Detector Coverage & Isotropy

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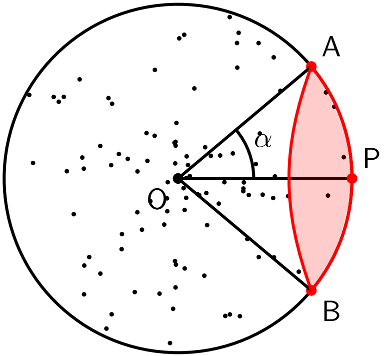
Contents

1) Rough Plan

Quantifying Isotropy - 1

- Goal: develop a simple model to measure the isotropy of a given configuration of points on a sphere.
- Consider a sphere with N points on it and a fixed opening angle α.



- Pick a direction $\overrightarrow{OP}$
- Count the number of points that lie inside the spherical cap of opening angle α around $\overrightarrow{OP}$
- n_P out of N points inside cap

Quantifying Isotropy - 2

- Select large number of directions $\overrightarrow{OP}$ (details TBD) and plot the n_P distribution $\rightarrow$ number of directions vs number of points in cap
- Isotropic point arrangement $\rightarrow$ expect similar n_P values for all Ps
- So we expect the distribution to be sharply peaked around $N \cdot \frac{A_{cap}}{A_{sphere}}$
- Quantify **isotropy** by **variance** of distribution (or maybe width, details TBD)
- Quantify **coverage** (point density) by **mean** of distribution (or maybe median, details TBD)

Quantifying Isotropy - 3

- As the distribution becomes less isotropic, expect normalized variance/width of the distribution to decrease
- Potential issue: dealing with long tails; will have to figure this out later
- Define α-dependent isotropy measure $I_\alpha = \frac{\mathrm{Var}[n]}{\bar{n}}$, where $\bar{n}$ is the mean of the distribution and $\mathrm{Var}[n]$ is the variance.
- Expect I_α to decrease as the distribution becomes less isotropic

Plan for Developing an Isotropy Criterion - 1

1. Select a bunch of runs with various active PMT configurations
2. Find a way to determine which runs are 'good' without directly checking for coverage and isotropy. For example, could base this on energy reconstruction for certain kinds of simulations
3. Select a set of directions
4. Fix a value for α
5. For each run, compute the mean, median and variance for the resulting n_P distributions

Plan for Developing an Isotropy Criterion - 2

1. The set of "sufficiently isotropic" PMT configurations is defined by an upper limit on $\mathrm{Var}[n]$, $\mathrm{Var}[n]_{max}$
2. The set of "sufficiently dense" PMT configurations is defined by a lower limit on $\bar{n}$, $\bar{n}_{min}$ (or maybe median)
3. The set of "good" runs as defined by the parameters of the n_P distribution is the intersection of the two
4. Find the values $(\frac{\mathrm{Var}[n]}{\bar{n}})_{max}$ and $\bar{n}_{min}$ that result in the greatest overlap between the "good" runs as defined by energy reconstruction, and the "good" runs defined by the parameters of the n_P distribution

Plan for Developing an Isotropy Criterion - 3

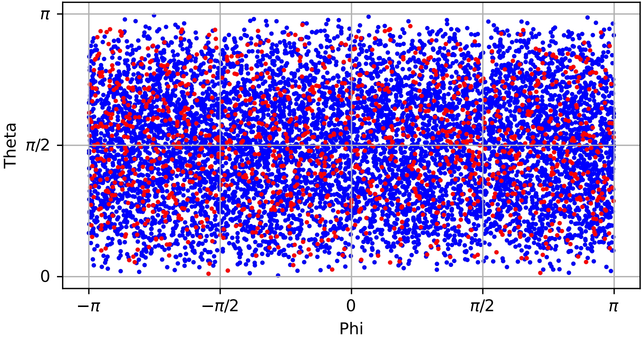
- Can apply this procedure for different values of α and get different values for $(\frac{\mathrm{Var}[n]}{\bar{n}})_{\mathsf{max}}(\alpha)$ and $\bar{n}_{\mathsf{min}}(\alpha)$
- Pick one α with the greatest overlap for most PMT configurations to compute check result more quickly
- Try with different sets of directions $\overrightarrow{OP}$ to see which one works best

- Question: If we find an external criterion, can we just use that if it's not too inefficient to compute its result?

