Your Name: ____________________ Group Members: ____________________ ____________________

Use the proofs of the following propositions as a guide.

Proposition 1. Let $a, b \in \mathbb{Z}$. If $a \mid b$ and $b \mid c$, then $a \mid c$.

Proof Since $a \mid b$ and $b \mid c$, there exist $d, e \in \mathbb{Z}$ such that $b = ae$ and $c = bf$. Combining these, we see

$$c = bf = (ae)f = a(ef),$$

so $a \mid c$. ∎

Proposition 2. Let $a, b, c, m, n \in \mathbb{Z}$. If $c \mid a$ and $c \mid b$ then $c \mid ma + nb$.

Proof Let $a, b, c, m, n \in \mathbb{Z}$ such that $c \mid a$ and $c \mid b$. Then by definition of divisibility, there exists $j, k \in \mathbb{Z}$ such that $cj = a$ and $ck = b$. Thus,

$$ma + nb = m(cj) + n(ck) = c(mj + nk).$$

Therefore, $c \mid ma + nb$ by definition. ∎

In-class Problem 1 Prove or disprove the following statements.

(a) If a, b, c, and d are integers such that if $a \mid b$ and $c \mid d$, then $a + c \mid b + d$.

(b) If a, b, c, and d are integers such that if $a \mid b$ and $c \mid d$, then $ac \mid bd$.

(c) If a, b, and c are integers such that if $a \nmid b$ and $b \nmid c$, then $a \nmid c$.

In-class Problem 2 Construct a truth table for $A \to B, \neg(A \to B)$ and $A \wedge \neg B$

Solution:

A	B	$A \Rightarrow B$	$\neg(A \Rightarrow B)$	$A \wedge \neg B$
T	T	T	F	F
T	F	F	T	T
F	T	T	F	F
F	F	T	F	F

In-class Problem 3 Prove that our two definitions of even are equivalent using the following outline:

Proposition 3. Let $n \in \mathbb{Z}$. Then there is some $k \in \mathbb{Z}$ such that $n = 2k$ if and only if $2 \mid n$.

Proof ($\Rightarrow$) Let $n \in \mathbb{Z}$. Assume that there is some $k \in \mathbb{Z}$ such that $n = 2k$. Thus, $2 \mid n$

Free Response: by definition of divides.

($\Leftarrow$) Let $n \in \mathbb{Z}$. Assume that $2 \mid n$. Then, there is some $k \in \mathbb{Z}$ such that $n = 2k$

Free Response: by definition of divides.

∎

In-class Problem 4 Prove that our two definitions of odd are equivalent using the following outline:

Proposition 4. Let $n \in \mathbb{Z}$. Then there is some $k \in \mathbb{Z}$ such that $n = 2k + 1$ if and only if $2 \nmid k$.

Proof ($\Rightarrow$) Let $n \in \mathbb{Z}$. Assume that there is some $k \in \mathbb{Z}$ such that $n = 2k + 1$. Then

Free Response: by the division algorithm, there exists unique $q, r \in \mathbb{Z}$ such that $n = 2q + r$ and $0 \leq r < 2$.

Thus, $2 \nmid k$.

($\Leftarrow$) Let $n \in \mathbb{Z}$. Assume that $2 \nmid k$. Then

Free Response: by the division algorithm, there exists unique $q, r \in \mathbb{Z}$ such that $n = 2q + r$ and $0 < r < 2$. Thus, $r = 1.1$

Learning outcomes:
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Thus, there is some $k \in \mathbb{Z}$ such that $n = 2k + 1$. $\blacksquare$