What do you know about surds?

Exercise 2: The Basics - Answers

Warm up

1. Express each of these as a multiple of the smallest possible surd.

Question		Working	Commentary
a)	$\sqrt{20}$	$\sqrt{20} = \sqrt{4 \times 5}$ $= \sqrt{4} \times \sqrt{5}$ $= 2\sqrt{5}$	The highest square number that divides into 20 is 4. The surd can be expressed as the product of two square roots. The square root of 4 is 2.
b)	$\sqrt{128}$	$\sqrt{128} = \sqrt{64 \times 2}$ $= 8\sqrt{2}$	The highest square number that divides into 128 is 64. This is quite easy to spot. The stage of writing $= \sqrt{64} \times \sqrt{2}$ is not really necessary once the square number has been identified. The square root of 64 is 8.
c)	$\sqrt{392}$	$\sqrt{392} = \sqrt{4 \times 98}$ $= \sqrt{4 \times 49 \times 2}$ $= 14\sqrt{2}$	The highest square number that divides into 392 is not that easy to spot. The simplification is therefore done in stages. One square number that divides into 392 is 4. $392 = 4 \times 98$ It is now easier to spot that the square number 49 also divides into 392. $392 = 4 \times 49 \times 2$ The square root of 4 is 2 and the square root of 49 is 7. The multiplier for the final surd is $2 \times 7 = 14$. Note: This could have been slightly shortened by realising that $392 = 2 \times 196$ and that the square root of 196 is 14.
d)	$\sqrt{243}$	$\sqrt{243} = \sqrt{9 \times 27}$ $= \sqrt{9 \times 9 \times 3}$ $= 9\sqrt{3}$	A quick test on the digit sum of 243 - $2 + 4 + 3 = 9$ – shows that it is divisible by the square number 9. $243 = 9 \times 27$ 9 also divides into 27 so $243 = 9 \times 9 \times 3$ The square root of 9×9 is 9. Note: This could have been done by spotting that $243 = 81 \times 3$ and that the square root of 81 is 9.

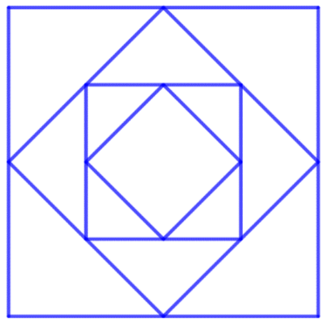
2. Express each of the following in a form without surds

Question		Working	Commentary
a)	$\left(\sqrt{2}\right)^8$	$\left(\sqrt{2}\right)^8 = \left(\sqrt{2}\right)^{2\times4}$ $= \left(\left(\sqrt{2}\right)^2\right)^4$ $= 2^4$ $= 16$	Index can be rewritten as 2×4. The expression can then be rewritten using the index law $(a^x)^y = a^{xy}$ The power of 2 can be applied to the square root giving 2. The remaining index is applied.

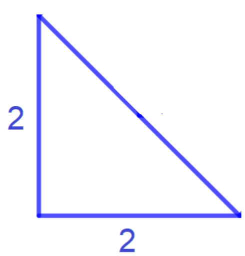
b)	$\left(\sqrt{2}\sqrt{3}\right)^4$	$\left(\sqrt{2}\sqrt{3}\right)^4 = \left(\sqrt{6}\right)^4$ $= \left(\left(\sqrt{6}\right)^2\right)^2$ $= 6^2$ $= 36$	The two roots can be written as one using $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$. A factor of 2 from the index can be used to square the root giving 6. The remaining index is applied.
c)	$\left(\frac{\sqrt{5}}{\sqrt{7}}\right)^4$	$\left(\frac{\sqrt{5}}{\sqrt{7}}\right)^4 = \left(\left(\frac{\sqrt{5}}{\sqrt{7}}\right)^2\right)^2$ $= \left(\frac{5}{7}\right)^2$ $= \frac{25}{49}$	A factor of 2 from the index can be used to square the root giving $\frac{5}{7}$. The remaining index is applied.
d)	$\left(\sqrt{\frac{3}{8}}\right)^6$	$\left(\sqrt{\frac{3}{8}}\right)^6 = \left(\left(\sqrt{\frac{3}{8}}\right)^2\right)^3$ $= \left(\frac{3}{8}\right)^3$ $= \frac{27}{512}$	A factor of 2 from the index can be used to square the root giving $\frac{3}{8}$. The remaining index is applied.

Moving on

3.



The sides of the large square in this diagram are 4 units long.
The vertices of each smaller square are at the mid-points of the sides of the next largest square.
Find the total length of all of the lines in the diagram.

