Homework #1 Rubrics

Proofs and writing

Strayer Exercise Set 1.1, Exercises 5, 10, 11. Ernst Problem 2.19, Problem 2.37, then either prove or provide a counterexample for the statements. Additional problem provided below.

Homework Problem 1 (Strayer Exercise 5)**.** Prove or disprove the following statements.

(a) If a, b, c, and d are integers such that if $a \mid b$ and $c \mid d$, then $a + c \mid b + d$.

(b) If a, b, c, and d are integers such that if $a \mid b$ and $c \mid d$, then $ac \mid bd$.

(c) If a, b, and c are integers such that if $a \nmid b$ and $b \nmid c$, then $a \nmid c$.

Rubric:

0 points Work does not contain enough of the relevant concepts to provide feedback.

1 points **Does not demonstrate understanding** Contains a reasonable attempt to prove each part, but does not meet the criteria for two points.

2 points **Needs revisions**

3 points **Demonstrates understanding**

4 points **Exemplary**

Solution: (a)

(b)

(c)

Homework Problem 2 (Strayer Exercise 10)**.** (a) Let $n \in \mathbb{Z}$. Prove that $3 \mid n^3 - n$.

(b) Let $n \in \mathbb{Z}$. Prove that $5 \mid n^5 - n$.

(c) Let $n \in \mathbb{Z}$. Is it true that $4 \mid n^4 - n$? Provide a proof or counter example.

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4 points **Exemplary**

Proof ■

Homework Problem 3 (Strayer Exercise 11)**.** Use the definition of even and odd from Strayer **not** Ernst.

(a) Let $n \in \mathbb{Z}$. Prove that n is an even integer if and only if $n = 2m$ with $m \in \mathbb{Z}$.

(b) Let $n \in \mathbb{Z}$. Prove that n is an odd integer if and only if $n = 2m + 1$ with $m \in \mathbb{Z}$.

(c) Prove that the sum and product of two even integers are even.

(d) Prove that the sum of two odd integers is even and that their product is odd.

(e) Prove that the sum of an even integer and an odd integer is odd and that their product is even.

(f) Prove that the sum of an even integer and an odd integer is odd and their product is even.

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4 points **Exemplary**

Proof ■

Homework Problem 4 (Ernst Problem 2.19)**.** Let A represent "6 is an even integer" and B represent "4 divides 6." Express each of the following compound propositions in an ordinary English sentence and then determine its truth value.

a. $A \wedge B$

b. $A \vee B$

c. $\neg A$

d. $\neg B$

e. $\neg(A \wedge B)$

f. $\neg(A \vee B)$

g. $A \Rightarrow B$

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4 points **Exemplary**

Solution:

Homework Problem 5 (Ernst Problem 2.37)**.** Let A and B represent the statements from Problem 2.19. Express each of the following in an ordinary English sentence.

(a) The converse of $A \Rightarrow B$

(b) The contrapositive of $A \Rightarrow B$

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3 points **Demonstrates understanding**

4 points **Exemplary**

Solution:

Homework Problem 6. For each of the following equation, find what real numbers x make the statement true. Prove your statement.

(a) $\lfloor x \rfloor + \lfloor x \rfloor = \lfloor 2x \rfloor$

(b) $\lfloor x+3 \rfloor = 3 + \lfloor x \rfloor$

(c) $\lfloor x+3 \rfloor = 3 + x$

Rubric:

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2 points **Needs revisions**

3 points **Demonstrates understanding**

4 points **Exemplary**

Solution: (a) If then $\lfloor x \rfloor + \lfloor x \rfloor = \lfloor 2x \rfloor$.

Proof ■

(b) If then $\lfloor x+3 \rfloor = 3 + \lfloor x \rfloor$

Proof ■

(c) If then $\lfloor x+3 \rfloor = 3 + x$

Proof ■