Your Name: ________________ Group Members: ________________ ________________

Use the first principle of mathematical induction to prove each statement.

Problem 1 *For all $n \in \mathbb{N}$, 3 divides $4^n - 1$.*

Proof *We proceed by induction. The base case is $n = 1$. Since ________, we are done.*

The induction hypothesis is that if $k \geq 1$ and $n = k$, then ________. We want to show that ________.

Complete the proof:

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Problem 2 *Let $p_1, p_2, \ldots, p_n$ be n distinct points arranged on a circle. Then the number of line segments joining all pairs of points is $\frac{n^2 - n}{2}$.*

Proof *We proceed by induction. The base case is $n = 1$. Since ________________ we are done.*

The induction hypothesis is that if $k \geq 1$ and $n = k$, then

We want to show that

Complete the proof:

■

Problem 3 *If n is a positive integer, then*

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}.$$

Proof *We proceed by induction. The base case is $n = 1$. Since ____________, we are done.*

The induction hypothesis is that if $k \geq 1$ and $n = k$, then

We want to show that

Complete the proof:

$\blacksquare$

Problem 4 *If n is an integer with $n \geq 5$, then*

$$2^n > n^2.$$

Proof *We proceed by induction. The base case is $n = 5$. Since ____________, we are done.*

The induction hypothesis is that if $k \geq 5$ and $n = k$, then ____________. We want to show that ____________.

Complete the proof:

$\blacksquare$

Recall the notation $\gcd(a, b) = (a, b)$.

Problem 5 *Let $a_1, a_2, \ldots, a_n \in \mathbb{Z}$ with $a_1 \neq 0$. Prove that*

$$(a_1, \ldots, a_n) = ((a_1, a_2, a_3, \ldots, a_{n-1}), a_n).$$

Proof *We proceed by induction. The base case is $n = 2$, since the statement we are trying to prove requires at least two inputs. Since*

we are done.

The induction hypothesis is that if $k \geq 2$ and $n = k$, then

We want to prove that

Complete the proof:

■

Problem 6 *Redo the following proofs using induction:*

Problem 6.1 *Let $n \in \mathbb{Z}$. Prove that $3 \mid n^3 - n$.*

Proof *We proceed by induction. The base case is $n = 1$. Since ____________, we are done. The induction hypothesis is that if $k \geq 1$ and $n = k$, then ____________. We want to show that ____________.*

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Problem 6.2 *Let $n \in \mathbb{Z}$. Prove that $5 \mid n^5 - n$.*

Proof *We proceed by induction. The base case is $n = 1$. Since ____________, we are done. The induction hypothesis is that if $k \geq 1$ and $n = k$, then ____________. We want to show that ____________.*

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