

Your Name: _____ Group Members: _____

Use the proofs of the following propositions as a guide.

Proposition 1. Let $a, b \in \mathbb{Z}$. If $a \mid b$ and $b \mid c$, then $a \mid c$.

Proof Since $a \mid b$ and $b \mid c$, there exist $d, e \in \mathbb{Z}$ such that $b = ae$ and $c = bf$. Combining these, we see

$$c = bf = (ae)f = a(e f),$$

so $a \mid c$. ■

Proposition 2. Let $a, b, c, m, n \in \mathbb{Z}$. If $c \mid a$ and $c \mid b$ then $c \mid ma + nb$.

Proof Let $a, b, c, m, n \in \mathbb{Z}$ such that $c \mid a$ and $c \mid b$. Then by definition of divisibility, there exists $j, k \in \mathbb{Z}$ such that $cj = a$ and $ck = b$. Thus,

$$ma + nb = m(cj) + n(ck) = c(mj + nk).$$

Therefore, $c \mid ma + nb$ by definition. ■

Problem 1 Prove or disprove the following statements.

- (a) If a, b, c , and d are integers such that if $a \mid b$ and $c \mid d$, then $a + c \mid b + d$.
- (b) If a, b, c , and d are integers such that if $a \mid b$ and $c \mid d$, then $ac \mid bd$.
- (c) If a, b , and c are integers such that if $a \nmid b$ and $b \nmid c$, then $a \nmid c$.

Problem 2 Construct a truth table for $A \rightarrow B$, $\neg(A \rightarrow B)$ and $A \wedge \neg B$

Pause for more lecture. If there is time, complete the following problem.

Problem 3 Prove that our two definitions of even are equivalent using the following outline:

Proposition 3. Let $n \in \mathbb{Z}$. Then there is some $k \in \mathbb{Z}$ such that $n = 2k$ if and only if $2 \mid n$.

Proof (\Rightarrow) Let $n \in \mathbb{Z}$. Assume that there is some $k \in \mathbb{Z}$ such that $n = 2k$. Thus, $2 \mid n$ _____

(\Leftarrow) Let $n \in \mathbb{Z}$. Assume that $2 \mid n$. Then, there is some $k \in \mathbb{Z}$ such that $n = 2k$ _____. ■

Problem 4 Prove that our two definitions of odd are equivalent using the following outline:

Proposition 4. Let $n \in \mathbb{Z}$. Then there is some $k \in \mathbb{Z}$ such that $n = 2k + 1$ if and only if $2 \nmid k$.

Proof (\Rightarrow) Let $n \in \mathbb{Z}$. Assume that there is some $k \in \mathbb{Z}$ such that $n = 2k + 1$. Then _____ Thus, $2 \nmid k$.

(\Leftarrow) Let $n \in \mathbb{Z}$. Assume that $2 \nmid k$. Then _____ Thus, there is some $k \in \mathbb{Z}$ such that $n = 2k + 1$. ■