

- 4.29. A wastewater with flow rate of 5,000 liters/d and containing soluble organic material only with a BOD_L of 6,000 mg/l is being treated by methane fermentation at 35 °C in a fixed-film reactor with a volume of 2,500 liters. The reactor effluent BOD_L is found to be 150 mg/l. Estimate the total biofilm surface area in the reactor. The wastewater organic matter is primarily acetic acid (a fatty acid). Assume the reactor acts like a deep-biofilm ($S_w = 0$) completely mixed system. Also, for acetate assume D equals 0.9 cm²/d, K is 50 mg/l, $D_f = 0.8 D$, L (boundary layer thickness) equals 0.01 cm, \hat{q} is 8.4 g BOD_L /g VS_a-d, $b = 0.1/d$, and X_f equals 20 mg VS_a/cm³.
- 4.30. In the biological oxidation of benzene by a fixed-film reactor, the flux into the deep biofilm ($S_w = 0$) was found to be 5 mg/cm² of biofilm surface area per day when the benzene concentration at the biofilm surface was 15 mg/l. What do you estimate the flux will be if the benzene concentration at the biofilm surface is increased to 50 mg/l?
- Rate coefficients for benzene are as follows: $Y = 0.6$ mg VS_a/mg benzene, $\hat{q} = 6$ mg benzene/mg VS_a/d, $K = 2$ mg/l, and $b = 0.1/d$.
- 4.31. (a) Draw figures showing substrate profiles from the bulk liquid through each of the following biofilm types: (1) deep biofilm, (2) shallow biofilm, and (3) fully penetrated biofilm.
- (b) For a given location of biofilm used for wastewater treatment and under steady-state conditions of operation, the bulk solution concentration of the electron donor is 5 mg/l and the concentration of electron donor at the biofilm surface is 2 mg/l. Other properties of the biofilm are a boundary layer thickness $L = 0.01$ cm, diffusion coefficients $D = 0.75$ cm²/d, D_f of 0.5 cm²/d, organism concentration $X_f = 30$ mg VS_a/cm³. Assuming the biofilm is flat as a plate and deep, estimate the electron donor flux into the biofilm.
- (c) Based upon the stoichiometric equation for the reaction occurring in the above biofilm, 0.6 g of electron acceptor is required for each g of electron donor consumed. If the electron acceptor concentration in the bulk solution is 3 mg/l, and the diffusion coefficient for the acceptor is 1.5 times that of the donor, what is the concentration of the acceptor at the biofilm surface?

REACTORS

Many different types of reactors are used in environmental engineering practice. Individual reactors are generally designed to emphasize *suspended growth* or *biofilms*. Reactors that make use of suspended growth are also called *suspended-floc*, *dispersed-growth*, or *slurry* reactors. Reactors that make use of biofilms also are called *fixed-film* reactors, *attached-growth* reactors, or *immobilized-cell* reactors. The reactor flow regimes may be similar for the suspended growth and biofilm reactors. At times, a series of reactors may be used, some of which may be suspended growth and others may be biofilm reactors. The engineer must understand the kinetics of substrate removal by the different microbial types and the fundamental properties of different reactor types in order to select the optimum reactor or series of reactors for a given waste-treatment problem and location.

Factors influencing the choice among the different reactor types can include: the physical and chemical characteristics of the waste being considered, the concentration of contaminants being treated, the presence or absence of oxygen, the efficiency of treatment and system reliability required, the climatic conditions under which the reactor will operate, the number of different biological processes involved in the overall treatment system, the skills and experience of those who will operate the system, and the relative costs at a given location and time for construction and operation of different possible reactor configurations.

In this chapter, the different reactor types and when they are most often used are discussed. This is followed by a presentation on how to construct mass balances for reactors and how to make use of mass balances to derive basic equations that describe the relationship between reactor size and treatment performance.

5.1 REACTOR TYPES

Typical reactors used in environmental applications are illustrated in Figure 5.1. Table 5.1 contains a summary of typical applications for each type. The three basic reactors may find application as either suspended growth or biofilm reactors.

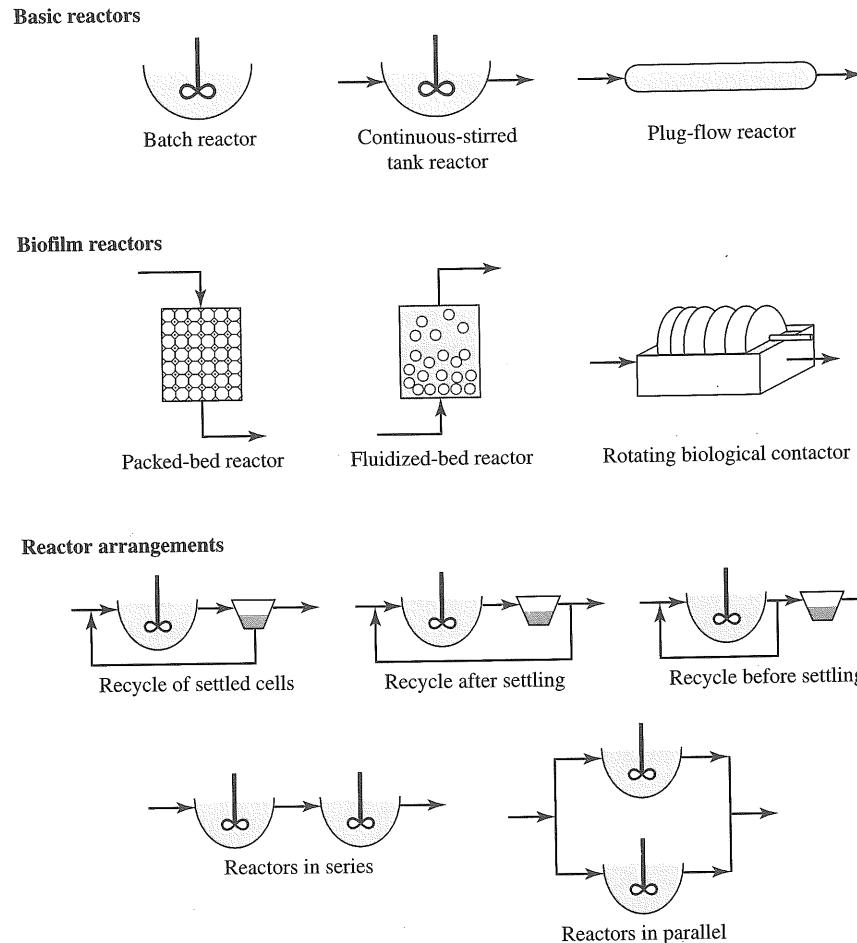


Figure 5.1 Various reactor types and arrangements

5.1.1 SUSPENDED-GROWTH REACTORS

The simplest suspended-growth reactor is the *batch reactor*. The reactor is filled with appropriate proportions of the liquid or slurry stream to be treated, the bacterial culture to be used, and required nutrients, such as nitrogen and phosphorus. Then, the reactor contents are stirred if needed to keep the reactor contents in suspension, and air or oxygen is introduced if the process is an aerobic one. The biochemical reactions then take place without new additions until the reaction is complete. Some or all of the contents are then removed, and new liquid or slurry stream and culture, etc., are added if the cycle is to be repeated. Batch reactors commonly are used in laboratory-scale basic investigations and treatability studies because of their ease of operation and the absence of mechanical pumps, which can be costly and difficult to maintain. Batch reactors are finding increasing use for treatment of slurries of soils in which

Table 5.1 Reactor types and their typical uses

Reactor Type	Typical Uses
Basic Reactors	
Batch	BOD test, high removal efficiency of individual wastewater constituents
Continuous-Flow Stirred-Tank (CSTR)	Anaerobic digestion of sludges and concentrated wastes, aerated lagoon treatment of industrial wastes, stabilization ponds for municipal and industrial wastes, part of activated sludge treatment of municipal and industrial wastewaters
Plug-Flow (PFR)	Activated sludge treatment of municipal and industrial wastes, aerated lagoon treatment of industrial wastes, stabilization ponds for municipal and industrial wastes, nitrification, high-efficiency removal of individual wastewater constituents
Biofilm Reactors	
Packed Bed	Aerobic and anaerobic treatment of municipal and industrial wastewaters, organic removal, nitrification, denitrification
Fluidized Bed	Aerobic treatment of low BOD concentration wastewaters, toxic organic biodegradation, anaerobic treatment, denitrification
Rotating Biological Contactor (RBC)	Aerobic treatment of municipal and industrial wastewaters, organic removal, nitrification
Reactor Arrangements	
Recycle	General aerobic and anaerobic treatment of municipal and industrial wastewaters, especially medium to low concentration BOD, organic removal, nitrification, denitrification
Series	BOD removal combined with nitrification or with nitrification and denitrification or combined with biological phosphorus removal, anaerobic staged treatment, stabilization pond treatment, sequential anaerobic and aerobic treatment of wastewaters such as for removal of specific toxic organic chemicals
Parallel	Generally used for redundancy and reliability in plant operation, especially with high overall wastewater flow rates
Hybrid	Used for combined forms of treatment such as organic removal and nitrification, or organic removal, nitrification, and denitrification, or organic, nitrogen, and phosphorus removal; anaerobic treatment of industrial wastewaters
Sequencing Batch	Useful for high-efficiency removal of individual constituents such as biodegradable but hazardous organics, combined removals of organics, nitrogen, and phosphorus; combination of aerobic and anaerobic processes with same microorganisms

difficult to degrade contaminants are present. The kinetics of contaminant removal in a batch reactor is similar to that of an ideal plug-flow reactor, a system that can lead to highly efficient removal of an individual contaminant. This recognition has led to the concept of *sequencing batch reactors*, a treatment system that employs several batch reactors operated in parallel. One can be filling, one can be emptying, and one or more can be treating. Thus, flow to the system of reactors can be continuous, even though treatment is batch. Indeed, with such batch operation, a single reactor may be

operated aerobically for a part of the time, such as to obtain nitrification of ammonia, and subsequently operated under anoxic conditions, such as to obtain denitrification.

The second basic reactor type is the *continuous-flow stirred-tank reactor (CSTR)*, which is also commonly called a *completely mixed reactor*. When a CSTR is used to culture organisms or to study basic biochemical phenomena in the laboratory, such reactors are also called *chemostats*. Here, the liquid or slurry stream is continuously introduced, and liquid contents are continuously removed from the reactor. Microbial culture may or may not be introduced to the reactor under normal operation. If operated properly, microorganisms that grow within the reactor continuously replace the microorganisms removed from the reactor in the effluent stream. The basic characteristic of the ideal CSTR is that the concentrations of substrates and microorganisms are the same everywhere throughout the reactor. In addition, the concentrations leaving in the effluent are the same as those in the reactor. This uniformity of concentration makes analysis of CSTRs comparatively simple. CSTRs are commonly used for aerobic and anaerobic treatment of highly concentrated organic mixtures, such as waste primary and biological sludges, as well as high strength industrial wastes.

The third basic reactor type is the *plug-flow reactor (PFR)*. This is sometimes referred to as a *tubular reactor* or a *piston-flow reactor*. As in the CSTR, the liquid or slurry stream continuously enters one end of the reactor and leaves at the other. However, in the ideal PFR, we envision that the flow moves through the reactor with no mixing with earlier or later entering flows. Hence, an element of the stream entering at one time moves down the reactor as a “plug.” The element thus moves downstream in the reactor in a discrete manner so that if one knows the flow rate to the reactor and its size, the location of the element at any time can be calculated. Unlike the CSTR, the concentrations of substrates and microorganisms vary throughout the reactor. An ideal PFR is difficult to realize in practice, because mixing in the direction of flow is impossible to prevent.

An ideal PFR has good and bad characteristics. Concentrations of substrates are highest at the entrance to the reactor, which tends to make rates there quite high. The high rate is bad when it exceeds the ability to supply sufficient oxygen in an aerobic system, results in excess organic acid production and pH problems in an anaerobic system, or causes substrate toxicity with some waste streams. However, if these problems are overcome, a PFR offers the advantage of highly efficient removal of individual contaminants, such as ammonium or trace organic contaminants. Because of the difficulty of obtaining ideal PFR conditions in a flow-through system, sequencing batch reactors, mentioned above, can be used as an alternative when the benefits of PFR treatment are desired. Even when PFRs do not achieve the ideal case, they still can provide many of the benefits of plug-flow. In addition, processes for in situ biodegradation of contaminants in groundwaters often operate similar to plug-flow processes. Here, mixing in the direction of flow (longitudinal direction) is generally small, making plug flow the natural outcome.

5.1.2 BIOFILM REACTORS

Biofilm reactors exhibit some of the characteristics of the three basic reactor types noted above, but most of the microorganisms are attached to a surface and, in this

manner, kept within the reactor. While microorganisms detach from the biofilm and may grow in the surrounding liquid, these suspended bacteria normally play a minor role in substrate removal. Three common biofilm reactor types are illustrated in Figure 5.1.

The most common biofilm reactor is a *packed bed*, in which the medium to which the microorganisms are attached is stationary. Historically, large rocks have been used as support media, but today it is more common to use plastic media or pea-sized stones. Both are lighter and offer greater surface area and pore volume per unit of reactor volume than do large rocks.

Commonly, packed-bed reactors are used for aerobic treatment of wastewaters and are known as *trickling filters* or *biological towers*. Here, the wastewater is distributed uniformly over the surface of the bed and allowed to trickle over the surface of the rock or plastic media, giving the packed-bed reactors some plug-flow character. The void space remains open to the passage of air so that oxygen can be transferred to the microorganisms throughout the reactor.

In other applications, the reactor media are submerged in the water. If the bed is not aerated, the packed-bed reactors can be used for denitrification to remove nitrate from water supplies or wastewaters or for anaerobic treatment of more concentrated industrial wastewaters through methane fermentation in an *anaerobic filter*. To do aerobic treatment, the submerged bed normally must be aerated. Backwashing of the filter is used here to prevent clogging by excessive bacterial growth.

The *fluidized-bed reactor* depends upon the attachment of microorganisms to particles that are maintained in suspension by a high upward flow rate of the fluid to be treated. In some cases, the fluidized bed is called an *expanded-bed reactor* or a *circulating-bed reactor*. The particles often are called *biofilm carriers*. The fluidized carriers may be sand grains, granular activated carbon (GAC), diatomaceous earth, or other small solids that are resistant to abrasion. The upward velocity of the fluid must be sufficient to maintain the carriers in suspension, and this depends upon the density of the carriers relative to that of water, the carrier diameter and shape, and the amount of biomass that is attached. Normally, biomass growth increases effective carrier size, but decreases its density, with the net result that carriers with higher amounts of biomass attached tend to be lighter and move higher in the reactor. This offers an advantage for cleaning carriers with excessive biological growth, as they move into the upper regions of the reactor, where they can be separated from the bed and cleaned. Once reintroduced, the cleaned carriers drop to the lower regions of the reactor until the biofilm regrows.

Fluidized-bed reactors can lie almost anywhere between a plug-flow and a completely mixed system. When the system is operated in the once-through mode, the fluid regime has strong plug-flow character. On the other hand, effluent recycle often is required to achieve high enough upward velocities for bed fluidization. With effluent recycle, the liquid regime of the fluidized bed is more like a CSTR. Fluidization of carriers and the mixing that results provide a uniform distribution of the fluid across the cross section of the reactor and also good mass transfer from the bulk fluid to the biofilm surface.

