What Are Computers, Really?

Clarissa Littler

6-23-2015

Fundamental Questions

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- What are the limits of computers?
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- Do all programming languages describe the same thing?
- What even are programs?

How We'll Answer Them

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- Build up a definition of computation independent of computers
- Sketch out mathematical models of computation
- Give examples of non-computable problems
- Discuss the implications and limits of our knowledge of computability

Computation Sounds Like Computer

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- Computation is what computers do
- A program *describes* a computation
- The limits of computation are limits of description
- What processes can be described in a finite way with a finite implementation?

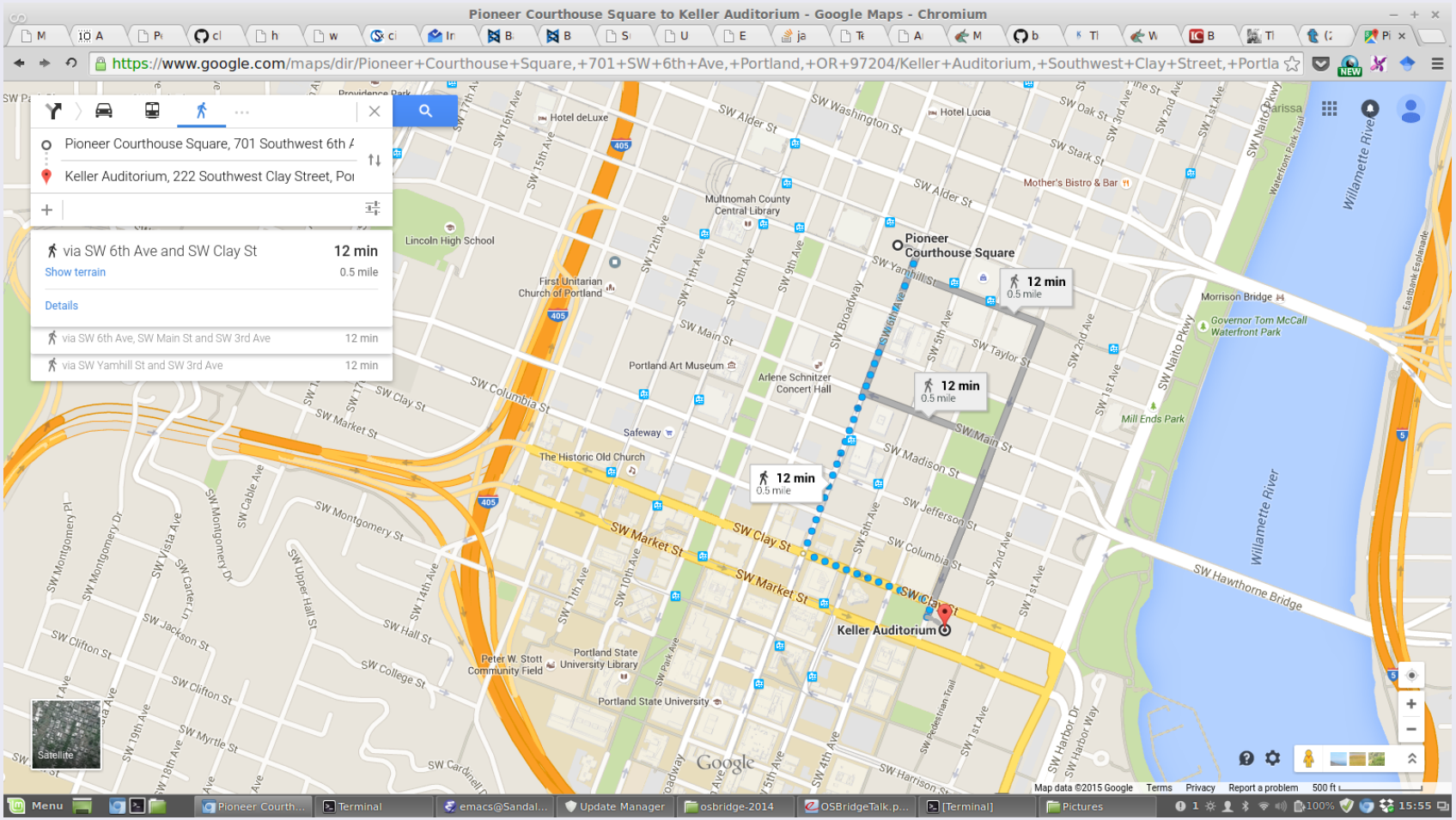
Examples of Finite Processes

Recipe as Finite Process

Cook celery and onion together til soft, then add frozen spinach and cook to get some of the moisture out and reduce volume add broth lentils cilantro and other spices, stir thoroughly, throw bay leaves on top.
Cook for 40 minutes
Turn off heat, wait til it stops bubbling and blend thoroughly.
Cook for 5-10 minutes after blending

Examples of Finite Processes

Another finite process



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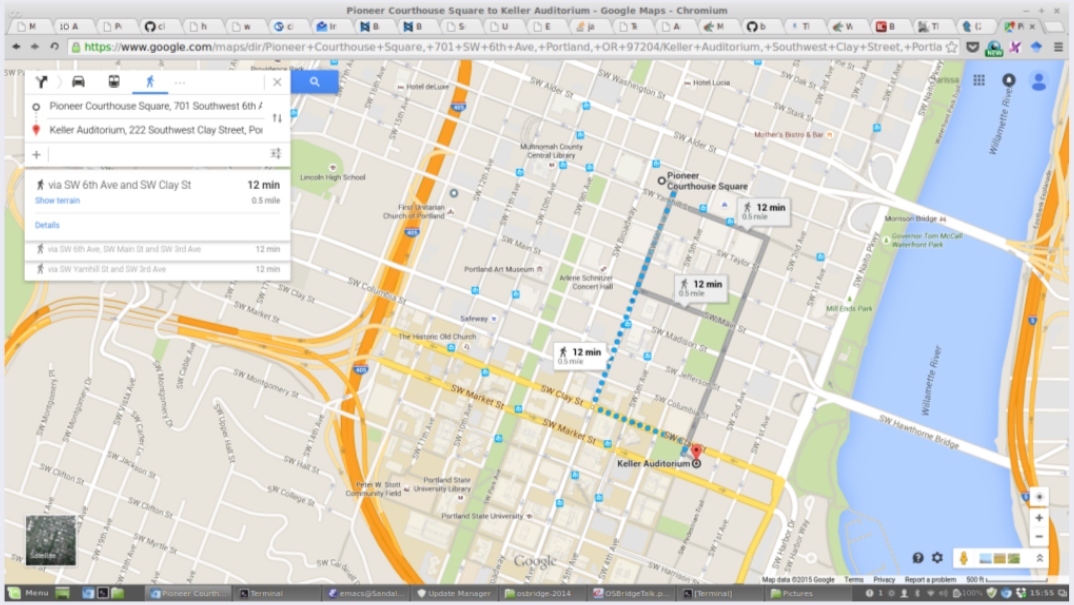

