

Your Name: _____ Group Members: _____

Problem 1 Find integral solutions to the Diophantine equation

$$8x_1 - 4x_2 + 6x_3 = 6.$$

- (a) Since $(8, -4, 6) = 2$, solutions exist
- (b) The linear Diophantine equation $8x_1 - 4x_2 = 4y$ has infinitely many solutions for all $y \in \mathbb{Z}$ by _____.
Substituting into the original Diophantine equation gives $4y + 6x_3 = 6$, which has infinitely many solutions by _____ since $(4, 6) = 2 \mid 6$. Find them.
- (c) For a particular value of y , the Diophantine equation $8x_1 - 4x_2 = 0$ has solutions, find them.
- (d) By inspection, $x_1 = 1, x_2 = 2$ is a particular solution. Then by Theorem 6.2, the solutions have the form

$$\begin{aligned} x_1 &= 1 + \text{_____}, & x_2 &= 2 - \text{_____}, & \text{or} \\ x_1 &= \text{_____}, & x_2 &= \text{_____}, & m \in \mathbb{Z}. \end{aligned}$$

- (e) Then $x_1 = \text{_____}, x_2 = \text{_____}, x_3 = \text{_____}$ for $m \in \mathbb{Z}$.