One major disadvantage of a fluidized bed is the need to carefully control bed fluidization. The upward fluid velocity must be sufficient for fluidization, but not so high that carriers are washed from the reactor. Depending on the type of fluidized

carriers used, biofilm detachment can be large due to abrasion and turbulence. This precludes using those types of carriers for microorganisms having low growth rates. Oxygen transfer also can be a problem with aerobic application for more concentrated wastewaters. Often, effluent recycle is used to oxygenate and to dilute the wastewater, as well as to maintain a constant upflow rate. Fluidized-bed reactors are used for denitrification and anaerobic wastewater treatment, processes that do not require oxygen transfer. This reactor appears to be especially good for rapid aerobic treatment of waters containing very low concentrations of organic contaminants, such as for the removal of aromatic hydrocarbons in contaminated groundwater.

A somewhat hybrid application of the fluidized-bed reactor with a dispersed-growth reactor is the *upflow anaerobic sludge bed reactor (UASBR)*, which is commonly used for anaerobic treatment of industrial wastewater. When properly operated with appropriate wastewaters, the microorganisms form granules that settle readily and serve as a biologically produced support media for additional biological growth. The rising gas bubbles, generated by rapid methanogenesis, fluidize the granules, thus effecting good mass transfer without mechanical mixing. In effect, the UASBR contains a fluidized bed of self-forming biofilm carriers.

The *rotating biological contactor (RBC)* is another approach for a biofilm reactor and, like the fluidized bed reactor, has good mixing and mass-transfer characteristics. Plastic media in a disk or spiral form are attached to a rotating shaft. Commonly used for aerobic treatment, the portion of the contactor in contact with air absorbs oxygen, and the portion within the liquid absorbs contaminants to be oxidized. Wastewater can enter from one end of the RBC and travel perpendicular to the contactors as illustrated in Figure 5.1, thus creating plug-flow character. Or, it can enter uniformly along the length of the reactor, and, in this manner, can create a completely mixed system. The RBC can be used for anoxic or anaerobic treatment as well, either by completely submerging the reactor or by placing a cover over it to exclude air.

5.1.3 REACTOR ARRANGEMENTS

Any of the three basic reactor types noted above can also be used with recycle. Recycle may involve the simple return of the effluent stream to the influent of the reactor, or it may return a concentrated stream of microorganisms after they are removed from the effluent by settling (most common), centrifugation (seldom used), membrane separation (a newer approach), or other means. Recycle has various purposes. With a CSTR, recycle of the effluent stream has little impact on reactor characteristics or performance, since the effluent stream is identical in concentration to that in the reactor itself. However, with a batch reactor or PFR, recycle dilutes chemicals in the influent stream. Dilution can be desirable to prevent excess oxygen demand, organic acid production, or substrate toxicity at the head-end of a PFR. In a biofilm reactor, the increased influent flow rate resulting from recycle can provide higher fluid velocities throughout the reactor, perhaps resulting in detachment of excess biofilm and better mass transfer kinetics. Recycle is frequently used with fluidized-bed reactors in order to maintain the required upward fluid velocity for fluidization.

Separation and recycle of microorganisms from the effluent stream, as illustrated by the left-hand drawing in Figure 5.1, are commonly practiced in the aerobic *acti-*

vated sludge process in order to maintain a large population of microorganisms within the reactor. The microorganisms are the catalysts that bring about contaminant removal, and reaction rates are proportional to the concentration of microorganisms present. Thus, organism capture and recycle provide significant benefits by greatly reducing the size of the treatment reactor, while maintaining a given efficiency of treatment. Settling tanks are most widely used for separation of microorganisms from the effluent stream for recycle, but because microorganisms do not always settle well, other separation approaches, mentioned above, are being explored.

Reactors are frequently combined in series or in parallel, as illustrated at the bottom in Figure 5.1. Reactors in series are used when different types of treatment are needed, such as organic oxidation followed by nitrification. In this example, the first reactor removes most of the organic material, and the second reactor is optimized for ammonium oxidation, or nitrification. When total-nitrogen removal is desired, a third reactor in series can be added to do denitrification. Reactors in series also are used to create plug-flow characteristics.

When connected in series, the reactors may be of the same or different types. For example, one might use a suspended-growth process for oxidizing organic matter and a biofilm process for nitrification. Another example is using a CSTR first to reduce the biodegradable toxicants below a toxic threshold, and then using a plug-flow reactor to achieve high efficiency of removal.

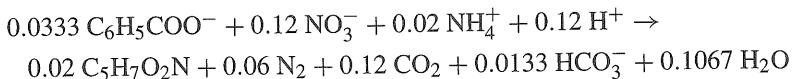
Reactors in parallel are used at most treatment plants to provide redundancy in the system so that some reactors can be out of service for maintenance, while others on a parallel track remain in operation. At larger treatment plants, parallel reactors also must be used, because the total flow to be treated far exceeds the capacity of the largest practical units available. Reactors in parallel also maintain more of a completely mixed nature, compared to the more plug-flow nature of reactors in series.

This section indicates that a few basic reactor systems can be operated in many different manners and connected together in many different ways. Each industrial or municipal wastewater treatment system has individual requirements. Frequently, many different overall system configurations could provide the required treatment. The design of choice for a given location depends not only upon the characteristics of the wastewater and the degree of treatment necessary, but also on local considerations, such as land availability, operator experience, designer experience, construction costs, and labor and energy costs. Municipalities frequently desire plants with higher construction costs and lower operating costs because of the difficulty in maintaining a constant supply of tax dollars for operation. On the other hand, industry generally prefers the opposite, because of the cost to borrow money and the likelihood that they will need to make periodic process changes when wastewater characteristics change. The engineer needs to work closely with the owners to determine the best reactor system to meet the particular needs, desires, and circumstances for each case.

5.2 MASS BALANCES

The *mass balance* is the key to design and analysis of microbiological processes. One type of mass balance is that provided by a balanced chemical equation such as

developed in Chapter 2. An example reproduced below is Equation 2.34 for benzoate ($C_6H_5COO^-$):



This equation indicates that for each 0.0333 mol of benzoate consumed by microorganisms in denitrification, 0.12 mol nitrate is required to serve as an electron acceptor, and 0.02 mol ammonium serves as a nitrogen source for bacterial growth. From this, 0.02 mol bacteria is produced, along with a defined amount of nitrogen and carbon dioxide gases, bicarbonate, and water. If the required amounts of nitrate and ammonium are not present in the wastewater, then they must be added to the treatment process. The 0.02 mol bacteria produced represents a waste product, which we call *sludge*, *waste biomass*, *waste biosolids*, or *excess biosolids*. The sludge must be removed from the system and disposed of in some acceptable manner. Knowing the quantity of waste sludge is essential for designing the sludge-disposal facilities. Thus, the mass balance given by such stoichiometric equations provides the critical information on what must be added to and removed from the process.

In reactor design, we also are interested in reaction rates, as they affect the size of the treatment system. Equations that relate reactor size to reaction rates, reaction stoichiometry, and the required treatment efficiency also depend on mass balances. An example of using mass balances for a chemostat system, one of the simplest treatment systems, was provided in Chapter 3. By following a similar approach, we can develop relevant equations for all of the biological treatment systems of interest. The rest of this chapter formalizes the mass-balance approach and provides several examples.

One of the first elements of a mass balance for a treatment system is a definition of the treatment system being addressed. For example, adding a settling tank and a recycle line to return the settled microorganisms expands the chemostat system to a solids-recycle system, such as activated sludge. Figure 5.2 shows the expanded system, which also has a separate sludge wasting line.

Next, a *control volume* must be defined. The dashed lines in Figure 5.2 illustrate three possible control volumes. The upper illustration shows a control volume around the entire treatment system, the lower left represents one around the reactor only, and the lower right shows one around the settling tank. In some cases, an appropriate control volume may be even smaller, such as a small volume within a plug-flow reactor. The choice of the control volume must be consistent with the goals of the analysis. The examples that follow illustrate why different choices are appropriate.

Once a control volume is selected, mass balances on components of interest are made. A component may enter and/or leave the control volume. For example, in Figure 5.2(a) the component may enter the reactor system only by way of the system influent stream, but may leave the system either by way of the system effluent stream or the sludge waste stream. The component may be destroyed or formed within the reactor system. If one considers a control volume around the reactor alone, as illustrated in Figure 5.2(b), then a given component can enter via the reactor influent

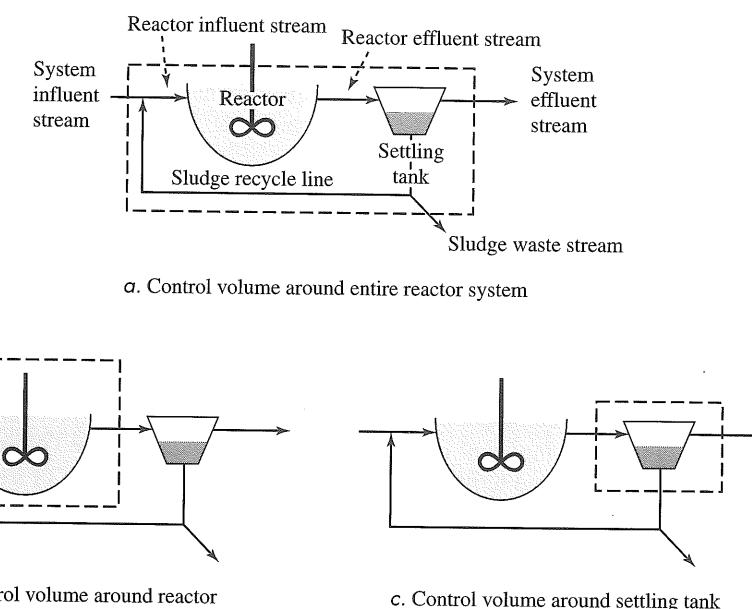


Figure 5.2 Possible control volumes around a CSTR with sludge recycle.

stream and can exit via the reactor effluent stream. In this case, the destruction or production of the component in the reactor alone is considered in the mass balance. Similarly, when the control volume is taken around the settling tank (Figure 5.2(c)), the component enters through the reactor effluent stream and leaves it either by way of the settling tank effluent stream or the sludge recycle line. Any destruction or production reactions in the settling tank are considered in the mass balance.

Mass balances made around different control volumes often lead to different kinds of information about reactor performance. The choice of a control volume depends upon the kind of information being sought. In the development of equations useful for a reactor system, mass balances on several different components of interest and around several different control volumes sometimes are required. In order to develop useful equations that are no more complicated than necessary, simplifying assumptions often are made. The limitations that result from the use of such assumptions must be understood, so that the derived equations are not used for situations where they do not apply.

A very important aspect of mass balances is that each component must have its own mass balance. The components may include the chemical oxygen demand (COD) of the waste stream, which is frequently used as a measure of waste strength; total organic carbon (TOC), another measure of waste strength; biomass; oxygen, a key electron acceptor; nitrate- and ammonium-nitrogen and phosphorus, macronutrients; or others. If one has a balanced stoichiometric equation for the reaction, such as represented by Equation 2.34, the consumption of one component, such as the

electron donor, can be used directly for other components, such as electron acceptor consumption or biomass production. A common mistake made by beginners is attempting to make a single mass balance equation that includes changes in mass of more than one component, such as COD and biomass. The rule that must be followed is: One mass balance for each component!

Once a reactor system, a control volume, and the components on which to do the mass balances are selected, then the mass-balance equations can be written. The mass balance always is defined in terms of rates of mass change in the control volume. In word form, that is,

$$\begin{aligned} \text{Rate of mass accumulation in control volume} &= \\ \text{rate(s) of mass in} - \text{rate(s) of mass out} + \text{rate(s) of mass generation} &= 0 \end{aligned} \quad [5.1]$$

The rate of mass accumulation always appears on the left side. Accumulation is the total mass of the component in the system, or the product of the volume times the concentration. The rate term takes the general mathematical form of $d(VC)/dt$, in which V is the volume of the control volume, C is the component's concentration, and d/dt is the differential with respect to time.

Mass in and mass out refer to mass that crosses the control-volume boundaries. Generation refers to the formation of the component of interest within the control volume. If generation is negative, then the component is destroyed rather than being formed within the control volume. Some components may be formed by some reactions and destroyed by others. For example, bacterial cells may be produced through consumption of an electron donor or "food" source. Here, the generation rate is positive. Endogenous respiration or predation may also destroy microorganisms, making generation negative. For the terms on the right side, more than one reaction or mechanism may work. Thus, each of the three terms can be plural. However, the accumulation is not plural; only one total mass accumulation exists for a component in a control volume.

Equation 5.1 may take many mathematical forms, depending upon the nature of the control volume, the manner in which mass flows into and out of the control volume, and what kind of reactions generate or destroy the component. The best way to learn how to apply Equation 5.1 is through a series of examples, which follows.

5.3 A BATCH REACTOR

For a batch reactor operated with mixing, as illustrated in Figure 5.1, the control volume consists of the entire reactor. Components in the reactor are distributed uniformly throughout the reactor so that the concentration of any component is the same at any location within the reactor at any time. We consider the case in which the reactor liquid volume, V , does not change with time. Then, only component concentrations change with time, or the rate of mass accumulation equals VdC/dt .

We select the components as the bacteria and their rate-limiting substrate, which is most frequently the electron donor, as our choice here. We assume that all other

bacterial requirements, such as electron acceptor and nutrients, are sufficiently high in concentration that they impose no limitations on organism growth rate. At time = 0, the reactor contains component concentrations of microorganisms (X^0 , mg/l) and rate-limiting substrate (S^0 , mg/l).

