Homework #2 Rubrics

Proofs and writing

Strayer Exercise Set 1.1, Exercises 7, 9, 15. Strayer Exercise Set 1.2, Exercises 23, 25.

Homework Problem 1 (Strayer Exercise 7)**.** Let $a, b \in \mathbb{Z}$ with $a \mid b$. Prove that $a^n \mid b^n$ for every positive integer n.

Rubric:

- **0 points** Work does not contain enough of the relevant concepts to provide feedback.
- **1 points** **Does not demonstrate understanding** Contains a reasonable attempt to prove each part, but does not meet the criteria for two points.
- **2 points** **Needs revisions**
- **3 points** **Demonstrates understanding**
- **4 points** **Exemplary**

Proof ■

Homework Problem 2 (Strayer Exercise 9)**.** Let a, m and n be positive integers with $a > 1$ Prove that $a^m - 1 \mid a^n - 1$ if and only if $m \mid n$. (*Hint:* For the "if" direction, write $n = md$ with d a positive integer and use the factorization $a^{md}d - 1 = (a^m - 1) \times (a^{m(d-1)} + a^{m(d-2)} + \cdot + a^m + 1)$.]

Rubric:

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- **4 points** **Exemplary**

Proof ■

Homework Problem 3 (Strayer Exercise 15)**.** The following exercises present two alternative versions of the division algorithm. Both versions allow negative divisors; as such, they are more general than Theorem 1.4

(a) Let a and b be nonzero integers. Prove that there exists a unique $q, r \in \mathbb{Z}$ such that

$$a = bq + r, \quad 0 \leq r < |b|.$$

(b) Find the unique q and r guarenteed by the division algorithm of part (a) above with $a = 47$ and $b = -6$.

(c) Let a and b be nonzero integers. Prove that there exist unique $q, r \in \mathbb{Z}$ such that

$$a = bq + r, \quad -\frac{|b|}{2} < r \leq \frac{|b|}{2}.$$

This algorithm is called the *absolute least remainders algorithm.*

(d) Find the unique q and r guarenteed by the division algorithm of part (c) above with $a = 47$ and $b = -6$.

Rubric:

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2 points **Needs revisions**

3 points **Demonstrates understanding**

4 points **Exemplary**

Solution: (a)

(b)

(c)

(d)

Homework Problem 4 (Strayer Exercise 23)**.** Prove or disprove the following conjecture, which is similar to Conjecture 1: **Conjecture:** There are infinitely many prime number p for which $p + 2$ and $p + 4$ are also prime numbers.

Rubric:

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1 points **Does not demonstrate understanding** Contains a reasonable attempt to prove each part, but does not meet the criteria for two points.

2 points **Needs revisions**

3 points **Demonstrates understanding**

4 points **Exemplary**

Solution:

Homework Problem 5 (Strayer Exercise 25)**.** (a) Prove that all odd prime numbers can be expressed as the difference of square of two successive integers.

(b) Prove that no prime numbers can be expressed as the difference of two fourth power integers (*Hint:* Use the factorization tool discussed in the final paragraph of this section.)

Rubric:

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2 points **Needs revisions**

3 points **Demonstrates understanding**

4 points **Exemplary**

Proof (a)

(b)

■

Homework Problem 6 (Ernst Problem 2.50)**.** Consider the following statement: If $x \in \mathbb{Z}$ such that x^2 is odd, then x is odd. The items below can be assembled to form a proof of this statement, but they are currently out of order. Put them in the proper order.

(a) Assume that x is an even integer.

(b) We will utilize a proof by contraposition.

(c) Thus, x^2 is twice an integer.

(d) Since $x = 2k$, we have that $x^2 = (2k)^2 = 4k^2$.

(e) Since k is an integer, $2k^2$ is also an integer.

(f) By the definition of even, there is an integer k such that $x = 2k$.

(g) We have proved the contrapositive, and hence the desired statement is true.

(h) Assume $x \in \mathbb{Z}$.

(i) By the definition of even integer, x^2 is an even integer.

(j) Notice that $x^2 = 2(2k^2)$.

Rubric:

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1 points **Does not demonstrate understanding** Contains a reasonable attempt to prove each part, but does not meet the criteria for two points.

2 points **Needs revisions**

3 points **Demonstrates understanding**

4 points **Exemplary**

Proof ■