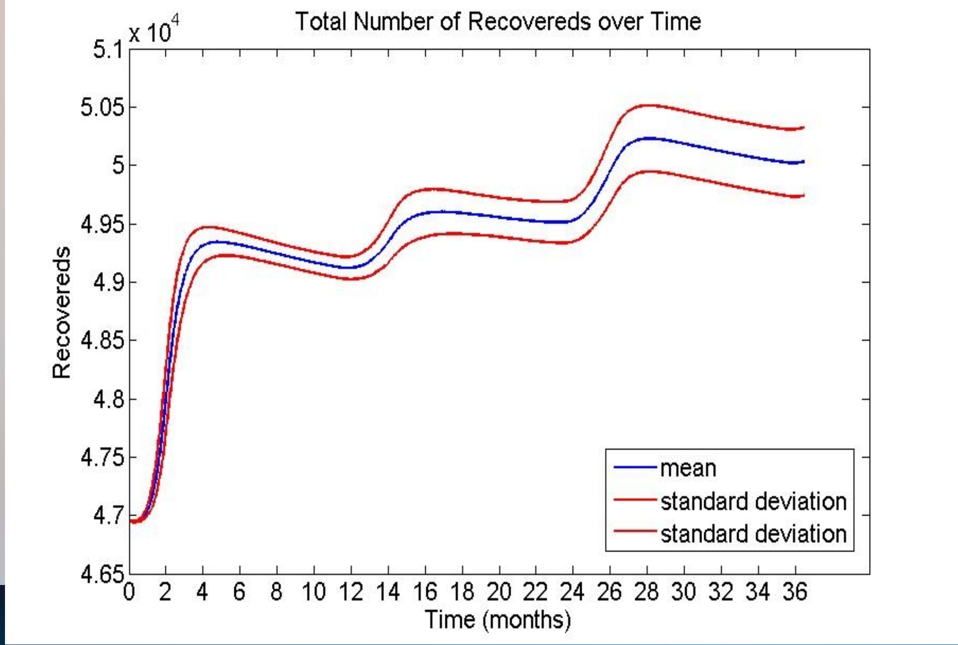


6. Results (cont.)









7. Extensions for the model – Future work

- Ro and Sensitivity analysis (identifying which parameters are important for the prediction imprecision)
- Spatial model
- Controlling the disease – Vaccination
- Waning or totally absent immunity

Thank you!

Questions?