What Are Computers, Really?

Clarissa Littler

6-23-2015

Fundamental Questions

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- What are the limits of computers?
- Are there problems a computer can never solve?
- Do all programming languages describe the same thing?
- What even are programs?

How We'll Answer Them

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- Build up a definition of computation independent of computers
- Sketch out mathematical models of computation
- Give examples of non-computable problems
- Discuss the implications and limits of our knowledge of computability

Computation Sounds Like Computer

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Computation Sounds Like Computer

- Computation is what computers do
- A program *describes* a computation
- The limits of computation are limits of description
- What processes can be described in a finite way with a finite implementation?

Examples of Finite Processes

Recipe as Finite Process

Cook celery and onion together til soft, then add frozen spinach and cook to get some of the moisture out and reduce volume add broth lentils cilantro and other spices, stir thoroughly, throw bay leaves on top.
Cook for 40 minutes
Turn off heat, wait til it stops bubbling and blend thoroughly.
Cook for 5-10 minutes after blending

Examples of Finite Processes

Another finite process

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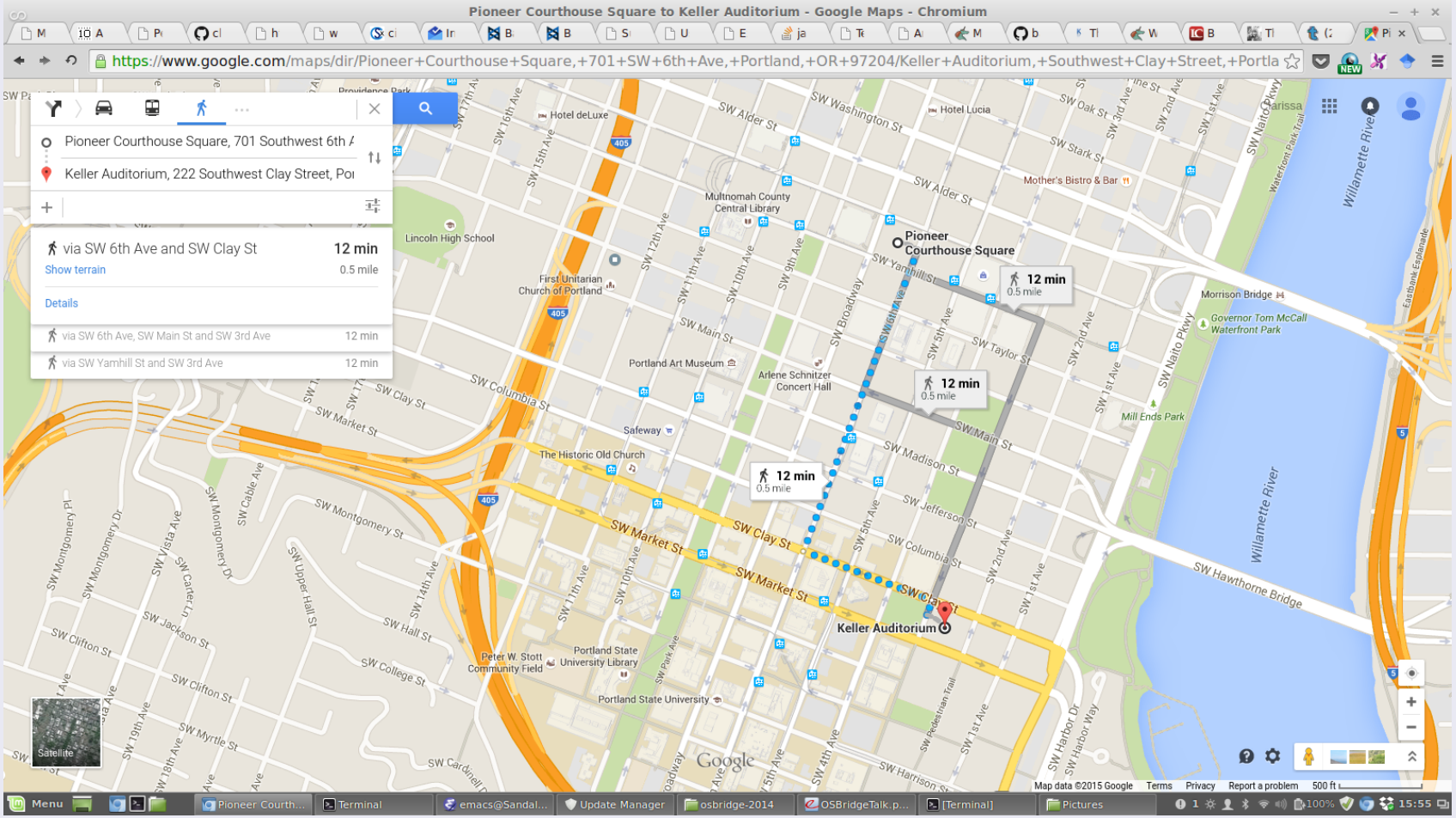
Another finite process



