

## Proofs and writing

Strayer Exercise Set 1.1, Exercises 5, 10, 11. Ernst Problem 2.19, Problem 2.37, then either prove or provide a counterexample for the statements. Additional problem provided below.

**Homework Problem 1** (Strayer Exercise 5). Prove or disprove the following statements.

- (a) If  $a, b, c$ , and  $d$  are integers such that if  $a \mid b$  and  $c \mid d$ , then  $a + c \mid b + d$ .
- (b) If  $a, b, c$ , and  $d$  are integers such that if  $a \mid b$  and  $c \mid d$ , then  $ac \mid bd$ .
- (c) If  $a, b$ , and  $c$  are integers such that if  $a \nmid b$  and  $b \nmid c$ , then  $a \nmid c$ .

### Rubric:

**0 points** Work does not contain enough of the relevant concepts to provide feedback.

**1 points Does not demonstrate understanding** Contains a reasonable attempt to prove each part, but does not meet the criteria for two points.

**2 points Needs revisions**

**3 points Demonstrates understanding**

**4 points Exemplary**

**Solution:** (a)

(b)

(c)

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**Homework Problem 2** (Strayer Exercise 10). (a) Let  $n \in \mathbb{Z}$ . Prove that  $3 \mid n^3 - n$ .

(b) Let  $n \in \mathbb{Z}$ . Prove that  $5 \mid n^5 - n$ .

(c) Let  $n \in \mathbb{Z}$ . Is it true that  $4 \mid n^4 - n$ ? Provide a proof or counter example.

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**4 points** Exemplary

*Proof*



**Homework Problem 3** (Strayer Exercise 11). Use the definition of even and odd from Strayer **not** Ernst.

- (a) Let  $n \in \mathbb{Z}$ . Prove that  $n$  is an even integer if and only if  $n = 2m$  with  $m \in \mathbb{Z}$ .
- (b) Let  $n \in \mathbb{Z}$ . Prove that  $n$  is an odd integer if and only if  $n = 2m + 1$  with  $m \in \mathbb{Z}$ .
- (c) Prove that the sum and product of two even integers are even.
- (d) Prove that the sum of two odd integers is even and that their product is odd.
- (e) Prove that the sum of an even integer and an odd integer is odd and that their product is even.
- (f) Prove that the sum of an even integer and an odd integer is odd and their product is even.

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**4 points** Exemplary

*Proof*



**Homework Problem 4** (Ernst Problem 2.19). Let  $A$  represent “6 is an even integer” and  $B$  represent “4 divides 6.” Express each of the following compound propositions in an ordinary English sentence and then determine its truth value.

- a.  $A \wedge B$
- b.  $A \vee B$
- c.  $\neg A$

- d.  $\neg B$
- e.  $\neg(A \wedge B)$
- f.  $\neg(A \vee B)$
- g.  $A \Rightarrow B$

**Rubric:**

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- 3 points Demonstrates understanding**
- 4 points Exemplary**

**Solution:**

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**Homework Problem 5** (Ernst Problem 2.37). Let  $A$  and  $B$  represent the statements from Problem 2.19. Express each of the following in an ordinary English sentence.

- (a) The converse of  $A \Rightarrow B$
- (b) The contrapositive of  $A \Rightarrow B$

**Rubric:**

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- 4 points Exemplary**

**Solution:**

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**Homework Problem 6.** For each of the following equation, find what real numbers  $x$  make the statement true. Prove your statement.

(a)  $\lfloor x \rfloor + \lfloor x \rfloor = \lfloor 2x \rfloor$

(b)  $\lfloor x + 3 \rfloor = 3 + \lfloor x \rfloor$

(c)  $\lfloor x + 3 \rfloor = 3 + x$

**Rubric:**

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**2 points** Needs revisions

**3 points** Demonstrates understanding

**4 points** Exemplary

**Solution:**    *Proof*

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*Proof*

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*Proof*

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