DS 302 Exploratory Data Analysis

1.1 $m(a+bX) = a + b \times m(X)$

$ma + mbx = a + b \times m(X)$

↑ ↓

constant $= a$ $\quad bm(X)$

$$a + b(m(X)) = a + b \times m(X)$$

1.2 $cov(X, a+bY) = b \times cov(X,Y)$

$cov(X,Y) = m((X - m(X))(Z - m(Z)))$ ← cov btwn 2 Random variables

$cov(X, a+bY) = cov(X,a) + cov(X, bY)$ ← Linearity of cov

↓

$cov(X,a) = 0$ ← constant w/ cov

$cov(X, bY) = b \times cov(X,Y)$

$cov(X, a+bY) = cov(X,a) + cov(X,bY) = 0 + b\,cov(X,Y)$

$= b \times cov(X,Y)$

$$cov(X, a+bY) = b \times cov(X,Y)$$

1.3 $cov(a+bx, a+bx) = b^2 cov(x,x)$ & in particular that $cov(x,x) = s^2$

Linearity → $cov(a+bx, a+bx) = cov(a,a) + cov(a,bx) + cov(bx,a) + cov(bx,bx)$

constants → $cov(a,a) = 0$, $cov(a,bx) = 0$, $cov(Bx,a) = 0$

scaled variable → $cov(Bx,Bx) = cov(a+bx, a+bx) = 0+0+0+b^2(cov,x,x)$

$cov(a+bx, a+bx) = b^2 cov(x,x)$

variance of covariance → $var(x) = cov(x,x) = \sigma^2$

1.4 A non-decreasing transformation of the Median is the median of the transformed variables because the median depends on order which would be changed.

Quantile, the transformation would alter

IQR, would alter

Range, it would alter

1.5 Consider a non decreasing transformation g. Is it always true that $m(g(X)) = g(m(X))$

No, for example if $g(X) = X^2$ then $mg(X^2) \neq m(g(X))^2$ because squaring stretches large values more than small values, shifting the median value