Your Name: ________________ Group Members: ________________ ________________

In-class Problem 1 Prove that

$$[a] = \{b \in \mathbb{Z} : 3 \mid (a-b)\}$$

is an equivalence relation on $\mathbb{Z}$.

Solution: Let $a, b \in \mathbb{Z}$. We must show that the relation is reflexive, symmetric, and transitive.

To show the relation is reflexive, we must show $a \in \{b \in \mathbb{Z} : 3 \mid (a-b)\}$. Since $3 \mid a - a = 0$, $a \in \{b \in \mathbb{Z} : 3 \mid (a-b)\}$.

To show the relation is symmetric, we must show that if $x \in \{b \in \mathbb{Z} : 3 \mid (a-b)\}$, then $a \in \{b \in \mathbb{Z} : 3 \mid (x-b)\}$. If $x \in \{b \in \mathbb{Z} : 3 \mid (a-b)\}$, then there exists $k \in \mathbb{Z}$ such that $3k = a - x$. Therefore, $-3k = b - a$ and $a \in \{b \in \mathbb{Z} : 3 \mid (x-b)\}$.

To show the relation is transitive, we must show that if $x \in \{b \in \mathbb{Z} : 3 \mid (a-b)\}$ and $y \in \{b \in \mathbb{Z} : 3 \mid (x-b)\}$, then $x \in \{b \in \mathbb{Z} : 3 \mid (a-b)\}$. If $x \in \{b \in \mathbb{Z} : 3 \mid (a-b)\}$, then there exists $k \in \mathbb{Z}$ such that $3k = a - x$. Similarly, if $y \in \{b \in \mathbb{Z} : 3 \mid (x-b)\}$, then there exists $m \in \mathbb{Z}$ such that $3m = x - y$. Therefore, $3(m+k) = a - k$ and $y \in \{b \in \mathbb{Z} : 3 \mid (a-b)\}$.

Since the relation is reflexive, symmetric, and transitive, it is an equivalence relation.

Learning outcomes:

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