Numerical Methods for Ordinary Differential Equations

Wylie Roberts, Hillary Spang, and Clancy Crawford

November 18, 2025

Abstract

Contents

1 Introduction

Write a brief description (background/significance) of what the project is about.

2 Problem Statement

State fully and precisely the mathematical problem. Explain meaning of all symbols used. Make clear what is given and what we are looking for.

2.1 Part I

Text introducing this subsection.

2.2 Part II

Text introducing this subsection.

2.3 Part III

Part 3 of this problem uses Euler's Method to Estimate The World's Population from 1950 to 2020. It uses a logistical growth differential equation to approximate the solution.

Given Data:

- p_0(in 1950) $= 2555$ million people
- $k_{rm} = 0.026$/yr
- $p_{max} = 12000$ million people
- t vs p graph for comparison:

Year	Population (millions)
1950	2555
1960	3040
1970	3708
1980	4454
1990	5276
2000	6079
2010	6922
2020	7753

Table 1: Given world population data from 1950–2020.

The differential equation is as follows:

$$\frac{dp}{dt} = k_{rm}\left(1 - \frac{p}{p_{\max}}\right)p$$

The Euler's Method is as follows:

$$p_{n+1} = p_n + h\,f(p_n)$$

3 Method/Analysis

Begin with naming or characterizing the method/approach to be used, perhaps explain the basic idea behind it, to what type of problems it applies, under what conditions, what it achieves, what are its main features, advantages, disadvantages. Justify why it is applicable to this problem, stating clearly any assumptions you need to make about the problem for the method to apply. Name some other methods/approaches one could use, and if/why your method may be preferable.

3.1 Part I

Text introducing this subsection.

3.2 Part II

Text introducing this subsection.

3.3 Part III

Part 3 of the project uses Euler's Method on the differential equation with the given data provided above to get a plot of the simulation. Euler's Method works by replacing the actual curve with an approximation of tangent lines over small steps. Euler's Method is advantageous because it is simple and accurate for our needs. It is disadvantageous because it is only accurate in the first order. This can lead to inaccuracy over a period of time. Other methods may include Improved Euler's Method and Implicit Method. Euler's Method is preferable in this instance because it is boundary of what is covered in MA305.

4 Solutions/Results

This section contains the presentation of your solution and results. Describe your implementation of the method(s) for this specific problem, any special features, numerical methods implementation strategy, choices of any parameters, stopping criteria, etc. Present the results in words and plots (annotate by hand if necessary), explain what they mean. Include your code in an Appendix.

4.1 Part I

Text introducing this subsection.

4.2 Part II

Text introducing this subsection.

4.3 Part III

Using Euler's Method and a step size of h = 1 from 1950 to 2020 with the initial population at p_0, the values corresponding to the time data are found. The plot below compares the Measured Population (in the millions) to the Year. It shows the given data points in comparison to Euler's Method. The general trend shows population growing upward with a discrepancy in the later years. This trend is continual with both plots. The Euler Method is more accurate as it gives a trend line versus just data points.

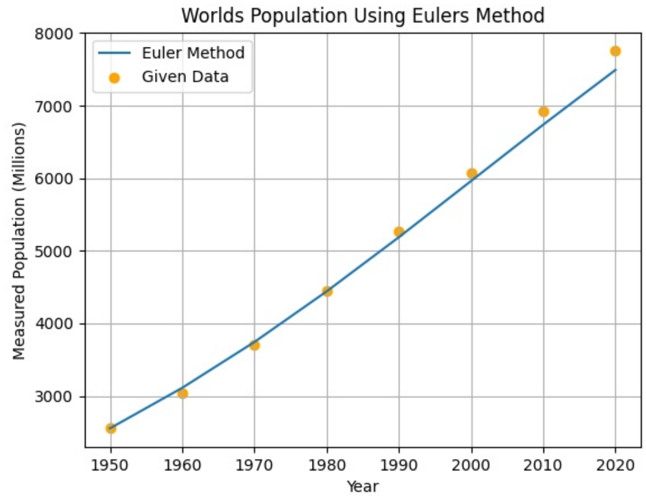

Figure 1: World population simulation using Euler's Method alongside given data.

5 Discussion/Conclusions

Interpret your solution physically, what we learn from it, comment on strengths and weaknesses of the solution method, any nice features you want to brag about, possible ways to improve it (e.g. how to make it more accurate, more efficient), as appropriate.