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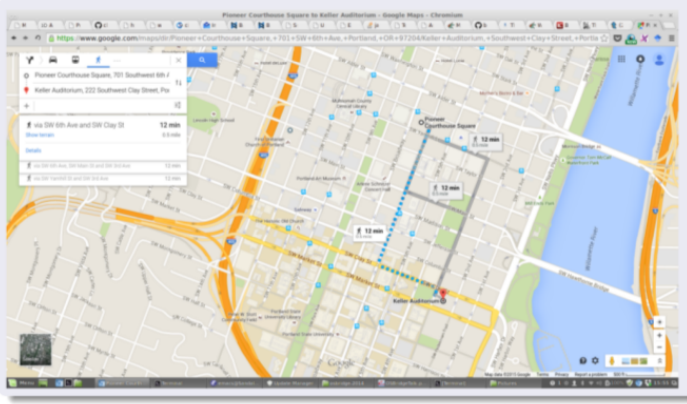

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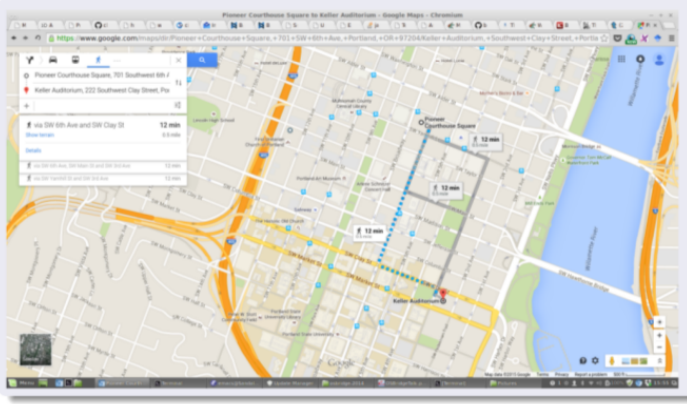

I'm done with this joke now, I promise.

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$$\begin{aligned} 5 + 10 &= 4 + 11 \\ &= 3 + 12 \\ &= 2 + 13 \\ &= 1 + 14 \\ &= 0 + 15 \\ &= 15 \end{aligned}$$

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Another Way

$$\begin{aligned} 5 + 10 &= 6 + 9 \\ &= 7 + 8 \\ &= 8 + 7 \\ &= 9 + 6 \\ &= 10 + 5 \\ &= 11 + 4 \\ &= 12 + 3 \\ &= 13 + 2 \\ &= 14 + 1 \\ &= 15 \end{aligned}$$

