Alternate diode voltage search method using Newton-Raphson

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Introduction

Calculating the diode voltage is difficult in even a simple circuit consisting of a voltage source of 1.45 V, a resistor of 4246.4 Ω, and diode with $I_S = 2$ fA. The diode current is $I_D = I_S\, e^{V_D/V_T}$ and is equal to the current through the resistor $(V_x - V_D)/R$.

Methods

Solution 1 with exponential term for V_D

Newton-Raphson can solve for the unknown V_D when setting these terms equal. The difference between these terms is zero and provides the function to solve

$$f(V_D) = I_S\, e^{V_D/V_T} - \tfrac{1}{R}(V_x - V_D),$$

with derivative

$$f'(V_D) = \tfrac{I_S}{V_T} e^{V_D/V_T} + \tfrac{1}{R}.$$

Solution 2 after taking logarithm

An interesting alternative solution that converges much quicker is taking the log of the terms to derive the solving function $g(V_D)$ as follows:

$$I_S\, e^{V_D/V_T} = \tfrac{1}{R}(V_x - V_D)$$

$$log(I_S) + \tfrac{1}{V_T} V_D = log(V_x - V_D) - log(R),$$

leading to

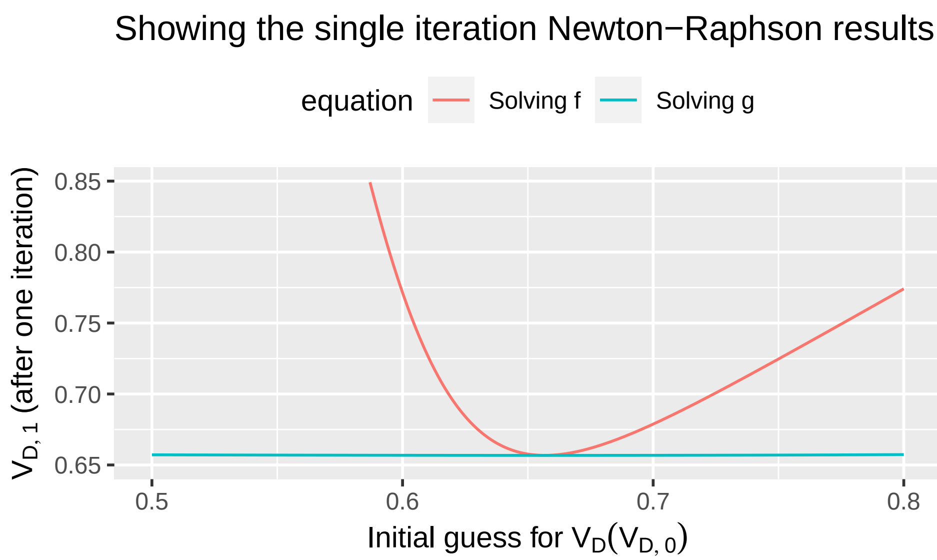
$$g(V_D) = log(V_x - V_D) - \tfrac{1}{V_T} V_D - log(I_S\, R)$$

and corresponding derivative

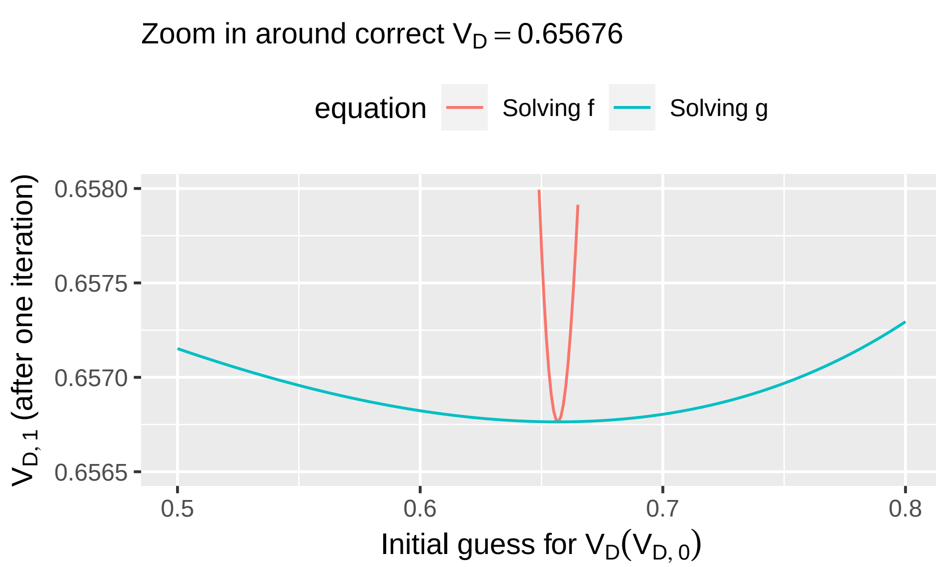
$$g'(V_D) = -\frac{1}{V_x - V_D} - \frac{1}{V_T}.$$

Results

Comparing the first iteration V_D values based on the initial guess $V_{D,0}$, noting that V_D is 0.65676.



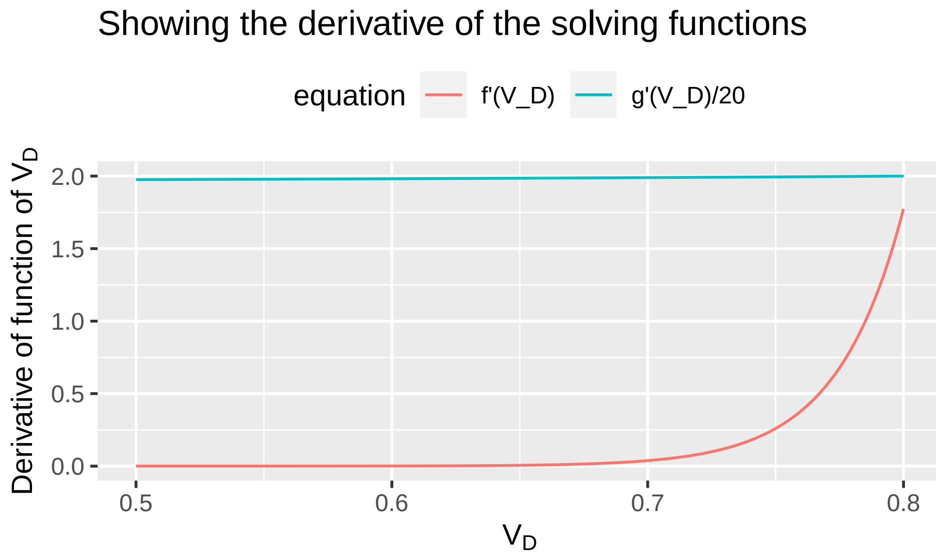
A zoom-in on the solution with $g(V_D)$ shows how accurate a single iteration is for the logarithm method.



Discussion

The Newton-Raphson algorithm converges much quicker if the derivative equation $f'(V_D)$ changes less over the range of V_D. E.g., an equation $f(V_D)$ with constant derivative (i.e., linear function) would get an exact solution after one iteration. Therefore the less the derivative $f'(V_D)$ changes versus V_D, the quicker the convergence.

Here is a plot of the derivatives $f'(V_D)$ and $g'(V_D)$ over the range of V_D.



As the derivative changes less drastically with $g(V_D)$, its solutions converge much quicker.