

# Lab 5: CE Amplifier Gain with Emitter Degeneration

Proper calculation for base width modulation

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## Introduction

This lab summarizes my own investigation of the voltage gain  $v_{out}/v_{in}$  for the Common Emitter amplifier with a finite Early voltage. This research sprung from my attempt to solve question 4 of Homework 6 when accounting for base width modulation.

I knew that  $r_o$  inclusion was not required but I wanted to solve for the gain anyway. Even despite the warning from Sedra and Smith who advise to ignore  $r_o$  in the small-signal model: “[...] note that we have not included  $r_o$ , because it would complicate the analysis significantly, which is not worth the trouble, given that  $r_o$  has little effect on the performance of discrete-circuit amplifiers.” (p. 467, 8th ed.)

A search for the gain found only this equation for the voltage gain,

$$\frac{v_{out}}{v_{in}} \approx -\frac{g_m(R_C \parallel r_o(1 + g_m R_E))}{1 + g_m R_E},$$

(see MIT Course link, slide 19 and another YouTube video with the same result) but as I'll show this equation isn't exactly correct. The derivation on YouTube neglects the base current's contribution to the voltage of the emitter resistor twice, and this simplification can lead to significant errors when either  $\frac{R_E}{r_\pi}$  or  $\frac{R_E}{\beta} r_o$  is not negligibly small. I confirmed that adding the missing emitter voltage contribution yields the same equation that I derive in this report.

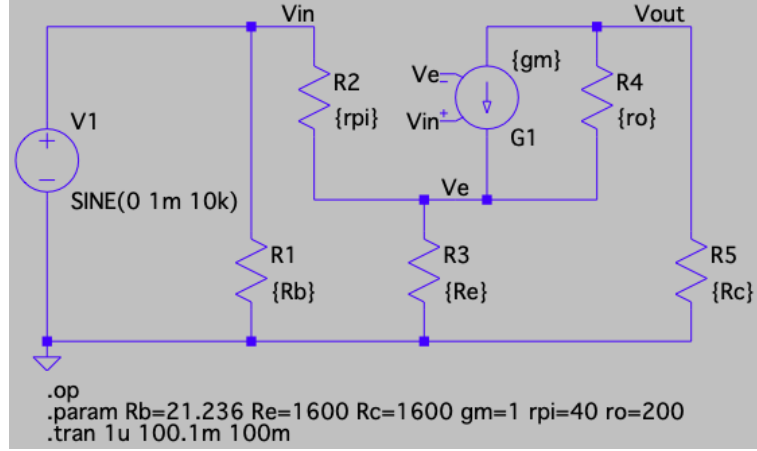
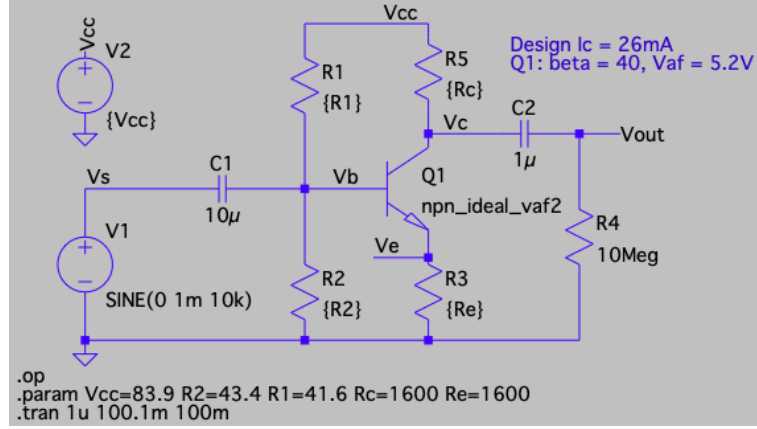
This report has three sections. In the first section, I derive  $A_v$  for a CE amplifier with emitter degeneration and base width modulation effects under a typical four resistor biasing scheme. In the second section, I present a summary for these CE amplifiers as they grow in complexity:

1. CE amplifier without  $R_E$  or base width modulation,
2. CE amplifier with base width modulation ( $V_A < \infty$ ),
3. CE amplifier with emitter degeneration ( $V_A = \infty$ ),
4. CE amplifier with emitter degeneration and base width modulation.

