STATS 100A: BASICS & EXAMPLES

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Some pictures are taken from the internet.
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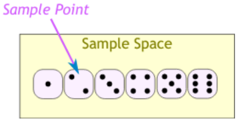
Sample space

Experiment $\rightarrow$ outcome $\rightarrow$ number

Example 1: Roll a die



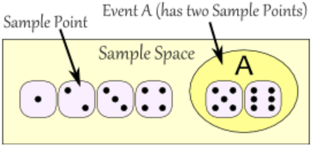
Sample space Ω: The set of all the outcomes (or sample points, elements).

Randomly sample an outcome from the sample space.

Event

Experiment $\rightarrow$ **outcome** $\rightarrow$ **number**

Example 1: Roll a die



Sample space Ω: The set of all the outcomes.

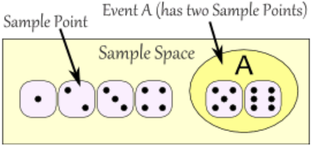
Event A:

(1) A **statement** about the outcome, e.g., bigger than 4.

(2) A **subset** of sample space, e.g., $\{5, 6\}$.

Counting equally likely possibilities

Experiment $\rightarrow$ **outcome** $\rightarrow$ **number**
Example 1: Roll a die



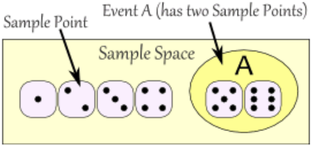
Assume the die is fair so that all the outcomes are **equally likely**.
Probability: defined on event:

$$P(A) = \frac{|A|}{|\Omega|} = \frac{2}{6} = \frac{1}{3}.$$

$|A|$ counts the size of A, i.e., the number of elements in A.

Random variable

Experiment $\rightarrow$ **outcome** $\rightarrow$ **number**
Example 1: Roll a die

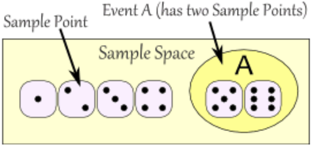


Random variable: Let X be the number:

$$P(X > 4) = \frac{1}{3}.$$

An event is a **math statement** about the random variable.
We can either use events or use random variables.
In Parts 2 and 3, we will focus on random variables.

Experiment $\to$ outcome $\to$ number
Example 1: Roll a die



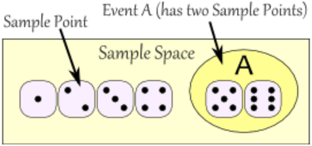
Conditional probability: Let B be the event that the number is 6. Given that A happens, what is the probability of B?

$$P(B|A) = \frac{1}{2}.$$

As if we randomly sample a number from A.
As if A is the sample space.

Conditional probability

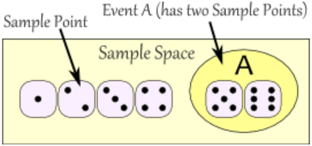
Experiment → outcome → number
Example 1: Roll a die



Random variable

$$P(X = 6|X > 4) = \frac{1}{2}.$$

Example 1: Roll a die



Complement

Statement: Not A

Subset: $A^c = \{1, 2, 3, 4\}$.

100A

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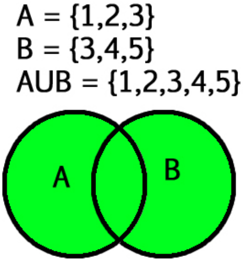
Basics
Population
Region
Coin
Markov
Reasoning

Example 1: Roll a die



Venn diagram

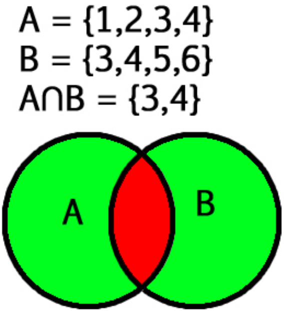
Union

Statement: A or B.

Subset: $A \cup B$.

