

Mathematical Analysis

By: Sijie Shang and Claire

Pseudocode:

Naive Pattern:

```
Def fibonacci_naive:
    If n == 1 or 2:
        Return 0
    If n == 3:
        Return 1
    i=1:
    For i in range (4, n):
        Result = (a + b + c)
        a = b
        b = c
        c = result
        i++
    Return result;
```

$$T(n) = 3 + n(1+1+1+1+1) = 5n+3 \rightarrow O(n)$$

Recursive Pattern:

```
Def fibonacci_recursive(n):
    If n == 1 or 2:
        Return 0
    If n == 3:
        Return 1
    Else:
        Return fibonacci_recursive(n-1) + fibonacci_recursive(n-2) +
fibonacci_recursive(n-3)
```

$$O(n) = 3^n$$

Recursive Pattern with Memorization

```
Def fibonacci_memorization(n):
    If f[n] is not None:
        Return F[n]
    Else:
        result = fibonacci_recursive(n)
```

```
F[n] = result
Return result
```

If the user never calculated the nth tribonacci before, then the time complexity is $O(3^n)$. If the user calculated the nth tribonacci before, then the time complexity is $O(1)$.

Dynamic programming

```
def fibonacci_dynamic(n):
    A = Array(max(n+1, 3), None)
    A[0] = 0
    A[1] = 0
    A[2] = 1
    for i in range(3, n+1):
        A[i] = A[i-3] + A[i-2] + A[i-1]
    return A[n]
```

Time Efficiency Class:

$$T(n) = 1 + 1 + 1 + 1 + n + 1$$

$$T(n) = n + 5$$

$$T(n) = n + 5 \in O(n + 5)$$

(trivial)

$$= O(\max(n, 5))$$

(dominated terms)

$$= O(n) \checkmark$$

Output

Naive Pattern

```
The 1 th tribonacci is 0
The 2 th tribonacci is 0
The 3 th tribonacci is 1
The 4 th tribonacci is 1
The 5 th tribonacci is 2
The 6 th tribonacci is 4
The 7 th tribonacci is 7
The 8 th tribonacci is 13
The 9 th tribonacci is 24
The 10 th tribonacci is 44
The 11 th tribonacci is 81
The 12 th tribonacci is 149
```

The 13 th tribonacci is 274
The 14 th tribonacci is 504
The 15 th tribonacci is 927
The 16 th tribonacci is 1705
The 17 th tribonacci is 3136
The 18 th tribonacci is 5768
The 19 th tribonacci is 10609
The 20 th tribonacci is 19513
The 21 th tribonacci is 35890
The 22 th tribonacci is 66012
The 23 th tribonacci is 121415
The 24 th tribonacci is 223317
The 25 th tribonacci is 410744

Recursive Pattern

The 1 th tribonacci is 0
The 2 th tribonacci is 0
The 3 th tribonacci is 1
The 4 th tribonacci is 1
The 5 th tribonacci is 2
The 6 th tribonacci is 4
The 7 th tribonacci is 7
The 8 th tribonacci is 13
The 9 th tribonacci is 24
The 10 th tribonacci is 44
The 11 th tribonacci is 81
The 12 th tribonacci is 149
The 13 th tribonacci is 274
The 14 th tribonacci is 504
The 15 th tribonacci is 927
The 16 th tribonacci is 1705
The 17 th tribonacci is 3136
The 18 th tribonacci is 5768
The 19 th tribonacci is 10609
The 20 th tribonacci is 19513
The 21 th tribonacci is 35890
The 22 th tribonacci is 66012
The 23 th tribonacci is 121415
The 24 th tribonacci is 223317
The 25 th tribonacci is 410744

Recursive with Memorize Pattern

The 1 th tribonacci is 0

The 2 th tribonacci is 0
The 3 th tribonacci is 1
The 4 th tribonacci is 1
The 5 th tribonacci is 2
The 6 th tribonacci is 4
The 7 th tribonacci is 7
The 8 th tribonacci is 13
The 9 th tribonacci is 24
The 10 th tribonacci is 44
The 11 th tribonacci is 81
The 12 th tribonacci is 149
The 13 th tribonacci is 274
The 14 th tribonacci is 504
The 15 th tribonacci is 927
The 16 th tribonacci is 1705
The 17 th tribonacci is 3136
The 18 th tribonacci is 5768
The 19 th tribonacci is 10609
The 20 th tribonacci is 19513
The 21 th tribonacci is 35890
The 22 th tribonacci is 66012
The 23 th tribonacci is 121415
The 24 th tribonacci is 223317
The 25 th tribonacci is 410744

Dynamic Pattern

The 1 th tribonacci is 0
The 2 th tribonacci is 0
The 3 th tribonacci is 1
The 4 th tribonacci is 1
The 5 th tribonacci is 2
The 6 th tribonacci is 4
The 7 th tribonacci is 7
The 8 th tribonacci is 13
The 9 th tribonacci is 24
The 10 th tribonacci is 44
The 11 th tribonacci is 81
The 12 th tribonacci is 149
The 13 th tribonacci is 274
The 14 th tribonacci is 504
The 15 th tribonacci is 927
The 16 th tribonacci is 1705
The 17 th tribonacci is 3136

The 18 th tribonacci is 5768
The 19 th tribonacci is 10609
The 20 th tribonacci is 19513
The 21 th tribonacci is 35890
The 22 th tribonacci is 66012
The 23 th tribonacci is 121415
The 24 th tribonacci is 223317
The 25 th tribonacci is 410744