Relating Specific Surface Area, Air Volume, and Ice Density

FOR A MONODISPERSE BUBBLE SIZE DISTRIBUTION

Total volume of air in the ice (units: m^3 m^{-3}): where r is the radius of the bubble with units of meters and n is the number concentration per unit volume.

$$V_{air} = \frac{4}{3}\pi r^3 n$$

Density of the ice with air bubbles (units: kg m^{-3}): where $\rho_{ice} = 917$ kg m^{-3} and $\rho_{air} = 1.025$ kg m^{-3}

$$\rho = (1 - V_{air})\rho_{ice} - V_{air}\rho_{air}$$

Specific Surface Area of the bubble ice system (units: m^2 kg^{-1}):

$$\alpha = \frac{4\pi r^2 n}{\rho}$$

FOR A LOG NORMAL SIZE DISTRIBUTION

Total volume of air in the ice (units: m^3 m^{-3}): where D is the diameter of the bubble with units of meters, N_0 is the total number concentration (units of m^{-3}) $\tilde{D}_n$ is the median diameter (units of m), $\tilde{\sigma}_g$ is the geometric standard deviation (unitless).

$$V_{air} = \int_0^\infty \frac{1}{6}\pi D^3 n(D) dD = \frac{1}{6}\pi N_0 \tilde{D}_n^3 exp\left(\frac{9\tilde{\sigma}_g^2}{2}\right)$$

SNICAR-AD uses the effective diameter (D_{eff}) and $\tilde{\sigma}_g = ln(\sigma_g)$ where $\sigma_g = 1.5$

$$\tilde{D_n} = \frac{D_{eff}}{exp(\frac{5}{2}\tilde{\sigma}_g^2)}$$

Making use of the relationship between the median and effective diameters, the volume of air in terms of the effective diameter is then

$$V_{air} = \frac{\pi}{6} N_0 D_{eff}^3 \frac{1}{exp(3\tilde{\sigma}_g^2)}$$

Using the known volume fraction of air (from equations above) we can find the number concentration (units of m^{-3})

$$N_0 = \frac{6}{\pi}\frac{V_{air}}{D_{eff}^3} exp(3\tilde{\sigma}_g^2)$$

The Specific Surface Area (SSA) of the bubble ice system (units: m^2 kg^{-1}):

$$\alpha = \frac{\int_0^\infty \pi D^2 n(D) dD}{\rho} = \frac{\pi N_0 \tilde{D}_n^2 exp(2\tilde{\sigma}_g^2)}{\rho}$$

Again utilizing the relationship between the median and effective diameters, the SSA becomes

$$\alpha = \frac{\pi N_0 D_{eff}^2}{\rho} \frac{1}{exp(3\tilde{\sigma}_g^2)}$$

We can substitute the equation for N_0 into the SSA equation because both the effective diameter and volume fraction are known

$$\alpha = \frac{6}{\rho} \frac{V_{air}}{D_{eff}}$$