Although changes in microorganism concentration and substrate concentration are interdependent, we must construct a separate mass balance on each of these components using the form of Equation 5.1. We begin with a mass balance for substrate,

$$\begin{aligned} \text{Mass rate of substrate accumulation in control volume} &= \\ \text{rate of mass in} - \text{rate of mass out} + \text{rate of mass generation} &= 0 \end{aligned} \quad [5.2]$$

While the microorganisms are consuming substrate, no substrate is added or removed from the batch reactor. Thus, over this time period the mass of substrate accumulating in the reactor equals the mass of substrate generated within the reactor. On the other hand, the substrate is consumed or destroyed by the microorganisms, and generation has a negative sign. In mathematical form, Equation 5.2 becomes:

$$V \frac{dS}{dt} = V r_{ut} \quad [5.3]$$

Commonly, the rate of substrate utilization is assumed to follow Monod kinetics, as given by Equation 3.6. With this substitution, we obtain

$$V \frac{dS}{dt} = V \left(-\frac{\hat{q}S}{K + S} X_a \right)$$

or,

$$\frac{dS}{dt} = -\frac{\hat{q}S}{K + S} X_a \quad [5.4]$$

We would like to integrate Equation 5.4 to determine how S changes with time in the reactor, but X_a also changes with time. In order to determine how X_a changes, we construct our second mass balance, this time on microorganisms in the reactor. The format of Equation 5.1 converts to

$$\begin{aligned} \text{Mass rate of organism accumulation in control volume} &= \\ \text{rate of mass in} - \text{rate of mass out} + \text{rate of mass generation} &= 0 \end{aligned} \quad [5.5]$$

With μ being the net specific growth rate of organisms (Equation 3.5), the mathematical form is similar to Equation 5.3:

$$V \frac{dX_a}{dt} = V(\mu X_a) \quad [5.6]$$

If we assume that the organism growth rate follows Monod kinetics and if decay, as

well as growth, is considered, then combining Equation 5.6 with Equation 3.5 gives

$$V \frac{dX_a}{dt} = V \left(\hat{\mu} \frac{S}{K + S} - b \right) X_a$$

or

$$\frac{dX_a}{dt} = \left(\hat{\mu} \frac{S}{K + S} - b \right) X_a \quad [5.7]$$

Here, we again see the interdependence between X_a and S , both of which vary with time. Thus, in order to solve for X_a and S as functions of time, we have to consider mass balance equations 5.4 and 5.7 together. We also need initial conditions, which have already been specified by:

$$X_a(0) = X_a^0 \quad S(0) = S^0 \quad [5.8]$$

Due to the nonlinear Monod forms, the system of Equations 5.4, 5.7, and 5.8 cannot be solved analytically. If it is to be solved, it must be done with a numerical solution, which can readily be accomplished with a computer program or spreadsheet. (This type of solution is demonstrated in the website chapter "Complex Systems.") However, an analytical solution can be obtained if organism decay is considered to be negligible (or not very important to the result), in which case b in Equation 5.7 can be taken to equal zero. This is a satisfactory assumption for cases of batch growth in which organism decay is a small factor while the microorganisms are growing rapidly. Ignoring decay will introduce errors whenever the cells do not grow very rapidly, such as after the exponential phase of batch growth and in most continuous processes.

In the absence of decay, the organism concentration at any time equals the initial concentration, X_a^0 , plus that which results from substrate consumption during that time, $Y\Delta S$, or:

$$X_a = X_a^0 + Y\Delta S \quad \text{or} \quad X_a = X_a^0 + Y(S^0 - S) \quad [5.9]$$

By substitution of Equation 5.9 into Equation 5.4, one ordinary differential equation is obtained:

$$\frac{dS}{dt} = -\frac{\hat{q}S}{K + S} \left[X_a^0 + Y(S^0 - S) \right] \quad [5.10]$$

This equation can be integrated, subject to the boundary conditions given by Equation 5.8, to yield:

$$t = \frac{1}{\hat{q}} \left\{ \left(\frac{K}{X_a^0 + YS^0} + \frac{1}{Y} \right) \ln(X_a^0 + YS^0 - YS) - \left(\frac{K}{X_a^0 + YS^0} \right) \ln \frac{SX_a^0}{S^0} - \frac{1}{Y} \ln X_a^0 \right\} \quad [5.11]$$

It would be desirable to have an equation that explicitly gives S as a function of t , but because of the complexity of the equation, this is not possible. Here, a computer spreadsheet can be very useful for solving for S when t is known.

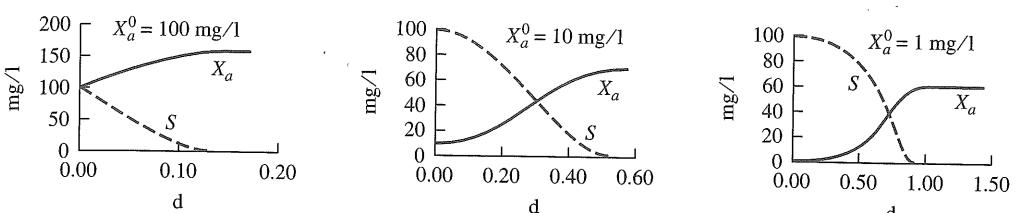


Figure 5.3 Change in S and X_a in a batch reactor with time for different concentrations of X_a^0 . Here, $\hat{q} = 10 \text{ mg/mg VSS}_a \text{ d}$, $K = 20 \text{ mg/l}$, $Y = 0.6 \text{ mg VSS}_a/\text{mg}$, $b = 0.0/\text{d}$, and $S^0 = 100 \text{ mg/l}$. Note the different time scales.

Figure 5.3 illustrates how the choice of X_a^0 affects bacterial growth and the substrate concentration over the course of the batch reaction. If one adds the highest initial organism concentration (100 mg VSS_a/l), substrate is consumed in a much shorter period of time than if the lowest concentration (1 mg/l) is added. For example, the time to reduce the substrate concentration to near zero is reduced by a factor of 8 (from 0.8 d to 0.1 d) with a 100-fold increase in initial organism concentration. For the lowest initial organism concentration, a *lag period* occurs before the onset of significant substrate utilization. This lag reflects the time needed for the small inoculum to grow to a concentration able to consume substrate at a rate that is noticeable. Significant removal of substrate occurs when X_a is about 10 mg VSS_a/l. No lag time is in evidence when the starting concentration is 10 mg/l or higher. The increase in microorganism concentration between time zero and just after the substrate is depleted is the same in all cases and equals YS^0 , which in this case equals 60 mg/l.

5.4 A CONTINUOUS-FLOW STIRRED-TANK REACTOR WITH EFFLUENT RECYCLE

The CSTR, which is the same as a chemostat, was thoroughly discussed in Chapter 3. It differs from a batch reactor in two profound ways. First, it has a continuous flow in and out, whereas the batch reactor has no flows in or out. Second, the CSTR can reach a steady state, where the change of mass accumulation becomes zero, or $VdC/dt = 0$. Steady state is not a relevant concept for a batch reactor, as long as components are reacting. Since the steady-state mass balances for the chemostat were thoroughly developed in Chapter 3, they need not be repeated here. The reader is advised to review the set up and solution of the mass balances in Chapter 3 in light of the information on mass balances presented in this chapter. One new case is presented here to illustrate a feature not discussed in Chapter 3 and as preparation for treating more complex reactor systems.

A CSTR with effluent recycle is illustrated in Figure 5.4. The difference between this case and the normal CSTR is that some of the effluent stream, containing effluent active microorganisms and substrate, is recycled back at a flow rate Q_r and introduced

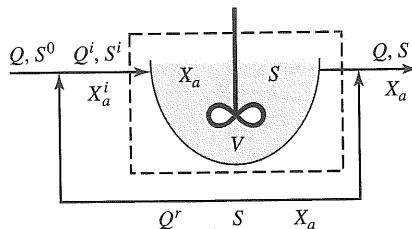


Figure 5.4 CSTR with effluent recycle

back into the reactor. The question is, how does this affect reactor performance? We will see that it does not affect reactor performance at all. In order to demonstrate this, we need to select the control volume, which might be taken around the entire system or just around the reactor itself as is the case in Figure 5.4. We can readily show that, in this case, it makes no difference to the results which is used as the control volume. But first, we need to make mass balances around the point where the influent and recycle flows come together in order to determine Q^i , S^i , and X^i . A mixing point has no reaction, and so the rate of mass flow into the mixing point equals the rate of flow out, or:

$$QS^0 + Q^r S = Q^i S^i \quad \text{and} \quad QX_a^0 + Q^r X_a^r = Q^i X_a^i$$

From which,

$$S^i = \frac{QS^0 + Q^r S}{Q^i} \quad \text{and} \quad X_a^i = \frac{QX_a^0 + Q^r X_a^r}{Q^i} \quad [5.12]$$

Also,

$$Q^i = Q + Q^r \quad [5.13]$$

Now, if we do the mass balance for substrate around the reactor control volume depicted in Figure 5.4, we obtain for the steady-state case

$$0 = Q^i S^i - Q^i S + r_{ut} V \quad [5.14]$$

Then, by making appropriate substitutions from Equations 5.12 and 5.13 and simplifying we obtain

$$0 = Q(S^0 - S) + r_{ut} V \quad [5.15]$$

Equation 5.15 is identical to Equation 3.15, the case for a chemostat without recycle. Thus, simple recycle for a CSTR does not change substrate removal compared with that obtained without recycle. A mass balance on microorganisms can be performed similarly, and the result is the same: Organism concentrations within the reactor and in the reactor effluent are not affected by effluent recycle, since the same mass flow that leaves the reactor returns to the reactor. We will see, however, that this is not the case with a plug-flow reactor, where concentrations are not the same everywhere.

5.5 A PLUG-FLOW REACTOR

With a PFR, the substrate and active-organism concentrations vary over the length of the reactor. Thus, the appropriate control volume is an incremental segment along the flow path in the reactor, as illustrated in Figure 5.5. Mass balances on substrate and on active microorganisms, following Equation 5.1, lead to:

Substrate

$$\Delta V \frac{\Delta S}{\Delta z} = QS - Q(S + \Delta S) + r_{ut} \Delta V \quad [5.16]$$

Active microorganisms

$$\Delta V \frac{\Delta X_a}{\Delta z} = QX_a - Q(X_a + \Delta X_a) + r_{net} \Delta V \quad [5.17]$$

where r_{ut} and r_{net} are the reaction rates for substrate (Equation 3.6) and active organisms (Equation 3.8). We consider the steady-state case, for which influent flow rate, substrate concentration, and active-organism concentration do not change with time. Thus, the left sides of Equations 5.16 and 5.17 are zero. If the reactor cross-sectional area (A) is constant throughout the reactor, then the area of the control volume is $A = \Delta V / \Delta z$, and the velocity of flow within the reactor is $u = Q/A$. With these substitutions and for steady-state conditions, Equations 5.16 and 5.17 become:

Substrate at steady state

$$u \frac{\Delta S}{\Delta z} = r_{ut} \quad [5.18]$$

Active microorganisms at steady state

$$u \frac{\Delta X_a}{\Delta z} = r_{net} \quad [5.19]$$

If we now let Δz approach zero and assume that the Monod reaction applies for substrate utilization (Equation 3.6) and that organism net growth represents growth and decay (Equation 3.8), then Equations 5.18 and 5.19 become:

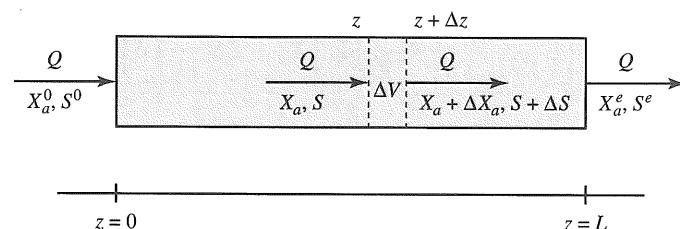


Figure 5.5 Control volume (ΔV) within a plug-flow reactor

Substrate at steady state with Monod kinetics

$$u \frac{dS}{dz} = -\hat{q} \frac{S}{K + S} X_a \quad [5.20]$$

Active microorganisms at steady state with growth and decay

$$u \frac{dX_a}{dz} = Y \hat{q} \frac{S}{K + S} X_a - b X_a \quad [5.21]$$

This series of equations cannot be solved analytically, and so numerical approaches must be used. However, if we again ignore organism decay ($b = 0$), then an analytical solution is possible. First, we combine Equations 5.20 and 5.21 to eliminate the Monod terms:

$$u \frac{dX_a}{dz} = -u Y \frac{dS}{dz} \quad [5.22]$$

We cancel u and take integrals to obtain

$$\int_{X_a^0}^{X_a} dX_a = -Y \int_{S^0}^S dS \quad [5.23]$$

Integrating gives

$$X_a = X_a^0 + Y(S^0 - S) \quad [5.24]$$

Substituting Equation 5.24 into Equation 5.20 gives a differential equation with only two variables, S and z :

$$u \frac{dS}{dz} = -\hat{q} \frac{S}{K + S} [X_a^0 + Y(S^0 - S)] \quad [5.25]$$

The ratio dz/u has dimensions of time and equals the differential time, dt , for an element of water to move along the reactor a distance dz . Substituting dt for dz/u in Equation 5.25 yields a differential equation that is exactly the same as Equation 5.10 for the batch reactor. Indeed, integration of Equation 5.25 results in an equation almost identical to Equation 5.11. The only difference is that t for the batch reactor is replaced by z/u in the integrated form for the plug-flow reactor:

$$\begin{aligned} \frac{z}{u} &= \frac{1}{\hat{q}} \left\{ \left(\frac{K}{X_a^0 + YS^0} + \frac{1}{Y} \right) \ln \left\{ X_a^0 + YS^0 - YS \right\} \right. \\ &\quad \left. - \left(\frac{K}{X_a^0 + YS^0} \right) \ln \frac{SX_a^0}{S^0} - \frac{1}{Y} \ln X_a^0 \right\} \end{aligned} \quad [5.26]$$

We obtain an expression for the effluent concentration from the batch reactor by letting $z = L$. We also note that L/u is equal to V/Q , the hydraulic detention time, θ , for the reactor. With these substitutions, the following solution is identical

to Equation 5.11 with θ replacing t :

$$\begin{aligned} \theta &= \frac{1}{\hat{q}} \left\{ \left(\frac{K}{X_a^0 + YS^0} + \frac{1}{Y} \right) \ln (X_a^0 + YS^0 - YS^e) \right. \\ &\quad \left. - \left(\frac{K}{X_a^0 + YS^0} \right) \ln \frac{S^e X_a^0}{S^0} - \frac{1}{Y} \ln X_a^0 \right\} \end{aligned} \quad [5.27]$$

We thus see that a PFR works exactly like a batch reactor. In practice, however, it is difficult to operate a PFR according to the assumptions involved. A PFR has no mixing or short-circuiting of the fluid along the flow direction. This is impossible to achieve in a real continuous-flow reactor. At a minimum, wall effects slow the fluid near the wall boundaries relative to the velocity near the middle. Aeration or mixing to keep the biomass in suspension introduces a large amount of mixing in all directions. Methods to achieve as much of a plug-flow character as possible include using a very long, narrow reactor and using many reactors in series. These measures help somewhat, but some mixing and short-circuiting are inevitable. If achieving the reaction kinetics represented by Equation 5.27 is of paramount importance, a batch reactor is the prudent choice, although it presents its own problems. For example, time is required to fill and empty a batch reactor, time that might otherwise be used for treatment. In order to minimize downtime, a batch reactor can be operated while it is filling.

5.6 A PLUG-FLOW REACTOR WITH EFFLUENT RECYCLE

One major difference between a PFR and a CSTR is that microorganisms must be introduced at the influent end of the PFR. If no organisms are introduced, then no microorganisms are present to carry out substrate removal, and the system fails to do any treatment. One way to introduce microorganisms into a PFR is to use effluent recycle. In this manner, a portion of the microorganisms in the effluent is brought back to the influent stream. The impact of this can be seen by constructing a mass balance similar to that constructed for the case of a PFR with recycle. A PFR with effluent recycle is depicted in Figure 5.6. As with the CSTR, mass balances to determine the flow rate and concentrations of substrate and microorganisms entering the reactor yield equations identical to Equations 5.12 and 5.13, that is,

$$S^i = \frac{Q S^0 + Q^r S}{Q^i} \quad \text{and} \quad X_a^i = \frac{Q X_a^0 + Q^r X_a^r}{Q^i}$$

and

$$Q^i = Q + Q^r$$

It should be noted that $X^r = X^e$ and $S = S^e$.