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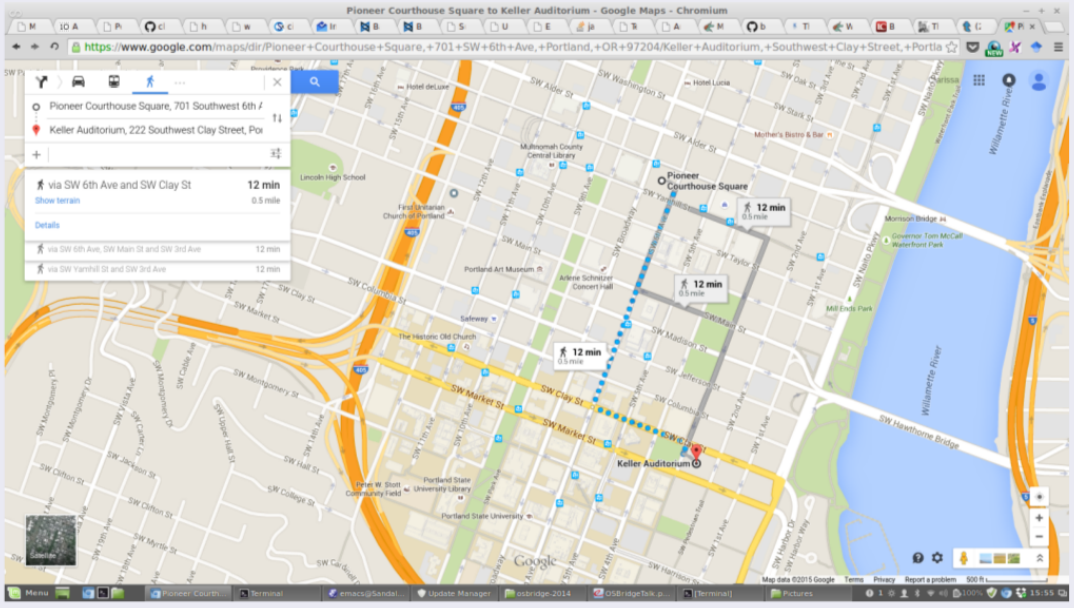

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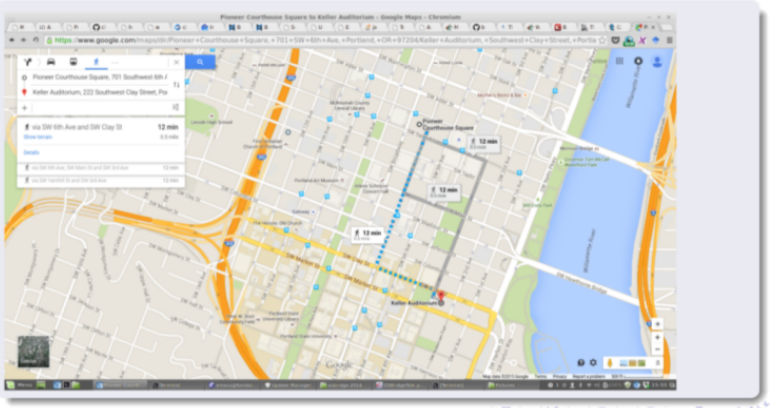

Examples of Finite Processes

I'm done with this joke now, I promise.

Examples of Finite Processes

Adding

$$
\begin{aligned}
5 + 10 &= 4 + 11 \\
&= 3 + 12 \\
&= 2 + 13 \\
&= 1 + 14 \\
&= 0 + 15 \\
&= 15
\end{aligned}
$$

Examples of Finite Processes

Adding

$$\begin{aligned} 5 + 10 &= 4 + 11 \\ &= 3 + 12 \\ &= 2 + 13 \\ &= 1 + 14 \\ &= 0 + 15 \\ &= 15 \end{aligned}$$

Another Way

$$\begin{aligned} 5 + 10 &= 6 + 9 \\ &= 7 + 8 \\ &= 8 + 7 \\ &= 9 + 6 \\ &= 10 + 5 \\ &= 11 + 4 \\ &= 12 + 3 \\ &= 13 + 2 \\ &= 14 + 1 \\ &= 15 \end{aligned}$$

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The following Haskell snippet that evaluates the sum of the integers from 1 to 10 is also a finite process

