Chapter 1 Linear Equations in Linear Algebra

INTRODUCTORY EXAMPLE: Linear Models in Economics and Engineering

1.1. Systems of Linear Equations
1.2. Row Reduction and Echelon Forms
1.3. Vector Equations
1.4. The Matrix Equation A**x=b**
1.5. Solution Sets of Linear Systems
1.6. Applications of Linear Systems (Lab 2 only)
1.7. Linear Independence
1.8. Introduction to Linear Transformations
1.9. The Matrix of a Linear Transformation

Standard 1. Determine the number of solutions to a system of linear equations. Give examples of systems with zero, one, and infinite solutions.

Standard 2. Solving a system of linear equations using an augment matrix in row echelon or reduced row echelon form

Standard 3. Perform algebraic operations with vectors, including addition, subtraction, scalar multiplication, and linear combinations. Represent a system of linear equations as a linear combination of vectors, and vice versa.

Standard 4. Compute the Span of a set of vectors and determine if a given vector is in the Span. Describe the Span geometrically.

Standard 5. Use the relationship between matrix equations, vector equations, and systems of equations to write or solve equations with given properties.

Standard 6. Understand and use the relationship between solutions to homogeneous and solutions to nonhomogeneous equations.

Standard 7. State and use the definition of linearly independent and dependent sets. Characterize linearly independent and dependent sets in terms of linear combinations, spans, and solutions to homogeneous equations.

Standard 8. State and use the definition of a linear transformation. Translate between the geometric and algebraic (matrix) representations of a linear transformation. Identify the domain and range of a linear transformation and determine if the transformation is one-to-one or onto.

Chapter 2 Matrix Algebra

INTRODUCTORY EXAMPLE: Computer Models in Aircraft Design

2.1. Matrix Operations
2.2. The Inverse of a Matrix
2.3. Characterizations of Invertible Matrices
2.6. The Leontief Input–Output Model (Bonus Lab Only)
2.7. Applications to Computer Graphics (Lab 4 only)
2.8. Subspaces of $\mathbf{R}^n$ (with 4.2—Standard 16)
2.9. Dimension and Rank (with 4.3, Standards 17-20)

Standard 9. Preform algebraic operations with matrices, including addition, subtraction, scalar multiplication, matrix multiplication, and transposition.

Standard 10. Use properties of matrix multiplication, inverses, and transposes to find a matrix with given information.

Standard 11. State and use several equivalent conditions for a matrix to be invertible. Compute the inverse if it exists.

Standard 12. Use facts about general matrices to solve a matrix equation.

Determinants

INTRODUCTORY EXAMPLE: Random Paths and Distortion

3.1. Introduction to Determinants
3.2. Properties of Determinants
3.3. Cramer's Rule, Volume, and Linear Transformations

Standard 13. Compute determinants and relate the determinant to a geometric description of a linear transformation.

Standard 14. Compute determinants using cofactor expansion and elementary row operations.

Chapter 4 Vector Spaces

INTRODUCTORY EXAMPLE: Space Flight and Control Systems

4.1. Vector Spaces and Subspaces
4.2. Null Spaces, Column Spaces, and Linear Transformations
4.3. Linearly Independent Sets; Bases
4.9. Applications to Markov Chains (Lab 6 only)

Standard 15. State the definition of a vector space and determine whether a set of vectors forms a vector space.

Standard 16. State the properties of a subspace of a vector space, and in particular for $\mathbb{R}^n$ and determine if a set of vectors forms a subspace of a vector space.

Standard 17. Determine whether a set of vectors forms a basis for a vector space. Find the dimension of a vector space or a subspace of a vector space.

Standard 18. Give basis and dimension for the column space and null space of a matrix.

Standard 19. Use the rank-nullity theorem to determine the dimension of the column and null spaces of a matrix as subspaces of $\mathbb{R}^n$.

Chapter 5 Eigenvalues and Eigenvectors

INTRODUCTORY EXAMPLE: Dynamical Systems and Spotted Owls

5.1. Eigenvectors and Eigenvalues
5.2. The Characteristic Equation
5.3. Diagonalization
5.4. Eigenvectors and Linear Transformations
5.6. Discrete Dynamical Systems

Standard 23. Find associated eigenvectors of a given matrix and its eigenvalues. Find the eigenspace of a matrix for a specified eigenvalue. Find and use the characteristic equation of a matrix to determine its eigenvalues (along with their multiplicities).

Standard 24. Diagonalize a given matrix and use properties of a diagonal matrix to find powers of a matrix.

Chapter 6 Orthogonality and Least Squares

INTRODUCTORY EXAMPLE: The North American Datum and GPS Navigation

6.1. Inner Product, Length, and Orthogonality
6.2. Orthogonal Sets
6.3. Orthogonal Projections
6.4. The Gram–Schmidt Process

Standard 20. Project a vector onto another vector or a span of vectors. Use facts about orthogonal vectors to solve vector equations.

Standard 21. Use the null space, column space, and row space to determine the number of solutions to a matrix equation.

Standard 22. State the definition of an orthonormal basis. Find an orthonormal basis for a subspace and give the orthogonal decomposition of a vector.

Chapter 7 Symmetric Matrices and Quadratic Forms

7.4. The Singular Value Decomposition (Lab 7 only)