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```
let f x = sum [1..x] in f 10
```

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- counting on your fingers

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Finite Description

- Finite length
- Finite alphabet

Digression: What Does Finite Mean?

Informal Intuition

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- the number of words in this talk
- number of other talks you'd rather be at

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Finite process produces output in finite time

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Only actions taking finite time can actually be finished because that's how our universe works.

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  - Servers
  - Interactive programs

Finite Resources

Finite processes only use finite resources

Examples of Resources

- scratch paper
- materials
- RAM
- disk space

Why Finite Resources

No computer and no physical process that we know of can use an infinite quantity, thus infinite resources shouldn't be allowed in computation.

Finite Descriptions

Finite processes are only allowed to have a finite number of steps in their description, i.e. a finite length as a string

Examples of Finite Descriptions

- Directions
- Recipes
- Programs
  - a `while` loop is finite!

Why Finite Directions?

Any process that has an infinite number of steps in its description would:

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Why Finite Directions?

Any process that has an infinite number of steps in its description would:

- take necessarily infinite time to process and run
  - not absolutely a bad thing, but likely so
- take infinite resources to store
  - this is absolutely bad

Finite Alphabet

Finite processes can only be written with a finite alphabet

Examples of Alphabets

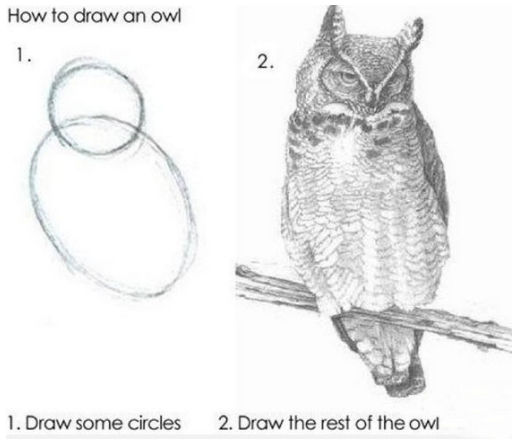
- 0 and 1
- the English alphabet (a-z, A-Z)
- unicode
- ASCII

Why a Finite Alphabet?

An infinite alphabet can't have an implementation that is, itself, a finite process.

What Next?

- These are rules of thumb
- But how do we actually specify a process?
- Most directions too broad:
  - driving directions
  - cooking directions
- Need instructions simple enough for a machine



Modelling Computation

