Modeling of a Private Monopolist Insurer : Theory and Practice

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Contents

Introduction

Introduction

- Modeling an insurer's strategy in a simplified scenario

- One contract vs Multiple contracts

Model and equations

- Same initial wealth W_0 for all agents
- θ : probability of experiencing a financial loss $L > 0$
- U : agent's attitude toward risk (concave and non decreasing)
- **Goal :** evaluate coverage if contract maximizes expected utility

- Single (non mandatory) contract of price P with indemnity R
- **Goal :** maximize profit

Model and Equations

- An agent of type θ evaluates the insurance offer by comparing it's expected utility with and without insurance.
- **Without insurance :**

$$V(\theta, 0) = \theta U(W_0 - L) + (1-\theta)U(W_0).$$

- **With contract $C = (P, R)$:**

$$V(\theta, C) = \theta U(W_0 + R - L - P) + (1-\theta)U(W_0 - P).$$

$$\text{Agent accepts} \iff G(\theta, P, R) := V(\theta, C) - V(\theta, 0) \geq 0.$$

Model and Equations

- We note $\theta_c \in [0, 1)$ the unique solution to $G(\theta_c, P, R) = 0$.

Setting $V(\theta, C) = V(\theta, 0)$ gives explicit solution :

$$P(\theta, R) = \frac{1}{\lambda} \ln \left(\frac{\theta e^{\lambda L} + (1-\theta)}{\theta e^{\lambda(L-R)} + (1-\theta)} \right), \tag{1}$$

- (1) gives the maximum premium that an agent of type θ is willing to pay for a contract offering indemnity R against a potential loss L

Model and Equations

- The insurer profit is given by

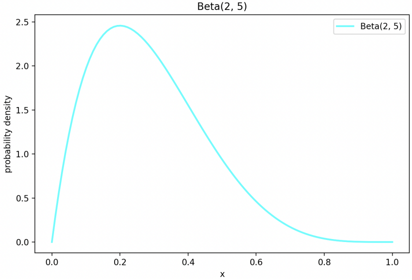
$$\Pi(\theta_c, R) = [P(\theta_c, R) - R \cdot A(\theta_c)] \cdot (1 - F(\theta_c))$$

$A(\theta)$ the average risk of the insured population.

- Existence and uniqueness of optimal (θ^*, R^*) is guaranteed

Numerical analysis

- We choose $L = 1, \lambda = 3,\ f \sim Beta(2, 5)$



$$\max_{\theta \in [0,1), R \in (0,L)} [P(\theta, R) - R \cdot A(\theta)] \cdot Q(\theta)$$

with `L-BFGS-B` algorithm.

- The measure of the insured population is $Q(\theta_c) := 1 - F(\theta_c)$

Single contract - Results

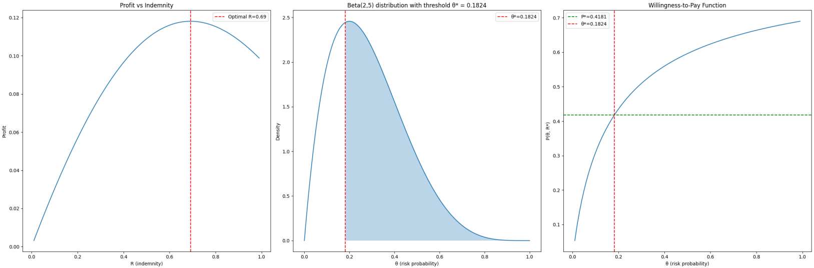
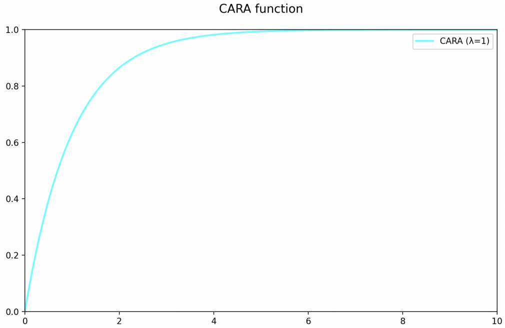


Figure – $R^* = 0.6913$, $\theta^* = 0.1824$ and $\Pi^*(\theta^*, R^*) = 0.1181$. The majority (69%) purchase the contract.

