

# Fibonacci Sequence

*Project on Fibonacci and Lucas sequence.*

**Exploration 1** Start by reviewing the Fibonacci Sequence from Strayer Appendix A.2:  $F_1 = 1, F_2 = 1, F_{k+1} = F_k + F_{k-1}$  for  $k \geq 3$ .

**Problem 1.1** Prove that  $\lim_{k \rightarrow \infty} \frac{F_{k+1}}{F_k} = \phi$  where  $\phi = \frac{1 + \sqrt{5}}{2}$ .

**Rubric.** 5 points if individual project, 3 points if presenting as a pair.

**Problem 1.2** Prove that for every positive integer  $k$ ,

$$F_1 + F_2 + \cdots + F_k = F_{k+2} - 1.$$

**Rubric.** 5 points if individual project, 3 points if presenting as a pair.

**Problem 1.3** (If presenting as a pair) Strayer Exercise Set A, Exercise 3.

**Rubric.** 6 points.

The following problems are from *Number Theory: A Lively Introduction with Proofs, Applications, and Stories* by Erica Flapan, Tim Marks, and James Pommersheim.

**Exploration 2** The following problems are from *Number Theory: A Lively Introduction with Proofs, Applications, and Stories* by Erica Flapan, Tim Marks, and James Pommersheim.

The Lucas numbers are similar to the Fibonacci numbers, where  $L_1 = 1, L_2 = 3, L_{k+1} = L_k + L_{k-1}$  for  $k \geq 3$ .

**Problem 2.1** (a) Make a table of the first 12 Lucas numbers. You do not need to present this part

(b) Use your results from part (a) to calculate the ratios of pairs of consecutive Lucas numbers. You do not need to present this part

(c) Make a conjecture about the value of  $\lim_{k \rightarrow \infty} \frac{L_{k+1}}{L_k}$ . You do not need to present this part

(d) Prove your conjecture is correct. You **do** need to present this part

Learning outcomes:  
Author(s): Claire Merriman

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**Problem 2.2** (*If presenting as a pair*)

- (a) Calculate  $L_1, L_1 + L_2, L_1 + L_2 + L_3, L_1 + L_2 + L_3 + L_4$ . You do not need to present this part
- (b) Make a conjecture about the relationship between  $L_1 + L_2 + L_3 + \cdots + L_n$  and the number  $L_{n+2}$ . You do not need to present this part
- (c) Prove your conjecture is correct. You **do** need to present this part

**Rubric.** 5 points if individual project, 3 points if presenting as a pair.

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