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- a way to perform the descriptions

The Search for a Model

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- Turing won the race!

Turing and His Automatic Machines

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- Turing was inspired by human computers

Human Computers

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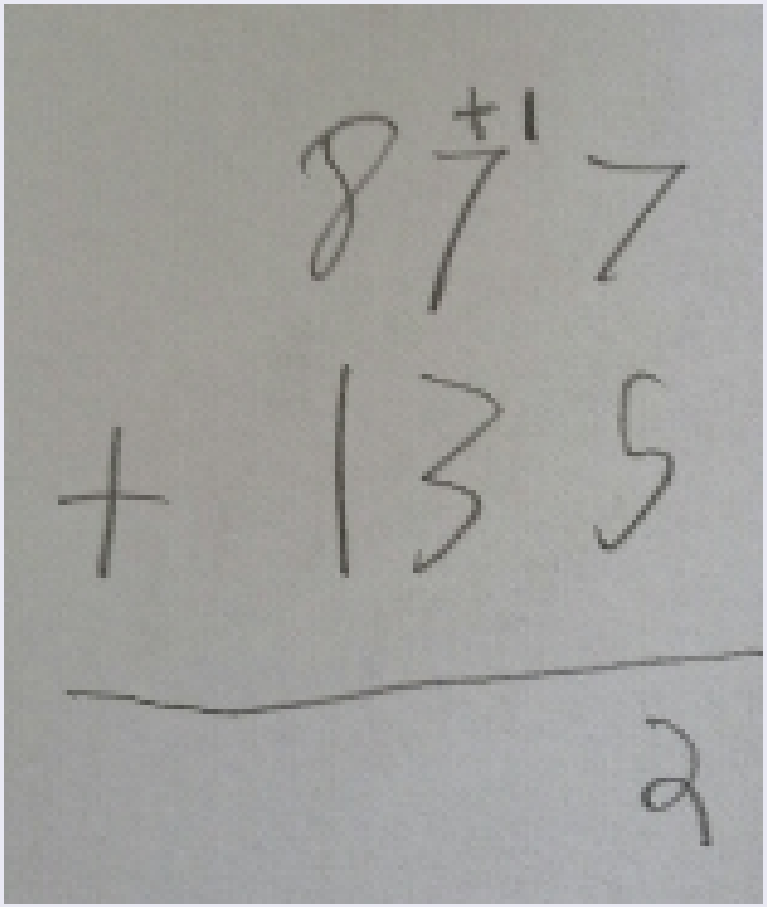
How Humans Compute

An Example



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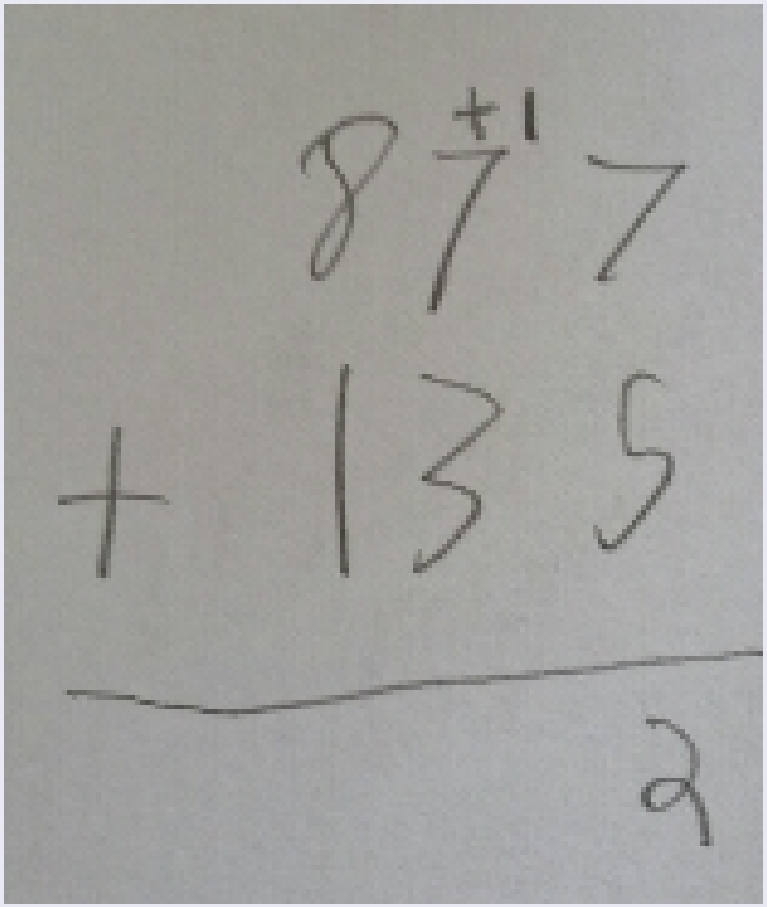

How We Think

- Finite scratch paper

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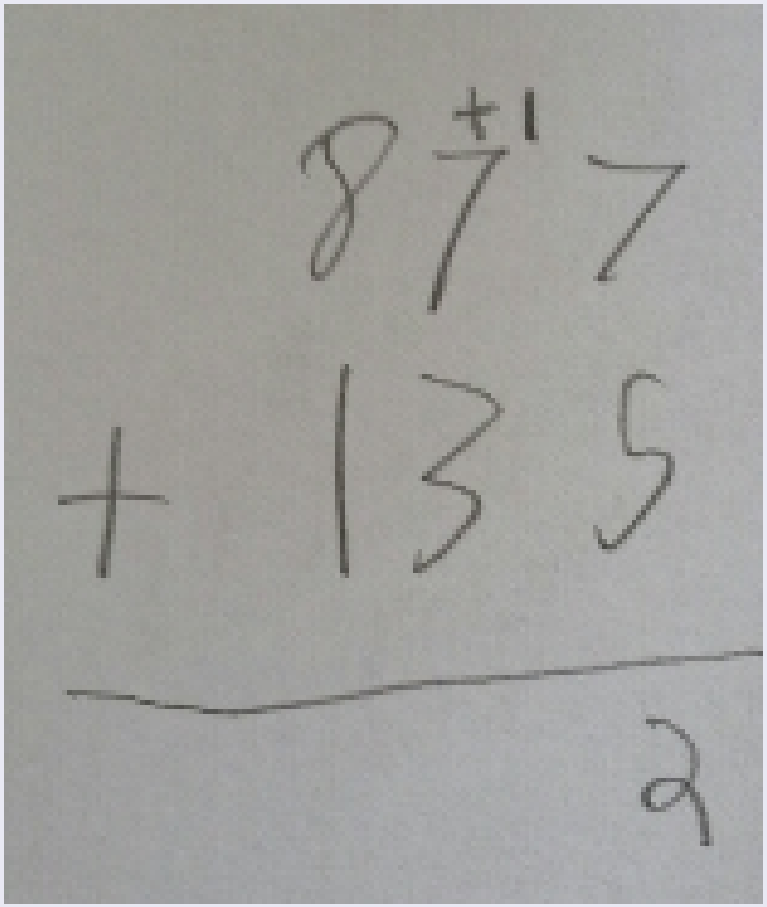

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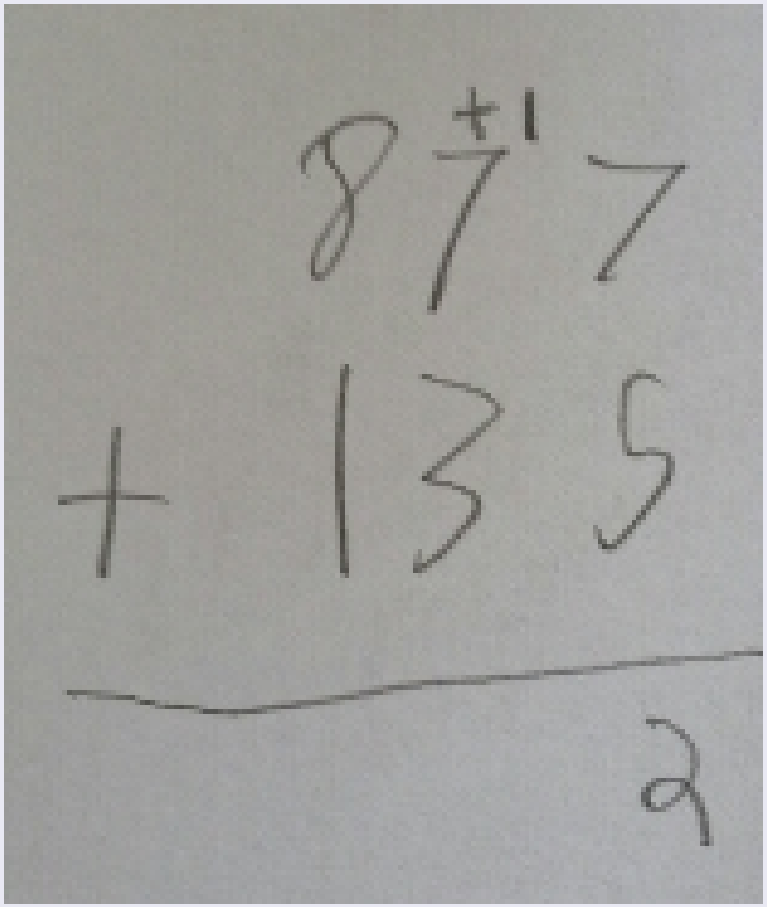

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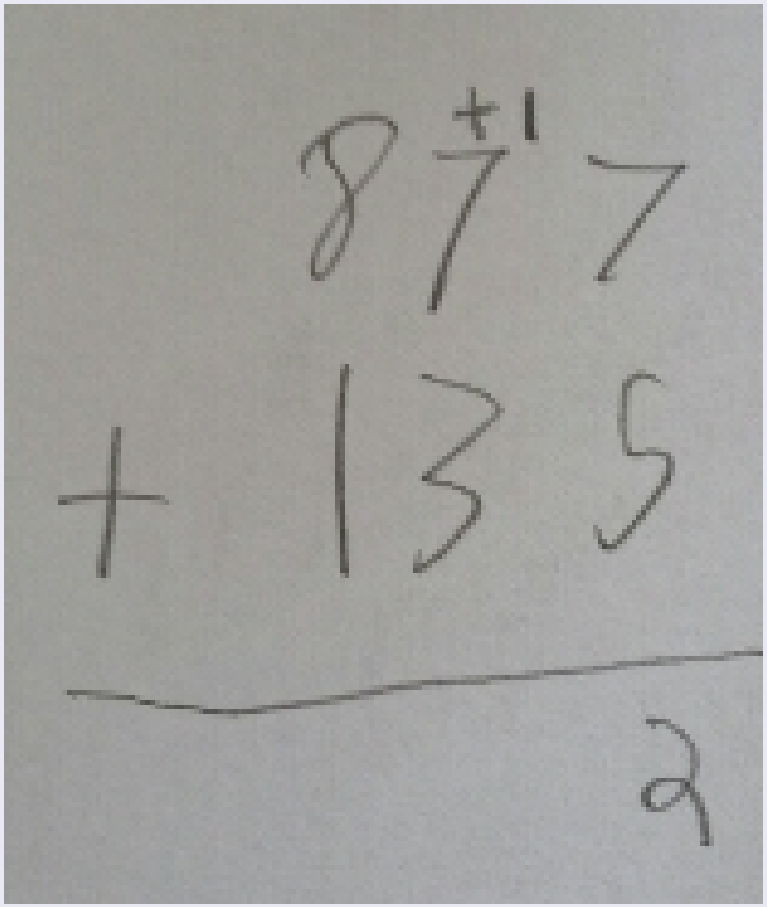
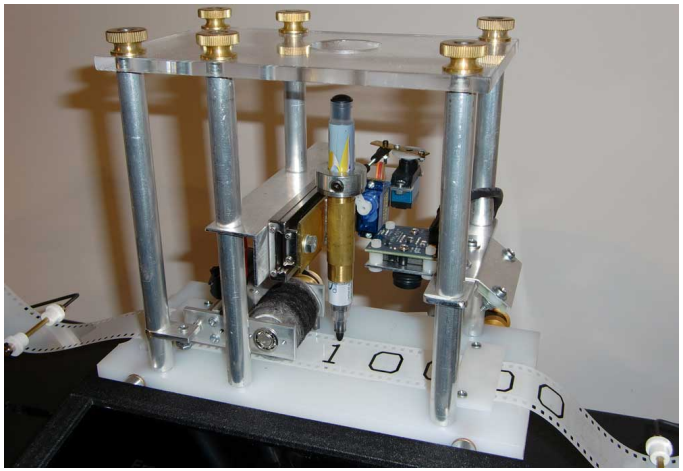


How We Think

- Finite scratch paper
- Finite steps
- You can look at it and pick up where I left off
- Only requires a finite number of brain states to perform

A Turing Machine

- Arbitrary amount of tape
- Reads and writes from only once cell at a time
- Only has a finite "alphabet" of symbols