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```
let f x = sum [1..x] in f 10
```

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- counting on your fingers

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- compiling code

Qualities of Finite Processes

Informal criterion for a "finite process"

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Finite Implementation

- Finite time

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- the number of words in this talk
- number of other talks you'd rather be at

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Finite process produces output in finite time

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Why Finite Resources

No computer and no physical process that we know of can use an infinite quantity, thus infinite resources shouldn't be allowed in computation.

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Why a Finite Alphabet?

An infinite alphabet can't have an implementation that is, itself, a finite process.

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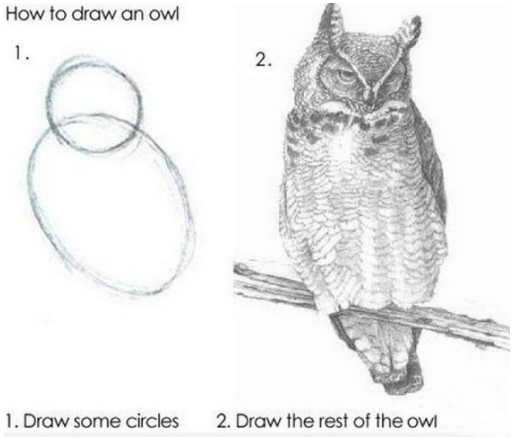

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What's in A Model?

- a rigorous way to describe computation
- a way to perform the descriptions

The Search for a Model

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- Turing (basically) won the race!

Turing and His Automatic Machines

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- Turing was inspired by human computers

Human Computers

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How Humans Compute

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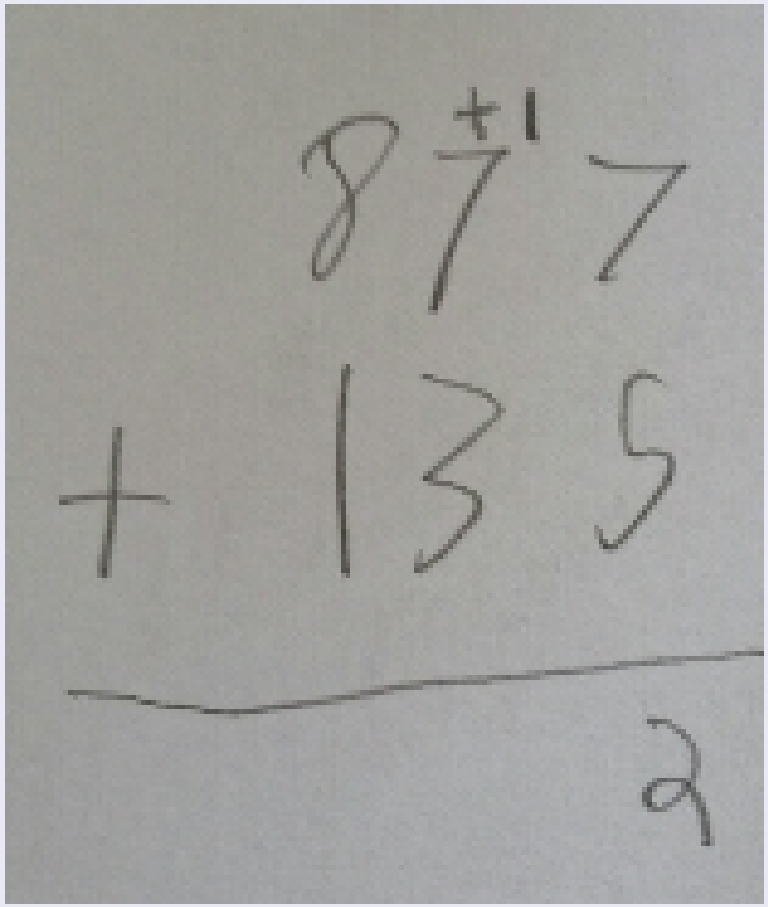

