GaborML_problem16_AW

March 11, 2024

Gabor ML - Problem 16

1 Libraries

[26]:
```
import pandas as pd
import numpy as np
import random
import time
import matplotlib.pyplot as plt
```

2 Functions

[27]:
```
#function that creates x and y data
def generate_xy(n, d):
    # create covariance matrix
    cov = np.identity(d)
    for i in range(d):
        cov[i,i] = 1/(i+1)
    # create x sample data
    x = np.random.multivariate_normal(np.zeros(d), cov, n)
    # initialize y data
    y = np.zeros(n)
    # standard normal error
    e = np.random.normal(0, 1, n)
    # determine y values by x values
    for i in range(n):
        y[i] = sum(x[i]) + e[i]
    return x, y

#function that gives parameters from OLS
def algorithm_OLS(x, y):
    #get n and d from x
    n = len(x)
    d = len(x[0])
    #follow the math from the hw description
    temp_matrix = np.zeros((d,d))
```

```
    for i in range(n):
        add_matrix = np.outer(x[i], x[i].T)
        temp_matrix = np.add(temp_matrix, add_matrix)
    temp_matrix = np.multiply(1/n, temp_matrix)
    temp_matrix = np.linalg.inv(temp_matrix)
    #follow the math from the hw description
    temp_vec = np.zeros(d)
    for i in range(n):
        add_vec = np.multiply(x[i], y[i])
        temp_vec = np.add(temp_vec, add_vec)
    temp_vec = np.multiply(1/n, temp_vec)
    #return w parameter
    w = np.dot(temp_matrix, temp_vec)
    return w

#function that gives parameters using GD - gradient descent
def algorithm_GD(x, y, method_learning_rate):
    #get n and d from x
    n = len(x)
    d = len(x[0])
    #create a list of what the learning rate will be at step_t
    if method_learning_rate == "fixed":
        step_at_t = [.1 for i in range(n)]
    else:
        step_at_t = [1/np.sqrt(i+1) for i in range(n)]

    #create a matrix of parameters at each step
    w = np.zeros((n,d))
    #with an initial vector
    w_0 = np.zeros(d)

    #determine parameters for each step
    for t in range(n):
        temp_vec_sum = np.zeros(d)
        #if we're at the first step, we'll use a zero vector as the initial␣
 ↪parameter values
        if t == 0:
            for i in range(n):
                temp1 = np.multiply(np.dot(w_0.T,x[i]),x[i])
                temp2 = np.multiply(x[i], y[i])
                temp3 = np.subtract(temp1, temp2)
                temp_vec_sum = np.add(temp_vec_sum, temp3)
            temp_vec_sum = np.multiply(step_at_t[t],temp_vec_sum)
            temp_vec_sum = np.multiply(2/n,temp_vec_sum)
            w[t] = np.subtract(w_0, temp_vec_sum)
        else:
            for i in range(n):
```

```
                temp1 = np.multiply(np.dot(w[t-1].T,x[i]),x[i])
                temp2 = np.multiply(x[i], y[i])
                temp3 = np.subtract(temp1, temp2)
                temp_vec_sum = np.add(temp_vec_sum, temp3)
            temp_vec_sum = np.multiply(step_at_t[t],temp_vec_sum)
            temp_vec_sum = np.multiply(2/n,temp_vec_sum)
            w[t] = np.subtract(w[t-1], temp_vec_sum)

    #calculate average of parameters
    w_average = np.zeros(d)
    for t in range(n):
        w_average = np.add(w_average, w[t])
    w_average = np.multiply(1/n, w_average)
    return w_average

#function that gives parameters using GD - gradient descent
def algorithm_SGD(x, y, method_learning_rate):
    #get n and d from x
    n = len(x)
    d = len(x[0])
    #create a list of what the learning rate will be at step_t
    if method_learning_rate == "fixed":
        step_at_t = [.1 for i in range(n)]
    else:
        step_at_t = [1/np.sqrt(i+1) for i in range(n)]

    #create a matrix of parameters at each step
    w = np.zeros((n,d))
    #with an initial vector
    w_0 = np.zeros(d)

    #determine parameters for each step
    for t in range(n):
        #if we're at the first step, we'll use a zero vector as the initial␣
 ↪parameter values
        if t == 0:
            temp1 = np.multiply(np.dot(w_0.T,x[t]),x[t])
            temp2 = np.multiply(x[t], y[t])
            temp_vec = np.subtract(temp1, temp2)
            temp_vec = np.multiply(2,temp_vec)
            temp_vec = np.multiply(step_at_t[t],temp_vec)
            w[t] = np.subtract(w_0, temp_vec)
        else:
            temp_vec = np.subtract(np.multiply(np.dot(w[t-1].T,x[t]),x[t]), np.
 ↪multiply(x[t], y[t]))
            temp_vec = np.multiply(2,temp_vec)
            temp_vec = np.multiply(step_at_t[t],temp_vec)
```

```
            w[t] = np.subtract(w[t-1], temp_vec)

    #calculate average of parameters
    w_average = np.zeros(d)
    for t in range(n):
        w_average = np.add(w_average, w[t])
    w_average = np.multiply(1/n, w_average)
    return w_average

#function that calculates the MSE
def mean_square_error(x, y, w):
    #get n and d from x
    n = len(x)
    #initialize
    mse = 0
    for i in range(n):
        mse += np.square(np.dot(w.T,x[i])-y[i])
    mse = mse/n
    return mse
```