- Has a finite number of states for deciding next move



Courte

of

`http://aturingmachine.com/hardware.php`

Historic Importance of Turing Machines

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- Turing's machines are obviously computable
  - Completely analogous to human processes

The Church-Turing Thesis

Original Formulation

There is no model of computation more expressive than Turing machines (equivalently, the lambda calculus). [4]

Equivalent Formulation

Equivalent formulation: no programming language can be more powerful than a Turing machine

Programs As Computations

- A programming language is a model of computation

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- The real meaning of Turing Complete

The Halting Problem

Formal Specification

Is there a Turing machine that, when fed the description of another Turing machine, determines if that machine will halt on a given input?

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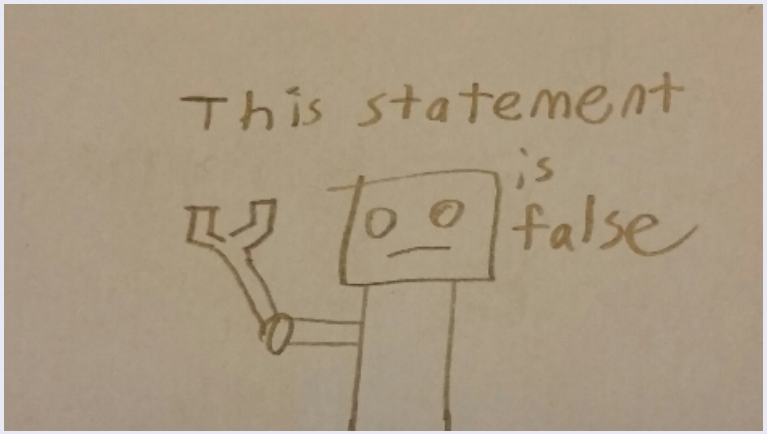
Informal Implication

Can you write a program that can detect if other programs have infinite loops?

Why Can't We Solve The Halting Problem?

Liar's Paradox