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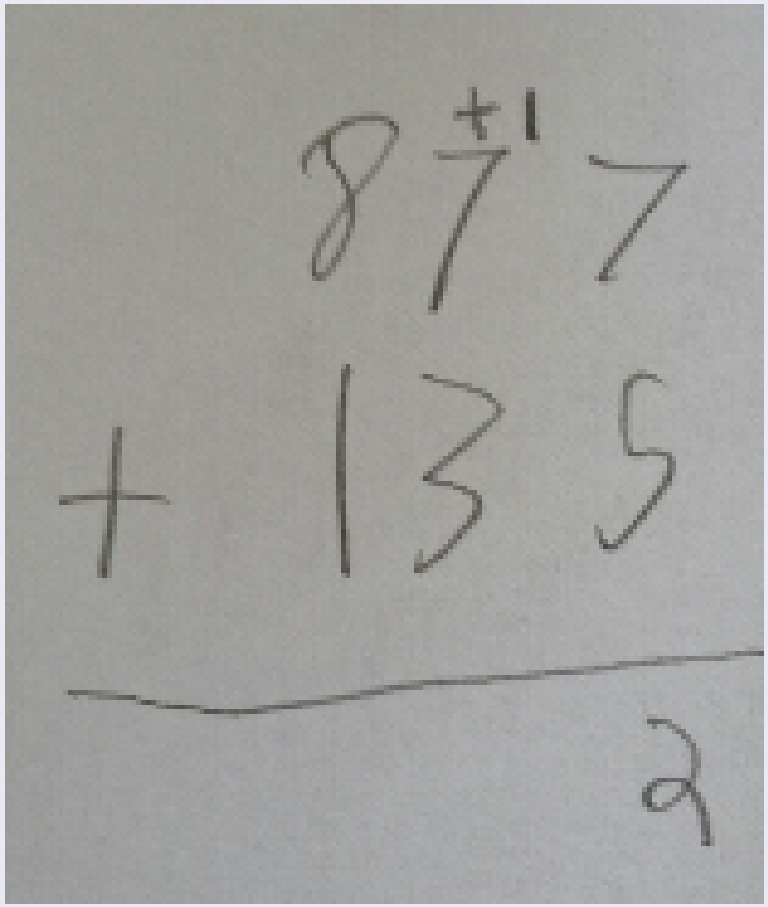

How We Think

- Finite scratch paper

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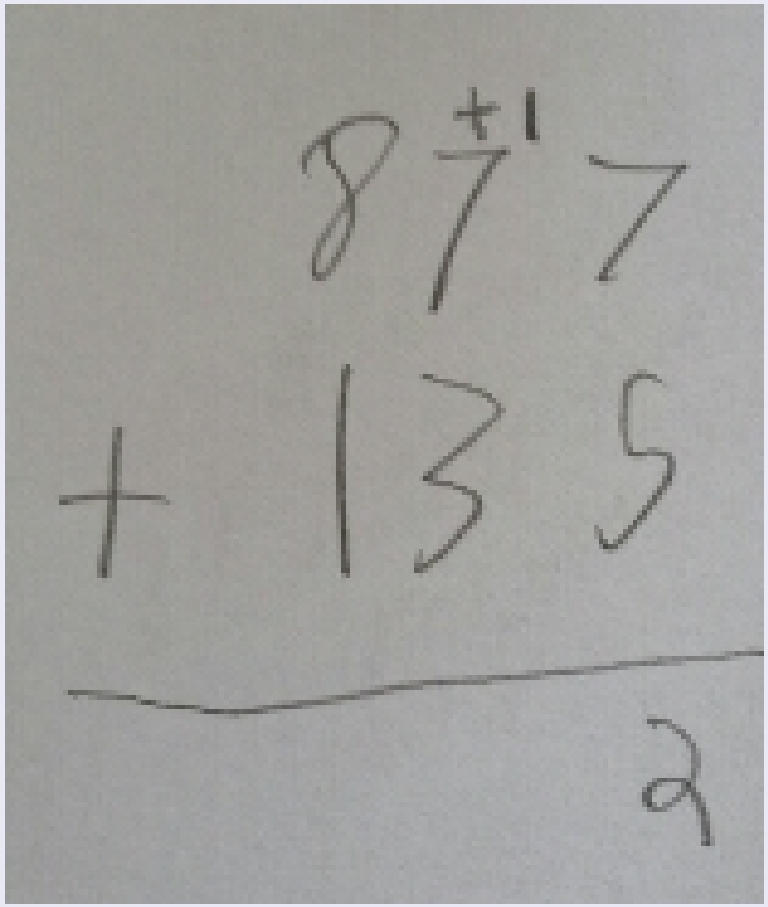

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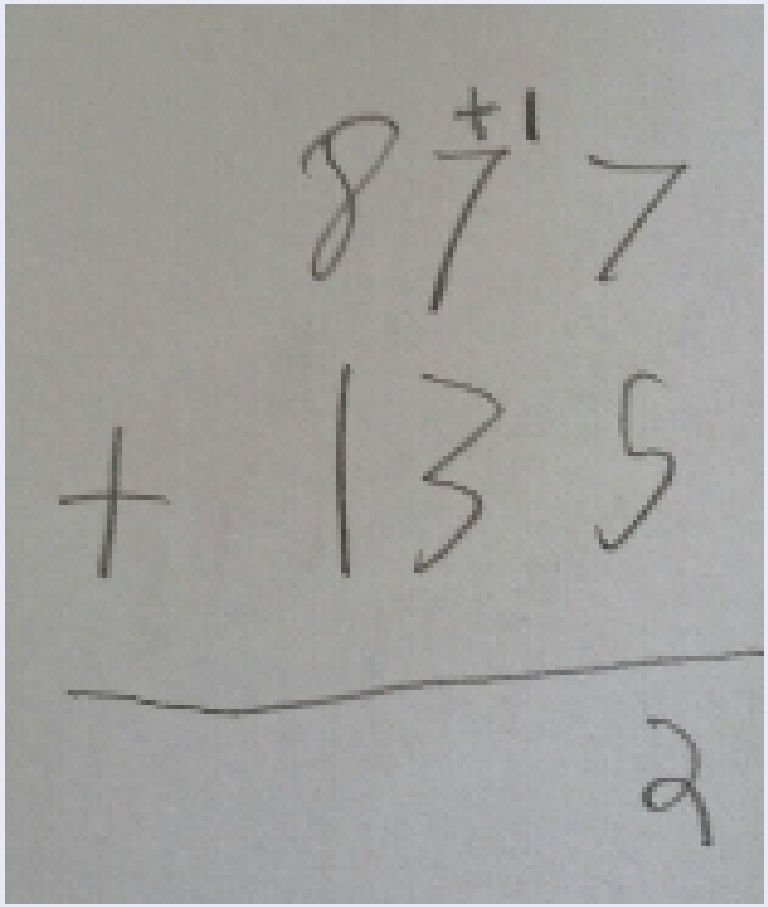

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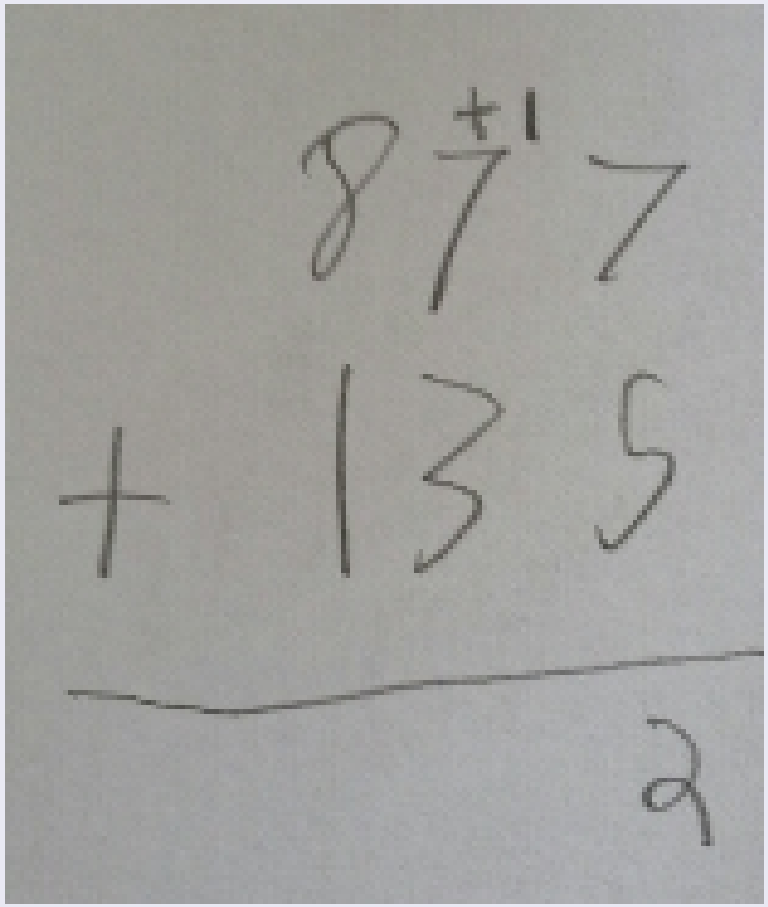

How We Think

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- Only requires a finite number of brain states to perform

A Turing Machine

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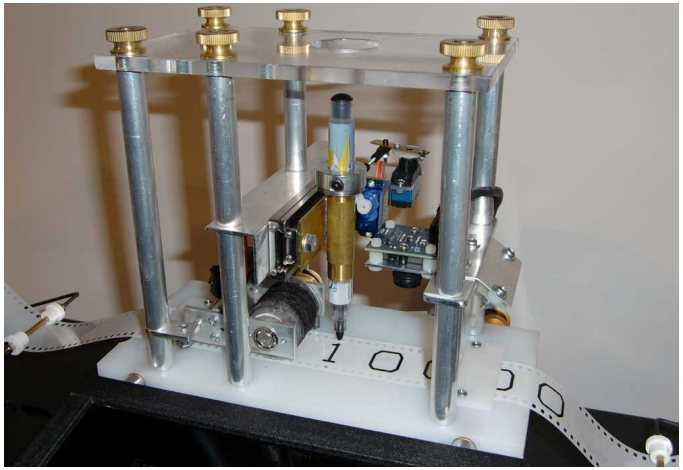

Courtesy of `http://aturingmachine.com/hardware.php`

Historic Importance of Turing Machines

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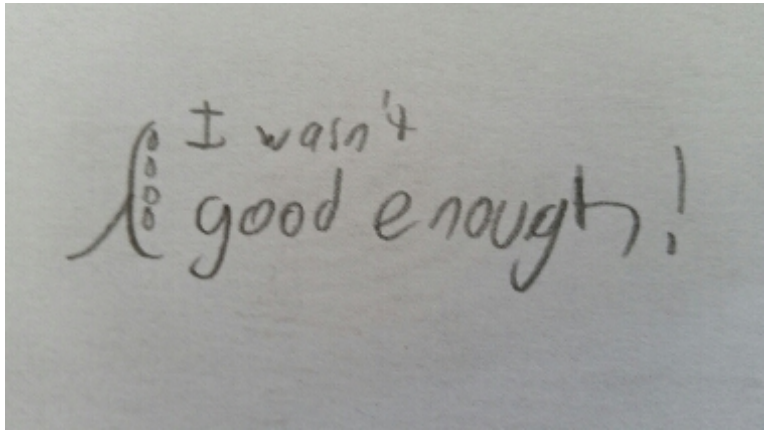

The Church-Turing Thesis

