THE FORMATION OF NON-AXISYMMETRIC VORTEX RINGS

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Summary In this study, we consider the formation of vortex rings from non-axisymmetric nozzles of different shapes, and compare the dynamics of the pinch-off process to those of axisymmetric vortex rings. By performing DPIV on several planes along the perimeter of the different nozzles, we investigate the relative importance of global parameters (such as equivalent diameter) and local quantities (such as the curvature and its rate of change) in predicting the pinch-off of non-axisymmetric vortex rings.

INTRODUCTION

The formation of axisymmetric vortex rings by the ejection of fluid through a circular nozzle or aperture has been the subject of numerous studies. As fluid is ejected, boundary layer separation at the aperture leads to roll-up and the formation of a vortex ring. Gharib *et al.* [1] found that vortex rings cannot grow indefinitely, as there is a physical limit to their size. When the non-dimensional *formation time* $\hat{T} = U_p t/D$ (where U_p is the piston velocity, and D is the diameter of the nozzle) exceeds approximately four, the vortex rings stops accepting additional vorticity and is said to have 'pinched off.' Gharib *et al.* explained this transition in terms of an energy maximization principle due to Kelvin [2] and Benjamin [3]. Subsequently, Krueger & Gharib [4] demonstrated that the pinch-off process has dynamical significance, as the efficiency of momentum transport in pulsed jets is optimized when the size of the vortex rings is maximized. Recent studies of jetting swimmers suggest that certain species may exploit these propulsive benefits in order to generate vortex rings optimally and achieve higher propulsive efficiencies [5].

However, naturally occurring vortex rings are rarely circular, so the extension of these results to non-axisymmetric geometries is of interest. Since the Kelvin-Benjamin principle does not apply to non-axisymmetric flows, the dynamics of the formation of non-axisymmetric vortex rings remain largely unknown. In this study, we considered the formation of vortex rings from non-axisymmetric nozzles of different shapes. By examining the formation process at different points along the contour of the different nozzles, we investigated the relative importance of global parameters such as nozzle equivalent diameter, and local quantities such as curvature in determining the limiting formation time for non-axisymmetric vortex rings.

METHODS

We considered the formation of vortices from elliptical nozzles using a piston-cylinder arrangement. Experiments were conducted using two elliptical nozzles with the same equivalent diameter ($D_{eq} = 4.45$cm) but different aspect ratios (two and four). The elliptical vortex generator was submerged in a water tank, and a vortex ring was generated by the motion of the elliptical piston, which was driven by flow delivered by a computer-controlled solenoid valve. Experiments were conducted at a Reynolds number of 2000 based on the piston speed and equivalent diameter of the nozzle.

The velocity field was mapped using two-component DPIV. A pulsed Nd:YAG laser sheet was used to illuminate a plane through the elliptical nozzle, and the flow was seeded with 13μm neutrally buoyant glass spheres. Images from an 18cm $\times$ 18cm test section were recorded by a CCD camera, and processed using the method of Willert and Gharib [6] with a separation of 33ms and an interrogation window size of 32 pixels with 50% overlap. The velocity and vorticity fields were calculated using an in-house code, and the resolution for both was 3mm $\times$ 3mm with an uncertainty of 1% and 3% respectively.

To determine the effect of local curvature on the formation time, the velocity field was recorded at several locations along the nozzle contour between the major and minor axes. At each location, the laser sheet was positioned such that is was normal to the nozzle edge, so as to minimize out-of-plane motions.

RESULTS

The velocity field was recorded at seven locations along the contour of the aspect-ratio-two nozzle, where the curvature ranged from $\kappa = 0.16\text{cm}^{-1}$ at the minor axis to $\kappa = 1.26\text{cm}^{-1}$ at the major axis. Using the data from these seven planes, we reconstructed an approximate 3D field of the vorticity magnitude. Figure 1 shows iso-surfaces of the reconstructed vorticity at two separate time instants. Initially, we observed the formation of an isolated vortex ring whose shape closely resembled the nozzle contour (Fig. 1(a)). However, since the self-induced speed of a vortex ring is proportional to the local curvature, the vortex ring underwent significant deformation in later stages of development (Fig 1.(b)). It is also noted in Fig. 1(b) that a trailing jet has developed behind the vortex ring, which has pinched off.