For Part 3, Euler's Method shows that the population trend provides growth from 1950 to 2020. However, the first-order nature of Euler's Method gives inaccuracy the longer the time period. As far as the scope of this class, Euler's Method is adequate.

References

[1] Heath, Michael T., Scientific Computing: An Introductory Survey, McGraw Hill, 2002.

A Python Codes

Text introducing this appendix. Subsections and further divisions can also be used in appendices.

```
#!/usr/bin/env python3
import math

"""
==========================================================================
MA305 - cw #: your name - date
Purpose: Find the number of trailing zeros in factorial(n).
==========================================================================
"""

n=input('Enter a positive integer:')
n=int(n)

count=0
for i in range(1,n+1):
    count += n//5**i #pow(5,i)

print(' Number of trailing zeros in factorial(',n,'):', count)
print(' Factorial (',n,')=',math.factorial(n))
```

```
#!/usr/bin/env python3
import math

"""
==========================================================================
MA305 - cw #: Part 2 - Hillary Spang - 11/17/25
Group: Wylie Roberts and Clancy Crawford
Purpose: Use Simpson's rule to approximate area of an irregularly shaped area - extrapolate to volume with h=3 in.
#From this volume data - calculate how many bags of mulch (3 ft^3 each) are needed to fill volume
==========================================================================
"""
#import packages
import numpy as np
import matplotlib.pyplot as plt
import math

#Given input data
x = np.linspace(0,26,27)
```

```
y = np.array([8,10,11,15,16,16,16,16,15.5,15.5,15.5,15,15,15,14.5,14.5,14,14,14,13.5,13,13,13,12

#Simpson's Rule
N = 27 #Define initial variables
a = 0
b = 26
h = (b-a)/N
y_odd = np.zeros(N)
y_even = np.zeros(N)

#iterate for odd half of simpson's rule
for i in range(0,N,2):
    xi = x[i]
    func_odd = y[i]
    y_odd[i] = func_odd

#iterate for even half
for i in range(1,N-1,2):
    xi = x[i]
    func_even = y[i]
    y_even[i] = func_even

#Bags of mulch calculation from Simpson's Rule
area = (h/3)*(y[0] + 4*np.sum(y_odd) + 2*np.sum(y_even) + y[N-1]) #Formula for area from Simpson's Approx.
volume = area * (3/12) #3 inch layer of mulch (in ft)
bags = math.ceil(volume/3) #Number of bags of mulch needed to complete task

#print results
print('Area of land: ',area,'ft^2')
print('Volume of mulch:',volume,'ft^3')
print('Bags needed:',bags)
```

```
#!/usr/bin/env python3
#Purpose: Euler's Method to predict world population from 1950-2020
#By: Clancy Crawford
#Group with: Wylie Roberts (Part 1) and Hillary Spang (Part 2)
#Date: 10/25/2025

import numpy as np
import math
import matplotlib.pyplot as plt


# Given Parameters
```

```
k_rm = 0.026 # per/yr
p_max = 12000 # million people
p0 = 2555 # million people in year start
yr_start = 1950
yr_end = 2020


# Given Function
def f(p):
    return k_rm*(1 - (p/p_max))*p

# Given Data for time
t_data = np.arange(yr_start, yr_end + 1, 1) #must go up by one then extract later
t_len = len(t_data)
#print(t_data)

# Data for p
p_data_given = np.array([2555, 3040, 3708, 4454, 5276, 6079, 6922, 7753], float)
p_data = np.zeros(t_len)
p_data[0] = p0

for i in range(t_len - 1):
    p_data[i+1] = p_data[i] + 1 * f(p_data[i])


#Extracting Values
t_data_actual = np.arange(yr_start, yr_end + 1, 10)
index = t_data_actual - yr_start
p_data_actual = p_data[index]

print()
print("Years (t):                    ", t_data_actual)
print("Population from Euler Method: ", p_data_actual)


# Plotting
plt.figure()
plt.plot(t_data_actual, p_data_actual, label="Euler Method")
plt.scatter(t_data_actual, p_data_given, color = 'orange', label = 'Given Data')
plt.xlabel('Year')
plt.ylabel('Measured Population (Millions)')
plt.title('World''s Population Using Euler''s Method')
plt.grid(True)
plt.legend()
plt.savefig('project_part3_plot.jpg')
plt.show()
```