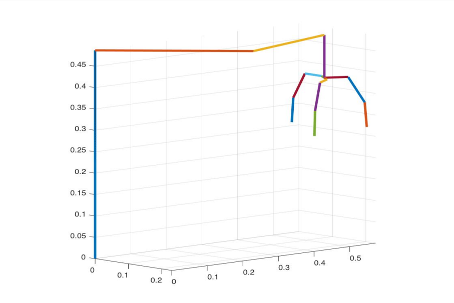
ROBOT INVERSE KINEMATICS USING SQP METHOD

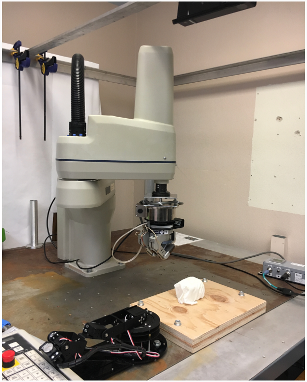
Xiaoqian Mu, Yuechuan Xue

December 5, 2017

Iowa State University

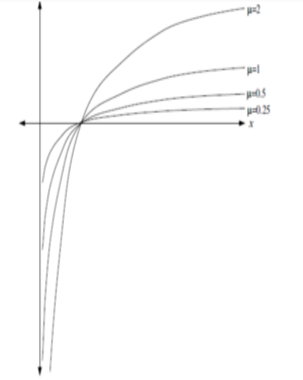
PROBLEM FORMULATION





In constrained optimization, the general aim is to transform the problem into an easier subproblem that can then be solved and used as the basis of an iterative process.

Some early methods translate the constrained problem to a basic unconstrained problem by using a **penalty function** for constraints that are near or beyond the constraint boundary. (Interior Point Method)



These methods are now considered relatively inefficient and have been replaced by methods that have focused on the solution of the KKT equations.

The KKT are necessary conditions for optimality for a constrained optimization problem. It is both necessary and sufficient for a global optimal point in a convex programming problem.

A series of algorithms attempt to compute the Lagrange multipliers directly.

Constrained quasi-Newton methods guarantee super-linear convergence by accumulating second-order information regarding the KKT equations using a quasi-Newton updating procedure.

A quadratic programming (QP) problem will be solved at each major iteration.

Principal idea: **Formulate a QP subproblem based on a quadratic approximation of the Lagrangian function.**

$$L(x, \lambda) = f(x) + \sum_{i=1}^{m} \lambda_i \cdot g_i(x)$$

At each major iteration, an approximation is made of the Hessian of Lagrangian function. The constraints are approximated linearly.

The solution of QP subproblem will be used to form a search direction for a line search procedure.

QP SubProblem:

$$\min \frac{1}{2} d^\top H_k d + \nabla f(x_k)^\top d$$

$$\begin{aligned}
\nabla g_i(x_k)^\top d + g_i(x_k) &= 0, i = 1, \cdots, m_e \\
\nabla g_i(x_k)^\top d + g_i(x_k) &\leq 0, i = m_e + 1, \cdots, m
\end{aligned}$$

The SQP implementation consists of three main stages:

- Updating the Hessian Matrix
- Quadratic Programming Solution
- Line Search and Merit Function

At each major iteration, a positive definite quasi-Newton approximation of the Hessian of Lagrangian function, H, is calculated using the **BFGS** method, where $\lambda_i, i = 1 \cdots m$, is an estimate of the Lagrange

$$C_k^{\mathrm{BFGS}} = \frac{q_k q_k^\top}{q_k^T s_k} - \frac{H_k p_k p_k^\top H_k^\top}{p_k^\top H_k p_k}$$

$$H_{k+1} = H_k + C_k^{\mathrm{BFGS}}$$

QUADRATIC PROGRAMMING SOLUTION

At each major iteraion, a QP problem of the following form is solved

$$\min \frac{1}{2} d^\top H_k d + c^\top d$$

$$A_i d = b_i, \quad i = 1, \cdots, m_e$$
$$A_i d = b_i, \quad i \leq m_e + 1, \cdots, m$$

The solution to the QP subproblem produces a vector d_k, which is used to form a new iterate

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \alpha \boldsymbol{d}_k$$

The step length parameter α_k is determined in order to produce a sufficient decrease in a **merit function**.

Merit Function:

$$\Psi = f(x) + \sum_{i=1}^{m_e} r_i \cdot g_i(x) + \sum_{i=m_e+1}^{m} r_i \cdot \max[0, g_i(x)]$$

where r_i is a penalty parameter.