For the aspect-ratio-four nozzle, the curvature ranged from $\kappa = 0.06\text{cm}^{-1}$ at the minor axis to $\kappa = 3.6\text{cm}^{-1}$ at the major

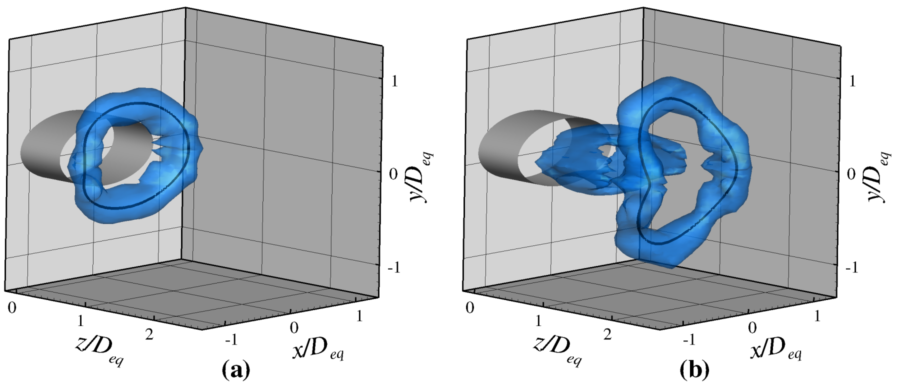

Figure 1. Iso-surfaces of 33% of the maximum vorticity at (a) $\hat{T} = U_p t/D_{eq} = 1.6$, and (b) $\hat{T} = 4.8$. The aspect-ratio-two elliptical nozzle is shown in grey, and the black line is a spline fit through the core center at each plane.

axis. This increased range of curvatures resulted in a more pronounced deformation of the vortex ring, and consequently reliable velocity fields were only obtained on the symmetry axes of the nozzle. However, these two data planes allowed us to compare the formation of the aspect-ratio-two and aspect-ratio-four elliptical vortex rings by comparing the critical formation time.

On all nine available data planes, the critical formation time based on equivalent diameter ($\hat{T} = U_p t/D_{eq}$) was determined by computing the total circulation emanating from the vortex generator as well as the final circulation in the vortex rings, and comparing the two to determine the formation time $\hat{T}$ at which the vortex ring ceased accepting vorticity [1]. Figure 2 shows the critical formation time as a function of the local curvature for data planes on both nozzles. From Fig. 2 it is evident that the critical formation time is not a function of the local curvature. Moreover, our results suggest that there is no significant difference in the critical formation tome for the aspect-ratio-two and aspect-ratio-four nozzles.

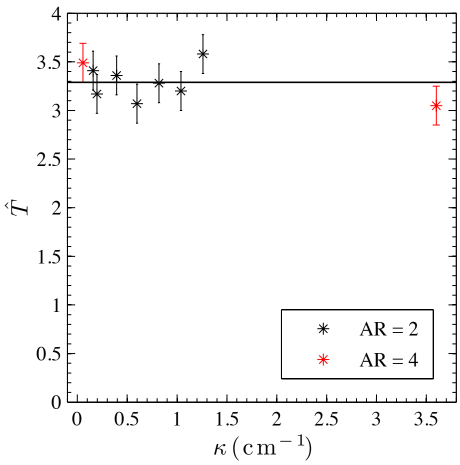

Figure 2. Critical formation time $\hat{T}$ as a function of curvature, for both elliptical nozzles. The average over all curvature points ($\hat{T} = 3.3$) is denoted by the solid black line.

CONCLUSIONS AND ON-GOING WORK

The present results suggest that, at least for elliptical nozzles, the critical formation time is not a spatially varying function of the local curvature. Instead, it is determined by the equivalent diameter of the nozzle, and is independent of the nozzle shape. However, this appears to contradict recent results by Domenichini [7] on the formation of vortices from slender orifices consisting of circular and rectilinear portions. In numerical simulations, unlike in the present experimental results, Domenichini found a noticeable difference in the pinch-off process between the circular and rectilinear portions of the orifice.

In order to resolve this discrepancy, current work is focused on conducting experiments on non-axisymmetric nozzles with two distinct curvatures. Examples include Domenichini's rectilinear/circular nozzles, as well as nozzles comprising circular arcs of distinct non-zero curvatures. The objective of these experiments is to determine if the critical formation time is determined exclusively by the equivalent diameter of the nozzle, with local parameters affecting only the shape and deformation of the vortex ring. Or, alternatively, whether local parameters such as the curvature and its rate of change play a role in defining a local critical formation time.

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