## 2 Scaled stability constant redefinition

Lines where `ZETA_N` or `I_ZETA_N` is mentioned:

**L455:**

```
hBL_neut_h_molec = ZETA_N * ((hBL_neut * ustar_h) / (5.0 * CS%kv_molec))
```

Equivalent to

$$\texttt{hBL\_neut\_h\_molec} = \frac{\xi_N}{k}\frac{u_*}{5\nu}\underbrace{\frac{ku_*}{f}}_{\texttt{hBL\_neut, L454}} = \frac{\xi_N u_*}{fh_\nu}$$

where $h_\nu = 5\nu/u_*$ (Eqn. 17, HJ99)

**L519:**

```
Gam_turb = I_VK * (ln_neut + (0.5 * I_ZETA_N - 1.0))
```

This is equivalent to Eqn. 15 in HJ99:

$$\Gamma_{\text{Turb}} = \frac{1}{k}\ln\left(\frac{u_*\xi_N\eta_*^2}{fh_\nu}\right) + \frac{1}{2\xi_N\eta_*} - \frac{1}{k} \qquad \text{(Eqn. 15, HJ99)}$$

with $\eta_* = 1$, knowing `ln_neut = log(hBL_neut_h_molec)` (L460) and the above definition for `hBL_neut_h_molec`.

**L530:**

```
n_star_term = (ZETA_N/RC) * (hBL_neut * VK) / (ustar_h)**3
```

As discussed previously (see original PR explanation doc and replace $\zeta_N$ with $\xi_N$, sorry for my Greek letter confusion), this is equivalent to

$$\eta_* = \left(1 + \frac{\xi_N u_*}{fL_O R_c}\right)^{-1/2} \qquad \text{(Eqn. 18, HJ99)}$$

if we use `ZETA_N`$= \xi_N/k$

**L539:**

```
Gam_turb = I_VK * ((ln_neut - 2.0*log(I_n_star)) + &
(0.5*I_ZETA_N*I_n_star - 1.0))
```

This is equivalent to that of L519 (and therefore Eqn. (15) in HJ99) except now $\eta_*$ is variable. Taking the reciprocal of $\eta_*$ and the negative sign cancel each other out via logarithm rules.

**L541:**

```
dG_dwB =  I_VK * ( -2.0 / I_n_star + (0.5 * I_ZETA_N)) * dIns_dwB
```

The purpose of this line is to take a derivative of `Gam_Turb` with respect to the surface buoyancy flux `wB_flux`. Note `dIns_dwB` is the derivative of `I_n_star` with respect to `wB_flux`. Manipulating Eqn. (15) of HJ99, we have

$$\frac{\partial\Gamma_{\text{Turb}}}{\partial\texttt{wB\_flux}} = \frac{\partial\Gamma_{\text{Turb}}}{\partial(1/\eta_*)}\frac{\partial(1/\eta_*)}{\partial\texttt{wB\_flux}} = \frac{1}{k}\left(-2\frac{1}{(1/\eta_*)} + \frac{k}{2\xi_N}\right)\texttt{dIns\_dwB}$$

which is the same as L541.

**L545:**

```
Gam_turb = I_VK * (0.5 * I_ZETA_N*I_n_star - 1.0)
```

This is a (rare) regime we assume the layer dominated by molecular viscosity is smaller than the assumed boundary layer, according to the code. This, I think, is when the terms in the log in Eqn. (15) of HJ99 (definition of $\Gamma_{\text{Turb}}$) are less than 1 and therefore the log is zero or negative. L545 is thus equivalent to Eqn. (15) of HJ99 but ignoring the log term.

**L546:**

```
dG_dwB = I_VK * (0.5 * I_ZETA_N) * dIns_dwB
```

As in L541, except removing the first term as the log term doesn't appear in L545.