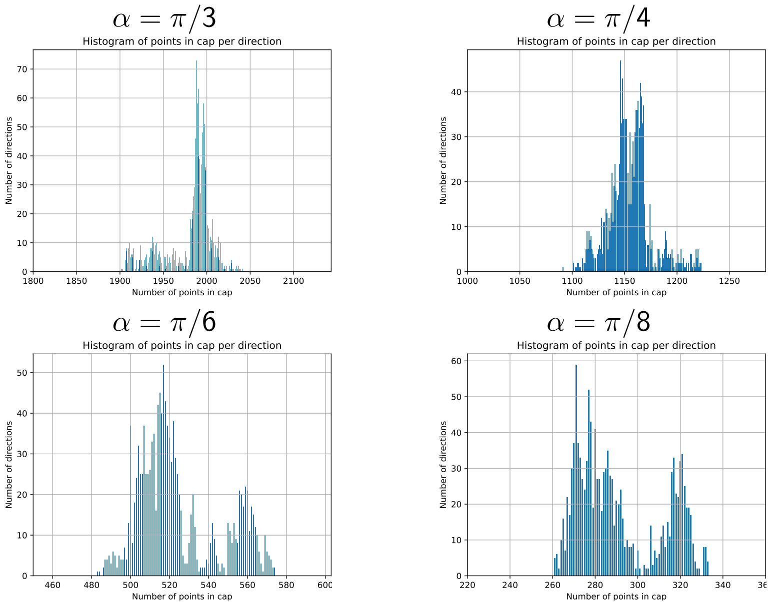
2) Isotropy and Cap Count Distributions

i) Random Points, Random Directions

- Generate a random set of points on sphere and a random set of directions[1]
- For this example: 1280 directions, 7847 points



n_P Distributions for Different α Values [2]

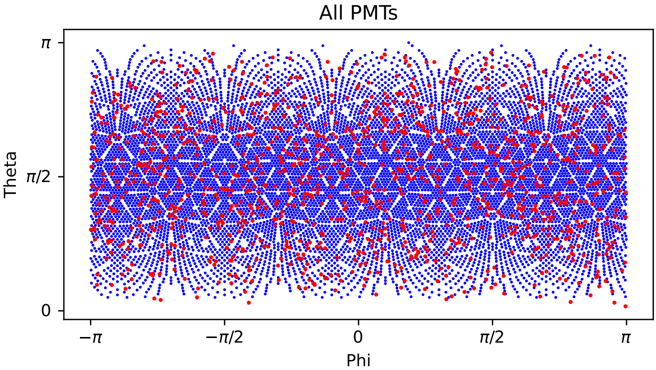


i) Random Points, Random Directions

- Distributions for $\alpha = \pi/3$ and $\alpha = \pi/4$ look as expected
- Other distributions not sharply peaked because points are randomly generated, not entirely isotropic
- Two peaks in distributions for lower α are just due to statistical fluctuations
- Best angle for obtaining nice distributions strongly dependent on points and directions chosen
- More plots here

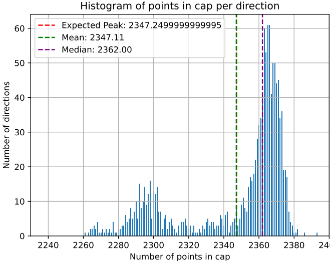
ii) True PMT Positions, Random Directions

- Compute the same kind of histogram using real PMT positions and random set of directions
- For this example: all PMTs (9389) on, 1253 directions

