GaborML_problem10_CM

February 27, 2024

Gabor ML - Problem 10

-Clarice Mottet

Outline:

Functions: -Generate linearly separably data -Generate non-linearly separably data -Perceptron algorithm

Uses: -test algorithm on different dimensions of data and types of data

[2]:
```
#Libraries

import pandas as pd
import numpy as np
import random
import matplotlib.pyplot as plt


path_out_ = r'/home/clarice/Documents/VSCode/Term2_Gabor_ML/homework3/
↪GaborML_problem10/outputs'
random.seed(123)
```

[3]:
```
#Functions

#Generate linearly separable data
def generate_ls_data(a, d, n):
    #a is a number in range [0,1]
    #d is dimension of sample data
    #n is number of sample points

    #x is a list of n numpy arrays
    #y is a list of n corresponding {-1,1}

    #determine y values with probability .5
    probability_y_equals_1 = .5
    y = [1 if random.random() > probability_y_equals_1 else -1 for _ in␣
↪range(n)]
    x = []
    #creates list of x iid sample vectors based on problem conditions
    for i in range(n):
        x_i = np.zeros(d)
        for j in range(d):
            #first element based on a
            if j == 0:
                if y[i] == 1:
                    x_i[j] = random.uniform(-1,-a)
                else:
                    x_i[j] = random.uniform(a,1)
            #other elements based on [-1,1]
            else:
                x_i[j] = random.uniform(-1,1)
        x.append(x_i)

    return x, y

# def generate_nls_data(m, d, n):
#     #create y data
#     probability_y_equals_1 = .5
#     y = [1 if random.random() >= probability_y_equals_1 else -1 for _ in␣
 ↪range(n)]

#     #create non-linearly separable data per problem description
#     mean = np.zeros(d)
#     mean[0] = m
#     cov = np.identity(d)

#     x_arr = np.random.multivariate_normal(mean, cov, n)
#     x = [x_arr[i] for i in range(n)]

#     return x, y

def generate_nls_data(m, d, n):
    #create y data
    probability_y_equals_1 = .5
    y = [1 if random.random() >= probability_y_equals_1 else -1 for _ in␣
 ↪range(n)]

    #create non-linearly separable data per problem description
    mean = np.zeros(d)
    mean[0] = m
    cov = np.identity(d)

    x_arr_zero = np.random.multivariate_normal(np.zeros(d), cov, n)
    x_arr_mean = np.random.multivariate_normal(mean, cov, n)
    x = [x_arr_zero[i] if y[i] == 1 else x_arr_mean[i] for i in range(n)]
```

```python
    return x, y


def perceptron_algorithm(x, y, learning_rate, max_epochs):
    #dimension requirements
    d = int(x[0].size)
    n = len(y)

    convergence_ind = 0
    steps_to_converge = max_epochs
    #random values between 0 and 1 (uniform)
    w = np.random.rand(d)
    b = 0.0

    #function vars
    update_log = np.zeros(max_epochs)
    num_epochs_taken = 0
    num_updates_per_epoch = 0

    while num_epochs_taken < max_epochs:
        for i in range(n):
            y_pred = np.sign(np.dot(w,x[i])+b)
            if y_pred != y[i]:
                w = np.add(w, learning_rate*(y[i]-y_pred)*x[i])
                b = b + learning_rate*(y[i]-y_pred)
                num_updates_per_epoch += 1
        #update vars for next epoch
        update_log[num_epochs_taken] = num_updates_per_epoch
        num_epochs_taken += 1
        num_updates_per_epoch = 0

        #check for convergence
        if update_log[num_epochs_taken-1] == 0:
            steps_to_converge = num_epochs_taken
            num_epochs_taken = max_epochs
            convergence_ind = 1

    return w, b, update_log, steps_to_converge, convergence_ind

def perceptron_algorithm_accuracy(x_test, y_test, w, b):
    n = len(y_test)
    error_count = 0
    for i in range(n):
        y_pred = np.sign(np.dot(w,x_test[i])+b)
        if y_test[i] != y_pred:
            error_count += 1
    error_rate = error_count/n

    return error_count, error_rate
```

