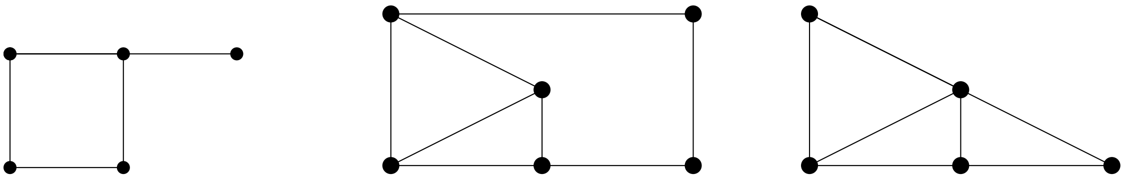

(a) A graph with 5 vertices, 5 edges, and 2 faces. (b) A graph with 6 vertices, 8 edges, and 4 faces. (c) A graph with 5 vertices, 7 edges, and 4 faces.

Figure 1: Examples of connected planar graphs

Exploration 1 *A mathematical* graph *is a set of vertices connected by edges. Each edge connects two (not necessarily distinct) vertices. A graph is* planar *if it can be drawn without any of the edges crossing. If it is possible to get from any vertex to any other vertex by traversing edges, the graph is* connected*. Some examples of connected planar graphs are in* **??**.

A connected planar graph divides the plane into disjoint enclosed areas called faces*. Note that the entire region outside of the graph is a face.*

Theorem 1 (Euler's Formula)**.** Call the number of vertices in a graph V, the number of edges E, and the number of faces F. For a connected planar graph,

$$V - E + F = 2.$$

The following problems are from Number Theory: A Lively Introduction with Proofs, Applications, and Stories *by Erica Flapan, Tim Marks, and James Pommersheim.*

Problem 1.1 (a) *The simplest graph consists of a single vertex with no edges. Verify that* **??** *is true for this graph.*

(b) *We can construct any planar graph by starting with a single vertex and repeatedly doing one of two moves:*

Move 1: *Starting at any vertex, draw a new edge that does not cross any existing edge, and then place a vertex at the end of your new edge.*

Move 2: *Connect any two existing vertices with a new edge that does not cross any existing edge.*

Show that in doing either move, the value of $V - E + F$ does not change.

(c) *Why does these facts lead to a proof that* **??** *holds true for all connected planar graphs?*

Exploration 2 *A polyhedron is a closed, three-dimensional shape with flat polygonal faces and straight edges.*

Problem 2.1 *Explain why we can draw the vertices, edges, and faces of polyhedra as a planar graph, so* **??** *is also true for polyhedra. See* **??** *for an example.*

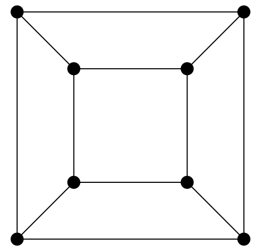

Figure 2: A cube drawn as a planar graph.

A regular polygon *is a convex polygon where every side is the same length and every angle is the same size. A* platonic solid *is a convex polyhedron where every is the same regular polygon, and the same number of faces meet at each vertex.*

Theorem 2. There are only five platonic solids.

Problem 2.2 *Let n be the number of edges for each face and m be the number of faces that meet at a vertex.*

(a) *Explain why $E = \frac{1}{2}(Fn)$ and $V = \frac{Fn}{m}$.*

(b) *Use **??** to finish the proof of **??**.*

Problem 2.3 *(If presenting as a pair) Prove that if every face of a polyhedron is a pentagon or a hexagon and if three faces meet at every vertex, then the polyhedron has exactly twelve pentagonal faces.*

Problem 2.4 *(If presenting as a pair) Prove that if every face of a polyhedron is a triangle and five or six faces meet at every vertex, then the polyhedron has exactly twelve vertices where five faces meet.*