Relations

Example 1: Roll a die



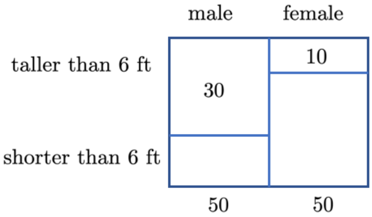
Intersection
Statement: A and B.
Subset: $A \cap B$.

Sample space is population

Experiment → outcome → number

Example 2: Sample a random person from a population of 100 people, 50 males and 50 females. 30 males are taller than 6 ft, 10 females are taller than 6 ft.

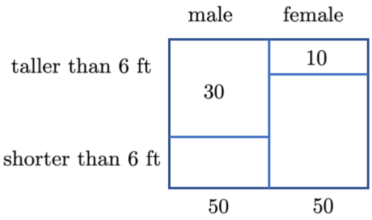
The sample space Ω is the population.



Events as sub-populations

Experiment $\rightarrow$ outcome $\rightarrow$ number

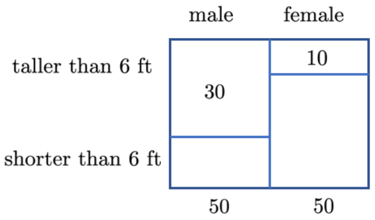
Example 2: Let A be the event that the person is male. Let B be the event that the person is taller than 6 feet (or simply the person is tall). A is the sub-population of males, and B is the sup-population of tall people.



Probability is population proportion

Experiment → outcome → number

Example 2: A male, B tall.



$$P(A) = \frac{|A|}{|\Omega|} = \frac{50}{100} = 50\%.$$

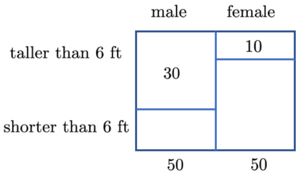
$$P(B) = \frac{|B|}{|\Omega|} = \frac{30+10}{100} = 40\%.$$

Probability = population proportion.

Conditional probability is proportion of sub-population

Experiment $\rightarrow$ outcome $\rightarrow$ number

Example 2: A male, B tall.



$$P(A|B) = \frac{|A \cap B|}{|B|} = \frac{30}{40} = 75\%.$$

Among tall people, what is the proportion of males?

$$P(B|A) = \frac{|A \cap B|}{|A|} = \frac{30}{50} = 60\%.$$

Among males, what is the proportion of tall people?

Conditional probability = proportion within sub-population.

Random variable as a function of outcome

Link between event and random variable.
Example 2: A male, B tall.
Let $\omega \in \Omega$ be a person. Let $X(\omega)$ be the gender of ω, so that $X(\omega) = 1$ if ω is male, and $X(\omega) = 0$ if ω is female. Let $Y(\omega)$ be the height of ω. Then

$$A = \{\omega : X(\omega) = 1\},\ B = \{\omega : Y(\omega) > 6\}.$$

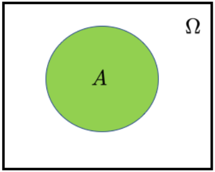
$$P(A) = P(\{\omega : X(\omega) = 1\}) = P(X = 1).$$

$$P(B) = P(\{\omega : Y(\omega) > 6\}) = P(Y > 6).$$

$$P(B|A) = P(Y > 6|X = 1),\ P(A|B) = P(X = 1|Y > 6).$$

Axiom 0

Equally likely scenario
A real population of people, under purely random sampling
or imagined population of equally likely possibilities

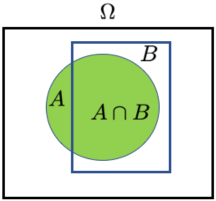


$$P(A) = \frac{|A|}{|\Omega|}.$$

Axiom 0.
Can always translate a problem into equally likely setting.

Conditional probability

Equally likely scenario



$$P(A|B) = \frac{|A \cap B|}{|B|} = \frac{|A \cap B|/|\Omega|}{|B|/|\Omega|} = \frac{P(A \cap B)}{P(B)}.$$

Physical: sample from B. B defines condition.
Mental: know that B happened, as if sample from B.
Axiom 4 or definition of conditional probability.