

Proofs and writing

Strayer Exercise Set 1.1, Exercises 7, 9, 15. Strayer Exercise Set 1.2, Exercises 23, 25.

Homework Problem 1 (Strayer Exercise 7). Let $a, b \in \mathbb{Z}$ with $a \mid b$. Prove that $a^n \mid b^n$ for every positive integer n .

Rubric:

0 points Work does not contain enough of the relevant concepts to provide feedback.

1 points Does not demonstrate understanding Contains a reasonable attempt to prove each part, but does not meet the criteria for two points.

2 points Needs revisions

3 points Demonstrates understanding

4 points Exemplary

Homework Problem 2 (Strayer Exercise 9). Let a, m and n be positive integers with $a > 1$. Prove that $a^m - 1 \mid a^n - 1$ if and only if $m \mid n$. (*Hint:* For the “if” direction, write $n = md$ with d a positive integer and use the factorization $a^{md} - 1 = (a^m - 1)(a^{m(d-1)} + a^{m(d-2)} + \cdots + a^m + 1)$.)

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Homework Problem 3 (Strayer Exercise 15). The following exercises present two alternative versions of the division algorithm. Both versions allow negative divisors; as such, they are more general than Theorem 1.4

(a) Let a and b be nonzero integers. Prove that there exists a unique $q, r \in \mathbb{Z}$ such that

$$a = bq + r, \quad 0 \leq r < |b|.$$

(b) Find the unique q and r guaranteed by the division algorithm of part (a) above with $a = 47$ and $b = -6$.

(c) Let a and b be nonzero integers. Prove that there exist unique $q, r \in \mathbb{Z}$ such that

$$a = bq + r, \quad -\frac{|b|}{2} < r \leq \frac{|b|}{2}.$$

This algorithm is called the *absolute least remainders algorithm*.

(d) Find the unique q and r guaranteed by the division algorithm of part (c) above with $a = 47$ and $b = -6$.

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Solution: (a)

(b)

(c)

(d)

Homework Problem 4 (Strayer Exercise 23). Prove or disprove the following conjecture, which is similar to Conjecture 1: **Conjecture:** There are infinitely many prime number p for which $p + 2$ and $p + 4$ are also prime numbers.

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- Homework Problem 5** (Strayer Exercise 25). (a) Prove that all odd prime numbers can be expressed as the difference of square of two successive integers.
- (b) Prove that no prime numbers can be expressed as the difference of two fourth power integers (*Hint*: Use the factorization tool discussed in the final paragraph of this section.)

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Proof (a)

(b)

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Homework Problem 6 (Ernst Problem 2.50). Consider the following statement: If $x \in \mathbb{Z}$ such that x^2 is odd, then x is odd. The items below can be assembled to form a proof of this statement, but they are currently out of order. Put them in the proper order.

- (a) Assume that x is an even integer.
- (b) We will utilize a proof by contraposition.
- (c) Thus, x^2 is twice an integer.
- (d) Since $x = 2k$, we have that $x^2 = (2k)^2 = 4k^2$.
- (e) Since k is an integer, $2k^2$ is also an integer.
- (f) By the definition of even, there is an integer k such that $x = 2k$.
- (g) We have proved the contrapositive, and hence the desired statement is true.
- (h) Assume $x \in \mathbb{Z}$.
- (i) By the definition of even integer, x^2 is an even integer.
- (j) Notice that $x^2 = 2(2k^2)$.

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Proof

