

Proofs and writing

Strayer Exercise Set 1.1, Exercises 5, 10, 11. Ernst Problem 2.19, Problem 2.37, then either prove or provide a counterexample for the statements. Additional problem provided below.

Homework Problem 1 (Strayer Exercise 5). Prove or disprove the following statements.

- (a) If a, b, c , and d are integers such that if $a \mid b$ and $c \mid d$, then $a + c \mid b + d$.
- (b) If a, b, c , and d are integers such that if $a \mid b$ and $c \mid d$, then $ac \mid bd$.
- (c) If a, b , and c are integers such that if $a \nmid b$ and $b \nmid c$, then $a \nmid c$.

Rubric:

0 points Work does not contain enough of the relevant concepts to provide feedback.

1 points Does not demonstrate understanding Contains a reasonable attempt to prove each part, but does not meet the criteria for two points.

2 points Needs revisions

3 points Demonstrates understanding

4 points Exemplary

Solution: (a)

(b)

(c)

Homework Problem 2 (Strayer Exercise 10). (a) Let $n \in \mathbb{Z}$. Prove that $3 \mid n^3 - n$.

(b) Let $n \in \mathbb{Z}$. Prove that $5 \mid n^5 - n$.

(c) Let $n \in \mathbb{Z}$. Is it true that $4 \mid n^4 - n$? Provide a proof or counter example.

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Proof



Homework Problem 3 (Strayer Exercise 11). Use the definition of even and odd from Strayer **not** Ernst.

- (a) Let $n \in \mathbb{Z}$. Prove that n is an even integer if and only if $n = 2m$ with $m \in \mathbb{Z}$.
- (b) Let $n \in \mathbb{Z}$. Prove that n is an odd integer if and only if $n = 2m + 1$ with $m \in \mathbb{Z}$.
- (c) Prove that the sum and product of two even integers are even.
- (d) Prove that the sum of two odd integers is even and that their product is odd.
- (e) Prove that the sum of an even integer and an odd integer is odd and that their product is even.
- (f) Prove that the sum of an even integer and an odd integer is odd and their product is even.

Rubric:

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Proof



Homework Problem 4 (Ernst Problem 2.19). Let A represent “6 is an even integer” and B represent “4 divides 6.” Express each of the following compound propositions in an ordinary English sentence and then determine its truth value.

- a. $A \wedge B$
- b. $A \vee B$
- c. $\neg A$

- d. $\neg B$
- e. $\neg(A \wedge B)$
- f. $\neg(A \vee B)$
- g. $A \Rightarrow B$

Rubric:

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- 2 points Needs revisions**
- 3 points Demonstrates understanding**
- 4 points Exemplary**

Solution:

Homework Problem 5 (Ernst Problem 2.37). Let A and B represent the statements from Problem 2.19. Express each of the following in an ordinary English sentence.

- (a) The converse of $A \Rightarrow B$
- (b) The contrapositive of $A \Rightarrow B$

Rubric:

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- 4 points Exemplary**

Solution:

Homework Problem 6. For each of the following equation, find what real numbers x make the statement true. Prove your statement.

(a) $\lfloor x \rfloor + \lfloor x \rfloor = \lfloor 2x \rfloor$

(b) $\lfloor x + 3 \rfloor = 3 + \lfloor x \rfloor$

(c) $\lfloor x + 3 \rfloor = 3 + x$

Rubric:

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Solution: *Proof*

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Proof

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Proof

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