Homework 1

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1 HW 1: Regular Languages (Due 10/21/2014T)

1.1 Problem 1

Give state diagrams of DFAs for the following languages

- $\{w|w$ contains the substringab and $ba\}$ ($\Sigma = \{$a,b$\}$)
- $\{w|w$ contains an even number of 0s or exactly three 1s$\}$ (hint: this is the union of two languages)
- $\{w|w$ is a binary multiple of 5$\}$ Note: there's a trick to this one. As a hint there should be a total of *five* states in your DFA and only one accept state. As another hint, think about reading a binary number from left to right and how you calculate the number as an iterative process. Clarification: *[2014-10-13 Mon]* Depending on if you want to accept the empty string in the language it could actually be 6 states. It's 5 states if you let the empty string count as the binary number 0, but you could also consider the empty string to be NaN.

1.2 Problem 2

Prove or disprove the following: let D be a DFA with k-states. If the language $L(D)$ is *finite* then there exists at least one string s of length at-most $k-1$ such that D does not accept s. Hint: what do we know about a DFA if its language is finite?

1.3 Problem 3

For any string $w = w_1 w_2 \ldots w_n$ then $w^R = w_n w_{n-1} \ldots w_1$ is the reverse of the string. For any language A, let $A^R = \{w^R | w \in A\}$. Prove that if A is regular then so is A^R.

1.4 Problem 4

Give NFAs for the following languages

- $\{w|w$ uses an arbitrary number of all but one of $\{a, b, c, d\}\}$ (Clarification: for the purposes of this language strings such as ab, a, ϵ are all **in** the language. Basically, any string that doesn't use all four letters of the alphabet is in the language.)
- $\{w|w$ ends with a zero$\}$ but you must use only two states

1.5 Problem 5

(from 1.38 in Sipser) An all-NFA M is a 5-tuple (Q, Σ, δ, q_0, F) that is like an NFA accept that the acceptance condition is that *every* possible state the NFA can be in at the end of processing the string must be an accept state. Prove that all-NFAs are equivalent in power to DFAs. Hint: what does the acceptance condition mean in terms of $\exists$ and $\forall$ and if there are *no* possible states you can be in?

1.6 Problem 6

Prove *using the pumping lemma* that the following languages aren't regular. Remember, your proof needs to be a proof by contradiction in the form we did in class.

- $\{ 0^n 1^m \mid m > n\}$
- { s | s has an even number of 0s and fewer 1s than pairs of 0s }