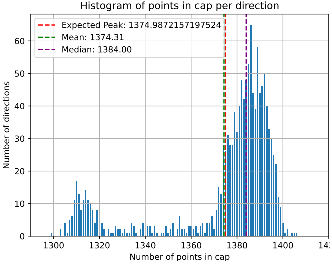


n_P Distributions for Different α Values [3]

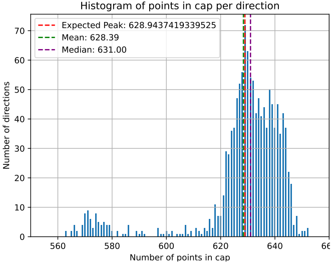
$\alpha = \pi/3$



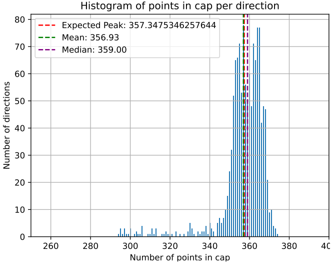
$\alpha = \pi/4$



$\alpha = \pi/6$



$\alpha = \pi/8$

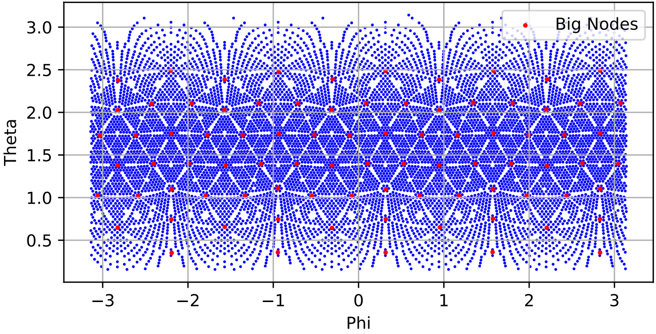


ii) True PMT Positions, Random Directions

- Long left tail or even second peak at lower n_P due to directions close to the top or the bottom of the detector where PMT density is lower
- Tail less significant & mean and median values closer to each other as α decreases
- Computed same types plots with other different sets of random directions - these properties seem to be consistent across the sets; check this link for comparison

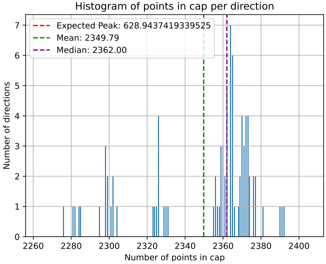
iii) True PMT Positions, Directions = 'Big Nodes'

- Compute the same kind of histogram using real PMT positions and the following set of directions (shown in red)
- Track number of points in each cap to confirm source of tails