Influence of the risk aversion

- Utility function : $U(W) = 1 - e^{-\lambda W}$



- $\lambda \to 0 : U(W) \approx W$, the agent is risk neutral
- λ increases : the agent becomes more risk-averse, "values certainty over risk", willing to pay a higher premium to avoid risk

Influence of the risk aversion

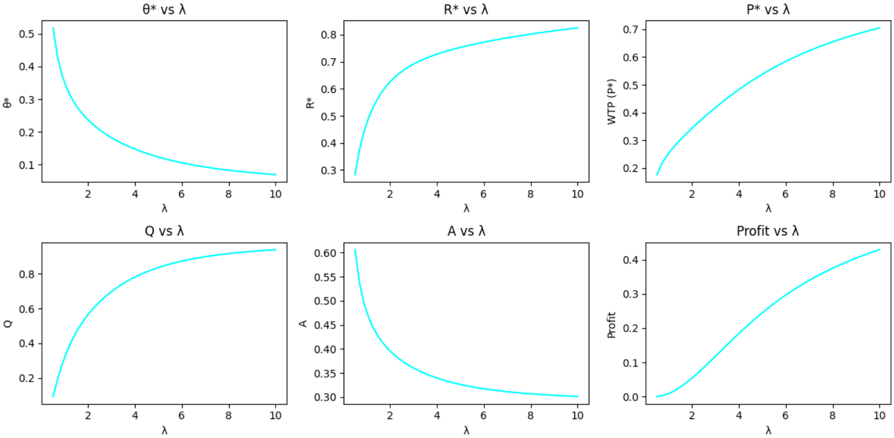


Figure – Evolution of variables with λ

Influence of the risk aversion

- For high λ : agents are already highly risk-averse

- At $\lambda \approx 4$, R^*, θ^* have weaker slopes

- if market's λ is approximately known, it gives the insurer an intuition on the risk situation

Multiple contracts

Generalizing to N contracts

- N contracts $(C_i)_{1 \leq i \leq N}$
- Population subdivided into $(\Theta_i)_{i \leq N}$ according to agent types
- The agents of types in $\Theta_i = [\theta_{i1}, \theta_{i+1}]$ are offered a contract (P_i, R_i)

- Agent chooses to buy it if and only if :

$$V(\theta, C_i) \geq V(\theta, 0)$$

Generalizing to N contracts

- Profit per segment $[\theta_i, \theta_{i+1}]$:

$$\Pi_i = (P(\theta_i, R_i) - R_i \cdot A_i) \cdot Q_i$$

with :

$$A_i = \frac{\displaystyle\int_{\theta_i}^{\theta_{i+1}} \theta' f(\theta')\, d\theta'}{F(\theta_{i+1}) - F(\theta_i)}$$

The optimization problem writes :

$$\max_{\substack{\theta_1 < \cdots < \theta_N \\ R_i \leq L}} \sum_{i=1}^{N} \Pi_i$$

