Homework #3 Rubrics

Proofs and writing

Strayer Exercise Set 1.3, Exercises 36, 37, 38, 39, 40, 51. Exercises 38, 39, and 40 will be graded as one problem. The generalized version of Lemmas from class on Friday are exercises immediately before Exercise 51. Strayer Exercise Set 1.5, Exercise 83, also on Paper 1. Strayer Exercise Set 6.1, Exercise 8.

Homework Problem 1 (Strayer Chapter 1, Exercise 36). (a) Do there exist integers x and y such that $x + y = 100$ and $(x, y) = 8$?

(b) Prove that there exist infinitely many pairs of integers x and y such that $x + y = 87$ and $(x, y) = 3$.

Rubric:

0 points Work does not contain enough of the relevant concepts to provide feedback.

1 points **Does not demonstrate understanding** Contains a reasonable attempt to prove each part, but does not meet the criteria for two points.

2 points **Needs revisions**

3 points **Demonstrates understanding**

4 points **Exemplary**

Proof (a)

(b)

■

Homework Problem 2 (Strayer Chapter 1, Exercise 37). (Transcribe problem statement)

Rubric:

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2 points **Needs revisions**

3 points **Demonstrates understanding**

4 points **Exemplary**

Proof ■

Homework Problem 3 (Strayer Chapter 1, Exercises 38-40). **Exercise 38** Let a and b be relatively prime integers. Prove that $(a+b, a-b)$ is either 1 or 2.

Proof ■

Exercise 39 Let a and b be relatively prime integers. Find all values of $(a+2b, 2a+b)$

Solution:

Exercise 40 Let $a, b \in \mathbb{Z}$ with $(a, 4) = 2$ and $(b, 4) = 2$. Find $(a+b, 4)$ and prove that your answer is correct.

Solution:

Rubric:

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Homework Problem 4. Let $a_1, \ldots, a_n \in \mathbb{Z}$ with $a_1 \neq 0$ and let $d = (a_1, \ldots, a_n)$. Show that $c \in \mathbb{Z}$ is a common divisor of $a_1, \ldots, a_n$ if and only if $c \mid d$.

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4 points **Exemplary**

Homework Problem 5 (Strayer Chapter 1, Exercise 51)**.** Let $a_1, \ldots, a_n \in \mathbb{Z}$ with $a_0 \neq 0$. Prove that

$$(a_1, \ldots, a_n) = ((a_1, a_2), a_3, \ldots, a_n).$$

Use this method to compute the greatest common divisor of each set of integers in Exercise 34.

Rubric:

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4 points **Exemplary**

Proof ■

Solution: (Exercise 34)

(a) $(18, 36, 63)$

(b) $(30, 42, 70)$

(c) $(0, 51, 0)$

(d) $(35, 55, 77)$

(e) $(36, 42, 54, 78)$

(f) $(35, 63, 70, 98)$

Homework Problem 6 (Strayer Chapter 1, Exercise 83)**.** Let $a, b \in \mathbb{Z}$ with $a, b > 0$ and $(a, b) = 1)$. Prove that the arithmetic progression

$$a, a + b, a + 2b, \ldots, a + nb, \ldots$$

contains infinitely many composite numbers.

Rubric:

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2 points **Needs revisions**

3 points **Demonstrates understanding**

4 points **Exemplary**

Proof ■

Homework Problem 7 (Strayer Chapter 6, Exercise 8)**.**

Rubric:

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1 points **Does not demonstrate understanding** Contains a reasonable attempt to prove each part, but does not meet the criteria for two points.

2 points **Needs revisions**

3 points **Demonstrates understanding**

4 points **Exemplary**

Proof ■