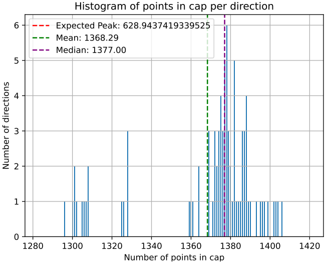


n_P Distributions for Different α Values

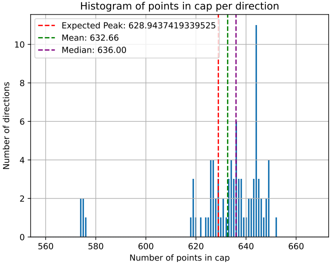
$\alpha = \pi/3$



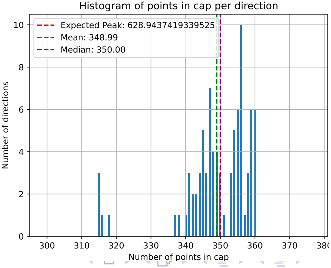
$\alpha = \pi/4$



$\alpha = \pi/6$



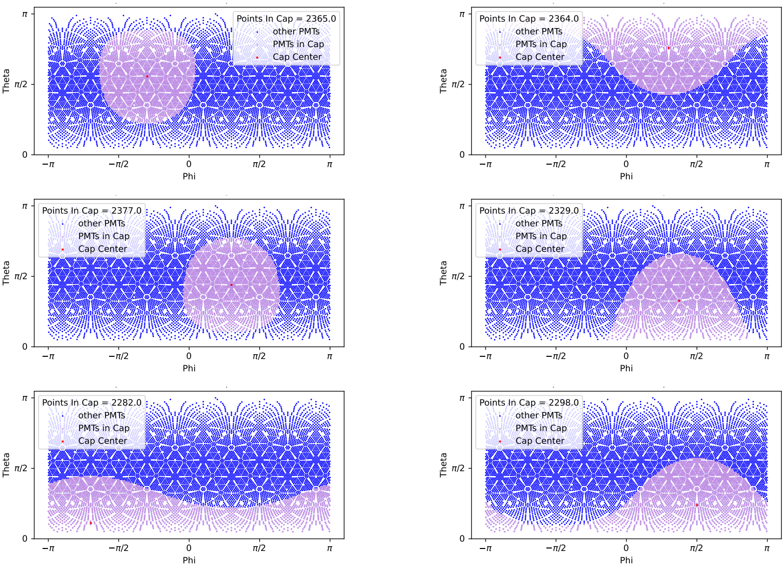
$\alpha = \pi/8$



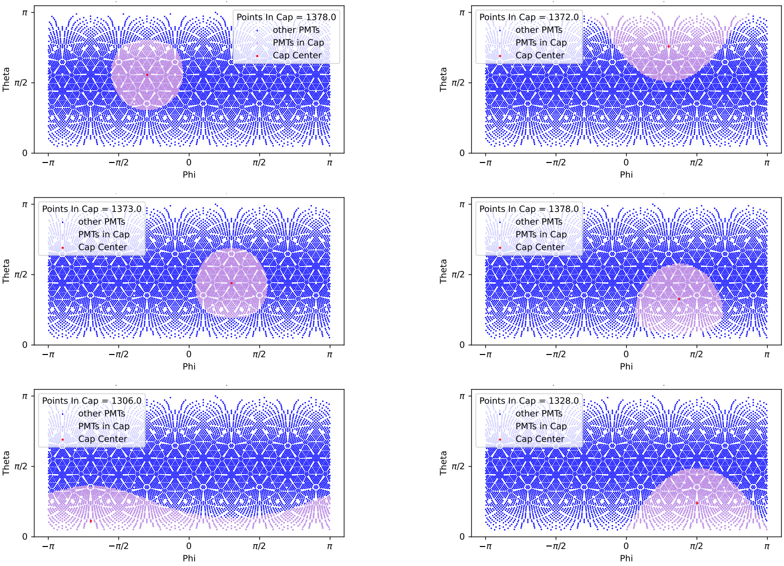
iii) True PMT Positions, Directions = 'Big Nodes'

- Less directions $\rightarrow$ less clear what's going on
- By eye: small α keeps distribution more narrow and tail less pronounced
- Plot points in cap for all directions $\overrightarrow{OP}$ to check source of tails [4]

