```
open import Codata.Musical.Notation
open import Data. Nat using (; suc; zero)
open import Relation.Binary.Core using (Rel)
open import Relation. Binary. Bundles using (Setoid)
open import Relation. Binary. Definitions using (Reflexive; Symmetric; Transitive)
open import Relation.Binary.PropositionalEquality using (__; subst; subst) renaming (sym to eqSym; trans to
import Level using (zero)
open import Data. Maybe using (Maybe; nothing; just)
open import Data. Maybe. Properties
open import Data. Bool using (Bool; true; false)
open import Data. Product
open import Data.Sum
open import Function.Base using (case_of_)
open import Relation. Nullary using (contradiction)
open import Data.Nat
open import nakata. Traces
open import nakata.Language
open import nakata.BigRel hiding (execDeterministic)
open exloop hiding (exloopincrementing; incrementingFrom; incrementingtrace; increasing; incrementingAlwa
open Trace
module latex.ProofProgram where
incrementing From: State \rightarrow Trace
incrementingFrom st = tcons \ st (tcons st (incrementingFrom (next st))))
incrementingtrace: Trace
incrementing trace = incrementing From\ start State
exloopincrementing : exec (Swhile ( _{-} \rightarrow 1) (Sassign 0 add1)) startState incrementingtrace
exloopincrementing = t startState
  where
    \mathsf{t}: (st:\mathsf{State}) \to \mathsf{exec} (\mathsf{Swhile} \ (\ \_ \to 1) \ (\mathsf{Sassign} \ 0 \ \mathsf{add} 1)) \ st \ (\mathsf{incrementingFrom} \ st)
    \mathsf{t} \; st = \mathsf{execWhileLoop}
        (tcons st ( (tcons st ( (tnil (update 0 (add1 st) st))))))
        (execseqCons st_{--} (execseqNil (execAssign (tcons (tnil)))))
```

{-# OPTIONS --guardedness #-}

```
postulate
  trace: Trace
  program : Stmt
  fromState: State
  proof: exec program fromState trace
data increasing : Id \rightarrow Val \rightarrow Trace \rightarrow Set where
  increasingCons : \{id : Id\} \{v : Val\} \{st : State\} \{tr \ tr : Trace\}

ightarrow st id v

ightarrow tr tcons st ( (tcons st ( tr)))
     \rightarrow (increasing id (suc v) tr)
     \rightarrow increasing id \ v \ tr
incrementingAlwaysIncrements: increasing 0 0 incrementingtrace
incrementingAlwaysIncrements = forever refl
  where
     open Setoid setoid using () renaming (refl to refl)
     open import Relation.Binary.PropositionalEquality
     open -Reasoning
     \mathsf{lem}: \{x: \} \to x+1 \ \mathsf{suc} \ x
     lem {zero} = refl
     lem {suc } x} = begin
       suc(x+1)
        \langle \rangle
        \mathrm{suc}\ x + \mathrm{suc}\ \mathrm{zero}
        ⟨ cong suc (lem) ⟩
        suc (suc x)
     \mathsf{lem}: \{v: \mathsf{Val}\} \to (st: \mathsf{State}) \to (st\; 0\;\; v) \to \mathsf{next}\; st\; 0\;\; \mathsf{suc}\; v
     lem \{v\} st x = begin
        next st 0
        \langle \rangle
        st \ 0 + 1
        \langle cong (+1) x \rangle
        v+1
        ( lem )
```

(execseqCons st_{--} ((execseqCons st_{--} ((execseqNil (t (next st)))))))

 $\operatorname{suc} v$

```
\begin{array}{l} \text{forever}: \{st: \mathsf{State}\} \ \{v: \mathsf{Val}\} \to (st \ 0 \ \ v) \to \mathsf{increasing} \ 0 \ v \ (\mathsf{incrementingFrom} \ st) \\ \text{forever} \ \{st\} \ x = \mathsf{increasingCons} \ x \ (\mathsf{tcons} \ (\ \mathsf{tcons} \ (\ \mathsf{refl})))) \ (\ \mathsf{forever} \ (\mathsf{lem} \ st \ x)) \\ \\ \begin{array}{l} \mathsf{postulate} \\ \mathsf{execDeterministic}: \ \{s: \mathsf{Stmt}\} \ \{st: \mathsf{State}\} \ \{tr \ tr: \mathsf{Trace}\} \\ \\ \to \mathsf{exec} \ s \ st \ tr \\ \\ \to \mathsf{exec} \ s \ st \ tr \\ \\ \to \mathsf{tr} \ tr \end{array}
```