Proof Idea

- Assume we have a program that can solve the halting problem

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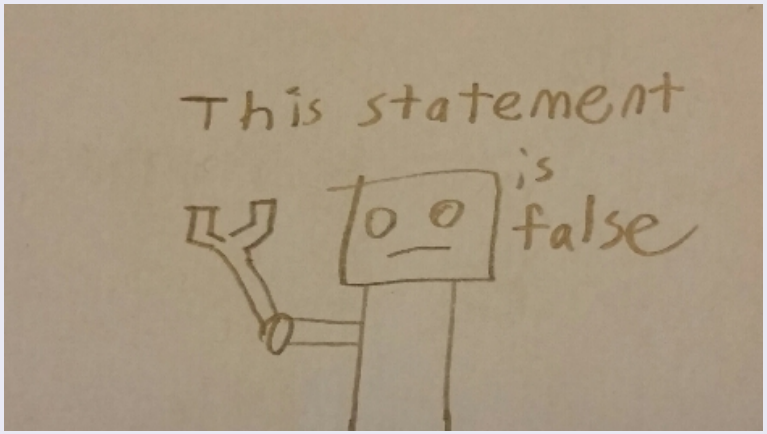

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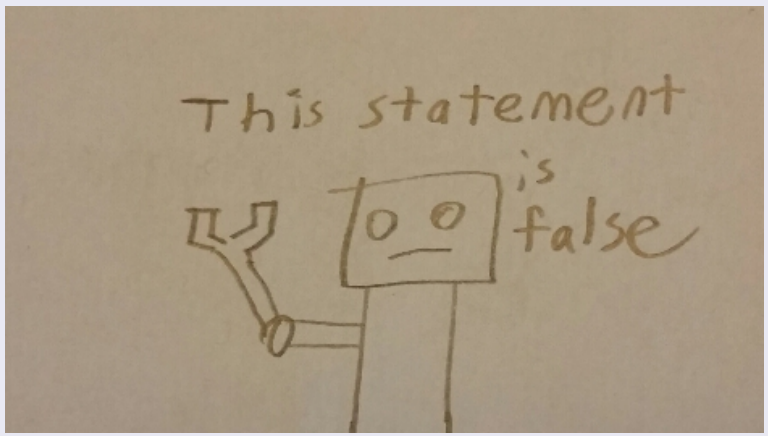

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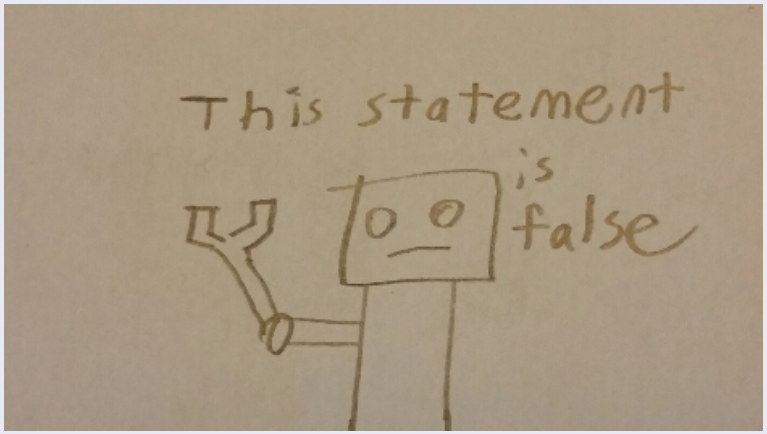

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  - halts if the input program loops when fed its own source code
- What does the liar say about itself?

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Full Employment Theorem

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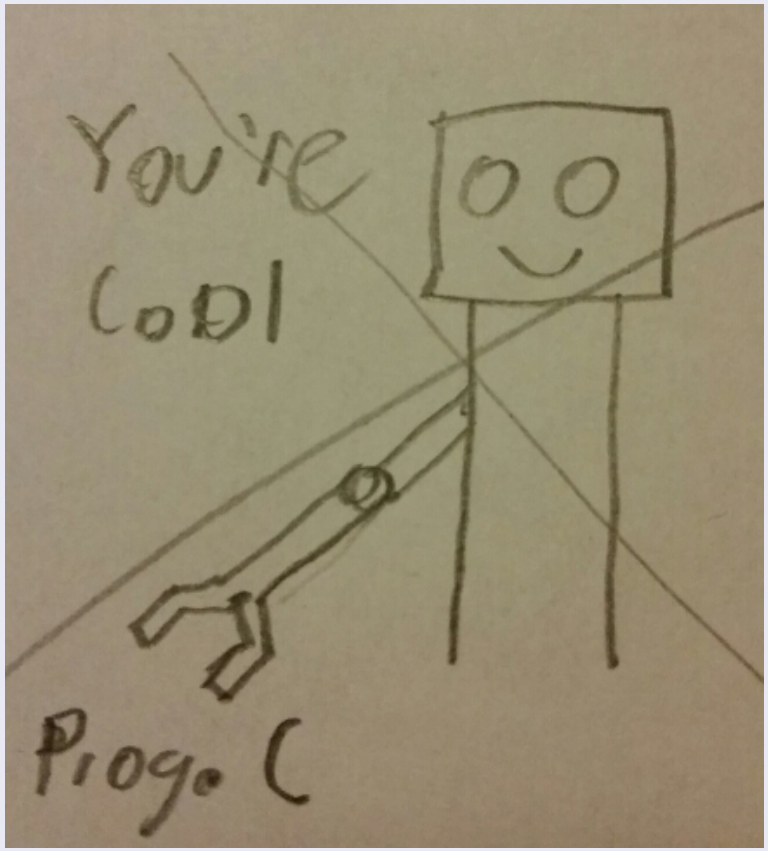

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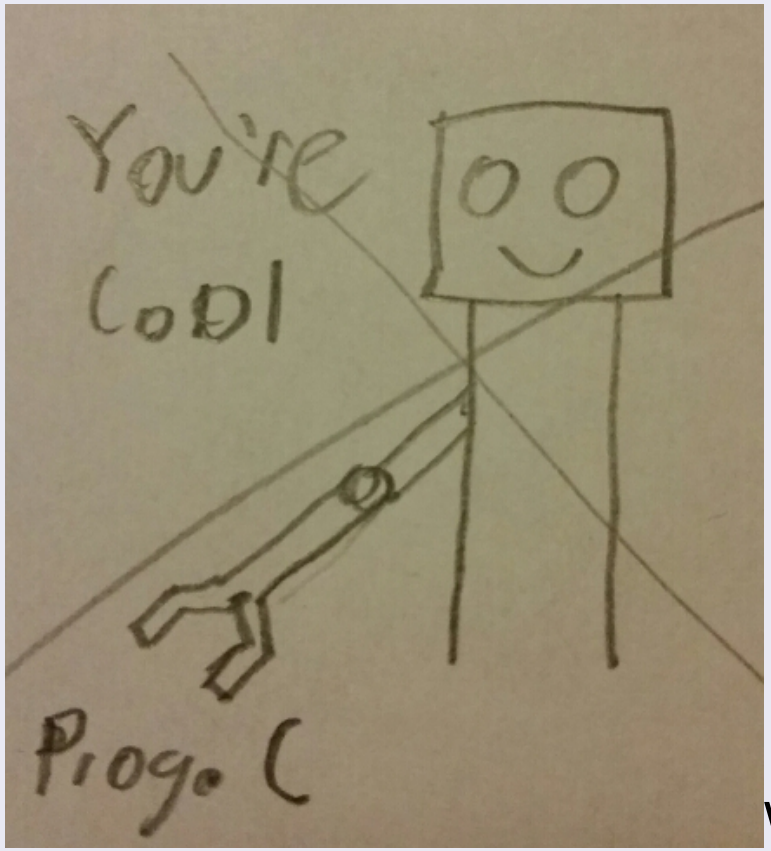

Why not?

Rice's Theorem

Semi-Formally

Non-trivial properties of all programs

Church-Turing as Physics

Strong Church-Turing Thesis

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  - reality must be "discrete"
  - real numbers are approximations at scale

Church-Turing as Cognition

- Are brains computable?

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- Does free will actually exist or is it an illusion?

