Quadratic reciprocity

Introducing quadratic reciprocity.

We are going to explore the relationship between $\left(\frac{p}{q}\right)$ and $\left(\frac{q}{p}\right)$. Let's look at an example:

Question 1 *We want to know if* 3 *is a quadratic residue modulo* 107. *It would be a lot easier to check if* 107 *is a quadratic residue modulo* 3. *We know that* $107 \equiv 2 \pmod 3$, *so* $\left(\frac{107}{3}\right) = -1$. *It would be nice if this also gave us* $\left(\frac{3}{107}\right)$.

Question 2 *Another example: Find* $\left(\frac{p}{5}\right)$ and $\left(\frac{5}{p}\right)$.

p	3	5	7	11	13
$\left(\frac{p}{5}\right)$	-1	0	-1	1	-1
$\left(\frac{5}{p}\right)$	-1	0	-1	1	-1

Question 3 *Another example: Find* $\left(\frac{p}{7}\right)$ and $\left(\frac{7}{p}\right)$.

p	3	5	7	11	13
$\left(\frac{p}{7}\right)$	-1	-1	0	1	-1
$\left(\frac{7}{p}\right)$	1	-1	0	-1	-1

This gives some evidence for our theorem:

Theorem 1. *Let* p *and* q *be odd primes with* $p \neq q$.

- *if* $p \equiv 1 \pmod 4$ *or* $q \equiv 1 \pmod 4$, *then* $\left(\frac{p}{q}\right) = \left(\frac{q}{p}\right)$

Learning outcomes:
Author(s):

- *if* $p \equiv q \equiv 3 \pmod 4$, *then* $\left(\dfrac{p}{q}\right) = -\left(\dfrac{q}{p}\right)$

Our goal for Friday is to prove this.