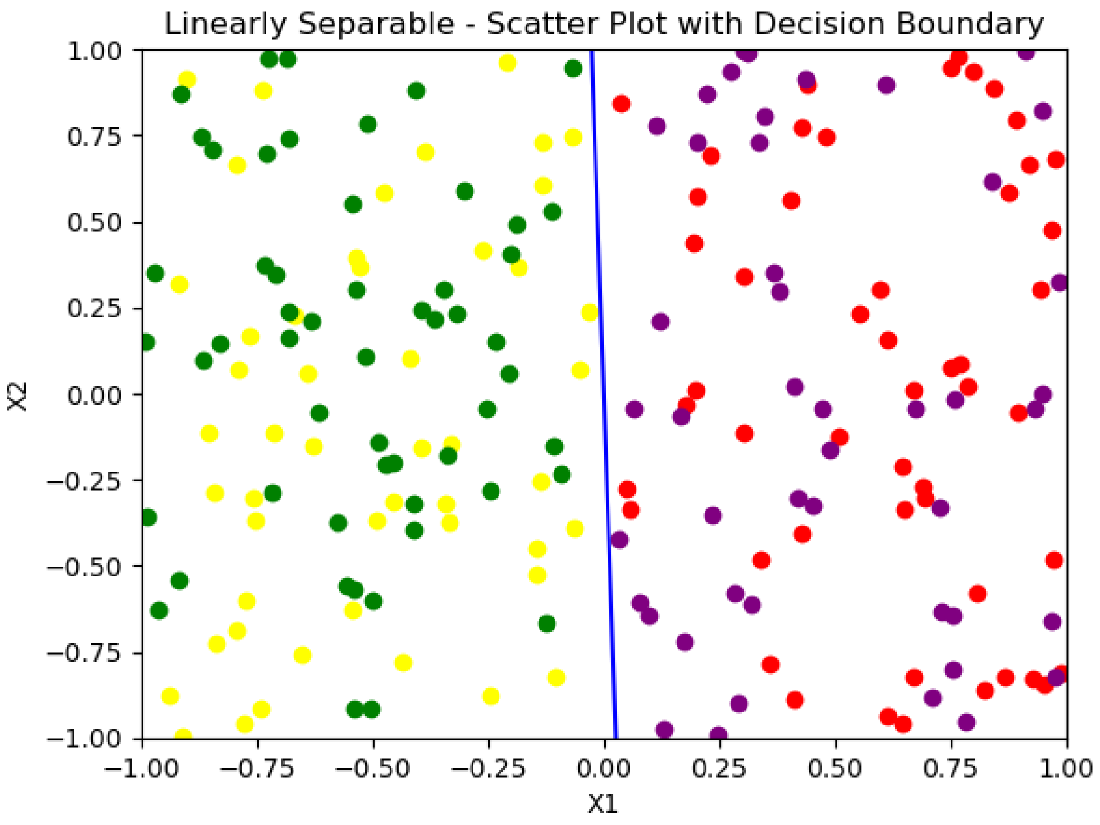
[19]:
```
#keep track of linearly separable data
# df_log_ls = pd.DataFrame()
```

[40]:
```
# #Linearly separable case

# a_values = [.0001, .001, .01, .1, .2]
# d_values = [2, 100, 250, 500, 1000]
# n_values = [20, 100, 1000, 5000, 7500]

# learning_rate = .1
# max_epochs = 200

# num_of_simulations = 20

# #create a log of testing different values
# for iter_ in range(num_of_simulations):
#     print(iter_)
#     for a in a_values:
#         for d in d_values:
#             for n in n_values:
#                 x, y = generate_ls_data(a, d, n)
#                 w, b, update_log, steps_to_converge, convergence_ind =␣
 ↪perceptron_algorithm(x, y, learning_rate, max_epochs)
#                 x_test, y_test = generate_ls_data(a, d, n)
#                 error_count, error_rate =␣
 ↪perceptron_algorithm_accuracy(x_test, y_test, w, b)
#                 df_ = pd.DataFrame([[iter_, a, d, n, steps_to_converge,␣
 ↪convergence_ind, error_rate]], columns =␣
 ↪['simulation_run','a_value','d_value','n_value','steps_to_converge','convergence_ind','erro
#                 df_log_ls = pd.concat([df_log_ls,df_], ignore_index = False)
#                 df_log_ls.reset_index(drop = True, inplace = True)
#                 df_log_ls.to_excel(path_out_+'//'+'df_log_linearly_separable.
 ↪xlsx',index = False)
```