3 Compare OLS, Gradient Descent, Stocastic Gradient Descent with fixed learning Rate

3.1 Calculate metrics for OLS, GradDesc, StoGradDesc

[28]:
```
#Determine algorithm performance when learning rate is fixed.

results_fixed = pd.DataFrame()
method_learning_rate = "fixed"
method_algorithm = ['OLS','GradDesc','StoGradDesc']
n_values = [10, 20, 100, 500]
d_values = [2, 5, 10, 50, 100]

#calculate average of the performance results over 5 runs
number_of_runs = 5
run_number = 0

#running through the n_values and d_values to compare algorithm methods
while run_number < number_of_runs:
    print(run_number)
    for n in n_values:
        for d in d_values:
            arr_train_x, arr_y_train = generate_xy(n, d)
            arr_test_x, arr_y_test = generate_xy(n, d)
            for method in method_algorithm:
                start = time.time()
```

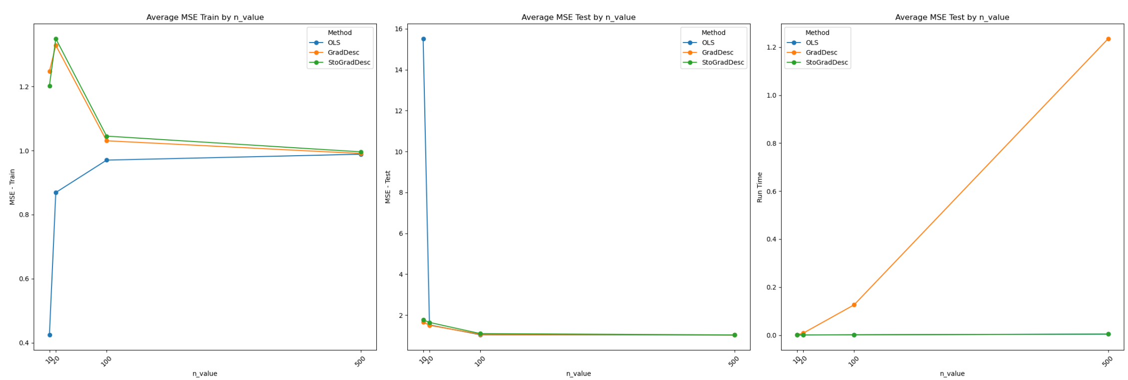
```
                if method == 'OLS':
                    w = algorithm_OLS(arr_train_x, arr_y_train)
                elif method == 'GradDesc':
                    w = algorithm_GD(arr_train_x, arr_y_train,␣
↪method_learning_rate)
                else:
                    w = algorithm_SGD(arr_train_x, arr_y_train,␣
↪method_learning_rate)
                end = time.time()
                mse_train = mean_square_error(arr_train_x, arr_y_train, w)
                mse_test = mean_square_error(arr_test_x, arr_y_test, w)
                run_time = (end - start)
                line_ = pd.DataFrame([[method, n, d, run_time, mse_train,␣
↪mse_test]], columns =␣
↪['method','n_value','d_value','run_time','mse_train','mse_test'])
                results_fixed = pd.concat([results_fixed, line_], ignore_index␣
↪= True)
                results_fixed.reset_index(drop = True, inplace = True)
    run_number += 1
```

```
0
1
2
3
4
```

