What Are Computers, Really?

Clarissa Littler

6-23-2015

• What are the limits of computers?

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- What even are programs?

• Give intuitive criterion for "computability" as finite process

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- Build up a definition of computation independent of computers
- Sketch out mathematical models of computation
- Give examples of non-computable problems
- Discuss the implications and limits of our knowledge of computability

Computation is what computers do

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- What processes can be described in a finite way with a finite implementation?

Recipe as Finite Process

Cook celery and onion together til soft, then add frozen spinach and cook to get some of the moisture out and reduce volume add broth lentils cilantro and other spices, stir thoroughly, throw bay leaves on top.

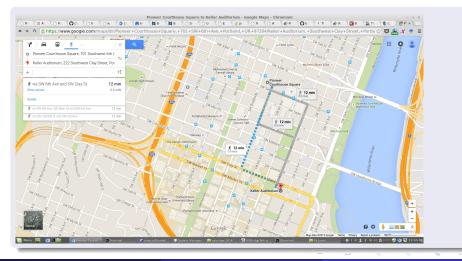
Cook for 40 minutes

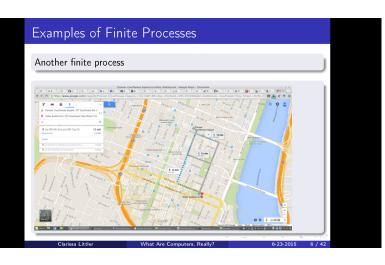
Turn off heat, wait til it stops bubbling and blend thoroughly.

Cook for 5-10 minutes after blending

Another finite process

Another finite process









I'm done with this joke now, I promise.

Adding

$$5 + 10 = 4 + 11$$

= $3 + 12$
= $2 + 13$
= $1 + 14$
= $0 + 15$
= 15

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Another Way

$$5 + 10 = 6 + 9$$

$$= 7 + 8$$

$$= 8 + 7$$

$$= 9 + 6$$

$$= 10 + 5$$

$$= 11 + 4$$

$$= 12 + 3$$

$$= 13 + 2$$

$$= 14 + 1$$

$$= 15$$

The following Haskell snippet that evaluates the sum of the integers from 1 to 10 is also a finite process