[92]:
```
#Plot linearly separable example case

a = .01
d = 2
n = 100

x, y = generate_ls_data(a, d, n)
# print("x",x)
```

```python
# print("y",y)

learning_rate = .1
max_epochs = 200

w, b, update_log, steps_to_converge, convergence_ind = perceptron_algorithm(x,␣
    ↪y, learning_rate, max_epochs)
print("convergence indicator",convergence_ind)
print("steps to converge",steps_to_converge)
# print("w",w)
# print("b",b)
x_test, y_test = generate_ls_data(a, d, n)
error_count, error_rate = perceptron_algorithm_accuracy(x_test, y_test, w, b)

print("error rate", error_rate)

# Scatter plot
for xi, yi in zip(x, y):
    if yi == 1:
        plt.scatter(xi[0], xi[1], color='yellow')
    else:
        plt.scatter(xi[0], xi[1], color='red')
#test data
for xi, yi in zip(x_test, y_test):
    if yi == 1:
        plt.scatter(xi[0], xi[1], color='green')
    else:
        plt.scatter(xi[0], xi[1], color='purple')

# To plot the line, we need two points that satisfy the equation. Let's choose␣
    ↪two x values:
x_values = np.array([-1,1])
# Calculate corresponding y values
y_values = (-b - w[0]*x_values) / w[1]

plt.xlim(-1,1)
plt.ylim(-1,1)
plt.plot(x_values, y_values, color='blue')  # Plotting the line
plt.xlabel('X1')
plt.ylabel('X2')
plt.title('Linearly Separable - Scatter Plot with Decision Boundary')
plt.show()
```

```
convergence indicator 1
steps to converge 3
error rate 0.0
```

