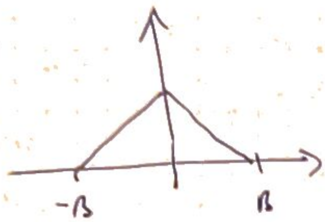
Théorème d'échantillonage de Shanon

$x(t) \xrightarrow{TF} X(f)$ est à support borné

$X(f) \neq 0 \quad |f| \leq B$



$$X(f) = \sum_{k=-\infty}^{+\infty} X_k \, e^{-2\pi j \frac{k}{2B} f}$$

$$X_k = \frac{1}{2B} \int_{-B}^{B} X(f) \, e^{2\pi j \frac{k}{2B} f} \, df$$

Rappel

$$x(t) = \int_{-B}^{B} X(f) \, e^{2\pi j f t} \, df$$

donc $X_k = \frac{1}{2B} \, x\left(\frac{k}{2B}\right)$

$T = \frac{1}{2B}$

$$X(f) = \frac{1}{2B} \sum_{k=-\infty}^{+\infty} x\left(\frac{k}{2B}\right) e^{-2\pi j \frac{k}{2B} f}$$

$$x(t) = \int_{-B}^{B} X(f) \, e^{2\pi j f t} \, df$$

$$= \frac{1}{2B} \sum_{k=-\infty}^{+\infty} x\left(\frac{k}{2a}\right) \int_{-B}^{B} e^{2\pi j f \left(t - \frac{k}{2B}\right)} \, df$$

$$= \frac{1}{2B} \sum_{k=-\infty}^{+\infty} x\left(\frac{k}{2B}\right) \left[\frac{e^{2\pi j f \left(t - \frac{k}{2B}\right)}}{2\pi j \left(t - \frac{k}{2B}\right)} \right]_{-B}^{B}$$

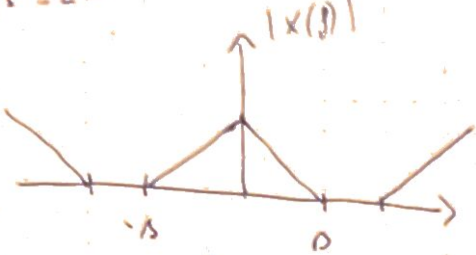
$$= \frac{1}{2B} \sum_{k=-\infty}^{+\infty} x\left(\frac{k}{2B}\right) \frac{e^{2\pi j B \left(t - \frac{k}{2B}\right)} - e^{-2\pi j B \left(t - \frac{k}{2B}\right)}}{2\pi j \left(t - \frac{k}{2B}\right)}$$

$$= \sum_{k=-\infty}^{+\infty} x\left(\frac{k}{2B}\right) \frac{\sin 2\pi j B \left(t - \frac{k}{2B}\right)}{t - \frac{k}{2B}} \times \frac{1}{2B}$$

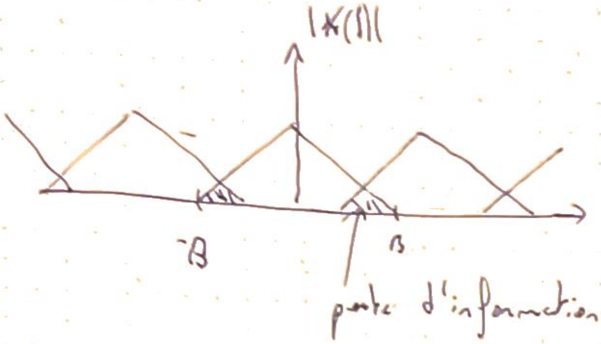
$$\boxed{= \sum_{k=-\infty}^{+\infty} x\left(\frac{k}{2B}\right) \operatorname{sinc} \left[2B \left(t - \frac{k}{2B}\right) \right]}$$

$f_s = 2B$

si $f > f_s = 2B$



si $f < f_s = 2B$



f_s est la plus petite fréquence d'échantillonage qui ne donne pas de perte d'informations.