Your Name: ______________ Group Members: ______________ ______________

Problem 1 (a) *Do there exist integers x and y such that $x + y = 100$ and $(x, y) = 8$?*

(b) *Prove that there exist infinitely many pairs of integers x and y such that $x + y = 87$ and $(x, y) = 3$.*

Scratch Work. Note that $87 =$ ________. To ensure that $(x, y) = 3$, not just $3 \mid x$ and $3 \mid y$, let $x = 3n$ where ____ $\nmid n$.

Proof *Let $x \in \mathbb{Z}$ with ______________________________.*
Let $y =$ ________. Then $3 \mid y$ by ______________. Then $(x, y) = 3$ since ________. Thus, there are infinitely many $x, y \in \mathbb{Z}$

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Problem 2 *Let a and b be relatively prime integers. Prove that $(a + b, a - b)$ is either 1 or 2.*

Hint: *From the back of Strayer: Let $(a + b, a - b) = d$ and note that $d \mid (a + b) + (a - b)$ and $d \mid (a + b) - (a - b)$.*

Hint: *Use Homework 3, Problem 2 which states $(ca, cb) = |c|(a, b)$ for all $a, b \in \mathbb{Z}$, not both 0.*