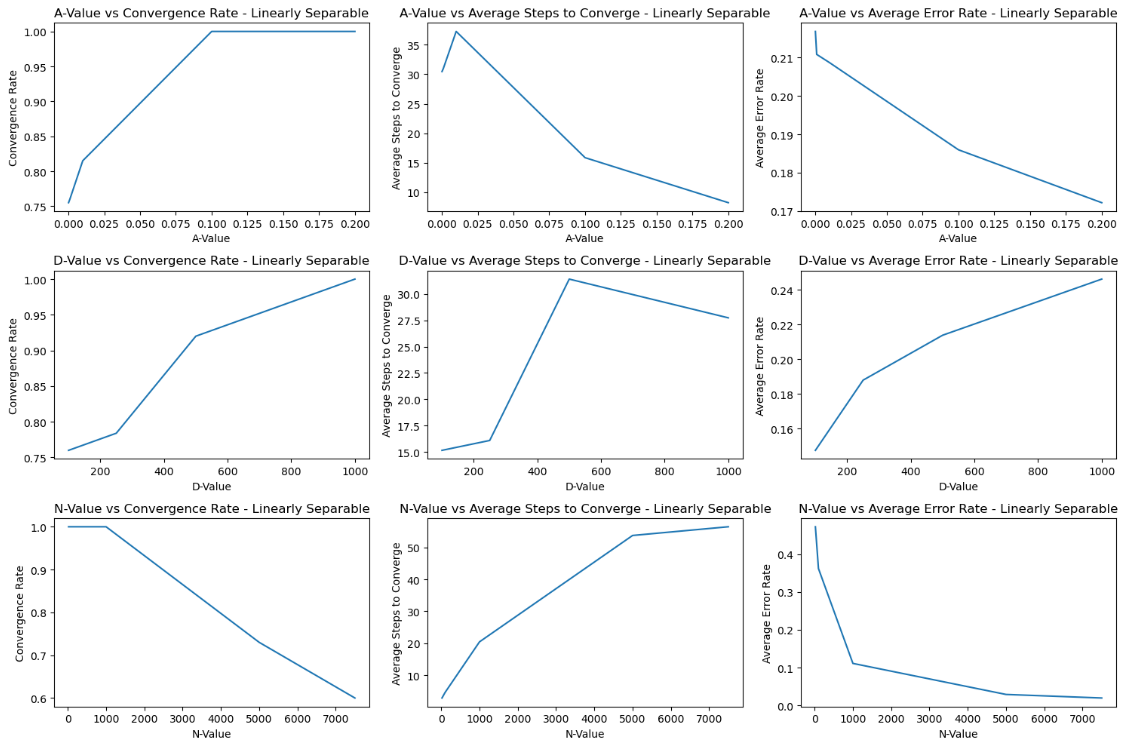


Above is a 2D example of 100 samples where the yellow dots represent the sample data points with Y=1, the red dots represent the sample data points with Y=-1, the green dots represent the test data points with Y=1 and the purple dots represent the test data points with Y=-1.

We see that both the training data and test data are linearly separable with a = .01 and that the algorithm was able to create a model that linearly separates the population.

[54]:
```
#Plot the Linearly Separable Results

df_log_ls = pd.read_excel(path_out_+'//'+'df_log_linearly_separable.xlsx')
df_log_ls = df_log_ls[df_log_ls['d_value']>2]

# Lists for iteration
list_groupbys = ['a_value', 'd_value', 'n_value']
list_groupbys_name = ['A-Value', 'D-Value', 'N-Value']
y_values = ['convergence_rate', 'average_steps_taken', 'average_error_rate']
list_calcs = ['Convergence Rate', 'Average Steps to Converge', 'Average Error␣
 ↪Rate']
type_ = 'Linearly Separable'

# Create a subplot layout
```

```
fig, axes = plt.subplots(len(list_groupbys), len(list_calcs), figsize=(15, 10))

for i, col in enumerate(list_groupbys):
    df_ = df_log_ls.copy()
    df_['convergence_rate'] = df_.groupby(by=[col])['convergence_ind'].
  ↪transform('mean')
    df_['average_error_rate'] = df_.groupby(by=[col])['error_rate'].
  ↪transform('mean')
    df_ = df_[df_['convergence_ind'] == 1]
    df_['average_steps_taken'] = df_.groupby(by=[col])['steps_to_converge'].
  ↪transform('mean')
    df_unique = df_.drop_duplicates(subset=[col])

    for j, calc in enumerate(list_calcs):
        y_val = y_values[j]
        axes[i, j].plot(df_unique[col].to_numpy(), df_unique[y_val].to_numpy())
        axes[i, j].set_xlabel(list_groupbys_name[i])
        axes[i, j].set_ylabel(calc)
        axes[i, j].set_title(f'{list_groupbys_name[i]} vs {calc} - {type_}')

plt.tight_layout()
plt.show()
```



Here we see the behavior of the algorithm with respect to various parameters.

The a parameter represents the extent to which the data is separable. An a value closer to zero increases the difficulty of separating the data given how we're defined the X iid population. For the a values closer to zero we see there are simulations that could not converge within 200 epochs (80% converged), we see that when the algorithm does converge it takes more steps to reach convergence (around 30), and we see that the error rate is higher. As the a values move further away from zero, we see the algorithm converge for each simulation, the number of steps it takes for the algorithm to converge is less (around 10) and the error rate decreases. This is in line with what we would expect to see with the a parameter determining the extent to which the data is linearly separable.

The d parameter represents the dimension of the sample data. We see as dimension increases, the probability of the algorithm converging increases, the average steps to converge peaks around dimension 500 and then decreases again, and we see the error rate increase. We'd expect the error rate to increase in line with the curse of dimensionality where the increase in dimension does not lead to a lower error rate. It is surprising the see the convergence rate increase as dimension increases.

The n parameter represents the size of the sample data the model is trained on. We see that up to the point where n is 1000 we see the model always converge (in less than 200 steps) but after that when n is 7500 it only converges around 75% of the time. For the cases where the algorithm does converge, we see it takes more steps for the algorithm to converge as the size of the sample data increases. We see the error rate of test data decrease exponentially as the size of the sample data increases. This is in line with what we would expect in terms of average steps taken and average error rate relating to the sample size.