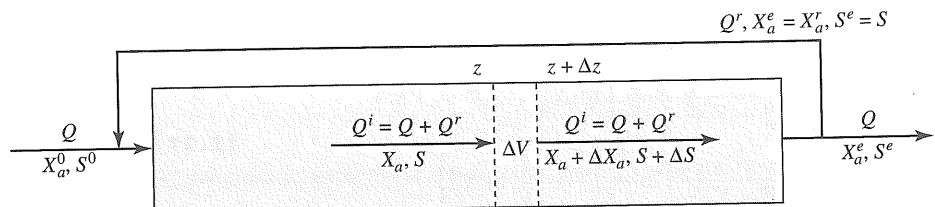


Figure 5.6 A plug-flow reactor (PFR) with effluent recycle

The reactions occurring throughout the reactor are obtained in exactly the same manner as with the simple PFR: by conducting a mass balance around a control volume taken within the reactor itself, as illustrated in Figure 5.6. Here, Equations 5.16 to 5.27 apply, but with Q^0 , X^0 , and S^0 now taken to equal Q^i , X^i , and S^i . For the case in which $b = 0$, an integrated form of an equation relating the effluent concentrations as a function of detention time can be obtained. In this manner, the effluent concentration of X_a can be obtained:

$$X_a^e = X_a^0 + Y(S^0 - S^e) \quad [5.28]$$

With this, the series of equations can be combined and integrated to give:

$$\frac{V}{Q^i} = \frac{1}{\hat{q}} \left\{ \left(\frac{K}{X_a^i + YS^i} + \frac{1}{Y} \right) \ln(X_a^i + YS^i - YS^e) - \left(\frac{K}{X_a^i + YS^i} \right) \ln \frac{S^e X_a^i}{S^i} - \frac{1}{Y} \ln X_a^i \right\} \quad [5.29]$$

Of interest is the impact of recycle on the performance of a PFR. We define the recycle ratio R , as

$$R = \frac{Q^r}{Q} \quad [5.30]$$

and the detention time, θ , as

$$\theta = \frac{V}{Q} = \frac{V(1+R)}{Q^i} \quad [5.31]$$

The above series of equations can be solved using a spreadsheet to determine S^e as a function of θ and R . The results, illustrated in Figure 5.7, were obtained in this manner and using rate coefficients typical of aerobic treatment of organic wastewaters. The effluent concentration is illustrated using an arithmetic scale in the upper graph and a logarithmic scale in the lower graph. The upper graph in Figure 5.7 illustrates that, for each individual recycle rate, there is a detention time below which the effluent concentration equals the influent concentration; in other words, no treatment takes place. This detention time is equivalent to the washout detention time discussed in Chapter 3, or θ_x^{\min} as given by Equation 3.26 for the CSTR. Using the influent substrate concentration and kinetic coefficients illustrated in Figure 5.7, the equivalent θ_x^{\min} for the CSTR is 0.2 days, or close to the value indicated for

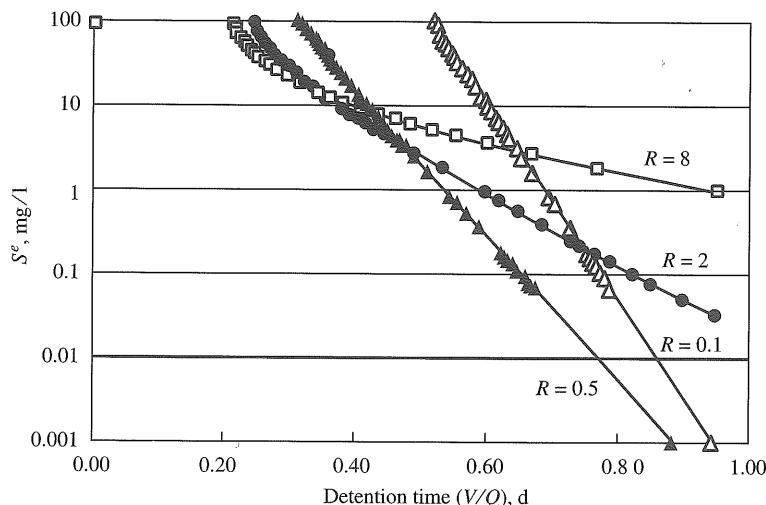
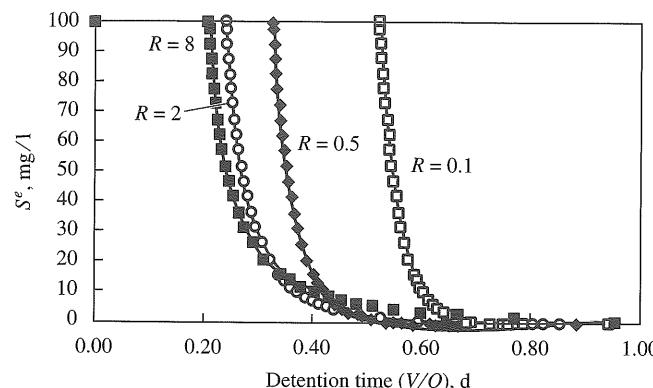


Figure 5.7 Effect of recycle ratio (R) and hydraulic detention time (θ) on effluent substrate concentration for a plug-flow reactor with effluent recycle. Case considered has $S^0 = 100 \text{ mg/l}$, $Y = 0.6 \text{ mg/mg}$, $K = 20 \text{ mg/l}$, $\hat{q} = 10 \text{ mg/mg-d}$, and $b = 0$.

$R = 8$ for plug-flow with recycle. In effect, as R increases in a plug-flow reactor with high recycle, the influent concentration to the reactor (S^i) decreases. When R approaches infinity, the PFR with recycle becomes identical to a CSTR in theoretical performance. Indeed, the performance with $R = 8$ is very close to that of a CSTR. The washout detention time is larger when R is less than 8.

Figure 5.7 helps illustrate the relative benefits of a CSTR and a plug-flow reactor with recycle. Considering that the PFR with recycle case with $R = 8$ is very similar to a CSTR's performance, one can see that the CSTR improves reliability when the

system is operated near washout. This would be a good choice if contaminant removal of 80 to 90 percent were satisfactory. However, if contaminant removal of 99.9 percent were required, the lower graph shows that operation as a PFR with a low recycle ratio is much more desirable. With the recycle system, there is an optimal recycle ratio that provides reasonably reliable performance at low detention times and highly efficient contaminant removal. An important lesson is that effluent recycle with a CSTR does not change system performance, but with a PFR, recycle is essential and has a great impact on performance.

As already noted, true plug-flow operation in a continuous flow system is not possible, as mixing always occurs in the direction of flow. The actual contaminant removal in an operational plug-flow reactor with recycle lies somewhere between that of the theoretical removal and that of a CSTR.

5.7 REACTORS WITH RECYCLE OF SETTLED CELLS

One of the most widely used suspended-growth reactors employs microorganism recycle from a settling tank, as depicted in Figure 5.2. The activated sludge treatment system used at a large majority of the municipal treatment plants in the United States is of this type, as are many used for the treatment of industrial wastewaters. Although many modifications to this process are described in Chapter 6, we develop the basic mass balances for a CSTR and a PFR with settling and microorganism recycle. The primary advantage of settling and recycle of microorganisms is that a much smaller reactor volume is required, because the biomass is captured and built up to a much higher concentration than is possible in a normal CSTR or PFR. As noted in Equation 3.6, the substrate-utilization rate is directly proportional to the active microorganism concentration. Any method that increases the concentration of microorganisms in the reactor also increases the volumetric reaction rate and, in this manner, decreases the required reactor volume. Counteracting the advantage of a smaller reactor volume is the cost of the settler and the recycling system. The comparison is between the cost of the smaller reactor plus a settler versus a larger reactor without a settler.

5.7.1 CSTR WITH SETTLING AND CELL RECYCLING

We begin with the simplest case, a CSTR with settling and microorganism recycle, as illustrated in Figure 5.2. The results we want can be obtained by constructing a mass balance around the whole reactor, as shown in the upper illustration. Figure 5.8 identifies in detail the items of interest in our mass balance. Since the settling tank is not likely to be 100-percent efficient in capturing microorganisms, some may be present in the effluent at concentration X_a^e . The microorganisms that settle in the settling tank form a thickened biological sludge that is removed for recycle back to the reactor. The concentration of microorganisms in this recycle line is X_a^r . Because we obtain a net growth of microorganisms, the net growth—called excess sludge or waste sludge—is removed from the system for subsequent sludge treatment and disposal.

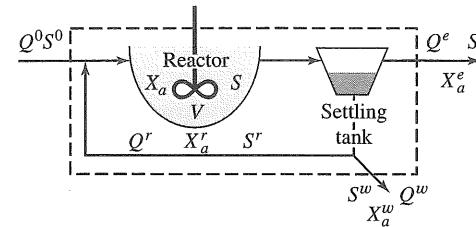


Figure 5.8 CSTR with sludge recycle

This sludge wasting of microorganisms is from the recycle line at a flow rate Q^w and a concentration X_a^w . Microorganism wasting may also occur directly from the reactor itself, but the approach we take is far more common.

In order to develop mass balances for the reactor, we again need some simplifying assumptions. Assumptions that we make here as a first exercise are: (1) biodegradation of the substrates takes place in the reactor only, no biological reactions take place in the settling tank, and the biomass in the settler is insignificant; (2) no active microorganisms are in the influent to the reactor ($X_a^0 = 0$); and (3) the substrate is soluble so that it cannot settle out in the settling tank. With these assumptions, we proceed with a mass balance for microorganisms around the control volume in Figure 5.8:

$$V \frac{dX_a}{dt} = 0 - (Q^e X_a^e + Q^w X_a^w) + [Y(-r_{ut})V - bX_a V] \quad [5.32]$$

Likewise, a mass balance for substrate gives:

$$V \frac{dS}{dt} = Q^0 S^0 - (Q^e S^e + Q^w S^w) + r_{ut} V \quad [5.33]$$

Equations 5.32 and 5.33 are general and can be used to describe the operation of the reactor under nonsteady- or steady-state conditions. In order to illustrate the general principles of operation of the treatment system, we consider here only the steady-state case. That is, we assume that the reactor has been operating continuously for some time with all flows and influent concentrations constant over time. At steady state, the changes in accumulation are zero, or:

$$\text{Steady-state conditions } V \frac{dX_a}{dt} = 0 \quad \text{and} \quad V \frac{dS}{dt} = 0 \quad [5.34]$$

We now consider the definition of solids retention time (θ_x) given by Equation 3.22, which is repeated here:

$$\theta_x = \frac{\text{active biomass in the system}}{\text{production rate of active biomass}}$$

Based upon our assumption for this exercise that no active biomass is in the settling tank, the active biomass in the system is only that in the reactor, or $X_a V$. Under steady-state conditions, the production rate of active biomass must just equal its rate

of removal from the control volume, either in the effluent ($Q^e X_a^e$) or in the waste sludge stream ($Q^w X_a^w$). Thus,

$$\theta_x = \frac{X_a V}{Q^e X_a^e + Q^w X_a^w} \quad [5.35]$$

We can now rearrange Equation 5.32 for the steady-state case to give:

$$\frac{Q^e X_a^e + Q^w X_a^w}{X_a V} = \frac{Y(-r_{ut})}{X_a} - b \quad [5.36]$$

Seeing the similarity between the left side of Equation 5.36 and the right side of Equation 5.35, we make the appropriate substitution to give an important result:

$$\frac{1}{\theta_x} = \frac{Y(-r_{ut})}{X_a} - b \quad [5.37]$$

Equation 5.37 is general for a CSTR with settling and recycle and can be applied whatever the form of the biological reaction, r_{ut} , may be. If we assume that it takes the usual form of the Monod reaction (Equation 3.6), then we obtain the following expression:

$$\frac{1}{\theta_x} = Y \frac{\hat{q} S}{K + S} - b \quad [5.38]$$

We solve this equation explicitly for S ,

$$S = K \frac{1 + b\theta_x}{\theta_x(Y\hat{q} - b) - 1} \quad [5.39]$$

Equation 5.39 is identical to Equation 3.24, which was developed for the chemostat without settling and recycle. So then, what is unique about the CSTR with settling and microorganism recycle? The answer is that the retention time of the microorganisms in the system (θ_x) is separated from the hydraulic detention time (θ). Thus, one can have a large θ_x , in order to obtain high efficiency of substrate removal, and at the same time have a small θ , which translates into a small reactor volume. For example, in a normal activated sludge biological treatment system used to treat municipal wastewaters, a typical solids retention time is 4 to 10 d, while the hydraulic detention time is only 4 to 10 h. Thus, while operating at the same treatment efficiency for substrate, the reactor can be one-twenty-fourth the volume that it would otherwise be if it were designed as a CSTR system without settling and recycle.

Now, we consider the concentration of active microorganisms in the reactor. We first solve Equation 5.37 for X_a ,

$$X_a = \theta_x \frac{Y(-r_{ut})}{1 + b\theta_x} \quad [5.40]$$

Next, we return to Equation 5.33, the mass-balance equation for substrate, and apply steady-state (Equation 5.34) to obtain r_{ut} as a function of S ,

$$-r_{ut} = \frac{Q^0 S^0 - Q^e S^e - Q^w S^w}{V} \quad [5.41]$$

For the CSTR, we see that the substrate concentration in the reactor, S , is equal to the concentration in the effluent S^e and in the waste sludge line, S^w , since no reaction occurs in the settling tank. Also, through mass balance, $Q^e + Q^w = Q^0$. With these substitutions into Equation 5.41, we obtain,

$$-r_{ut} = \frac{Q^0 (S^0 - S)}{V} = \frac{(S^0 - S)}{\theta} \quad [5.42]$$

Like Equation 5.37, Equation 5.42 is another general representation, this time of the utilization rate in terms of reactor characteristics and performance. Substituting Equation 5.42 into Equation 5.40 gives:

$$X_a = \frac{\theta_x}{\theta} \frac{Y(S^0 - S)}{1 + b\theta_x} \quad [5.43]$$

Equation 5.43 indicates that the active biomass concentration in the reactor depends on the ratio of solids retention time to the hydraulic detention time, θ_x/θ . We call this important ratio the *solids-concentration ratio*. We might note that for a CSTR without settling and recycle, $\theta_x/\theta = 1$, and Equation 5.43 becomes identical to Equation 3.25. For an activated sludge system treating municipal wastewaters with a solids-concentration ratio of about 24 and the same θ_x , the concentration of active organisms in the reactor is about 24 times higher than it would be without biomass recycling.

Also crucially important is the quantity of waste sludge (or biosolids) produced in the system, as it must be removed continually in order to maintain the steady-state conditions. In addition, the waste sludge must be properly disposed of. Therefore, the rate of sludge wasting is essential for operating the treatment system and for determining the total cost of construction and operation for the system.

We can see from Figure 5.8 that, at steady state, the mass rate of *active biomass production* (r_{abp} , M/T) must just equal the rate at which biomass leaves the system from the effluent stream and the waste stream:

$$r_{abp} = Q^e X_a^e + Q^w X_a^w \quad [5.44]$$

Substituting this equation into Equation 5.35 and rearranging yields:

$$r_{abp} = \frac{X_a V}{\theta_x} \quad [5.45]$$

With the addition of Equation 5.45, we now have a series of equations that allow us to design a treatment system consisting of a CSTR with settling and recycle. The important equations are summarized in Table 5.2.

