

HW 4: Computable Reductions

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1. Show that the language ALL_{TM} , defined as $\{M \mid \text{where } M \text{ is a TM and } L(M) = \Sigma^*\}$, is undecidable.
2. A **useless state** in a Turing machine is defined as a state that is never entered by the machine on any input. Consider the problem of detecting if a Turing machine has a useless state. Formulate this problem *as a language* and show that the language is undecidable.
3. If $A \leq_m B$ and B is a regular language, is A necessarily a regular language? Justify your answer.
4. Let B be a decidable language with $B \neq \emptyset$ and $B \neq \Sigma^*$, then if A is decidable define a computational reduction $A \leq_m B$.
5. (from Sipser) Let Double-SAT be the language $\{ \phi \mid \phi \text{ has at least two satisfying assignments} \}$. Show, by polynomial time reduction, that Double-SAT is NP-complete.