3.2 Compare method by sample size - fixed learning rate

[34]:
```
#look at the fixed learning rate results

df = results_fixed.copy()
df = df[df['d_value']<50]
parameter = 'n_value'
method_algorithm = ['OLS','GradDesc','StoGradDesc']

#calculate average metrics for n_values
for col in ['mse_train', 'mse_test', 'run_time']:
    df[col] = df.groupby(by = ['method',parameter])[col].transform('mean')
df.drop_duplicates(subset = ['method',parameter], inplace = True)

# Creating the plot with adjustments for clear visibility of 'n_value'
fig, axes = plt.subplots(nrows=1, ncols=3, figsize=(24, 8))
for j, metric in enumerate(['mse_train', 'mse_test', 'run_time']):
    ax = axes[j]
    for method in method_algorithm:
        subset = df[(df['method'] == method)]
```

```
        ax.plot(subset[parameter].to_numpy(), subset[metric].to_numpy(), marker='o', linestyle='-', label=method)

    if j == 0:
        ax.set_title(f'Average MSE Train by {parameter}')
        ax.set_ylabel('MSE - Train')
        ax.set_xlabel(parameter)
    elif j == 1:
        ax.set_title(f'Average MSE Test by {parameter}')
        ax.set_ylabel('MSE - Test')
        ax.set_xlabel(parameter)
    else:
        ax.set_title(f'Average MSE Test by {parameter}')
        ax.set_ylabel('Run Time')
        ax.set_xlabel(parameter)
    ax.legend(title='Method')
    ax.set_xticks(subset[parameter])
    ax.tick_params(axis='x', rotation=45)

plt.tight_layout()
plt.show()
```

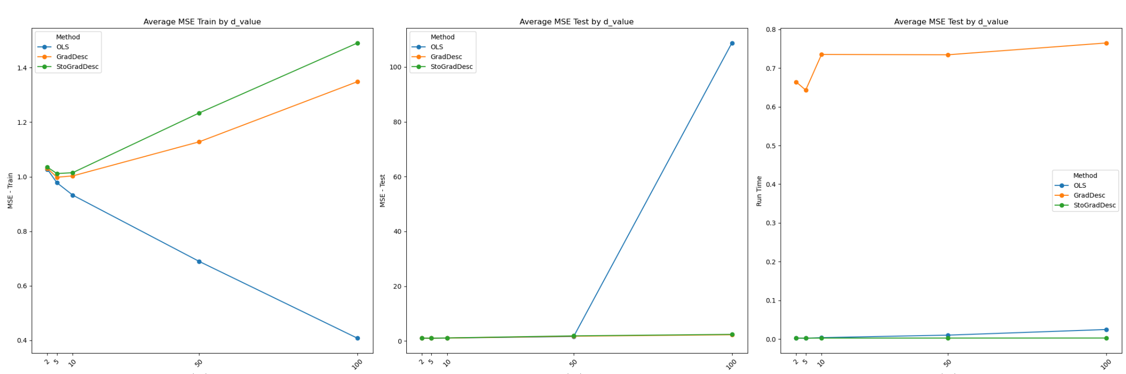


Fixed learning rate When the sample size increases, the MSE for the training and testing data is around 1 but for smaller sample sizes the MSE is higher which we would expect. The MSE for OLS is super high when the sample size is small for the testing data yet the lowest MSE for the training data therefore overfitting to the training data. The highest run time is for Gradient Descent which we would expect since it uses all the training data in each update.

3.3 Compare method by dimension - fixed learning rate

[37]:
```
#look at the fixed learning rate results

df = results_fixed.copy()
df = df[df['n_value']>=100]
```

```python
parameter = 'd_value'
method_algorithm = ['OLS','GradDesc','StoGradDesc']

#calculate average metrics for n_values
for col in ['mse_train', 'mse_test', 'run_time']:
    df[col] = df.groupby(by = ['method',parameter])[col].transform('mean')
df.drop_duplicates(subset = ['method',parameter], inplace = True)

# Creating the plot with adjustments for clear visibility of 'n_value'
fig, axes = plt.subplots(nrows=1, ncols=3, figsize=(24, 8))
for j, metric in enumerate(['mse_train', 'mse_test', 'run_time']):
    ax = axes[j]
    for method in method_algorithm:
        subset = df[(df['method'] == method)]
        ax.plot(subset[parameter].to_numpy(), subset[metric].to_numpy(), marker='o', linestyle='-', label=method)

    if j == 0:
        ax.set_title(f'Average MSE Train by {parameter}')
        ax.set_ylabel('MSE - Train')
        ax.set_xlabel(parameter)
    elif j == 1:
        ax.set_title(f'Average MSE Test by {parameter}')
        ax.set_ylabel('MSE - Test')
        ax.set_xlabel(parameter)
    else:
        ax.set_title(f'Average MSE Test by {parameter}')
        ax.set_ylabel('Run Time')
        ax.set_xlabel(parameter)
    ax.legend(title='Method')
    ax.set_xticks(subset[parameter])
    ax.tick_params(axis='x', rotation=45)

plt.tight_layout()
plt.show()
```



Fixed learning rate

We see that as the dimension of the data grows, the MSE of the training data decreases for OLS but we see the opposite behavior of the MSE on the testing data, further showing us that OLS is overfitting to the training data. For gradient descent and stocastic gradient descent we see the MSE increase as the dimension increases, which is what we would expect given the curse of dimensionality but we see it stay approximately the same for the testing data MSE. Again we see the gradient descent takes the longest to run regardless of dimension of the sample data.