The equations in Table 5.2 can be used for a CSTR without a settler by letting $\theta_x = \theta$. Because inert biomass and total volatile solids are particles, like active biomass, the concentrations X_i and X_v take the same form as for a chemostat, but are multiplied by the solids-concentration factor, as shown in Equations 5.46 and 5.47. The minimum value of the mean cell residence time (θ_x^{\min}) and its limiting value [$\theta_x^{\min}]_{lim}$] are identical to the case without settling (Equations 3.26 and 3.27) and are listed in Table 5.2. The total biological-solids production rate is analogous to Equation 5.45 and is shown as Equation 5.48 in Table 5.2. The student should be

Table 5.2 Summary of applicable equations for a CSTR with settling and recycle of microorganisms (operating at steady state, treating a soluble substrate, and with no input of active biomass)

Hydraulic Detention Time (θ):

$$\theta = \frac{V}{Q^0} \quad [3.20]$$

Solids Retention Time, SRT (θ_x):

$$\theta_x = \frac{X_a V}{X_a^e Q^e + X_a^w Q^w} \quad [5.35]$$

SRT at which microorganism washout results (θ_x^{\min}), and the limit thereto:

$$\theta_x^{\min} = \frac{K + S^0}{S^0(Y\hat{q} - b) - Kb} \quad S \rightarrow S^0 \quad [3.26]$$

$$[\theta_x^{\min}]_{\lim} = \frac{1}{Y\hat{q} - b} \quad S \rightarrow \infty \quad [3.27]$$

Reactor or Effluent Substrate Concentration ($S = S^e$):

$$S = K \frac{1 + b\theta_x}{\theta_x(Y\hat{q} - b) - 1} \quad [5.39]$$

Reactor Minimum Substrate Concentration (S_{\min}):

$$S_{\min} = K \frac{b}{Y\hat{q} - b} \quad \theta_x \rightarrow \infty \quad [3.28]$$

Reactor Active Microorganism Concentration (X_a):

$$X_a = \theta_x \frac{X(-r_{ut})}{1 + b\theta_x} \quad [5.40]$$

$$X_a = \frac{\theta_x}{\theta} \frac{Y(S^0 - S)}{1 + b\theta_x} \quad [5.43]$$

Reactor Inert Microorganism Concentration (X_i):

$$X_i = \frac{\theta_x}{\theta} [X_i^0 + X_a(1 - f_d)b\theta] \quad [5.46]$$

Reactor volatile suspended solids concentration (X_v):

$$X_v = X_i + X_a \quad [5.47]$$

$$X_v = \frac{\theta_x}{\theta} \left[X_i^0 + \frac{Y(S^0 - S)(1 + (1 - f_d)b\theta_x)}{1 + b\theta_x} \right] \quad [5.47]$$

Active Biological Sludge Production Rate (r_{abp}):

$$r_{abp} = \frac{X_a V}{\theta_x} \quad [5.45]$$

Total Biological Solids Production Rate (r_{tbp}):

$$r_{tbp} = \frac{X_v V}{\theta_x} \quad [5.48]$$

able to derive these equations using mass balance as performed above and using the definitions in Chapter 3.

One aspect of using the θ_x for design of a CSTR that many find baffling at first is that the effluent concentration, S , as given by Equations 3.24 and 5.39, is independent

of the influent concentration S^0 . Only one operational variable affects S , and it is θ_x . All other parameters in the equations are coefficients. How can this be? Why is it that when one operates a CSTR at a constant θ_x , the effluent concentration remains the same, regardless of the influent concentration? The answer is related to the fact that as the influent concentration increases, so does the concentration of active microorganisms in the reactor (see Equations 3.25 and 5.43). The increased biomass that results is sufficient to consume the additional substrate that is added to the reactor. Another way to view this is that the organisms' growth rate (μ) and θ_x are equal to the inverse of each other. By maintaining θ_x constant, μ is also held constant. If μ is constant, then S , the concentration to which the active microorganisms are exposed, must be constant, since growth rate is a direct function of S (Equation 3.9).

For those who are still troubled by this concept, we can develop another equation without using θ_x as the master variable. We proceed this time by considering the control volume around the reactor, as illustrated in Figure 5.4. At steady state, a mass balance for substrate leads to

$$V \frac{dS}{dt} = Q^i S^i - Q^i S + r_{ut} V = 0 \quad [5.49]$$

If Monod kinetics apply, then substituting Equation 3.6 and rearranging give

$$\frac{\hat{q}S}{K + S} X_a = \frac{Q^i(S^i - S)}{V} \quad [5.50]$$

We can define V/Q^i to equal the hydraulic detention time in the reactor itself (θ_r) and solve Equation 5.50 for S . Furthermore, we can do a similar mass balance around the entire reactor in Figure 5.4, including the recycle line. We would obtain

$$S = \frac{S^i}{1 + \frac{\hat{q}X_a\theta_r}{K + S}} = \frac{S^0}{1 + \frac{\hat{q}X_a\theta}{K + S}} \quad [5.51]$$

Here, we see that S is directly proportional to S^i or S^0 , providing X_a remains constant. Actually though, we have not solved the above equations explicitly for S , as S is also present in the denominator on the right side of the equations. Frequently, with highly efficient wastewater treatment, S is much less than K , so that on the right side S can be eliminated:

$$S = \frac{S^i}{1 + \frac{\hat{q}X_a\theta_r}{K}} = \frac{S^0}{1 + \frac{\hat{q}X_a\theta}{K}} \quad (S \ll K) \quad [5.52]$$

The result indeed shows S to be exactly proportional to S^i and to S^0 , a result that should be satisfying to those who are not comfortable with the use of the θ_x concept. The important point is that either approach is based on the same principles. Which approach is more useful in a given situation? Equations similar to Equation 5.51 and 5.52 are frequently used to design biological treatment systems. One major problem with using Equation 5.51 in practice, however, is that X_a is very difficult, and often impossible, to measure with the usual wastewaters that contain many other forms of suspended solids. This is not a problem when Equations 3.24 or 5.39 are used, since

prior knowledge of X_a is not needed. The key here is to control θ_x , which can be achieved through controlling the hydraulic detention time in a system without cell settling and recycle, or by controlling the rate at which suspended solids leave the system (Equation 5.35).

5.7.2 EVALUATION OF ASSUMPTIONS

Now we return to the assumptions we made in the development of Equations 5.32 to 5.52. The first assumption was that no biological reactions take place in the settling tank and that no significant biomass is present in that tank. Is either part of this assumption acceptable for the design of real treatment systems? If not, what other assumption should be made? The answer is that it depends on the particular circumstances; no general, a priori answer can be given.

The settling tank often contains a considerable mass of microorganisms; indeed, it might be as much as in the reactor. Clearly, having a large amount of biomass in the settling tank affects the total biomass accumulation, which is represented by $X_a V$ in the exercise. The assumption that no biological reactions occur in the settling tank must be considered at the same time. If the bacteria in the settling tank do not consume substrate, grow, or decay, then they do not affect the mass balances on substrate and biomass. Quantitatively, Equations 5.32 to 5.52 are still valid when no reactions occur in the settling tank.

If the microorganisms in the settling tank grow or decay significantly, the mass balances given in the exercise are in error, as are many of the subsequent equations through 5.52. For illustration, we make the other extreme assumption: that is, we assume that the rates of substrate utilization, biomass synthesis, and biomass decay are the same in the settling tank as in the reactor. Further, we assume that the average biomass concentration in the settling tank is equal to the biomass concentration in the reactor. The effect on the mass balances is that the volume term V in Equations 5.32 to 5.52 includes the combined volumes of the reactor and the settler. Hence, the solids retention time is still given by Equation 5.35, as long as V in that equation equals $V_{\text{reactor}} + V_{\text{settler}}$. The Table 5.2 equations all apply by making the simple substitution for V in all instances in which it occurs.

Is one assumption about system volume superior? The answer is not clear. The truth probably lies somewhere between the two extremes. For example, organism growth due to substrate utilization may occur to some extent in the settler, as there is some substrate carryover from the reactor. However, the carryover likely is small compared to the substrate-input rate to the reactor. How about biomass decay? It probably continues in the settler at a rate similar to that in the reactor, providing that the electron acceptor required for decay is present. In an aerobic system, the acceptor is oxygen, which can become depleted in the settler.

We consider a third case in which microorganism growth from substrate utilization is zero, but the decay rate remains the same as in the reactor. Applying these “between the extremes” assumptions to the mass balances results in equations similar to those in Table 5.2, but with b increased to account for the decay occurring in the

settling tank. If we assume that the average biomass concentration in the settling tank is equal to the biomass concentration in the reactor, the effect is that all the equations in Table 5.2 remain valid with $V = V_{\text{reactor}}$, but an effective decay coefficient (b_{eff}) is required. The effective decay coefficient equals b multiplied by the ratio of the total volume divided by the reactor volume: $b_{\text{eff}} = b(V_{\text{reactor}} + V_{\text{settler}})/V_{\text{reactor}}$.

The second assumption made in the exercise is that the incoming wastewater contains no active biomass. Because most wastewaters contain bacteria, we might be tempted to conclude that active biomass is input to the treatment system. Normally, this conclusion is not warranted, and the original assumption is accurate. In many cases, the bacteria carried with the incoming water are not the same microorganisms that are selected and accumulated in the biological-treatment system. For example, enteric bacteria contained in sewage are not going to be important in an aerobic process operated at temperatures far below the temperature of the human digestive systems, under aerobic conditions, and with a long solids retention time. In such a case, the input bacteria become part of the organic substrate, or the BOD_L .

The third assumption of the exercise is that the substrate is soluble and does not settle out with the solids in the settling tank. In many situations, the input BOD_L is comprised of a significant fraction of BOD_L that is suspended. In order for the suspended BOD_L to be utilized, it must be hydrolyzed. Nearly complete hydrolysis of suspended BOD_L makes the assumption valid. Complete hydrolysis is possible when the solids retention time is long enough. For short SRTs (e.g., 2 d, Example 3.6), hydrolysis is incomplete, and the assumption of a soluble substrate is inaccurate. When the input BOD_L contains a significant fraction of suspended BOD_L and hydrolysis is incomplete, suspended BOD_L and its hydrolysis kinetics must be included in the mass-balance equations.

This discussion reflects the normal fact that models of complex processes are simplifications of reality. They contain simplifying assumptions that involve uncertainties. However, other uncertainties are inherent to the design process. These include predictions of future waste flow rates and composition, as well as changes in temperature and other characteristics of the wastewater. The prudent engineer generally accounts for these uncertainties by making appropriately conservative choices in design. However, overly conservative design comes at a financial cost. Financial risk must be weighed against the risk of system failure in performance. In order to balance these risks well, the engineer must understand the assumptions made and the limitations in the development of the models used for design.

5.7.3 PLUG-FLOW REACTOR WITH SETTLING AND CELL RECYCLE

Suspended-growth biological treatment systems often have significant plug-flow character, especially if they are large and use long, narrow tanks. For the plug-flow reactor, Equation 5.29 applies, but requires a value of X_a^i , which is difficult to obtain. However, the concentration of microorganisms maintained in the reactor is generally quite high with settling and recycle, and the change in microorganism concentration from

the inlet to the outlet of the reactor tends to be small. In this case, we can assume X_a to be constant (\bar{X}_a) throughout the reactor, making integration of Equation 5.20 for the steady-state case much easier. The result is

$$\theta_r = \frac{1}{\hat{q}\bar{X}_a} \left[K \ln \left(\frac{S^i}{S} \right) + (S^i - S) \right] \quad [5.53]$$

We would like to relate treatment efficiency to θ_x , since it is much easier to control θ_x than it is to measure X_a . We construct a mass balance for active microorganisms around the entire reactor, which provides the same results as for the CSTR with settling and recycle, repeated here for convenience:

$$\frac{1}{\theta_x} = \frac{Y(-\bar{r}_{ut})}{\bar{X}_a} - b$$

Similarly, Equations 5.42 and 5.43 also apply to the plug-flow case,

$$-\bar{r}_{ut} = \frac{Q^0(S^0 - S)}{V} = \frac{(S^0 - S)}{\theta}$$

$$\bar{X}_a = \frac{\theta_x}{\theta} \frac{Y(S^0 - S)}{1 + b\theta_x}$$

where we recognize that \bar{r}_{ut} and \bar{X}_a are average values for the reactor.

Substituting Equation 5.42 into 5.37, we obtain

$$\frac{1}{\theta_x} = \frac{YQ^0(S^0 - S)}{\bar{X}_a V} - b \quad [5.54]$$

Recognizing that $\theta_r = V/(Q^0 + Q')$, substituting this value into Equation 5.53, solving for $\bar{X}_a V$, and then substituting this into Equation 5.54 give

$$\frac{1}{\theta_x} = \frac{\hat{q}Y(S^0 - S)}{(S^0 - S) + eK} - b \quad [5.55]$$

in which

$$e = (1 + R) \ln[(S^0 + RS)/(1 + R)S] \quad [5.56]$$

When $R < 1$, e approximately equals $\ln(S^0/S)$, so that Equation 5.55 becomes

$$\frac{1}{\theta_x} = \frac{\hat{q}Y(S^0 - S)}{(S^0 - S) + K \ln \frac{S^0}{S}} - b, \quad R < 1 \quad [5.57]$$

Similarities between the plug-flow Equations 5.55 and 5.57 and the CSTR Equation 5.38 are apparent. However, in the plug-flow case, θ_x depends on S^0 and S , while S^0 is not involved with a CSTR.

One of the outcomes of using Equations 5.55 or 5.57 is that the computed value of S becomes vanishingly small when θ_x is not close to the washout value [θ_x^{\min}]. This means that the biomass is in positive growth near the inlet of the plug-flow reactor, but it is in negative growth (decay) near the outlet end. Thus, the specific growth rate

calculated as the reciprocal of θ_x is an average value for the whole reactor. The very low S value, less than S_{\min} , is possible because the cells are growing faster than the average μ near the inlet, and this balances the negative μ near the outlet.

5.8 USING ALTERNATE RATE MODELS

The last parts of Chapter 3 describe a range of alternative rate expressions that can be used for r_{ut} instead of the Monod expression, Equation 3.6. For example, when competitive inhibition occurs, Equation 3.62, along with 3.66, should be used. Or, when hydrolysis controls the removal of a particulate substrate, Equation 3.57 may be appropriate. When the situation requires a different rate expression, the correct form of r_{ut} replaces the Monod form in the mass-balance equations, such as Equation 5.3 for a batch reactor, Equation 3.15 for a CSTR, and Equation 5.18 for a plug-flow reactor. In addition, the same rate form should be used for new biomass synthesis, since $\mu_{syn} = -Yr_{ut}$.

5.9 LINKING STOICHIOMETRIC EQUATIONS TO MASS-BALANCE EQUATIONS

Chapter 2 presents procedures for writing balanced stoichiometric equations for biological reactions. The mass-balance equations developed so far in this chapter relate only to suspended solids and substrate. We want to tie these two together so that mass balances can be made on electron acceptors and nutrients as well. In this section, we link the full stoichiometry to the mass balances.