Working	Commentary
Let the squares be called A, B, C and D in order of size from largest to smallest.	Labelling the squares helps to identify them.
Perimeter of A $= 4 \times 4 = 16$ units	Square A (the largest square) has side lengths of 4 units so its perimeter is 16 units.
The side lengths of C are half of the side lengths of A. Perimeter of C $= \frac{1}{2} \times 16 = 8$ units	Moving 2 squares in to square C. Since the vertices of C are at the midpoints of the edges of square B, it follows that the side length of C is half that of A (look at the diagram). The perimeter of C is therefore half that of A.
Side length of B 	In the same way, the side lengths of square D are half of those of square B. The side length of square B can be found using a right-angled triangle and Pythagoras's theorem.
Length $= \sqrt{4+4}$ $= \sqrt{8}$ $= 2\sqrt{2}$	This needs a little surds work: $\sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2}$
Perimeter of B $= 4 \times 2\sqrt{2} = 8\sqrt{2}$ units	The perimeter of B is 4 sets of this i.e. $8\sqrt{2}$ The perimeter of D is half of this i.e. $4\sqrt{2}$
The side lengths of D are half of the side lengths of B. Perimeter of D $= \frac{1}{2} \times 8\sqrt{2} = 4\sqrt{2}$ units	The total length of the lines is then found by adding the perimeters together.
Total length of lines $= 16 + 8\sqrt{2} + 8 + 4\sqrt{2}$ $= 24 + 12\sqrt{2}$ units	

4. Write each of the following in the form $a + b\sqrt{c}$ where a and b are rational and $\sqrt{c}$ is the smallest surd possible

Question		Working	Commentary
a)	$1 - \frac{2}{\sqrt{3}}$	$1 - \frac{2}{\sqrt{3}} = 1 - \frac{2\sqrt{3}}{\sqrt{3}\sqrt{3}}$ $= 1 - \frac{2\sqrt{3}}{3}$ $= 1 - \frac{2}{3}\sqrt{3}$	The number at the start means that it is almost in the correct form. The fraction needs to be rationalised. This involves multiplying both the top and the bottom by $\sqrt{3}$. This gives $2\sqrt{3}$ at the top and 3 at the bottom. The final step is to separate out the fraction from the surd so it looks like the required form (even though the previous version is also in the correct form).
b)	$\sqrt{\frac{3}{2}} + \sqrt{\frac{2}{3}}$	$\sqrt{\frac{3}{2}} + \sqrt{\frac{2}{3}} = \frac{\sqrt{3}}{\sqrt{2}} + \frac{\sqrt{2}}{\sqrt{3}}$ $= \frac{3+2}{\sqrt{6}}$ $= \frac{5}{\sqrt{6}}$ $= \frac{5\sqrt{6}}{6}$ $= \frac{5}{6}\sqrt{6}$	The first step rewrites the expression using $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ The next step is to use the common denominator $\sqrt{2} \times \sqrt{3} = \sqrt{6}$ The first fraction is multiplied by $\frac{\sqrt{3}}{\sqrt{3}}$ to do this and the second is multiplied by $\frac{\sqrt{2}}{\sqrt{2}}$. The fraction simplifies to $3 + 2$ in the numerator as $\sqrt{3} \times \sqrt{3} = 3$ and $\sqrt{2} \times \sqrt{2} = 2$ The resulting fraction is multiplied by $\frac{\sqrt{6}}{\sqrt{6}}$ to rationalise the denominator. The final result is then written so it looks like the required form. Note: $a = 0$ for this question
c)	$(2 - \sqrt{5})^2$	$(2 - \sqrt{5})^2 = (2 - \sqrt{5})(2 - \sqrt{5})$ $= 4 - 2\sqrt{5} - 2\sqrt{5} + 5$ $= 9 - 4\sqrt{5}$	The expression is rewritten to ensure that no multiplications are missed. Each term in the first bracket is multiplied by each term in the second. Like terms are simplified.

