Quadratic reciprocity

Introducing quadratic reciprocity.

We are going to explore the relationship between $\left(\frac{p}{q}\right)$ and $\left(\frac{q}{p}\right)$. Let's look at an example:

Question 1 *We want to know if* 3 *is a quadratic residue modulo* 107. *It would be a lot easier to check if* 107 *is a quadratic residue modulo* 3. *We know that* $107 \equiv \boxed{2} \pmod 3$, *so* $\left(\frac{107}{3}\right) = \boxed{-1}$. *It would be nice if this also gave us* $\left(\frac{3}{107}\right)$.

Question 2 *Another example: Find* $\left(\frac{p}{5}\right)$ *and* $\left(\frac{5}{p}\right)$.

p	*3*	*5*	*7*	*11*	*13*
$\left(\frac{p}{5}\right)$	$\boxed{-1}$	$\boxed{0}$	$\boxed{-1}$	$\boxed{1}$	$\boxed{-1}$
$\left(\frac{5}{p}\right)$	*-1*	*0*	*-1*	*1*	*-1*

Question 3 *Another example: Find* $\left(\frac{p}{7}\right)$ *and* $\left(\frac{7}{p}\right)$.

p	*3*	*5*	*7*	*11*	*13*
$\left(\frac{p}{7}\right)$	$\boxed{-1}$	$\boxed{-1}$	*0*	$\boxed{1}$	$\boxed{-1}$
$\left(\frac{7}{p}\right)$	$\boxed{1}$	$\boxed{-1}$	*0*	*-1*	*-1*

This gives some evidence for our theorem:

Theorem 1. *Let* p *and* q *be odd primes with* $p \neq q$.

- *if* $p \equiv 1 \pmod 4$ *or* $q \equiv 1 \pmod 4$, *then* $\left(\frac{p}{q}\right) = \left(\frac{q}{p}\right)$

Learning outcomes:
Author(s):

- *if* $p \equiv q \equiv 3 \pmod 4$, *then* $\left(\frac{p}{q}\right) = -\left(\frac{q}{p}\right)$

Our goal for Friday is to prove this.