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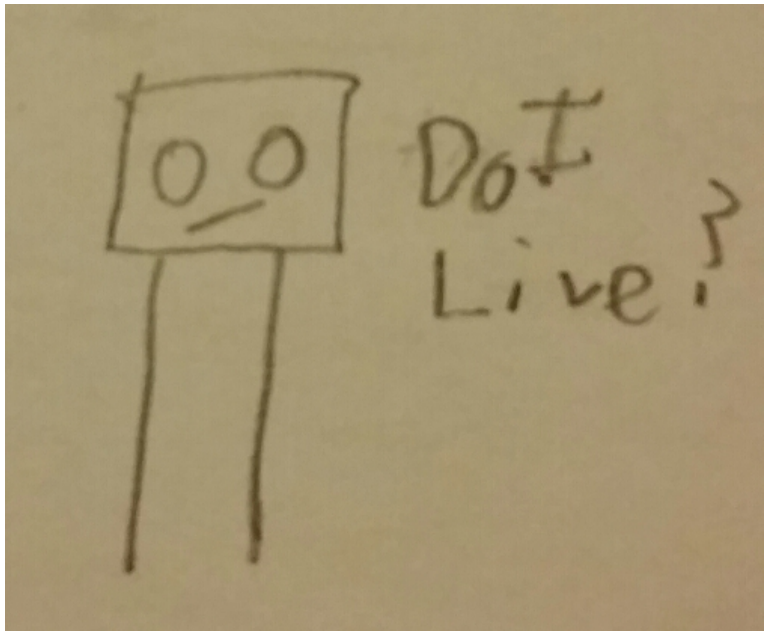
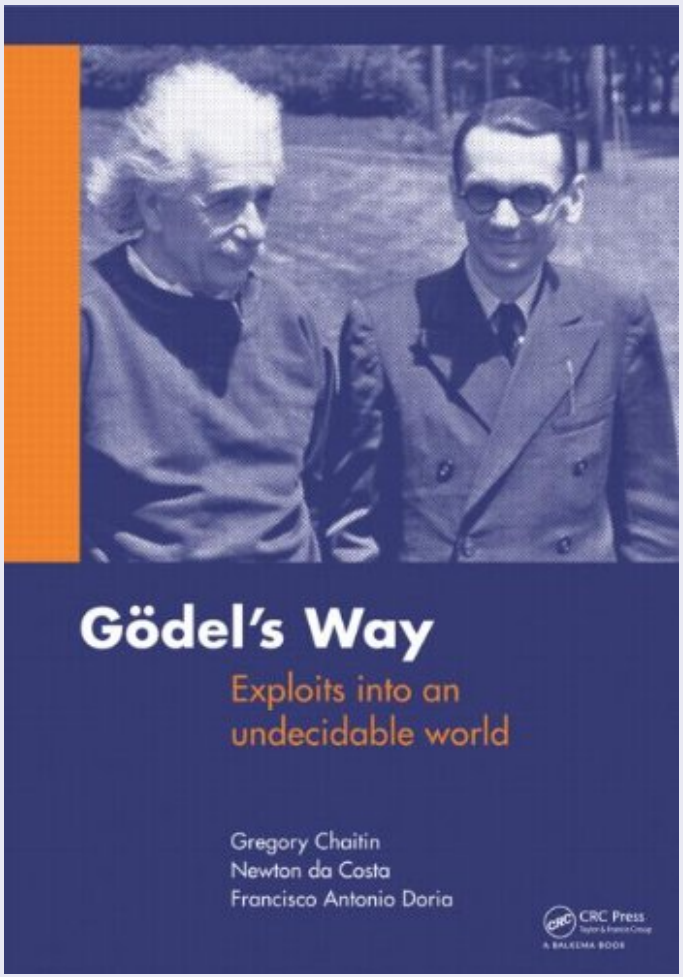


Skirting Computability

Goedel's Way



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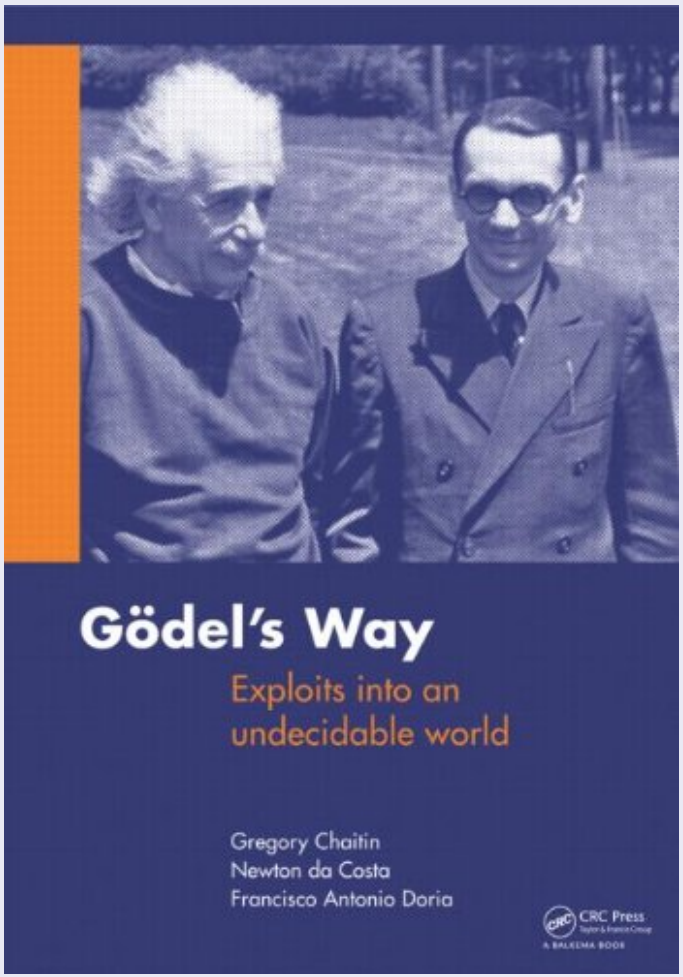

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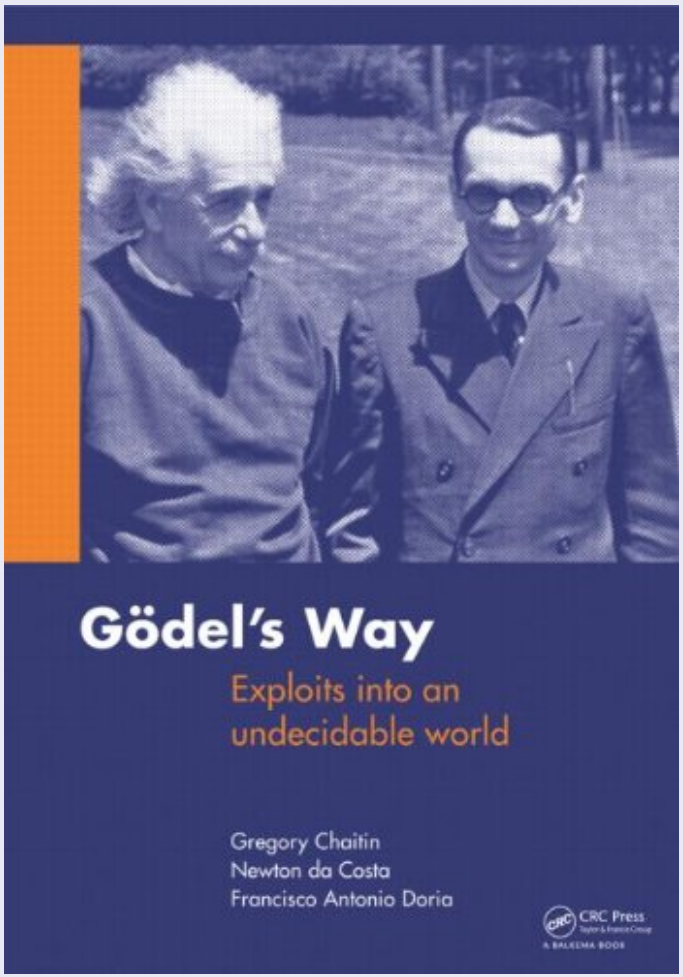

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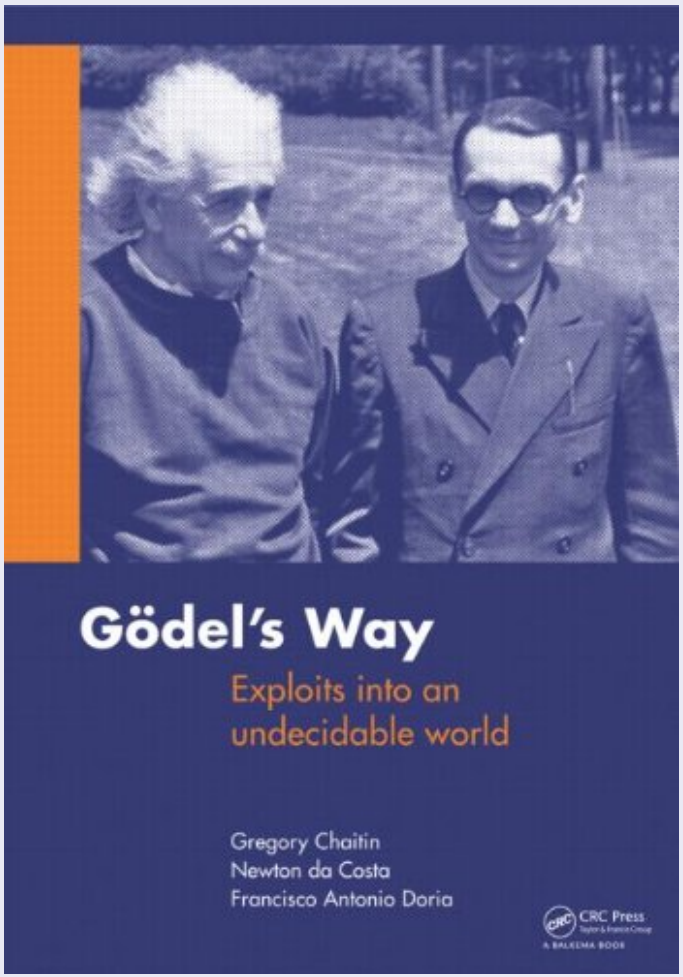
- Is computation (in the Church-Turing sense) complete?
- Is physics computable?
  - If yes, then no

Skirting Computability

Goedel's Way



Hyper-Turing Computation

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- Is physics computable?
  - If yes, then no
  - If no, then maybe [1]

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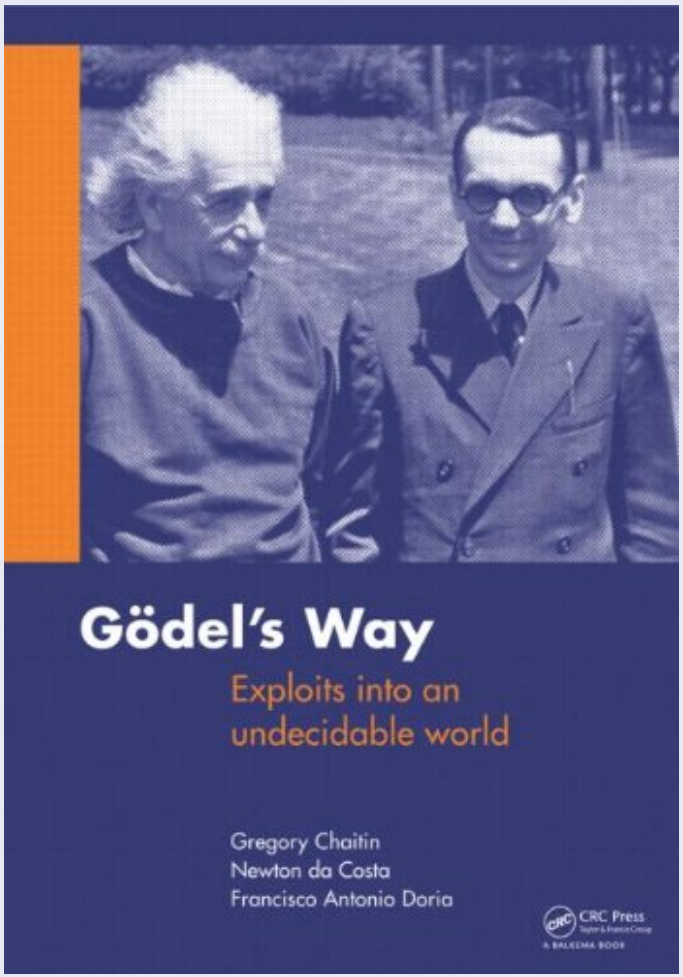

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- Computation has limits
- The limits of computation are understood
- How computation relates to the laws of the universe?
  - Much more unknown

Any Questions?

Bibliography


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