d)	$\dfrac{6\sqrt{2}}{\sqrt{3}} - \dfrac{8}{3}\sqrt{\dfrac{3}{2}}$	$\dfrac{6\sqrt{2}}{\sqrt{3}} - \dfrac{8\sqrt{3}}{3\sqrt{2}} = \dfrac{36-24}{3\sqrt{6}}$ $= \dfrac{12}{3\sqrt{6}}$ $= \dfrac{4}{\sqrt{6}}$ $= \dfrac{4\sqrt{6}}{6}$ $= \dfrac{2}{3}\sqrt{6}$	The first step rewrites the $\sqrt{\frac{3}{2}}$ using $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ The next step is to use the common denominator $\sqrt{3} \times 3\sqrt{2} = 3\sqrt{6}$ The first fraction is multiplied by $\frac{3\sqrt{2}}{3\sqrt{2}}$ to do this and the second is multiplied by $\frac{\sqrt{3}}{\sqrt{3}}$. The fraction simplifies to $36 - 24$ in the numerator as $6\sqrt{2} \times 3\sqrt{2} = 18 \times 2$ and $8\sqrt{3} \times \sqrt{3} = 8 \times 3$ 12 is divisible by 3 so some cancelling can take place. The resulting fraction is multiplied by $\frac{\sqrt{6}}{\sqrt{6}}$ to rationalise the denominator. The final result is then written so it looks like the required form. Note: $a = 0$ for this question

5. Rationalise the denominator of each of the following

Question		Working	Commentary
a)	$\dfrac{2}{5\sqrt{3}}$	$\dfrac{2}{5\sqrt{3}} = \dfrac{2\sqrt{3}}{5\sqrt{3}\sqrt{3}}$ $= \dfrac{2}{15}\sqrt{3}$	Both the top and the bottom of the fraction are multiplied by $\sqrt{3}$ $5\sqrt{3} \times \sqrt{3} = 5 \times 3$ The final expression is written in the form $a\sqrt{b}$

b)	$\dfrac{\sqrt{3}}{2-\sqrt{2}}$	$\dfrac{\sqrt{3}}{2-\sqrt{2}} = \dfrac{\sqrt{3}(2+\sqrt{2})}{(2-\sqrt{2})(2+\sqrt{2})}$ $= \dfrac{\sqrt{3}(2+\sqrt{2})}{4+2\sqrt{2}-2\sqrt{2}-2}$ $= \dfrac{\sqrt{3}(2+\sqrt{2})}{2}$ $= \dfrac{2\sqrt{3}+\sqrt{6}}{2}$ $= \sqrt{3}+\dfrac{\sqrt{6}}{2}$	Both top and bottom of the fraction are multiplied by $(2+\sqrt{2})$. The change of sign to the term in the original denominator ensures that the irrational $\sqrt{2}$ terms will cancel out in the denominator. The top and bottom of the fraction are carefully multiplied out. The final step is splitting the fractions terms into separate terms over the denominator and cancelling where possible.
c)	$\dfrac{3-\sqrt{5}}{\sqrt{5}+5}$	$\dfrac{3-\sqrt{5}}{\sqrt{5}+5} = \dfrac{(3-\sqrt{5})(-\sqrt{5}+5)}{(\sqrt{5}+5)(-\sqrt{5}+5)}$ $= \dfrac{-3\sqrt{5}+15+5-5\sqrt{5}}{-5+5\sqrt{5}-5\sqrt{5}+25}$ $= \dfrac{20-8\sqrt{5}}{20}$ $= \dfrac{20}{20}-\dfrac{8\sqrt{5}}{20}$ $= 1-\dfrac{2\sqrt{5}}{5}$	Both top and bottom of the fraction are multiplied by $(-\sqrt{5}+5)$. The change of sign to the $\sqrt{5}$ in the denominator ensures that all $\sqrt{5}$ terms will cancel out. Note that the same effect could be achieved by multiplying by $(\sqrt{5}-5)$ and this would also give the correct result. The top and bottom of the fraction are carefully multiplied out. The final step is splitting the fractions terms into separate terms over the denominator and cancelling where possible.
d)	$\dfrac{2+3\sqrt{7}}{5-2\sqrt{7}}$	$\dfrac{2+3\sqrt{7}}{5-2\sqrt{7}} = \dfrac{(2+3\sqrt{7})(5+2\sqrt{7})}{(5-2\sqrt{7})(5+2\sqrt{7})}$ $= \dfrac{10+4\sqrt{7}+15\sqrt{7}+42}{25+10\sqrt{7}-10\sqrt{7}-28}$ $= \dfrac{52+19\sqrt{7}}{-3}$ $= -\dfrac{52}{3}-\dfrac{19}{3}\sqrt{7}$	Both top and bottom of the fraction are multiplied by $(5+2\sqrt{7})$. The change of sign to the $2\sqrt{7}$ in the denominator ensures that all $\sqrt{7}$ terms will cancel out. The top and bottom of the fraction are carefully multiplied out. The final step is splitting the fractions terms into separate terms over the denominator. No cancelling is possible.