Optimization is done with `differential_evolution`

Numerical results

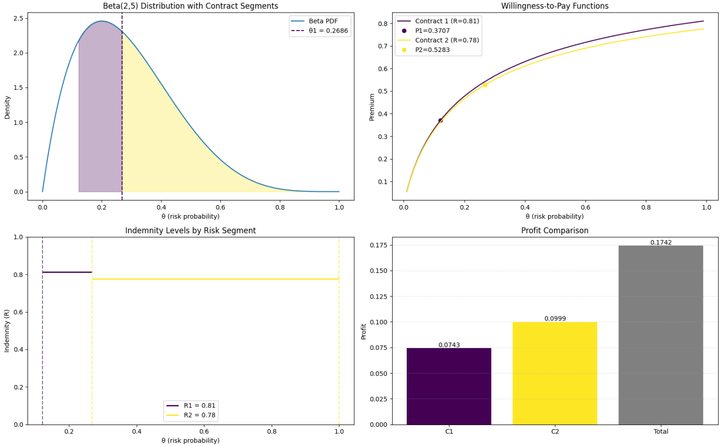


Figure – Results for $N = 2$ contracts

Numerical results

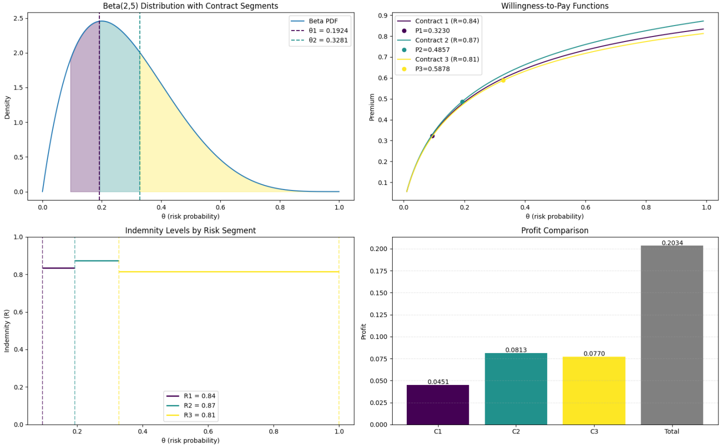


Figure – Results for $N = 3$

Numerical results

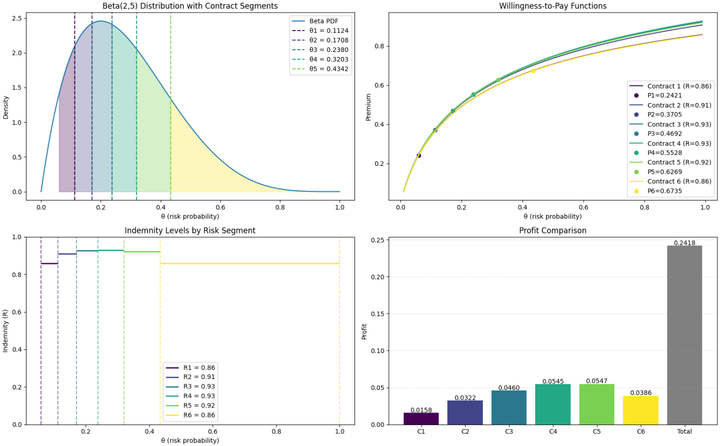
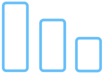


Figure – Results for $N = 6$

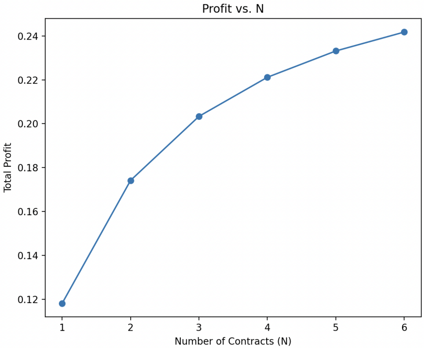
Numerical results

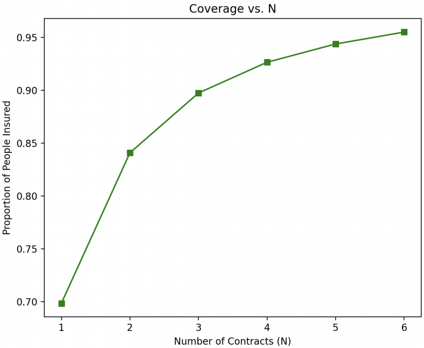
- More contract options have a higher chance to suit different needs of the population



- Total profit increases with N
- More people are covered

Numerical results





- As N increases, more people are covered and the insurer makes a greater profit.

Continuum of contracts

Continuum of contracts

- $N \to \infty$, where the contract function is continuous :

$$R : [0,1] \longrightarrow \mathbb{R}$$

- Each agent of type θ is offered a contract with indemnity $R(\theta)$

$$\max_{R(\cdot)} \Pi[R(\cdot)] = \int_0^1 \left[P(\theta, R(\theta)) - R(\theta)\theta\right] f(\theta)\, d\theta,$$

Continuum of contracts

- $\mathcal{L}(\theta, R) = P(\theta, R(\theta)) - R(\theta)\theta$

$$\partial_R P(\theta, R(\theta)) - \theta = 0$$

- which gives : $\dfrac{\theta e^{\lambda(L-R)}}{\theta e^{\lambda(L-R)} + 1 - \theta} - \theta = 0$

- Solving the first-order condition yields $R^*(\theta) = L$: maximum reimbursement to all individuals, i.e. **full insurance coverage**.

Numerical results

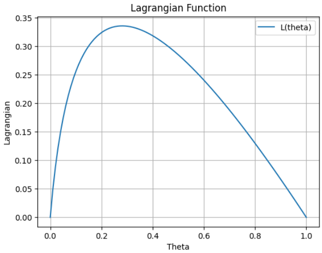


Figure – Lagrangian integrand $\mathcal{L}(\theta, R)$ evaluated at $R^*(\theta) = L$

- Optimal profit $\Pi^* = 0.29$.

Conclusion

Conclusion

- A simple yet realistic model of a private monopolist insurer
- Theoretical and numerical results
- Generalization to more than a single contract and beyond

Merci !