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```
let f x = sum [1..x] in f 10
```

Many more examples exist in the wild including:

counting on your fingers

- counting on your fingers
- long division

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- sorting your vinyl collection with a bucket sort

- counting on your fingers
- long division
- sorting your vinyl collection with a bucket sort
- compiling code

Informal criterion for a "finite process"

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Finite Implementation

• Finite time

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- Finite time
- Finite resources

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Finite Description

Finite length

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Finite Description

- Finite length
- Finite alphabet

Digression: What Does Finite Mean?

Informal Intuition

• A quantity is finite when it is "measurable"

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Examples

how massive our Sun is

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- the number of lines of code in your program
- the number of words in this talk
- number of other talks you'd rather be at

Finite process produces output in finite time

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Examples of Finite Time

- Counting on your fingers
- Sorting vinyl
- Walking to a friend's house
- Boiling ramen

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Only actions taking finite time can actually be finished because that's how our universe works.

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Finite processes only use finite resources

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Examples of Resources

scratch paper

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- scratch paper
- materials

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Examples of Resources

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Why Finite Resources

No computer and no physical process that we know of can use an infinite quantity, thus infinite resources shouldn't be allowed in computation.

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Any process that has an infinite number of steps in its description would:

 take necessarily infinite time to process and run

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 - this is absolutely bad

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Examples of Alphabets

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Why a Finite Alphabet?

An infinite alphabet can't have an implementation that is, itself, a finite process.

What Next?

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Modelling Computation

What Is a Model?

A model of computation is a mathematically precise formulation of computation.

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• a rigorous way to describe computation

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What Is a Model?

A model of computation is a mathematically precise formulation of computation.

What's in A Model?

- a rigorous way to describe computation
- a way to perform the descriptions

1930s logicians searched for theorem proving algorithms

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 - Proofs in 1st order logic

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- Needed a model of computation for this
- Turing (basically) won the race!

Turing and His Automatic Machines

 Turing's 1936 paper "On Computable Numbers, with an Application to the [Decision Problem]" [6]

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- Automatic machines weren't actually stand-ins for modern computers
- Turing was inspired by human computers

• Turing's day computers were people

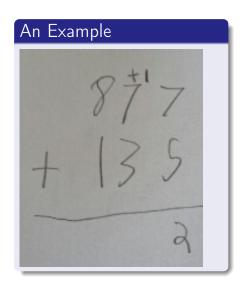
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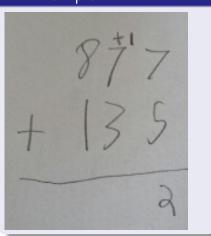
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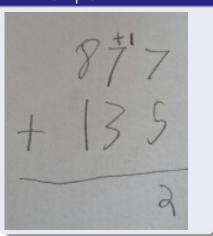
An Example



How We Think

Finite scratch paper

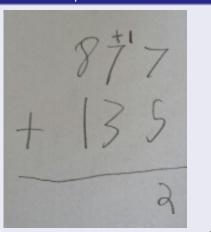
An Example



How We Think

- Finite scratch paper
- Finite steps

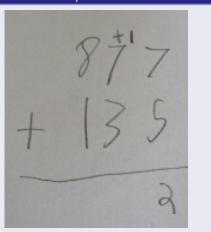
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How We Think

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- Only requires a finite number of brain states to perform

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Courtesy of http://aturingmachine.com/hardware.php

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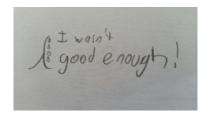
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Original Formulation

There is no model of computation more expressive than Turing machines/lambda calculus. [4]

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Equivalent Formulation

No programming language can be more powerful than a Turing machine

• A programming language is a model of computation

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- The real meaning of Turing Complete

The Halting Problem

Formal Specification

Is there a Turing machine that can tell if another Turing machine will halt on a given input?

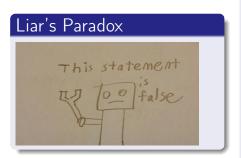
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Informal Implication

Can you write a program that can detect if other programs have infinite loops?



Proof Idea

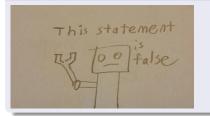
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Liar's Paradox This statement

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- Assume we have a program that can solve the halting problem
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- What does the liar say about itself?

Virus Scanners

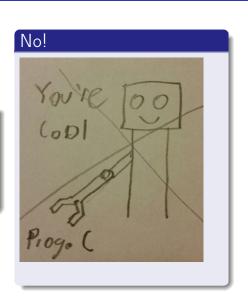
Full Employment Theorem

Is there a program that can perfectly detect if another program is carrying a viral payload?

Virus Scanners

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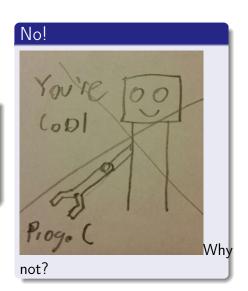
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Examples

- Virus scanner
- Programs that contain infinite loops
- Programs that fit a specification
 - Testing is always going to be hard

Strong Church-Turing Thesis

The laws of physics are inherently computable and there is no physical process that cannot be computed by some algorithm.

• is this actually true?

Strong Church-Turing Thesis

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 - reality must be "discrete"
 - real numbers are approximations at scale

Church-Turing as Cognition

• Are brains computable?

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Church-Turing as Cognition

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- Does free will actually exist or is it an illusion?

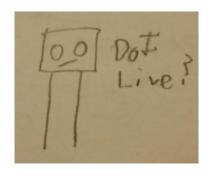
 Can we make a machine intelligence comparable to our own?

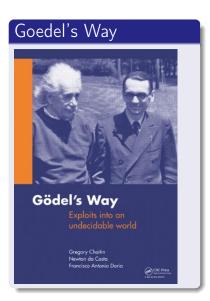
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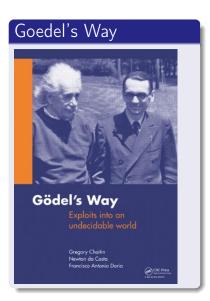
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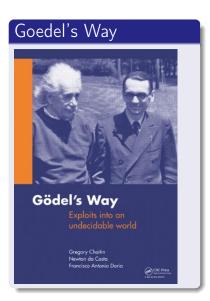
Hyper-Turing Computation

 Is computation (in the Church-Turing sense) complete?



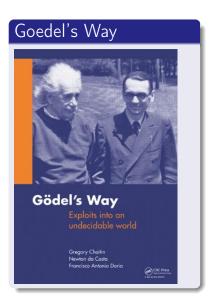
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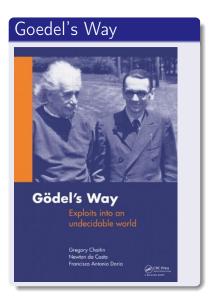
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- The mathematics of finite methods
- Computation has limits
- The limits of computation are understood
- How computation relates to the laws of the universe?
 - Much more unknown

Any Questions?

Bibliography

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