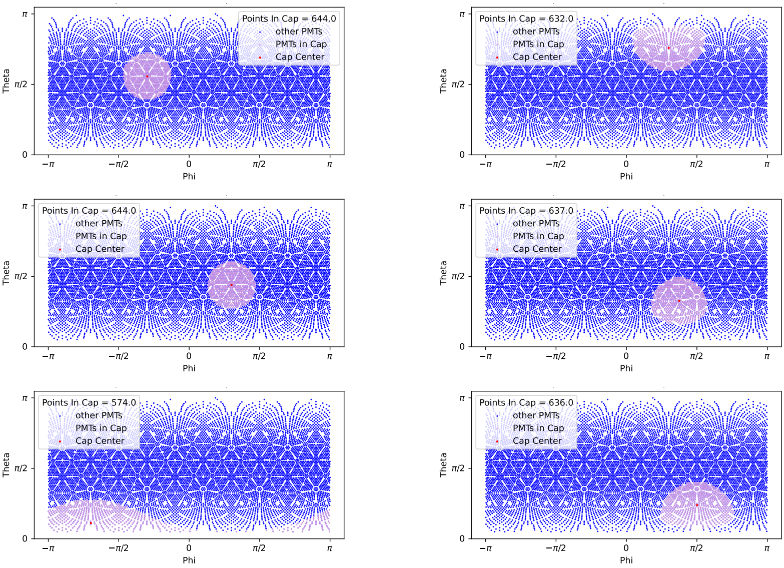
Points per Cap, $\alpha = \pi/3$



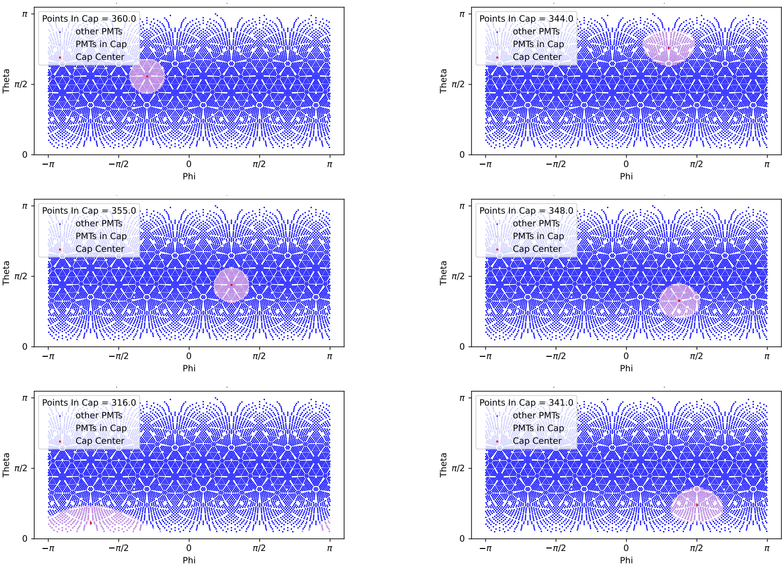
Points per Cap, $\alpha = \pi/4$



Points per Cap, $\alpha = \pi/6$



Points per Cap, $\alpha = \pi/8$



3) Conclusions

Conclusions

- Directions around top and bottom of the detector cover significantly less PMTs
- Median captures peak better,but close to mean for low α
- Smaller values for α seem to lead to nicer distributions *rightarrow* no second peak at lower n_P, tail less significant
- Probably ok to only look at $\alpha \leq \pi/6$ from now on

4) Next Steps

Next Steps

- Compute variances for distributions from iii)
- Rerun everything with different PMT configurations, compute associated quantities for resulting distributions
- Rerun everything with bigger set of directions e.g. add midpoints between big nodes
- Exclude directions around poles?
- External criterion for "good" PMT configurations? Based on simulations?

Notes - 1

[1] Points on the sphere are generated as follows: First, generate points in unit cube by drawing from the uniform distribution $U[0, 1]$ in each direction. These should be somewhat uniformly spread out throughout the volume. Then, select points within the unit sphere. Now we have uniformly spread out points within the sphere volume. Lastly, rescale all vectors from the center of the sphere to the points to have length 1. Now, we should have a set of somewhat-isotropically-distributed points on the surface of the unit sphere.

Notes - 2

[2] These plots can be found here; two more sets of plots generated in the same way (but with different points and different directions) can be found in the other two folders with similar names.

[3] Expected peak $N \cdot \frac{A_{\text{cap}}}{A_{\text{sphere}}}$ These plots can be found here; more plots generated in the same way (true PMT positions, all PMTs and different sets of random directions) can be found in folders run 1 - run 4.

[4] Can check cap plots for all nodes here