Central Difference Approximation

$$f'(a) = \frac{f(a+h) - f(a-h)}{2h}$$

Taylor approximation of $f(a+h)$

$$f(a+h) = f(a) + f'(a)(a+h-a) + \frac{1}{2}f''(a)(a+h-a)^2 + \frac{1}{6}f'''(a)(a+h-a)^3 + \dots$$

$$f(a+h) = f(a) + f'(a)h + \frac{1}{2}f''(a)h^2 + \frac{1}{6}f'''(a)h^3 + \dots$$

Taylor approximation of $f(a-h)$

$$f(a-h) = f(a) + f'(a)(a-h-a) + \frac{1}{2}f''(a)(a-h-a)^2 + \frac{1}{6}f'''(a)(a-h-a)^3 + \dots$$

$$f(a-h) = f(a) - f'(a)h + \frac{1}{2}f''(a)h^2 - \frac{1}{6}f'''(a)h^3 + \dots$$

|error| = |exact value - approximation| $\leq ch^r$

$$= f'(a) - \frac{1}{2h}(f(a) + f'(a)h + \frac{1}{2}f''(a)h^2 + \frac{1}{6}f'''(a)h^3 + \dots$$
$$-(f(a) - f'(a)h + \frac{1}{2}f''(a)h^2 - \frac{1}{6}f'''(a)h^3 + \dots))$$

$$= f'(a) - \frac{1}{2h}(2f'(a)h + \frac{2}{6}f'''(a)h^3 + \dots)$$
$$= \frac{1}{6}f'''(a)h^2$$

The rate of convergence is h^2