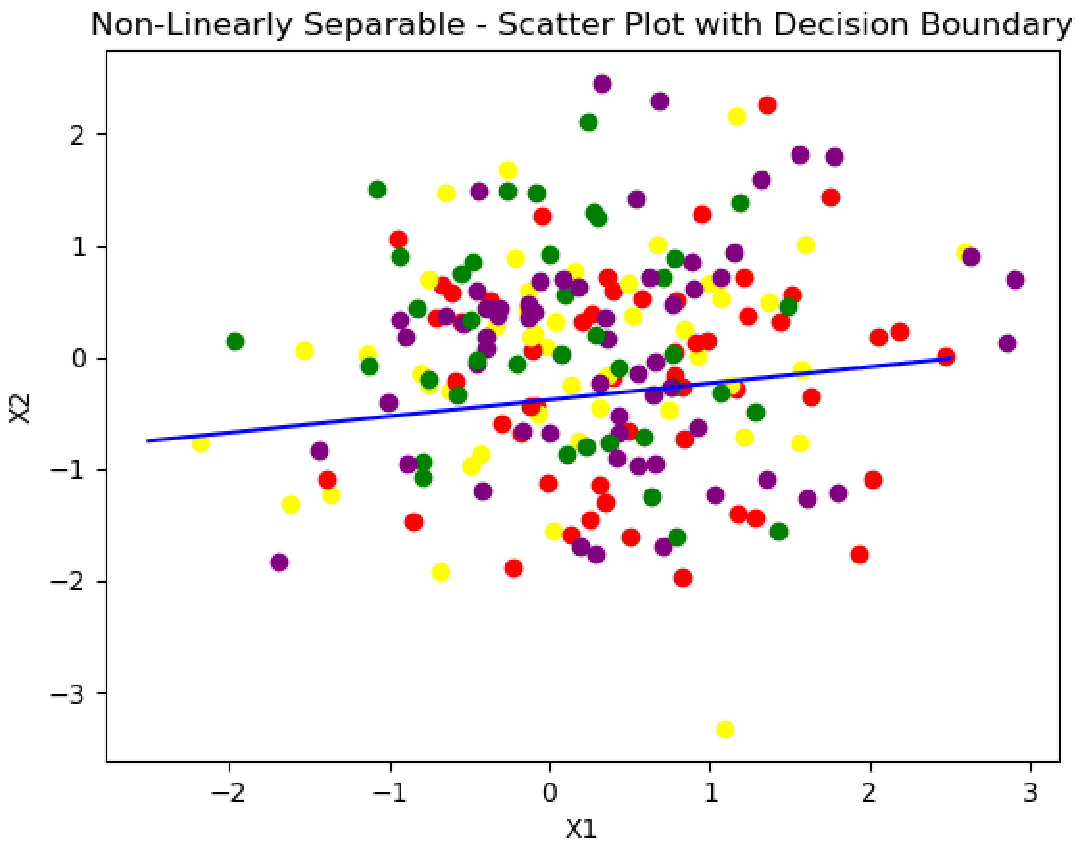
[13]:
```
#create log of non-linearly separable data
# df_log_nls = pd.DataFrame()
```

