MAT-255– Number Theory   Spring 2024   In Class Work January 22

Your Name: _____   Group Members:_____   _____

br Use the division algorithm on $a = 47, b = 6$ and $a = 281, b = 13$.

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br Let a and b be nonzero integers. Prove that there exists a unique $q, r \in \mathbb{Z}$ such that

$$a = bq + r, \quad 0 \le r < |b|.$$

(a) Use the division algorithm to prove this statement as a corollary. That is, use the *conclusion* of the division algorithm as part of the proof. Use the following outline:

   (i) Let a and b be nonzero integers. Since $|b| > 0$, the division algorithm says that there exist unique $p, s \in \mathbb{Z}$ such that _____ and _____.

   (ii) There are two cases:

      i. When _____, the conditions are already met, and $r =$ _____ and $q =$ _____.

      ii. Otherwise, _____, $r =$ _____ and $q =$ _____.

   (iii) Since both cases used that the p, s are unique, then q, r are also unique

(b) Use the *proof* of the division algorithm as a template to prove this statement. That is, repeat the steps, adjusting as necessary, but do not use the conclusion.

   (i) In the proof of the division algorithm, we let $q = \left\lfloor \dfrac{a}{b} \right\rfloor$. Here we have two cases:

      i. When _____, $q =$ _____ and $r =$ _____.

         as in the proof of the division algorithm.

      ii. When _____, $q =$ _____ and $r =$ _____.

   (ii) Summarizing these statements, rewrite q, r in terms of a and b, as in the original proof of the division algorithm.

   (iii) Now use your scratch work and follow the outline of the proof of the division algorithm to provide a new proof *without referencing the division algorithm.*

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