Induction

This section is included as a review of proof by induction.

Learning Objectives. By the end of class, students will be able to:

- Construct a proof by induction .

If you have not seen proof by induction or need a review, see Ernst Section 4.1 and Section 4.2

Instructor Notes:

Read Strayer Appendix A.1: The First Principle of Mathematical Induction or Ernst Section 4.1 and Section 4.2

Turn in Strayer Exercise Set A, Exercise 1a. If n is a positive integer, then

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

In-class Problem 1 Theorems in Ernst Section 4.1
Theorem 1 (Ernst Theorem 4.5). For all $n \in \mathbb{N}$, 3 divides $4^n - 1$.

Solution: We proceed by induction. When $n = 1$, $3 \mid 4^n - 1 = 3$. Thus, the statement is true for $n = 1$.

Now assume $k \geq 1$ and the desired statement is true for $n = k$. Then the induction hypothesis is

$$3 \mid 4^k - 1.$$

By the definition of **??**, there exists $m \in \mathbb{Z}$ such that $3m = 4^k - 1$. In other words, $3m + 1 = 4^k$. Multiplying both sides by 4 gives $12m + 4 = 4^{k+1}$. Rewriting this equation gives $3(4m + 1) = 4^{k+1} - 1$. Thus, $3 \mid 4^{k+1} - 1$, and the desired statement is true for $n = k + 1$. By the (first) principle of mathematical induction, the statement is true for all positive integers, and the proof is complete.

In-class Problem 2 (Strayer Exercise 1) . Use the first principle of mathematical induction to prove each statement.

(a) If n is a positive integer, then

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}.$$

(b) If n is an integer with $n \geq 5$, then

$$2^n > n^2.$$

Learning outcomes:
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