The first step is to develop a relationship between f_s , the critical component of the stoichiometric equation, and biomass synthesis, as expressed in the reactor mass balances. In order to do this, we make use of volatile suspended solids Equation 3.31 for the case without settling and cell recycle or Equation 5.47 (Table 5.2) with settling and cell recycle. Our concern is only with biological solids production, which equals the net production of active cells (X_a) and inactive biomass (X_i). We exclude any inactive solids that were present in the influent stream (X_i^0), as they do not represent suspended solids produced during substrate metabolism. Thus, we can define the biomass solids concentration within the reactor (X_b) with

Reactor without settling and cell recycle:

$$X_b = Y(S^0 - S) \left[\frac{1 + (1 - f_d)b\theta_x}{1 + b\theta_x} \right] \quad [5.58]$$

Reactor with settling and cell recycle:

$$X_b = \frac{\theta_x}{\theta} \left\{ Y(S^0 - S) \left[\frac{1 + (1 - f_d)b\theta_x}{1 + b\theta_x} \right] \right\} \quad [5.59]$$

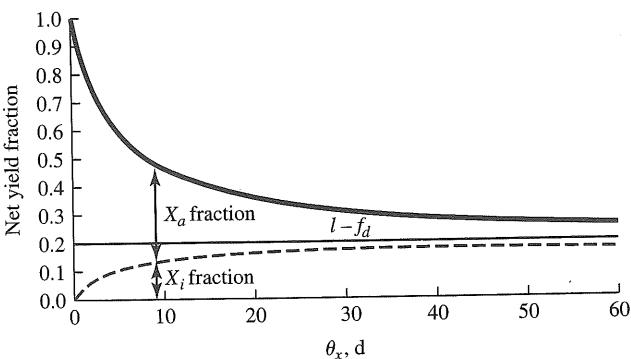


Figure 5.9 Relationship between the net yield fraction and θ_x for the case where f_d equals 0.8 and $b = 0.2 \text{ d}^{-1}$. The fractions of the net yield that are represented by active microorganisms and by inactive microorganisms are also indicated.

We next recognize that the last bracketed term in these equations is dimensionless and represents a fractional adjustment to the true yield (Y). We define the term in brackets as the *net yield fraction* (NYF). Indeed, if the NYF is plotted as a function of θ_x , a curve as shown in Figure 5.9 results. For Figure 5.9, f_d equals the typical value of 0.8. We see that the NYF varies from 1, when θ_x equals zero, to $(1 - f_d)$, or in this case 0.2, as θ_x approaches infinity. Also shown in Figure 5.9 are the fractions of net produced biomass that equal X_a and X_i . In Equations 5.58 and 5.59, the fraction of active biomass is represented by $[1/(1 + b\theta_x)]$, and the inert biomass is $(1 - f_d)b\theta_x/(1 + b\theta_x)$.

Finally, we convert Y , which is expressed as mass of bacterial cells per unit mass of substrate consumed, into electron equivalent units. The equivalent expression for Y is then f_s^0 as discussed in Chapter 2. The result is that the net yield of active plus inactive biomass is related to the bracketed modifier in Equations 5.58 and 5.59 as indicated previously in Equation 3.32

$$f_s = f_s^0 \left[\frac{1 + (1 - f_d)b\theta_x}{1 + b\theta_x} \right]$$

Example 5.1

ALL STOICHIOMETRIC RATES FOR A CSTR A CSTR with settling and recycle is being used for aerobic treatment of a wastewater containing 600 mg/l of acetate at a flow rate of $15 \text{ m}^3/\text{s}$, θ_x is 6 d, and X_v is 2,000 mg/l. Determine the reactor volume, biological sludge production rate, oxygen demand rate, and requirements for the biological nutrients nitrogen and phosphorus. Assume there are no suspended solids or nutrients in the influent stream. The appropriate coefficients are: $Y = 0.55 \text{ g cells/g acetate}$, $b = 0.15 \text{ d}^{-1}$, $\hat{q} = 12 \text{ g acetate/g cells-d}$, $K = 10 \text{ mg acetate/l}$, and $f_d = 0.8$.

First, Equation 5.39 and θ_x give the effluent acetate concentration:

$$S = 10 \frac{1 + 0.15(6)}{6[0.55(12) - 0.15] - 1} = 0.5 \text{ mg/l}$$

A rearrangement of Equation 5.47 makes it possible to determine the hydraulic detention time, θ , from S , θ_x , and X_v :

$$\theta = \frac{6}{2,000} \left[\frac{0.55(600 - 0.5)(1 + (1 - 0.8)(0.15)(6))}{1 + 0.15(6)} \right] = 0.61 \text{ d}$$

The reactor volume can then be determined from Q^0 and θ :

$$V = \theta Q^0 = 0.61 \text{ d} \left(15 \frac{\text{m}^3}{\text{s}} \right) \left(\frac{(3,600)(24)\text{s}}{\text{d}} \right) = 791,000 \text{ m}^3$$

Now, we develop a stoichiometric equation for the reaction. First, we need to determine f_s^0 , which is related to Y by units conversion:

$$f_s^0 = \frac{0.55 \text{ g cells}}{\text{g acetate}} \times \frac{59 \text{ g acetate}}{\text{mol acetate}} \times \frac{\frac{1}{8} \text{ mol acetate}}{\text{e}^- \text{ eq acetate}} \times \frac{\text{mol cells}}{113 \text{ g cells}} \times \frac{\text{e}^- \text{ eq cells}}{\frac{1}{20} \text{ mol cells}}$$

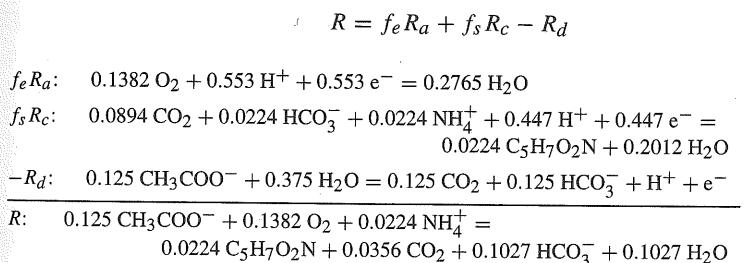
$$f_s^0 = 0.72 \frac{\text{e}^- \text{ eq cells}}{\text{e}^- \text{ eq acetate}}$$

We can then determine f_s from Equation 5.60 and f_e by difference:

$$f_s = 0.72 \left[\frac{1 + (1 - 0.8)(0.15)(6)}{1 + (0.15)(6)} \right] = 0.447$$

$$f_e = 1 - f_s = 1 - 0.447 = 0.553$$

We construct the overall stoichiometric equation for this reaction by using, from Table 2.3, Reaction 0-1 (acetate) for R_d , and Reaction 0-20 (cell synthesis) for R_c ; and from Table 2.2, Reaction I-14 (oxygen) the electron acceptor for R_a :



This equation indicates that, for each $0.125(59) = 7.38 \text{ g acetate consumed}$, $0.1382(32) = 4.42 \text{ g O}_2$ is consumed, and $0.0224(113) = 2.53 \text{ g biomass is produced}$. The nitrogen required is $0.0224(14) = 0.314 \text{ g ammonium-nitrogen}$. Since the phosphorus requirement is about 1 g per 6 g nitrogen, the phosphorus requirement is $0.314/6 = 0.052 \text{ g}$.

The acetate consumed per day can be obtained from the flow rate and the concentration of acetate consumed:

$$\text{Acetate consumption rate} = Q^0(S^0 - S)$$

$$= 15 \frac{\text{m}^3}{\text{s}} (600 - 0.5) \frac{\text{mg}}{1} \times \frac{10^3 \text{l}}{\text{m}^3} \times \frac{\text{g}}{10^3 \text{mg}} \times \frac{3,600(24)\text{s}}{\text{d}} = 777(10^6) \text{ g acetate/d}$$

From the rate of substrate consumption, we compute all the other consumptions and productions:

$$\text{Oxygen consumption rate} = \frac{4.42}{7.38} \times 777(10)^6 = 465(10)^6 \text{ g O}_2/\text{d}$$

$$\text{Biomass production rate} = \frac{2.53}{7.38} \times 777(10)^6 = 266(10)^6 \text{ g biomass/d}$$

$$\text{Nitrogen required} = \frac{0.314}{7.38} \times 777(10)^6 = 33(10)^6 \text{ g NH}_3\text{-N/d}$$

$$\text{Phosphorus required} = \frac{0.052}{7.38} \times 777(10)^6 = 5.5(10)^6 \text{ g P/d}$$

Finally, we can compare the results of the above equation for biomass production rate, as determined from stoichiometry, with that given by Equation 5.48 (Table 5.2):

$$r_{tbp} = \frac{X_v V}{\theta_x}$$

$$= \frac{2000 \frac{\text{mg}}{1} (791,000 \text{ m}^3) \left(\frac{10^{31}}{\text{m}^3}\right) \left(\frac{\text{g}}{10^3 \text{ mg}}\right)}{6 \text{ d}} = 264(10)^6 \text{ g biomass/d}$$

Within round-off error, the biomass production rates obtained by the two different methods are the same, which they should be. This provides an overall check on the two approaches. In addition, it is possible to compute the same nitrogen and phosphorus requirements by multiplying r_{tbp} by the N and P contents of the biomass, as described by Equation 3.43. The same oxygen consumption rate can be obtained by mass balance on oxygen demand, as described by Equation 3.48.

5.10 ENGINEERING DESIGN OF REACTORS

The discussions to this point have been concerned with the development of mathematical equations that relate substrate removal, biological sludge production, reactor size, and reaction stoichiometry to important operational variables, such as hydraulic detention time and solids retention time. The question remains as to what are the criteria that must be considered when an engineer designs a treatment system to perform a given task. First, the engineer must specify the required efficiency of contaminant removal. This is usually based upon regulatory requirements, which may be specified as general minimum levels of performance that are applicable to all treatment systems, or they may be levels of performance that are specifically selected for the treatment system to be designed. The level of performance may be specified in different ways. For example, a minimum BOD_5 removal of 85 percent by the biological treatment process may be required. In another case, the specifications may be that the effluent BOD_5 and total suspended solids (TSS) concentrations shall not exceed 30 mg/l. Often, instead of setting absolute levels of performance such as these, some variation in performance may be allowed. For example, instead of an absolute requirement that the effluent BOD_5 or TSS concentrations not exceed 30 mg/l, the

requirement may recognize that these values may be exceeded at times. One example is a statement that the 30-d average not exceed 30 mg/l. This would mean that if the concentration did exceed 30 mg/l on some days, then the concentration would have to be below 30 mg/l on other days so that the average requirement could be met. The 85-percent removal and 30-d effluent average of 30 mg/l for BOD_5 and TSS typically are minimum requirements for municipal wastewater treatment in the United States, as specified by the Federal Water Pollution Control Act Amendments of 1972 (Public Law 92-500). States may and do adopt more stringent requirements when justified in order to protect receiving waters.

Should the engineer design the treatment system to just barely meet the effluent concentration requirement? No, because the engineer must guarantee reliability in performance. Just as the structural engineer must provide a factor of safety when designing a building or bridge to take a certain loading, environmental engineers must guarantee the reliability of the treatment plants they design by applying a factor of safety. Thus, treatment plants should be designed for treatment efficiency with operational reliability. How might safety factors be assigned?

The suggested approach for applying a safety factor to treatment reactor design (Christensen and McCarty, 1975) is to select a design θ_x (i.e., θ_x^d) as a multiple of the limiting value of θ_x . The multiple is the safety factor (SF):

$$\theta_x^d = \text{SF} \left[\theta_x^{\min} \right]_{\text{lim}} \quad [5.60]$$

Typical values of SF can be inferred from the designs developed from long-term empirical practice. Since the inception of suspended-growth reactors, such as the aerobic activated sludge system, reliable designs evolved through years of trial-and-error experience, rather than from fundamentals of reactor design, since such fundamentals were not initially known. The long history of trial and error eventually resulted in designs for municipal wastewater treatment plants, for which wastewater characteristics do not vary greatly, that gave an acceptable degree of reliability in operation. Applying the more fundamental understanding, we can calculate implied safety factors using Equation 5.60, and they are listed in Table 5.3 for the typical activated sludge systems. Parallel safety factors apply to anaerobic systems, although "conventional" safety factors are somewhat lower (i.e., 10 to 30).

Historically, activated sludge treatment plants are described as "conventional," "high-rate," or "low-rate." In general, "conventional" means medium-sized treatment systems that are expected to operate reliably with fairly constant supervision by

Table 5.3 Implied safety factors for typical biological treatment design loadings

Loading	Implied SF
Conventional	10–80
High Rate	3–10
Low Rate	>80

Table 5.4 Factors to consider when selecting a safety factor for a given design loading

Expected temperature variations
Expected wastewater variations
Flow rate
Wastewater concentration
Wastewater composition
Possible presence of inhibitory materials
Level of operator skill
Efficiency required
Reliability required
Potential penalties for noncompliance
Confidence in design coefficients

reasonably skilled operators. High-rate systems are plants for which the skill of the operator is unusually high and where system oversight is exceptional. An alternate view of high-rate operation is that the removal efficiency and high reliability are not as critical. The low-rate systems are generally used when operator attention is quite limited. Examples are small “extended-aeration” activated sludge plants that may be used in shopping centers or apartment complexes, where operators are present for only short periods of time and mainly to insure that the equipment such as pumps and aerators are working properly.

Table 5.3 illustrates two key points about safety factors for activated sludge processes. First, the safety factor for design is larger for systems whose operators are less skilled and attend to the process for less time. Second, the SF multiplier is much larger than 1. Even for high-rate design, the safety factor is at least 3, and most of the time it is 5 or larger. For low-rate systems, the safety factor often exceeds 100.

Within each range, the values for θ_x^d vary significantly. Here, the designing engineer must use judgment in the selection of an appropriate safety factor. Table 5.4 lists several of the factors that should be taken into consideration in making such judgments. The engineer should work with the community and regulators to determine the most appropriate safety factors to use in a given situation. Higher safety factors increase the degree of reliability of operation, but also give higher construction costs. Designs with lower safety factors require more continuous supervision and operators with increased skill, which add to the operational costs. The appropriate balance for a given situation depends upon many local factors that must be evaluated carefully by the design engineer.

An additional design decision by the engineer for a suspended-growth system with recycle is the concentration of suspended solids that is to be maintained in the treatment reactor. This depends to a large degree upon the settling characteristics of the floc and the degree of recycle to be provided. In general, using a higher suspended solids concentration makes the reactor smaller (and less expensive) for a given θ_x^d .

Table 5.5 Typical values for total suspended solids concentration (X) in aerobic suspended-growth reactors with settling and recycle

Floc Type	X (mgSS/L)
Normal	1,500–3,000
Poor compaction or low recycle rate	500–1,500
Good compaction or high recycle rate	3,000–5,000

However, higher suspended-solids concentrations may require larger settling tanks, because of the increased load of suspended solids to the settling tank. The proper balance requires an approach to optimize the size for the combined reactor-settling-tank system, which is described in detail in Chapter 6. Some rules of thumb that can be useful in preliminary design calculations are provided in Table 5.5.