[15]:
```
#Non-linearly separable case

# m_values = [0, .5, 2, 4]
# d_values = [2, 100, 250, 500, 1000]
# n_values = [20, 100, 1000, 5000, 7500]

# learning_rate = .1
# max_epochs = 100

# num_of_simulations = 20

# #create a log of testing different values
# for iter_ in range(num_of_simulations):
#     print(iter_)
#     for m in m_values:
#         for d in d_values:
#             for n in n_values:
#                 x, y = generate_nls_data(m, d, n)
#                 w, b, update_log, steps_to_converge, convergence_ind =␣
↪perceptron_algorithm(x, y, learning_rate, max_epochs)
```

```
#                   x_test, y_test = generate_nls_data(m, d, n)
#                   error_count, error_rate =␣
 ↪perceptron_algorithm_accuracy(x_test, y_test, w, b)
#                   df_ = pd.DataFrame([[iter_, m, d, n, steps_to_converge,␣
 ↪convergence_ind, error_rate]], columns =␣
 ↪['simulation_run','m_value','d_value','n_value','steps_to_converge','convergence_ind','erro
#                   df_log_nls = pd.concat([df_log_nls,df_], ignore_index = False)
#                   df_log_nls.reset_index(drop = True, inplace = True)
#                   df_log_nls.to_excel(path_out_+'//
 ↪'+'df_log_nonlinearly_separable_update.xlsx',index = False)
```

```
[12]: m = .5
d = 2
n = 100

x, y = generate_nls_data(m, d, n)
# print("x",x)
# print("y",y)

learning_rate = .1
max_epochs = 100

w, b, update_log, steps_to_converge, convergence_ind = perceptron_algorithm(x,␣
 ↪y, learning_rate, max_epochs)

print("convergence indicator",convergence_ind)
print("steps to converge",steps_to_converge)

# print("w",w)
# print("b",b)

x_test, y_test = generate_nls_data(m, d, n)

error_count, error_rate = perceptron_algorithm_accuracy(x_test, y_test, w, b)

print("error rate", error_rate)

# Scatter plot
for xi, yi in zip(x, y):
    if yi == 1:
        plt.scatter(xi[0], xi[1], color='yellow')
    else:
        plt.scatter(xi[0], xi[1], color='red')
#test data
for xi, yi in zip(x_test, y_test):
    if yi == 1:
        plt.scatter(xi[0], xi[1], color='green')
    else:
        plt.scatter(xi[0], xi[1], color='purple')

# To plot the line, we need two points that satisfy the equation. Let's choose␣
↪two x values:
# x_values_train = np.array([min(x, key=lambda v: v[0])[0], max(x, key=lambda v:
↪ v[0])[0]])
# x_values_test = np.array([min(x_test, key=lambda v: v[0])[0], max(x_test,␣
↪key=lambda v: v[0])[0]])
# x_values = np.array([min(x_values_train[0],x_values_test[0]),␣
↪max(x_values_train[1],x_values_test[1])])
x_values = np.array([-2.5,2.5])
# Calculate corresponding y values
y_values = (-b - w[0]*x_values) / w[1]

plt.plot(x_values, y_values, color='blue')  # Plotting the line
plt.xlabel('X1')
plt.ylabel('X2')
plt.title('Non-Linearly Separable - Scatter Plot with Decision Boundary')
plt.show()
```

```
convergence indicator 0
steps to converge 100
error rate 0.52
```

