Your Name: ____________ Group Members: ____________ ____________

Problem 1 *Prove that a positive integer can be written as the difference of two squares of integers if and only if it is not of the form $4n+2$ for some $n \in \mathbb{Z}$.*

Proof ($\Rightarrow$) *We will show that if a positive integer can be written as the difference of two squares of integers, then it is not of the form $4n+2$ for some $n \in \mathbb{Z}$.*

($\Leftarrow$) *We will show that any positive integer not of the form $4n+2$ for some $n \in \mathbb{Z}$ can be written as the difference of two squares of integers.*

First, we will show that if a and b are positive integers that can be written as the difference of two squares of integers, then so can ab.

Now we will show that every odd prime can be written as the difference of two squares of integers. Let p be an odd prime. Then $p = x^2 - y^2 = (x-y)(x+y)$ when $x =$ ____________ and $y =$ ____________. Therefore every odd number can be written as the difference of two squares since

It remains to show that every positive integer of the form $4n$ for some $n \in \mathbb{Z}$ can be written as the difference of two squares of integers. Why is this the only remaining case?

Similar to the odd prime case, $4n = x^2 - y^2 = (x - y)(x + y)$ when $x =$ __________ and $y =$ __________.

$\blacksquare$