Original Formulation

There is no model of computation more expressive than Turing machines/lambda calculus. [4]

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Equivalent Formulation

No programming language can be more powerful than a Turing machine

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Formal Specification

Is there a Turing machine that can tell if another Turing machine will halt on a given input?

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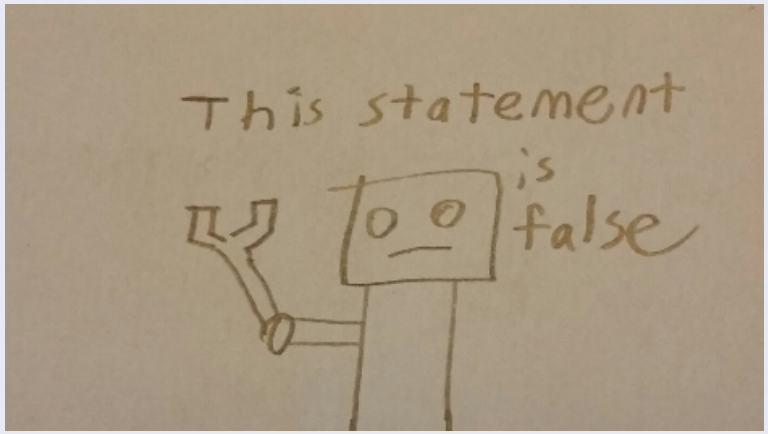
Informal Implication

Can you write a program that can detect if other programs have infinite loops?

Why Can't We Solve The Halting Problem?

Liar's Paradox



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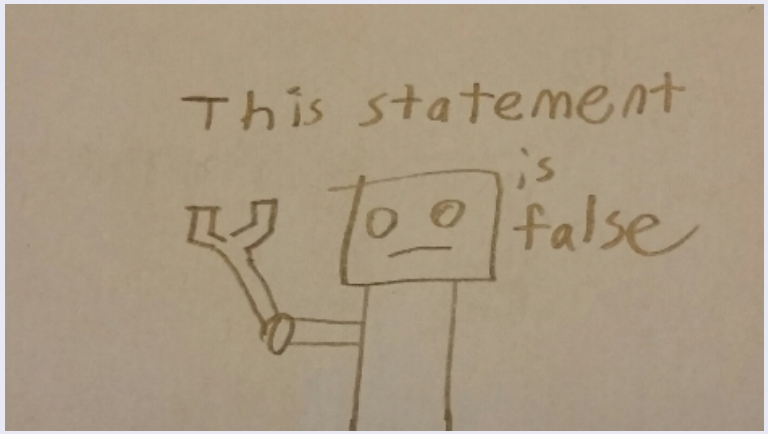

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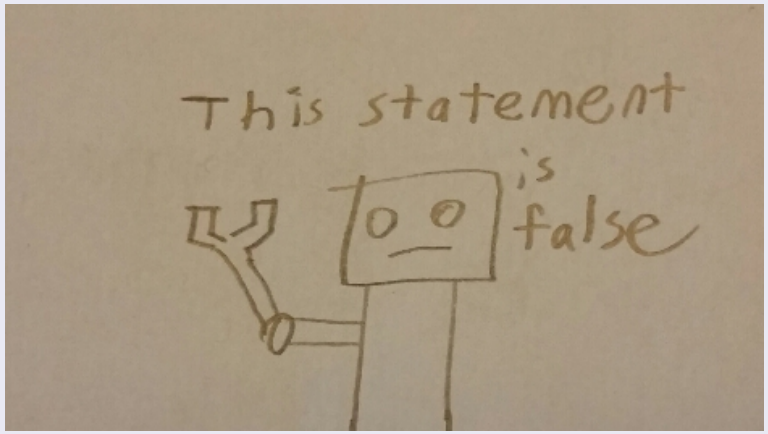

