Symbolic logic

This section is included for students who have not seen symbolic logic and truth tables or need a review.

Learning Objectives. By the end of class, students will be able to:

- Prove basic mathematical statements using definitions and direct proof
- Use truth tables to understand compound propositions
- Prove statements by contradiction
- Use the greatest integer function .

Read: Read Ernst Chapter 1 and Section 2.1. Also read Strayer Introduction and Section 1.1 through the proof of Proposition 1.2 (that is, pages 1-5).

Turn in: From Ernst: Problem 2.6 and 2.8

If you have not seen proof by induction or need a review, see Ernst Chapter 1 and Section 2.1 and Section 2.2 through Example 2.21. Problem 2.17 is also provided below:

In-class Problem 1 Determine whether each of the following is a proposition. Explain your reasoning.

- All cars are red.
- Every person whose name begins with J has the name Joe.
- $x^2 = 4$.
- There exists a real number x such that $x^2 = 4$.
- For all real numbers x, $x^2 = 4$.
- $\sqrt{2}$ is an irrational number.
- p is prime.
- Is it raining?
- It will rain tomorrow.
- Led Zeppelin is the best band of all time.

In-class Problem 2 Construct a truth table for $A \Rightarrow B, \neg(A \Rightarrow B)$ and $A \wedge \neg B$

Solution:

A	B	$A \Rightarrow B$	$\neg(A \Rightarrow B)$	$A \wedge \neg B$
True	True	(True ✓/ False)	(True/ False ✓)	(True/ False ✓)
True	False	(True/ False ✓)	(True ✓/ False)	(True ✓/ False)
False	True	(True ✓/ False)	(True/ False ✓)	(True/ False ✓)
False	True	(True ✓/ False)	(True/ False ✓)	(True/ False ✓)

This is the basis for *proof by contradiction.* We assume both A and $\neg B$, and proceed until we get a contradiction. That is, A and $\neg B$ cannot both be true.

Learning outcomes:
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Definition (Proof by contradiction)**.** Let A and B be propositions. To prove A implies B by contradiction, first assume the B is false. Then work through logical steps until you conclude $\neg A \wedge A$.

All definitions are 'biconditionals but we normally only write the "if."

We say that two definitions are *equivalent* if definition A is true if and only if definition B is true.