Settling characteristics of suspended flocs can vary significantly from location to location, from one type of design to another (e.g., CSTR vs. PFR), and with time at a given plant. A changeover from good settling floc to poor settling floc can cause failure of the treatment system. This is the most common cause of failure of activated sludge treatment plants. Designing for a smaller suspended solids concentration in the reactor generally increases the reliability of the plant, but results in larger reactor size and higher capital costs. This is another area where good judgment by the design engineer is critical. Joint decisions among the client, the regulators, and the engineer are recommended.

SELECTION OF DESIGN CRITERIA Select an appropriate volume for a CSTR reactor with settling and recycle to achieve an average 90-percent nitrification of ammonium in a municipal wastewater following biological treatment for BOD removal. The following wastewater characteristics and organism parameters apply:

$$Q^0 = 10^4 \text{ m}^3/\text{d}$$

$$\text{Influent BOD}_5 = 0 \text{ mg/l}$$

$$S^0 \text{ (Influent } \text{NH}_4^+ \text{-N)} = 25 \text{ mg/l}$$

$$X_i^0 = 18 \text{ mg VSS/l}$$

$$Y = 0.34 \text{ g VSS/g } \text{NH}_4^+ \text{-N}$$

$$\hat{q} = 2.7 \text{ g } \text{NH}_4^+ \text{-N/g VSS-d}$$

$$K = 1.0 \text{ mg } \text{NH}_4^+ \text{-N/l}$$

$$b = 0.15/\text{d}$$

Example 5.2

First, we compute the baseline value of $[\theta_x^{\min}]_{\lim}$ from Equation 3.27:

$$[\theta_x^{\min}]_{\lim} = \frac{1}{Y\hat{q} - b} = \frac{1}{0.34(2.7) - 0.15} = 1.3 \text{ d}$$

We arbitrarily select an intermediate safety factor from Table 5.3 of 15. (Factors as listed in Table 5.4 need to be weighed in selecting an SF for an actual design.) So, the design SRT becomes

$$\theta_x^d = \text{SF} [\theta_x^{\min}]_{\text{lim}} = 15(1.3) = 20 \text{ d}$$

Second, we must select a concentration of total suspended solids. We choose a typical value for conventional treatment, 2,000 mg/l, from Table 5.5. We rearrange Equation 5.47 to select the hydraulic detention time for the reactor:

$$\begin{aligned}\theta &= \frac{\theta_x^d}{X_v} \left[X_i^0 + \frac{Y(S^0 - S)(1 + (1 - f_d)b\theta_x^d)}{1 + b\theta_x^d} \right] \\ \theta &= \frac{20}{2,000} \left[18 + \frac{0.34(25 - 0)(1 + (1 - 0.8)(0.15)(20))}{1 + 0.15(20)} \right] = 0.21 \text{ d}\end{aligned}$$

The reactor volume is then simply the product of the flow rate and detention time:

$$V = Q^0 \theta = 10^4(0.21) = 2,100 \text{ m}^3$$

Third, we must check whether or not the ammonium removal is sufficient. This requires that the effluent NH_4^+ -N concentration be computed using Equation 5.39:

$$S = K \frac{1 + b\theta_x^d}{\theta_x^d(Y\hat{q} - b) - 1} = 1.0 \frac{1 + 0.15(20)}{20(0.34(2.7) - 0.15) - 1} = 0.28 \text{ mg NH}_4^+ \text{-N/l}$$

The efficiency of ammonia-nitrogen removal is much greater than 90 percent:

$$\text{Removal efficiency} = 100 \frac{S^0 - S}{S^0} = 100 \frac{20 - 0.28}{20} = 98.6 \text{ percent} >> 90 \text{ percent}$$

The average NH_4^+ -N removal efficiency is thus much greater than required, providing another degree of safety in the design. For this case, as with most designs in practice, reactor volumes tend to be governed more by reliability considerations than by considerations of removal efficiency per se.

5.11 REACTORS IN SERIES

Reactors connected in series (Figure 5.1) are commonly used in wastewater treatment. There are different reasons for doing so. Two of the most common treatment requirements for municipal wastewaters are removal of organic material (BOD) and transformation of nitrogen, such as the oxidation of ammonium to nitrate. Organic removal and nitrification are carried out by two different groups of microorganisms, but both reactions can be carried out together in a single aerobic reactor. However, the treatment can be easier to control if two reactors are connected in series: the first for organic removal and the second for nitrification. Here, a different type of reactor may be used for each process. For example, a CSTR with settling and recycle may be used for organic removal and then followed by a biofilm reactor for nitrification. The

reverse has also been used, a biofilm reactor for organic removal and a suspended-growth system for nitrification. A CSTR is often adequate for organic removal, but a plug-flow reactor tends to be more efficient for nitrification. This combination can be used through reactors connected in series. In other cases, the same type of reactor may be used both for the first stage of organic removal and the second one of nitrification.

Another use of series reactors is when different electron acceptors are desired in the overall treatment scheme. As an example, nitrogen removal, rather than just transformation of ammonium to nitrate, may be required. Two reactors in series might accomplish this: the first an aerobic reactor to accomplish organic removal and ammonium oxidation and the second reactor operated under anoxic (the absence of oxygen) conditions to obtain conversion of the nitrate produced in the first reactor to N_2 through denitrification. It is also increasingly common to have a first anoxic stage followed by an aerobic stage. Here, nitrate may be produced in the aerobic stage, and through recycle, the nitrate is brought back to the first stage through recycle of second stage effluent to achieve a mixture of untreated wastewater BOD and nitrate to achieve BOD oxidation by denitrification in the first anoxic stage. These various schemes for nitrogen transformation and removal are discussed in more detail in Chapters 9 and 10.

Staged reactors also can be used to achieve highly efficient removal of toxic organic chemicals. The first stage here might be a CSTR with settling and recycle, and the second stage could be some form of plug-flow reactor, such as a suspended-growth reactor with settling, but no cell recycle. This system is illustrated in Figure 5.10. Here, excess biomass is removed for disposal only from the second settling tank. We assume that the concentration of the contaminant in the wastewater is S^0 , and that this concentration is quite toxic to microorganisms. Thus, we do not want to expose the organisms to this high concentration. For example, phenol is known to be quite toxic to microorganisms at concentrations above 200 to 300 mg/l, but concentrations in some industrial wastewaters may be in the range of several thousands of milligrams per liter. In addition, phenol can be toxic to some aquatic species at concentrations in the low mg/l range, it can impart taste and odor to fish at even smaller concentrations, and it causes taste and odor in drinking water at a concentration of about 40 $\mu\text{g/l}$. Indeed, chlorination for disinfection can form chlorinated phenols that cause even more severe taste and odors problems in drinking water. The question is how can we

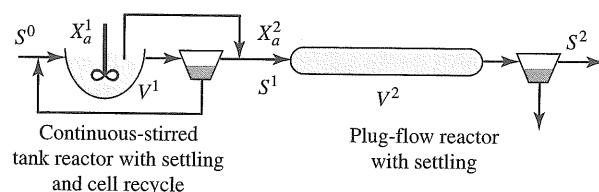


Figure 5.10

Possible two-stage reactor system for treatment of high concentrations of toxic chemicals such as phenol to meet low concentration standards

design a treatment system to treat a wastewater with a phenol concentration of, say, 4,000 mg/l and reduce the concentration to 20 µg/l? We consider the reactor system illustrated in Figure 5.10 as one possibility. We also assume that no other organic substrates are present, and inorganic nutrients are available.

First, we use aerobic treatment and consider Haldane kinetics, since we are dealing with a substrate that is self-inhibitory. We assume the following wastewater treatment conditions apply for the first reactor:

$$\begin{aligned} Q^0 &= 10^4 \text{ m}^3/\text{d} \\ S^0 (\text{phenol}) &= 4,000 \text{ mg/l} \\ \theta_x &= 8 \text{ d} \\ \hat{q} &= 6 \text{ d}^{-1} \\ K &= 2 \text{ mg/l} \\ K_{IS} &= 120 \text{ mg phenol/l} \\ Y &= 0.35 \text{ g cells/g phenol} \\ b &= 0.2 \text{ d}^{-1} \\ X_a^1 &= 1,500 \text{ mgVSS}_a/\text{l} \end{aligned}$$

For the first stage reactor, we can use Equation 5.37 for a CSTR with settling and recycle of microorganisms to determine the effluent concentration of phenol (S^1), the active organism concentration (X_a^1), and the reactor volume (V^1). For Haldane kinetics, we need to combine Equations 3.62, 3.64, and 3.65 to obtain r_{ut} in Equation 5.37:

$$\begin{aligned} \frac{1}{\theta_x} &= \frac{Y(-r_{ut}^1)}{X_a^1} - b = \frac{Y \hat{q} S^1}{K + S^1 + (S^1)^2/K_{IS}} - b \\ \frac{1}{8} &= \frac{0.35(6) S^1}{2 + S^1 + (S^1)^2/120} - 0.2 \end{aligned}$$

from which $S = 0.37 \text{ mg phenol/l}$. Then,

$$\begin{aligned} \theta^1 &= \frac{\theta_x^1 Y(S^0 - S^1)}{X_a^1 1 + b\theta_x^1} = \frac{8}{1,500} \frac{0.35(4,000 - 0.37)}{1 + 0.2(8)} = 2.87 \text{ d} \\ V^1 &= \frac{Q^0}{\theta^1} = \frac{10^4}{2.87} = 3,480 \text{ m}^3 \end{aligned}$$

We see that CSTR treatment alone, on the average, reduces the phenol concentration below 1 mg/l. In order to reduce the concentration to near our goal of 20 µg/l, θ_x would need to be increased to near infinity. However, if we use a plug-flow reactor for the second stage, then our goal can be achieved. Since the phenol concentration is now well below the K_{IS} value of 120 mg/l, we can assume the simpler Monod kinetics for further analyses.

One problem with a second-stage plug-flow reactor is that the phenol concentration in the effluent from the CSTR settling is much too low to support adequate

biomass for phenol destruction. We can remedy this if we transfer the excess biological solids produced in the first stage reactor directly into the second-stage reactor, as illustrated in Figure 5.10. We might take these organisms from the recycle line of the first-stage reactor, but we will obtain better control if draw it directly from the first-stage reactor as shown. The quantity to be withdrawn daily can be determined from the equations in Table 5.2. Since our design θ_x for the first stage is 8 d, we can accomplish the required transfer of excess biological solids by transferring one-eighth of the stage-one reactor mixed liquor contents each day to the second-stage reactor: $3,480 \text{ m}^3/8 \text{ d} = 435 \text{ m}^3/\text{d}$. When we mix the remaining wastewater from stage one ($10^4 \text{ m}^3/\text{d} - 435 \text{ m}^3/\text{d}$ or $9,565 \text{ m}^3/\text{day}$) with the excess microorganisms produced in stage one, the active microorganism concentration in the combined wastewater entering the stage two reactor is $435 \text{ m}^3/\text{d} (1,500 \text{ mg/l})/10^4 \text{ m}^3/\text{d} = 65 \text{ mg/l}$. If we assume at this point that the second-stage reactor does not have organism recycle, then we can use the desired S^2 phenol concentration of 0.02 mg/l directly to determine θ_x^2 , which here also equals θ^2 for the reactor. For this simple plug-flow reactor, Equation 5.27 applies:

$$\begin{aligned} \theta^2 &= \frac{1}{\hat{q}} \left\{ \left(\frac{K}{X_a^2 + YS^1} + \frac{1}{Y} \right) \ln(X_a^2 + YS^1 - YS^2) \right. \\ &\quad \left. - \left(\frac{K}{X_a^2 + YS^1} \right) \ln \frac{S^2 X_a^2}{S^1} - \frac{1}{Y} \ln X_a^2 \right\} \\ \theta^2 &= \frac{1}{6} \left\{ \left(\frac{2}{65 + 0.35(0.37)} + \frac{1}{0.35} \right) \ln(65 + 0.35(0.37) - 0.35(0.02)) \right. \\ &\quad \left. - \left(\frac{2}{65 + 0.35(0.37)} \right) \ln \frac{0.02(65)}{0.37} - \frac{1}{0.35} \ln(65) \right\} \\ \theta^2 &= 0.016 \text{ d} \end{aligned}$$

from which,

$$V = \theta^2 Q^0 = 0.016(10^4 \text{ m}^3) = 160 \text{ m}^3$$

We see here that the volume of the second-stage reactor is quite small compared with the first-stage reactor and is able to do a job that the first-stage CSTR could not do, even if infinite in size. We have not applied any safety factor to the second-stage reactor design, which should be done. Perhaps we can optimize the system by reducing θ_x for the first-stage and shifting more of the load to the second stage. We could also have cell recycle in the second stage to increase organism concentration within the reactor. No matter how we optimize the system, this preliminary analysis illustrates important benefits of the two-stage system. In the first reactor, the microorganisms never experience a high phenol concentration and thus are not subject to its toxicity. This would not be true if a plug-flow reactor were used here. In addition, the inefficiency of the CSTR is circumvented by use of the second-stage plug-flow reactor. A similar argument can be made for operating anaerobic systems in two stages. A first stage with a CSTR helps avoid problems associated with organic acid production

and low pH with high organic concentrations, but a second-stage plug-flow reactor permits high removal efficiencies with a much smaller overall reactor size.

5.12 BIBLIOGRAPHY

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5.13 PROBLEMS

- 5.1. You have the following parameters:

$$\begin{array}{ll} f_s^0 = 0.7 & Y = 0.5 \text{ g VSS}_a/\text{g} \\ \hat{q} = 10 \text{ mg/mg VSS}_a\text{-d} & X_a = 1,500 \text{ mg VSS}_a/\text{l} \\ K = 10 \text{ mg/l} & Q = 2,500 \text{ m}^3/\text{d} \\ f_d = 0.8 & b = 0.1/\text{d} \end{array}$$

Design safety factor (SF) = 30. What is the design value of f_s ?

- 5.2. Calculate the hydraulic detention time (θ) of a CSTR with settled-solids recycle if:

$$\begin{array}{l} Y = 0.6 \text{ g VSS}_a/\text{g} \\ \hat{q} = 20 \text{ g/g VSS}_a\text{-d} \\ K = 20 \text{ mg COD/l} \\ b = 0.25/\text{d} \\ f_d = 0.8 \\ S^0 = 10,000 \text{ mg COD/l} \\ X_v = 4,000 \text{ mg/l} \\ X_v^0 = 0 \\ \theta_x = 6 \text{ d} \\ Q = 1,000 \text{ m}^3/\text{d} \end{array}$$

Ignore SMP formation and biomass in the settler. Discuss the practicality of treating this rather high-strength, soluble wastewater by this method.

- 5.3. In the laboratory you are operating two CSTRs in parallel. Both have solids recycle, the same total volume (100 liters), and the same influent conditions:

$$\begin{array}{l} Q = 400 \text{ l/d} \\ S^0 = 1,000 \text{ mg BOD}_L/\text{l} \\ X_v^0 = 0 \end{array}$$

Both are being operated with the same SRT, 10 d. Both have the same effluent VSS concentration, 15 mg/l. However, system A is aerobic, while system B

is anaerobic. Pertinent parameters are:

	A	B
\hat{q} , mg BOD _L /mg VSS _a -d	16	10
K , mg BOD _L /l	10	10
b , d ⁻¹	0.2	0.05
Y , mg VSS _a /mg BOD _L	0.35	0.14

Estimate and compare the total effluent-BOD_L concentrations for systems A and B. Assume the effluent SMP contributes to the BOD_L.