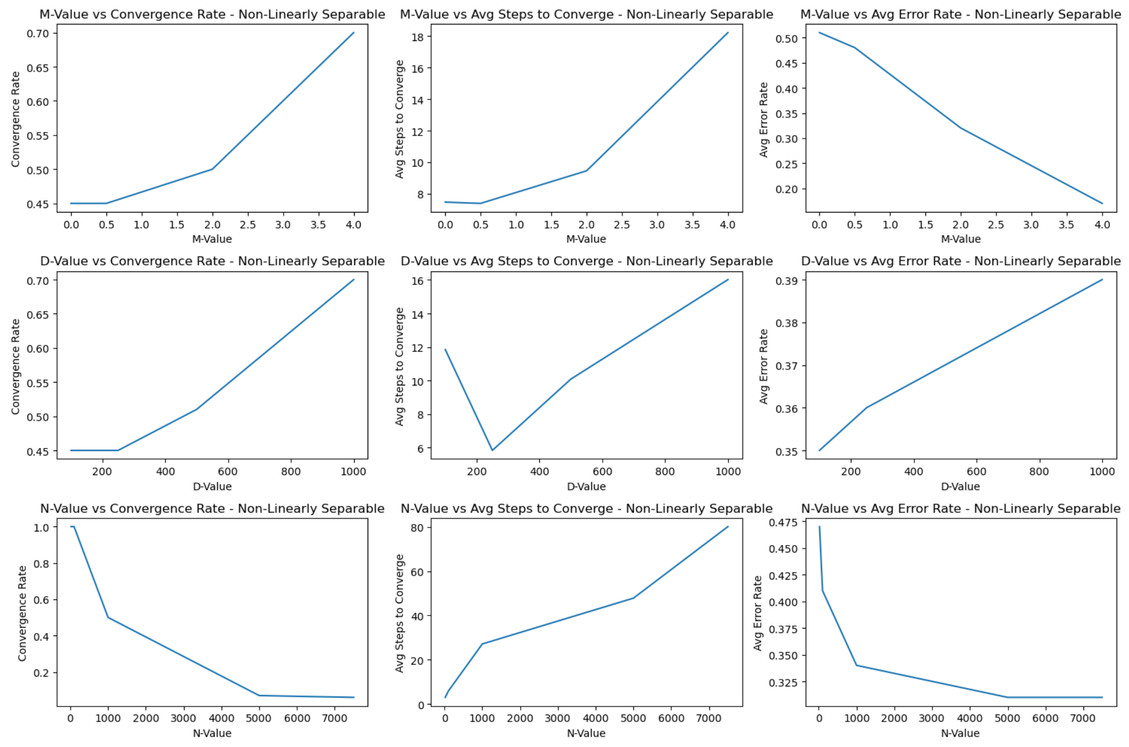


Again we have a 2D example of 100 samples where the yellow dots represent the sample data points with Y=1, the red dots represent the sample data points with Y=-1, the green dots represent the test data points with Y=1 and the purple dots represent the test data points with Y=-1.

We see that the data is not linearly separable (the red and yellow dots overlap and the green and purple dots overlap), the algorithm is not able to converge to a model that linearly separates the data and the model created performs as well as a guess (error rate = .5).

[5]:
```
#Plot the Non-Linearly Separable Results

df_log_nls = pd.read_excel(path_out_+'//'+'df_log_nonlinearly_separable_update.
 ↪xlsx')
df_log_nls = df_log_nls[df_log_nls['d_value']>2]

# Lists for iteration
list_groupbys = ['m_value', 'd_value', 'n_value']
list_groupbys_name = ['M-Value', 'D-Value', 'N-Value']
y_values = ['convergence_rate', 'average_steps_taken', 'average_error_rate']
list_calcs = ['Convergence Rate', 'Avg Steps to Converge', 'Avg Error Rate']
type_ = 'Non-Linearly Separable'
```

```python
# Create a subplot layout
fig, axes = plt.subplots(len(list_groupbys), len(list_calcs), figsize=(15, 10))

for i, col in enumerate(list_groupbys):
    df_ = df_log_nls.copy()
    df_['convergence_rate'] = df_.groupby(by=[col])['convergence_ind'].
 ↪transform('mean').round(2)
    df_['average_error_rate'] = df_.groupby(by=[col])['error_rate'].
 ↪transform('mean').round(2)
    df_ = df_[df_['convergence_ind'] == 1]
    df_['average_steps_taken'] = df_.groupby(by=[col])['steps_to_converge'].
 ↪transform('mean').round(2)
    df_unique = df_.drop_duplicates(subset=[col])

    for j, calc in enumerate(list_calcs):
        y_val = y_values[j]
        axes[i, j].plot(df_unique[col].to_numpy(), df_unique[y_val].to_numpy())
        axes[i, j].set_xlabel(list_groupbys_name[i])
        axes[i, j].set_ylabel(calc)
        axes[i, j].set_title(f'{list_groupbys_name[i]} vs {calc} - {type_}')

plt.tight_layout()
plt.show()
```



Here we have graphs for how the perceptron algorithm runs with non-linearly separable data.

When looking at the m parameter representing the extent to which we can “separate” the data we see that the larger value for m (and therefore the data is more separable) the algorithm is better able to find a way to linearly separate the data. As the we increase the value of m (the data is more separable) it takes fewer steps for the algorithm to converge (if it does) and that the rate of miss classifications on test data decreases. This aligns with what we would expect to see that an increase in the margin (so great that we are infact able to separate the data) leads to a better ability to yield a classifier and better results when performing on test data.

With regard to the d value - dimension of the data, similar to the m-parameter, we’re able to yield more results of convergence when the dimension of the data increases. The difference between the behavior of the algorithm with regard to the dimension v.s. the margin is that as we increase the dimension, we see that the error rate increases with we would expect as a result of the “curse of dimensionality”.

For the n parameter representing the number of samples we have our algorithm create a classifier on, we see kind of “flip-floppy” behavior meaning the rate of converging to a classifier decreases as the number of samples increases, yet the error rate on test data decreases as the number of samples increases. So while it is more challenging to create a classifier on non-separable data when the sample size is large, when that classifier is created, it performs better out of sample than a classifier based on fewer data points.

Overall it is strange to see the frequency with which we are able to linearly separate data that we were expecting to not be able to linearly separate.