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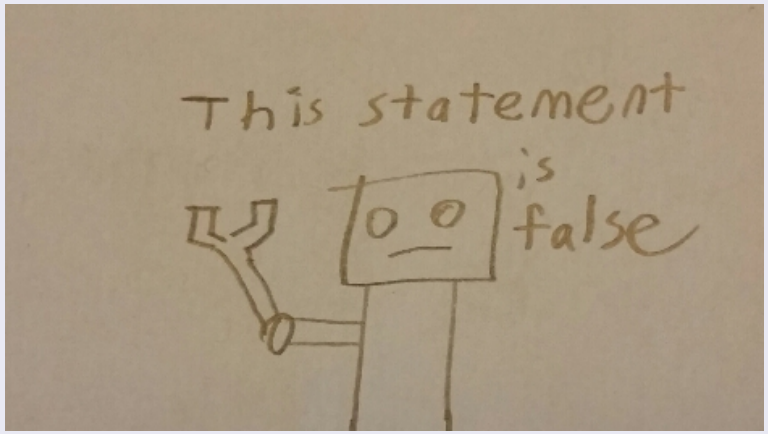

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  - halts if the input program loops when fed its own source code
- What does the liar say about itself?

Virus Scanners

Full Employment Theorem

Is there a program that can perfectly detect if another program is carrying a viral payload?

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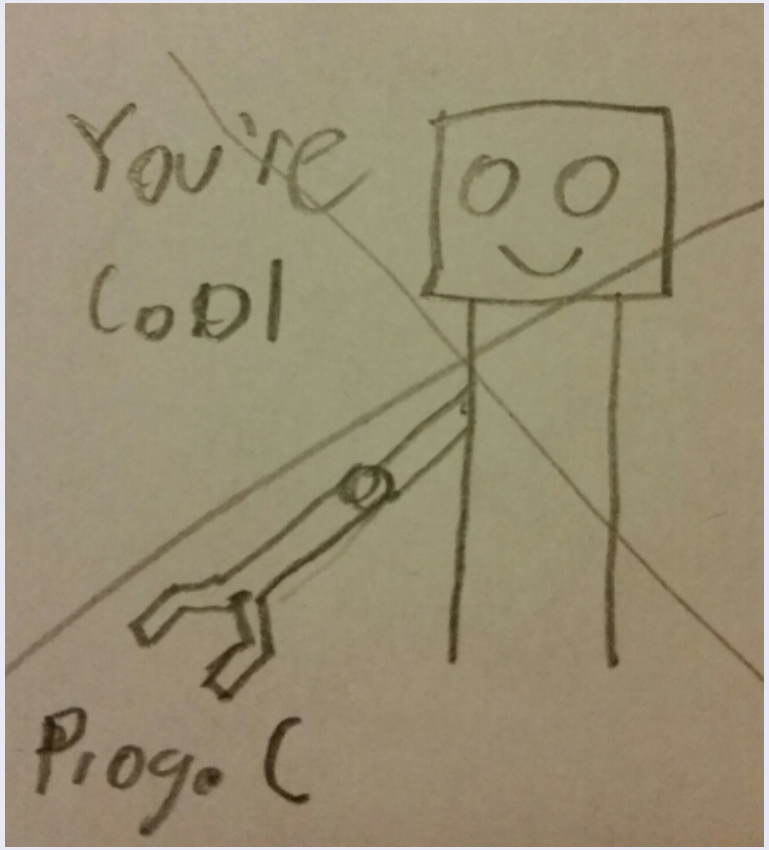

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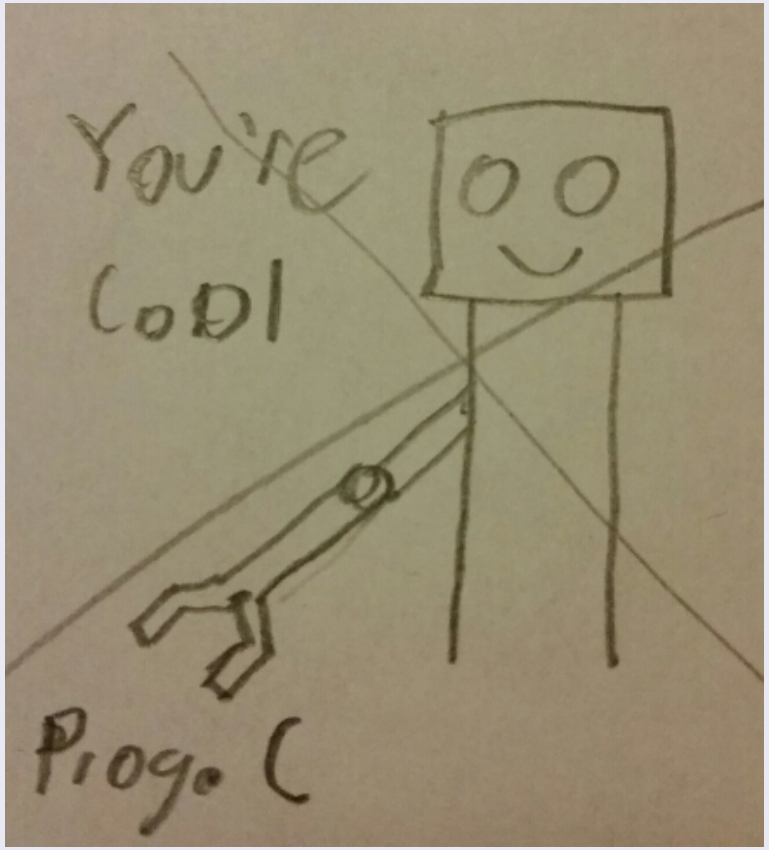

Why not?

Rice's Theorem

Semi-Formally

- Non-trivial classifications of programs can't be computed by a program.

Rice's Theorem

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Church-Turing as Physics