4 Compare OLS, Gradient Descent, Stocastic Gradient Descent with dynamic learning Rate

4.1 Calculate metrics for OLS, GradDesc, StoGradDesc

[39]:
```
#Determine algorithm performance when learning rate is fixed.

results_dynamic = pd.DataFrame()
method_learning_rate = "dynamic"
method_algorithm = ['OLS','GradDesc','StoGradDesc']
n_values = [10, 20, 100, 500]
d_values = [2, 5, 10, 50, 100]

#calculate average of the performance results over 5 runs
number_of_runs = 5
run_number = 0

#running through the n_values and d_values to compare algorithm methods
while run_number < number_of_runs:
    print(run_number)
    for n in n_values:
        for d in d_values:
            arr_train_x, arr_y_train = generate_xy(n, d)
            arr_test_x, arr_y_test = generate_xy(n, d)
            for method in method_algorithm:
                start = time.time()
                if method == 'OLS':
                    w = algorithm_OLS(arr_train_x, arr_y_train)
                elif method == 'GradDesc':
                    w = algorithm_GD(arr_train_x, arr_y_train,␣
 ↪method_learning_rate)
                else:
                    w = algorithm_SGD(arr_train_x, arr_y_train,␣
 ↪method_learning_rate)
                end = time.time()
                mse_train = mean_square_error(arr_train_x, arr_y_train, w)
```

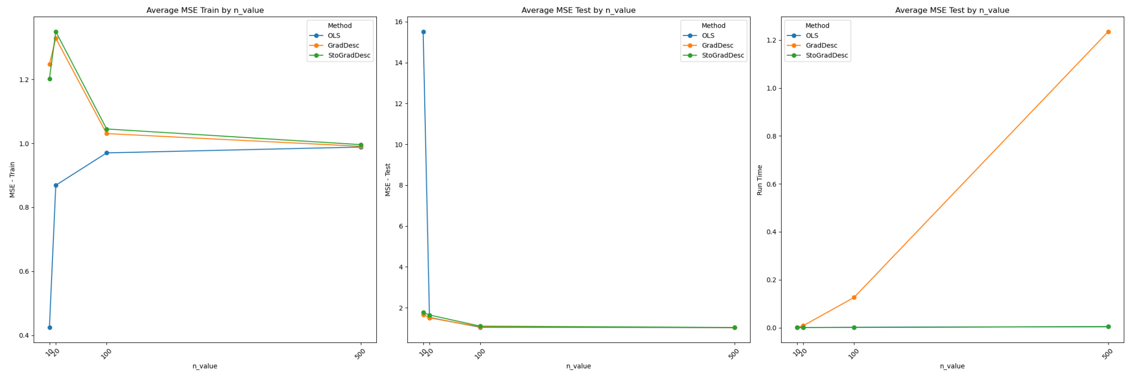
```
                mse_test = mean_square_error(arr_test_x, arr_y_test, w)
                run_time = (end - start)
                line_ = pd.DataFrame([[method, n, d, run_time, mse_train,␣
 ↪mse_test]], columns =␣
 ↪['method','n_value','d_value','run_time','mse_train','mse_test'])
                results_dynamic = pd.concat([results_fixed, line_],␣
 ↪ignore_index = True)
                results_dynamic.reset_index(drop = True, inplace = True)
    run_number += 1
```

```
0
1
2
3
4
```

