SAMPLE FINAL EXAM FOR "SET THEORY AND LOGIC THROUGHOUT MATHEMATICS"

You have up to 30 minutes to complete the exam. Take up to 15 minutes to decide on a topic and prepare your answers, and then use the remainder of the time to present your answers. You may not use outside sources/materials during the exam. If you have any questions/need any clarification, please feel free to ask.

Choose either (A) or (B), and answer all parts included therein.

(A) (Zorn's Lemma)

(1) Define "partial order", and give some natural examples of partial orders.

(2) State Zorn's Lemma.

(3) Give at least one other principle that is equivalent to Zorn's Lemma over the axioms of ZF.

(4) Describe an application of Zorn's Lemma, including a basic outline of the proof of the result.

(B) (Categoricity of algebraically closed fields)

(1) Define "field", "characteristic of a field", and "algebraically closed field".

(2) Given a cardinal κ, describe what it means for a first-order theory T to be "κ-categorical".

(3) Give a broad outline of the proof of the fact that the theory of algebraically closed fields of a fixed characteristic is κ-categorical for every uncountable cardinal κ.

(4) Explain why the theory of algebraically closed fields of a fixed characteristic is not $\aleph_0$-categorical.