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The laws of physics are inherently computable and there is no physical process that cannot be computed by some algorithm.

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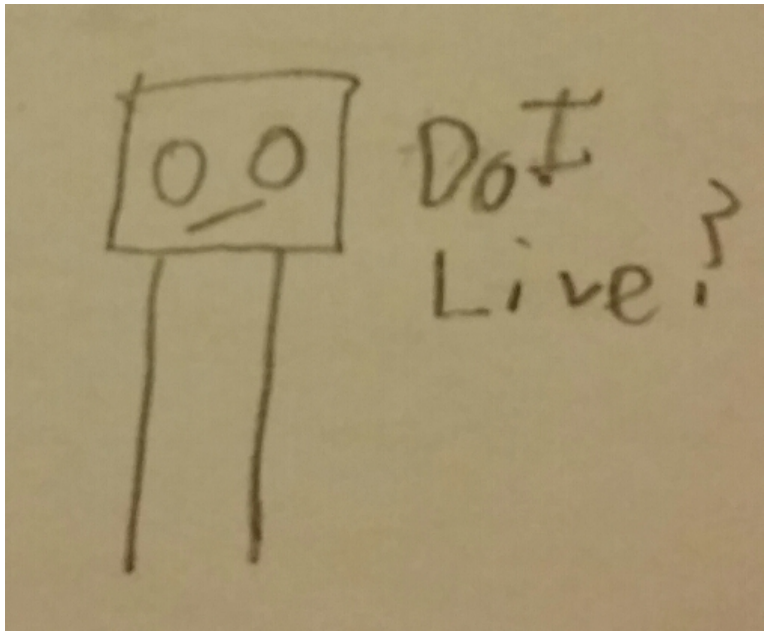
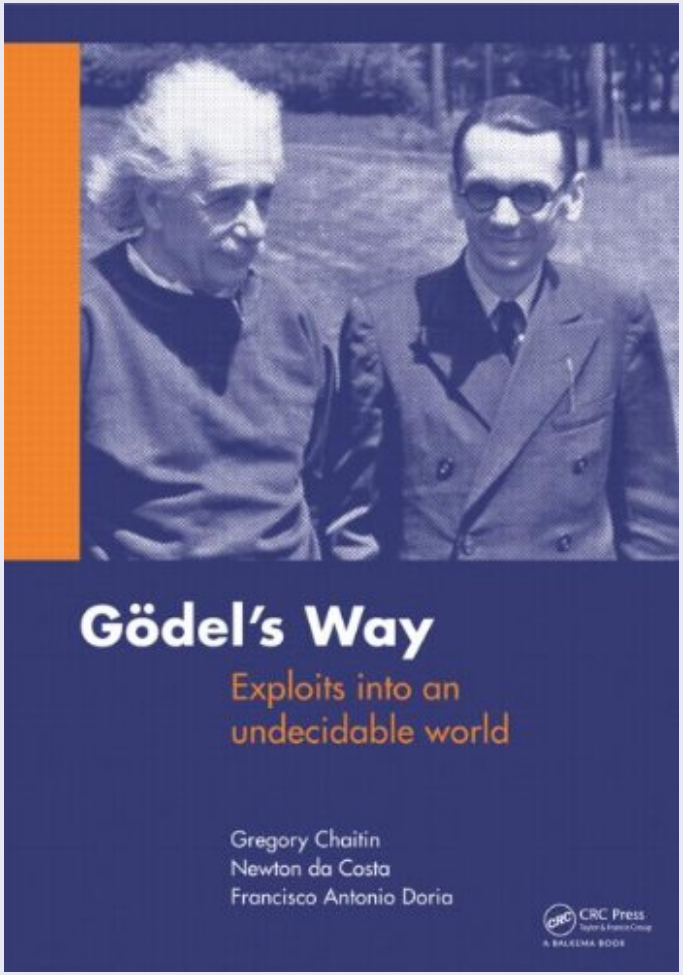


Skirting Computability

Goedel's Way



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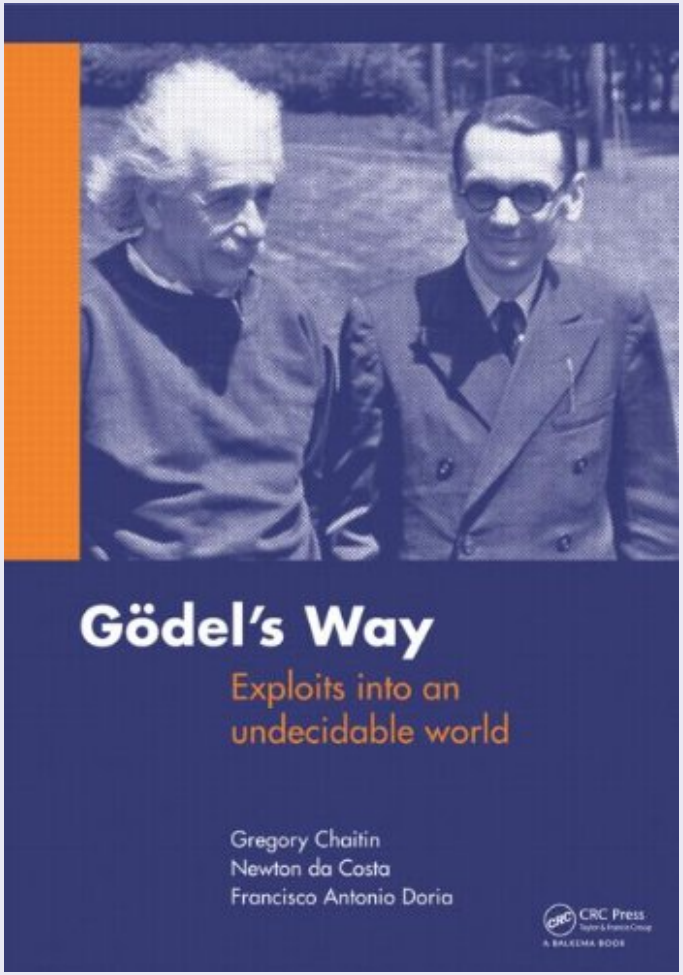

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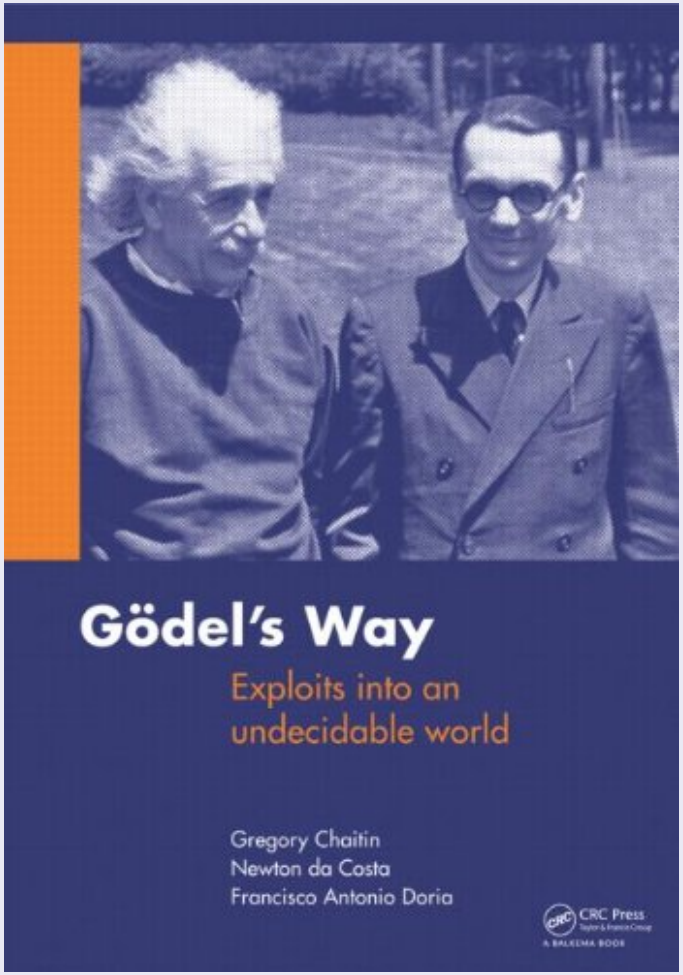

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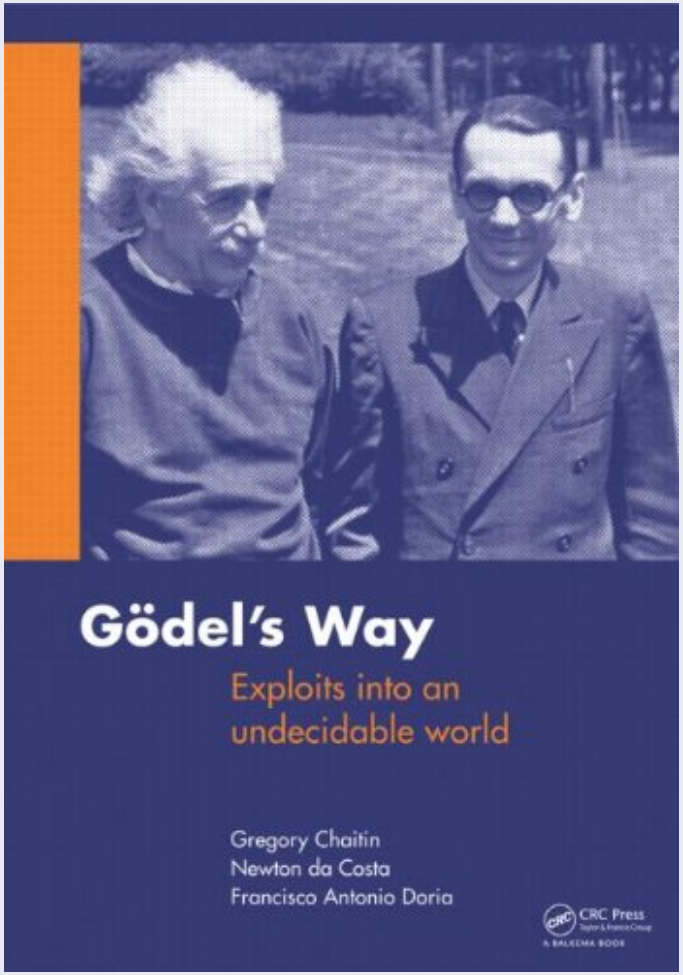

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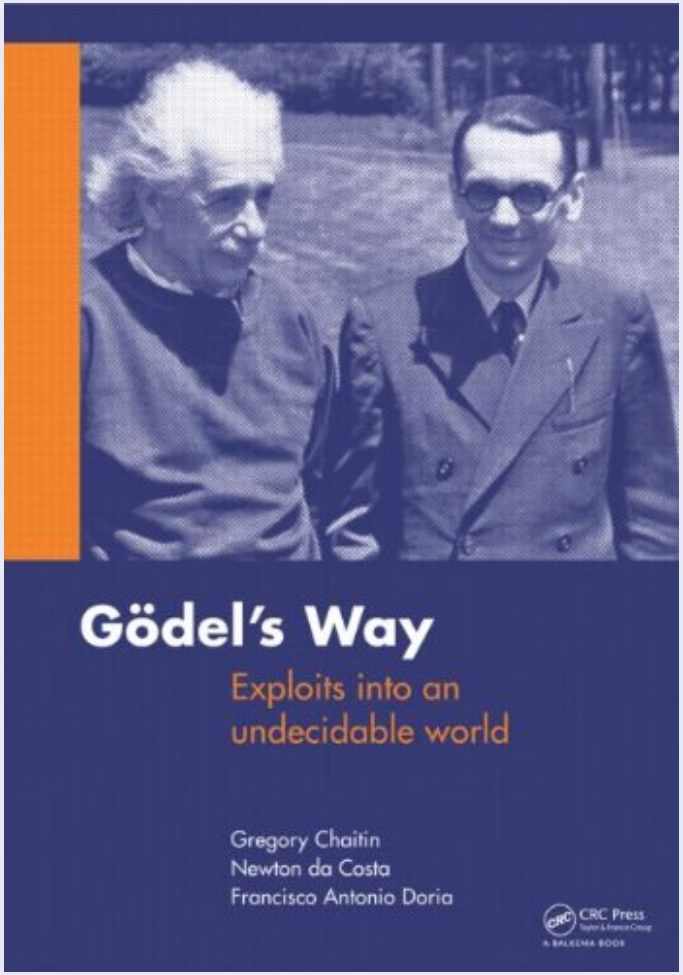

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    - Much more unknown

Any Questions?

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