4.2 Compare method by sample size - dynamic learning rate

[41]:
```
#look at the fixed learning rate results

df = results_dynamic.copy()
df = df[df['d_value']<50]
parameter = 'n_value'
method_algorithm = ['OLS','GradDesc','StoGradDesc']

#calculate average metrics for n_values
for col in ['mse_train', 'mse_test', 'run_time']:
    df[col] = df.groupby(by = ['method',parameter])[col].transform('mean')
df.drop_duplicates(subset = ['method',parameter], inplace = True)

# Creating the plot with adjustments for clear visibility of 'n_value'
fig, axes = plt.subplots(nrows=1, ncols=3, figsize=(24, 8))
for j, metric in enumerate(['mse_train', 'mse_test', 'run_time']):
    ax = axes[j]
    for method in method_algorithm:
        subset = df[(df['method'] == method)]
        ax.plot(subset[parameter].to_numpy(), subset[metric].to_numpy(),␣
 ↪marker='o', linestyle='-', label=method)

    if j == 0:
        ax.set_title(f'Average MSE Train by {parameter}')
        ax.set_ylabel('MSE - Train')
        ax.set_xlabel(parameter)
    elif j == 1:
        ax.set_title(f'Average MSE Test by {parameter}')
        ax.set_ylabel('MSE - Test')
        ax.set_xlabel(parameter)
```

```
    else:
        ax.set_title(f'Average MSE Test by {parameter}')
        ax.set_ylabel('Run Time')
        ax.set_xlabel(parameter)
    ax.legend(title='Method')
    ax.set_xticks(subset[parameter])
    ax.tick_params(axis='x', rotation=45)

plt.tight_layout()
plt.show()
```

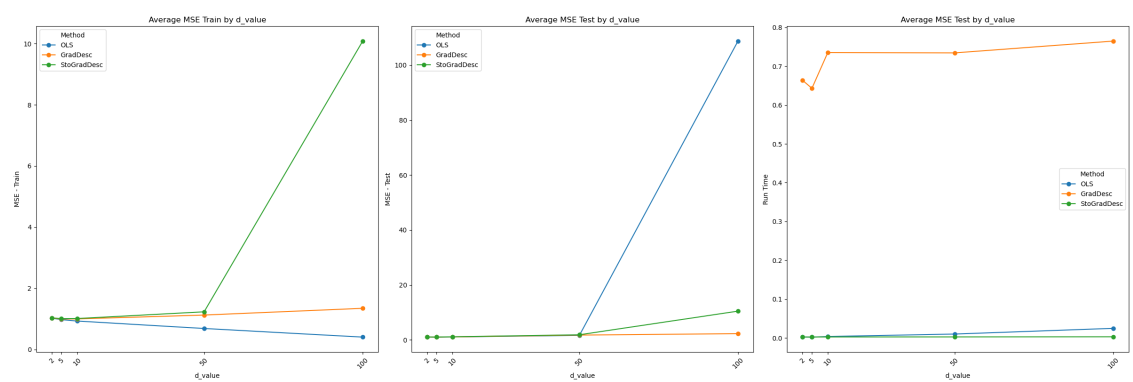


Here we see similar behavior for the algorithms to the fixed learning rate as the sample size increases except it appears the gradient descent and stocastic gradient descent perform slightly worse on the testing data when the dynamic learning rate is used.

4.3 Compare method by dimension - dynamic learning rate

[43]:
```
#look at the dynamic learning rate results

df = results_dynamic.copy()
df = df[df['n_value']>=100]
parameter = 'd_value'
method_algorithm = ['OLS','GradDesc','StoGradDesc']

#calculate average metrics for n_values
for col in ['mse_train', 'mse_test', 'run_time']:
    df[col] = df.groupby(by = ['method',parameter])[col].transform('mean')
df.drop_duplicates(subset = ['method',parameter], inplace = True)

# Creating the plot with adjustments for clear visibility of 'n_value'
fig, axes = plt.subplots(nrows=1, ncols=3, figsize=(24, 8))
for j, metric in enumerate(['mse_train', 'mse_test', 'run_time']):
    ax = axes[j]
    for method in method_algorithm:
```

```
        subset = df[(df['method'] == method)]
        ax.plot(subset[parameter].to_numpy(), subset[metric].to_numpy(), marker='o', linestyle='-', label=method)

    if j == 0:
        ax.set_title(f'Average MSE Train by {parameter}')
        ax.set_ylabel('MSE - Train')
        ax.set_xlabel(parameter)
    elif j == 1:
        ax.set_title(f'Average MSE Test by {parameter}')
        ax.set_ylabel('MSE - Test')
        ax.set_xlabel(parameter)
    else:
        ax.set_title(f'Average MSE Test by {parameter}')
        ax.set_ylabel('Run Time')
        ax.set_xlabel(parameter)
    ax.legend(title='Method')
    ax.set_xticks(subset[parameter])
    ax.tick_params(axis='x', rotation=45)

plt.tight_layout()
plt.show()
```



Here we see stochastic gradient descent performs a lot worse when the dimension increases for the training data and the test data with the dynamic learning rate. It could be that as the learning rate decreases, it doesn't coincide with the parameters converging to a solution. We don't see the same thing with gradient descent but gradient descent uses the all of the training data to update the parameters so its more robust against the learning rate decreasing.