The third section presents comprehensive LTspice simulations that confirm the derivations.

## Voltage gain for the CE amplifier with emitter degeneration and $V_A < \infty$

Here is a schematic of the amplifier under consideration (DC then AC):



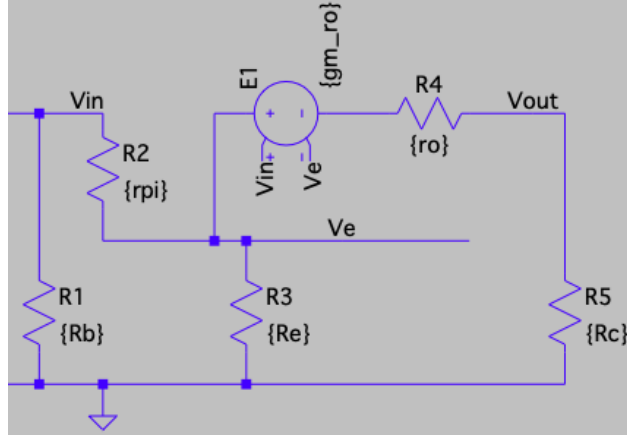
There are two important equations to derive the voltage gain. The first is KCL at the emitter node using the collector current through  $R_C$ :

$$v_{in} = v_{\pi} + \left( \frac{v_{\pi}}{r_{\pi}} - \frac{v_o}{R_C} \right) R_E$$

After a Thevenin transform of the VCCS to a VCVS (with voltage  $-g_m r_o v_{\pi}$  and resistance  $r_o$ ), KVL yields:

$$v_o + g_m r_o v_{\pi} = -\frac{v_o}{R_C} r_o + \left( \frac{v_{\pi}}{r_{\pi}} - \frac{v_o}{R_C} \right) R_E,$$

noting that on RHS the first term is the  $r_o$  voltage using collector current and the second term is the same  $R_E$  voltage used in the first equation. Here is a schematic of the KVL loop using the Thevenin source:



The first equation yields this substitution for  $v_p i$ :

$$v_\pi = \left( v_i + \frac{R_E}{R_C} v_o \right) \frac{r_\pi}{r_\pi + R_E}$$

After substituting  $v_\pi$  and some reduction, the voltage gain becomes

$$\frac{v_o}{v_i} = \frac{-R_C(\beta r_o - R_E)}{r_\pi(r_o + R_C + R_E) + R_C R_E + (\beta + 1)r_o R_E}$$

## Summary of CE properties

$R_E$	$r_o$	$A_v$
0	$\infty$	$-\frac{\beta R_C}{r_\pi}$
$R_E$	$\infty$	$-\frac{\beta R_C}{r_\pi + (\beta + 1)R_E}$
0	$r_o$	$-\frac{\beta R_C}{r_\pi \left(1 + \frac{R_C}{r_o}\right)}$
$R_E$	$r_o$	$-\frac{\left(\beta - \frac{R_E}{r_o}\right) R_C}{r_\pi \left(1 + \frac{R_C}{r_o}\right) + R_E \frac{r_\pi + R_C}{r_o} + (\beta + 1)R_E}$

resistance	value as function of $R_E$ and $r_o$
$R_{in}$	$R_1 \parallel R_2 \parallel \left\{ \frac{r_\pi + (\beta + 1)R_E + \frac{1}{r_o}(r_\pi R_C + r_\pi R_E + R_C R_E)}{1 + \frac{1}{r_o}(R_C + R_E)} \right\}$
$R_{out}$	$R_C \parallel \left\{ r_o \frac{r_\pi + R_B + (\beta + 1)R_E}{r_\pi + R_B + R_E} + (r_\pi + R_B) \parallel R_E \right\}$