Your Name: ____________________ Group Members: ____________________ ____________________

In-class Problem 1 (Chapter 1, Exercise 29) Let n be a positive integer with $n \neq 1$. Prove that if n^2+1 is prime, then n^2+1 can be written in the form $4k+1$ with $k \in \mathbb{Z}$.

Hint: *Try showing the statement is true for all odd integers greater than* 1.

Solution: Assume that n is a positive integer, $n \neq 1$, and n^2+1 is prime. If n is odd, then n^2 is odd, which would imply $n^2+1=2$, the only even prime. However, $n \neq 1$ by assumption. Thus, n is even.

By definition of even, there exists $j \in \mathbb{Z}$ such that $n = 2k$ and $n^2 = 4j^2$. Thus, $n^2+1 = 4k+1$ when $k = j^2$.

In-class Problem 2 (Chapter 1, Exercise 33) Prove or disprove the following conjecture, which is similar to the Twin Prime Conjecture:

Conjecture 1. There are infinitely many prime number p for which $p+2$ and $p+4$ are also prime numbers.

Hint: *Show that the only prime where $p+2$ and $p+4$ are also prime is $p=3$.*

In-class Problem 3 Without looking up the proof, prove Proposition 1.10: Let $a, b \in \mathbb{Z}$ with $(a,b) = d$. Then $\left(\frac{a}{d}, \frac{b}{d}\right) = 1$.

Learning outcomes:
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