Your Name: ______________ Group Members: ______________ ______________

In-class Problem 1 Repeat the proof from last class to prove

Theorem 1 (Theorem 3.2). Let m and n be positive integers where $(m,n)=1$. Then $\phi(mn)=\phi(m)\phi(n)$.

Proof Let m and m be relatively prime positive integers. A number a is relatively prime to mn if and only if a is relatively prime to ______ and ______.

We can partition the positive integers less that or equal to mn into

$$\begin{aligned}
1 &\equiv ______ \equiv ______ \equiv \cdots \equiv ______ \pmod{m}\\
2 &\equiv ______ \equiv ______ \equiv \cdots \equiv ______ \pmod{m}\\
&\vdots\\
m &\equiv ______ \equiv ______ \equiv \cdots \equiv ______ \pmod{m}
\end{aligned}$$

For any b in the range $1,2,3,\ldots,m$, define s_b to be the number of integers a in the range $1,2,\ldots,mn$ such that $a\equiv b \pmod{m}$ and $\gcd(a,mn)=1$. Thus, when $(b,m)=1$, $s_b=\phi($______$)$ and when $(b,m)>1$, $s_b=$ ______.

We have seen that $\phi(mn)=s_1+s_2+\cdots+s_m$, that when $(b,m)=1$, $s_b=$ ______, and that when $(b,m)>1$, $s_b=$ ______. Since there are $\phi($______$)$ integers b where $(b,m)=1$. Thus, we can say that $\phi(mn)=$ __________. ■

In-class Problem 2 Complete the proof of Theorem 3.2 by proving

Proposition 1. If m,n, and i are positive integers with $(m,n)=(m,i)=1$, then the integers

$$i, m+i, 2m+i, \ldots, (n-1)m+i$$

form a complete system of residues modulo n.

Learning outcomes:

Author(s): Claire Merriman

This blank is asking for a function, not a value.