Mathematical definitions and notation

- Formally define even and odd
- Complete basic algebraic proofs

Definition 1. *We will use the following number systems and abbreviations:*

- *The* integers, *written* $\mathbb{Z}$, *is the set* $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$.
- *The* natural numbers, *written* $\mathbb{N}$. *Most elementary number theory texts either define* $\mathbb{N}$ *to be the positive integers or avoid using* $\mathbb{N}$. *Some mathematicians include* 0 *in* $\mathbb{N}$.
- *The* real numbers, *written* $\mathbb{R}$.
- *The* integers modulo n, *written* $\mathbb{Z}_n$. *We will define this set in Strayer Chapter 2, although Strayer does not use this notation.*

We will also use the following notation:

- *The symbol* $\in$ *means "element of" or "in." For example,* $x \in \mathbb{Z}$ *means "x is an element of the integers" or "x in the integers."*

This first section will cover basic even, odd, and divisibility results. These first few definitions and results will use algebraic proofs, before we cover formal proof methods.

Definition 2 (Even and odd, multiplication definition)**.** *An integer n is* even *if $n = 2k$ for some $k \in \mathbb{Z}$. That is, n is a* multiple *of* 2.

An integer n is odd if $n = 2k + 1$ for some $k \in \mathbb{Z}$.

Now, the preceding definition is standard in an introduction to proofs course, but it is not the only definition of even/odd. We also have the following definition that is closer to the definition you are probably used to:

Definition 3 (Even and odd, division definition)**.** *Let $n \in \mathbb{Z}$. Then n is said to be* even *if 2 divides n and n is said to be* odd *if 2 does not divide n.*

Note that we need to define *divides* in order to use the second definition. We will formally prove that these definitions are *equivalent,* but for now, let's use the first definition.

Theorem 1. *If n is an even integer, then n^2 is even.*

Problem 1 *Prove this theorem.*

Proof *If n is an even integer, then by definition, there is some $k \in \mathbb{Z}$ such that $n = 2k$. Then*

$$n^2 = (2k)^2 = 2(2k^2).$$

Since $2(k^2)$ is an integer, we have written n^2 in the desired form. Thus, n^2 is even. ■

Proposition 1. *The sum of two consecutive integers in odd.*

For this problem, we need to figure out how to write two consecutive integers.

Learning outcomes:

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Proof Let $n, n+1$ be two consecutive integers. Then their sum is $n + n + 1 = 2n + 1$, which is odd by Even and odd, multiplication definition. ■