- 5.4. You are to treat a wastewater that is contaminated with 100 mg/l of SO₄²⁻ as S. You will do the treatment biologically by sulfate reduction, in which SO₄²⁻-S is reduced to H₂S-S, which is stripped to a gas phase. You do not need to be concerned with the stripping. The electron donor will be acetate (CH₃COO⁻), which you will need to add, say, in the form of vinegar. You know the following about the wastewater: flow rate = 1,000 m³/d, sulfate = 100 mg/l as S, volatile suspended solids (X_v^0) = 0.

From previous tests and theory, you have estimated the following parameters for the rate-limiting substrate, which will be acetate (expressed as COD):

$$\begin{array}{l} f_s^0 = 0.08 \\ \hat{q} = 8.6 \text{ g COD/g VSS}_a\text{-d} \\ b = 0.04/\text{d} \\ K = 10 \text{ mg COD/l} \\ f_d = 0.8 \\ \text{N-source} = \text{NH}_4^+\text{-N} \end{array}$$

You also have selected these design criteria:

- Use a solids-recycle system with a CSTR reactor
- Have $\theta_x = 10$ d and $\theta = 1$ d
- Have an effluent sulfate concentration of 1 mg S/l

You must provide the following crucial design information.

- (a) Compute the effluent concentration of acetate (in mg/l of COD).
- (b) Compute (using stoichiometric principles) the input concentration of acetate (in mg/l as COD) needed to take the effluent sulfate concentration from 100 to 1 mg S/l. [Note that the formula weight of S is 32 g.]
- (c) Compute the concentrations (in mg VSS/l) of active, inert, and total volatile suspended solids.
- (d) Compute the waste-sludge mass flow (in kg VSS/d) and volumetric flow rate (in m³/d) if the effluent VSS is 5 mg/l and the recycle VSS is 5,000 mg/l.

- (e) Compute the recycle ratio R .
 - (f) Based on the (normal) multisubstrate approach, determine the effluent concentration of SMP.
 - (g) Do you see any problems with the design? If yes, explain the cause of any problems and ways to overcome them.
- 5.5. This question addresses a novel system to treat a sulfide-bearing wastewater by autotrophic bacteria that aerobically oxidize sulfides (i.e., H_2S and HS^-) to SO_4^{2-} , which is harmless. The electron acceptor is O_2 , and ample NH_4^+ is present as the N source. For these sulfide oxidizers, you may utilize the following stoichiometric and kinetic parameters:

$$f_s^0 = 0.2 e^-$$

$$Y = 0.28 \text{ g VSS}_a/\text{mg S}$$

$$\hat{q} = 5 \text{ mg S/mg VSS}_a\text{-d}$$

$$b = 0.05/\text{d}$$

$$K = 2 \text{ mg S/l}$$

$$f_d = 0.8$$

For the questions that follow, you should assume that the sulfides are not lost by any mechanism other than microbially catalyzed metabolism, that no intermediate sulfur species form, and that you can use the total sulfide concentration as rate-limiting. The influent has:

$$Q = 1,000 \text{ m}^3/\text{d}$$

$$S^0 = 100 \text{ mg S/l}$$

$$X_v^0 = 0.$$

You will use a CSTR reactor with a solids-recycle system. Take the following steps.

- (a) Determine the design θ_x if a safety factor of 10 is employed. Please round your θ_x value to two significant digits.
- (b) You have determined from judgment that the mixed liquor volatile suspended solids concentration (X_v) must be no less than 1,000 mg/l. If you accept this minimum X_v value, what are the hydraulic residence time (hours) and volume of the system (m^3)?
- (c) Compute the sludge wasting flow rate (in m^3/d) and the sludge recycle flow rate (also in m^3/d) if the effluent VSS (X_v^e) is 15 mg VSS/l and the underflow has 8,000 mg VSS/l.
- (d) What is the total effluent COD from organic materials? For this part, we need to make some reasonable assumptions about SMP. For the formation of SMP, you may assume: $k_1 = 0.04 \text{ mg COD}_p/\text{mg COD}_s$ and $k_2 = 0.09 \text{ mg COD}_p/\text{mg VSS-d}$. Note carefully the units! As a first approximation and because the sulfide oxidizers are autotrophs, you should assume that neither the UAP nor the BAP is biodegraded. In other words,

there are no heterotrophs in the system. (This is not going to be true in reality, but it keeps the problem from getting too complicated.)

- (e) What is the rate of O_2 supply needed (in kg O_2/d)?
- 5.6. Compare the cell concentration that will be present after 24 h through batch growth on a nongrowth limiting concentration of acetate for the following four cases, assuming that Y is the only variable between the organisms. The initial cell concentration in all cases is 10 mg/l. Assume other constants of interest are $\hat{q} = 12 \text{ mg acetate/mg VSS}_a\text{-d}$, $K = 2 \text{ mg/l}$, and $b = 0.1/\text{d}$.
 - (a) $Y = 0.6 \text{ mg VSS}_a/\text{mg acetate}$ (aerobic growth)
 - (b) $Y = 0.45 \text{ mg VSS}_a/\text{mg acetate}$ (denitrification)
 - (c) $Y = 0.06 \text{ mg VSS}_a/\text{mg acetate}$ (sulfate reduction)
 - (d) $Y = 0.04 \text{ VSS}_a/\text{mg acetate}$ (methane fermentation)
- 5.7. A CSTR reactor is used for treatment of $50 \text{ m}^3/\text{d}$ of wastewater with COD_b of 10,000 mg/l by methane fermentation. Assume $Y = 0.04 \text{ mg VSS}_a/\text{mg BOD}_L$, $\hat{q} = 8 \text{ mg BOD}_L/\text{per VSS}_a\text{-d}$, $K = 200 \text{ mgBOD}_L/\text{l}$, and $b = 0.05/\text{d}$. What reactor detention time will result in the maximum BOD_L removal per day per unit volume of reactor? (You may find it useful to use a spreadsheet or programmable calculator, as a trial and error solution is likely to be needed). Show a graph of BOD_L removal per day per unit reactor volume versus reactor detention time.
- 5.8. The characteristics of a wastewater were found to be the following:

$$Q = 150,000 \text{ m}^3/\text{d}$$

$$X = 350 \text{ mg/l}$$

$$X_v = 260 \text{ mg/l}$$

$$\text{COD (total)} = 880 \text{ mg/l}$$

$$\text{COD (soluble)} = 400 \text{ mg/l}$$

$$BOD_L (\text{total}) = 620 \text{ mg/l}$$

$$BOD_L (\text{soluble}) = 360 \text{ mg/l}$$

Estimate the following: S^0 , Q^0 , X^0 , and X_v^0 .

- 5.9. Compare the relative volumes of the following reactors required to achieve (1) 85 percent removal and (2) 98 percent removal of ammonium by oxidation to nitrate for the following wastewater characteristics:

$$Q^0 = 50 \text{ m}^3/\text{d} \quad \hat{q} = 2.5 \text{ g NH}_4^+ \text{-N/g VSS}_a\text{-d}$$

$$S^0 = 60 \text{ mg NH}_4^+ \text{-N/l} \quad K = 1 \text{ mg/l}$$

$$X_i^0 = 20 \text{ mg/l} \quad Y = 0.34 \text{ g VSS}_a/\text{g NH}_4^+ \text{-N}$$

$$b = 0$$

- (a) CSTR
- (b) CSTR with settling and solids recycle, with $\theta_x/\theta = 10$.

- 5.10. For the wastewater and biological characteristics in problem 5.9, determine the reactor volumes and ammonium removal efficiencies for the case where the safety factor used in design for both reactors is the same or 10:

- (a) CSTR
(b) CSTR with settling and biomass recycle, and with $X_v = 1,000 \text{ mgVSS/l}$

- 5.11. You have a given wastewater to treat and are evaluating different reactor types for treatment and the sensitivity of effluent substrate concentration to various changes in operation. Let's assume for each reactor type you have already selected a given reactor volume that results in satisfactory treatment. What effect on effluent substrate concentration (S^e) will an increase in each of the factors listed below have? Use + for an increase in S^e , - for a decrease, 0 for no change, and ? for undetermined.

Factor Increased	Reactor Type			
	CSTR with Recycle	CSTR with Cell Settling and Recycle (θ_x Kept Constant)	Plug Flow with Recycle	Fixed-Film Completely Mixed with Recycle
Q^0				
S^0				
X_i^0				
X_a^0				
R (recycle rate)				
Y				

- 5.12. You are designing a CSTR with cell settling and recycle for treatment of an organic industrial wastewater. You have selected a reactor for design that results in a given substrate concentration in the effluent. You now wish to do a sensitivity analysis to determine the effect of certain changes on the volume of the reactor, while keeping the effluent concentration unchanged. Fill out the following table to indicate how an increase in each of the variables in the left-hand column will affect reactor volume. Assume all other factors listed in the left-hand column remain constant. Use: + = increase, - = decrease, 0 = no change, and i = need more information to tell.

Variable Increased	Effect on Reactor Size
K	
\hat{q}	
Q^0	
S^0	
X_i^0	
X_a	

- 5.13. You have designed a CSTR with settling and solids recycle to treat a waste with flow rate (Q^0) of $100 \text{ m}^3/\text{d}$, and have assumed that X_i^0 is zero. Your design θ_x is 6 d, resulting in a reactor volume V of 20 m^3 . However, you have now found that X_i^0 for the waste will actually be $200 \text{ mgVSS}_i/\text{l}$. If in your design you wish to maintain the previously selected θ_x and mixed liquor suspended solids concentration ($X = 2,000 \text{ mg SS/l}$), what change, if any, in reactor volume (in m^3) is required?
- 5.14. Determine the volume of an aerobic CSTR with settling and solids recycle for BOD removal under the following conditions (assume typical values for aerobic noncarbohydrate BOD from Chapter 3):
- (a) $\text{SF} = 40$
 - (b) $Q^0 = 10^3 \text{ m}^3/\text{d}$
 - (c) $X_i^0 = 300 \text{ mgVSS}_i/\text{l}$
 - (d) Influent $\text{BOD}_L = 200 \text{ mg/l}$
 - (e) Effluent $\text{BOD}_L = 5 \text{ mg/l}$
 - (f) $X_v = 2,000 \text{ mgVSS/l}$
- 5.15. A $200 \text{ m}^3/\text{d}$ stream flows from an abandoned mine and contains 10 mg/l of dissolved Fe(II) (formula weight = 56 g) and no suspended solids. When oxidized in the stream to Fe(III), the brown precipitate that forms degrades the stream. You wish to consider biological oxidation of the Fe(II) to remove at least 95 percent of the iron before discharge to the stream. It must be considered that the Fe(III) produced in the reactor precipitates there forming Fe(OH)_3 , which becomes part of the reactor suspended solids and is removed from the system for disposal along with the waste biological solids. What reactor volume do you suggest be used for a CSTR with settling and solids recycle? Assume the reactor total suspended solids concentration (X) equals $3,000 \text{ mgSS/l}$ and that $[\theta_x^{\min}]_{\text{lim}}$ for Fe(II) oxidation is 2.2 d.
- 5.16. Consider a suspended-growth reactor with settling and solids recycle and with $X_a = 1,200 \text{ mgVSS}_a/\text{l}$ and detention time $\theta = 4 \text{ h}$. A contaminant has an

influent concentration S^0 to the reactor of 0.5 mg/l, and is decomposed with kinetic coefficients based upon the total X_a concentration of $\hat{q} = 0.05 \text{ mg/mg VSS}_{a\text{-d}}$ and $K = 3 \text{ mg/l}$. Estimate the effluent concentration S^e from the reactor if the reactor were (1) completely mixed and (2) plug-flow.

- 5.17. A wastewater has a concentration of benzene equal to 30 mg/l, and no significant concentration of other biodegradable organic materials are present. The regulatory agencies require that the wastewater be treated to reduce the benzene concentration to 0.01 mg/l. Assume for benzene, that the following rate coefficients apply for aerobic treatment: $Y = 0.9 \text{ g VSS}_a/\text{g benzene}$, $b = 0.2/\text{d}$, $\hat{q} = 8 \text{ g benzene/g VSS}_{a\text{-d}}$, and $K = 5 \text{ mg benzene/l}$.
- What is the minimum concentration of benzene that you could expect to achieve from biological treatment in a CSTR with settling and solids recycle?
 - Assuming the above does not meet regulatory compliance for benzene, describe another biological approach that is likely to have better potential for meeting the requirements. Describe the approach and indicate why it may be better than a CSTR. Calculations are not required to support your answer; a description of the concept is all that is needed.

THE ACTIVATED SLUDGE PROCESS

The activated sludge process surely is the most widely used biological process for the treatment of municipal and industrial wastewaters. Normally, the activated sludge process is strictly aerobic, although anoxic variations are coming into use for denitrification, the subject of Chapter 10. In simple terms, the activated sludge process consists of a reactor called the *aeration tank*, a *settling tank*, *solids recycle* from the settler to the aeration tank, and a *sludge wasting line*. The aeration tank is a suspended-growth reactor containing microbial aggregates, or *flocs*, of microorganisms termed the *activated sludge*. The microorganisms consume and oxidize input organic electron donors collectively called the *BOD*. The activated sludge is maintained in suspension in the reactor through mixing by aeration or other mechanical means. When the slurry of treated wastewater and microbial flocs pass to the settling tank, the flocs are removed from the treated wastewater by settling and returned to the aeration tank or wasted to control the solids retention time (SRT). The clear effluent is discharged to the environment or sent for further treatment. Capturing the flocs in the settler and recycling them back to the reactor are the keys to the activated sludge process, because they lead to a high concentration of microorganisms in the reactor. Thus, the sludge is "activated" in the sense that it builds up to a much higher concentration than could be achieved without the settler and recycle. The high biomass concentration allows the liquid detention time to be small, generally measured in hours, which makes the process much more cost effective. Wasting the sludge through the separate sludge-wasting line makes the solids retention time (SRT or θ_x) separate from and much larger than the hydraulic detention time (θ).

In 1914, E. Ardern and W. T. Lockett (1914) discovered the activated sludge process in England. They noted that aeration of sewage led to formation of flocculent suspended particles. They discovered that the time to remove organic contaminants (and to achieve nitrification, the subject of Chapter 9) was reduced from days to hours when these flocculent particles were held in the system. They referred to the suspended particles, more specifically the resulting sludge from settling to collect the particles, as being "activated," and so was born the activated sludge process. By 1917, the Manchester Corporation had brought a 946-m³/d continuous-flow plant into operation, and, in the same year, Houston, Texas, completed the construction of a 38,000-m³/d plant. Many activated sludge plants were soon constructed at larger cities throughout England and the United States (Sawyer, 1965).

It is interesting to realize that successful application of the process occurred even though an understanding of how the process actually worked was lacking. The early literature contains many articles