

# What Are Computers, Really?

Clarissa Littler

6-23-2015

# Fundamental Questions

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- What are the limits of computers?
- Are there problems a computer can never solve?
- Do all programming languages describe the same thing?
- What even **are** programs?

# How We'll Answer Them

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- Build up a definition of computation independent of computers
- Sketch out mathematical models of computation
- Give examples of non-computable problems
- Discuss the implications and limits of our knowledge of computability

# Computation Sounds Like Computer

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- Computation is what computers do
- A program *describes* a computation
- The limits of computation are limits of **description**
- What processes can be **described** in a finite way with a finite **implementation**?

# Examples of Finite Processes

## Recipe as Finite Process

Cook celery and onion together til soft, then add frozen spinach and cook to get some of the moisture out and reduce volume add broth lentils cilantro and other spices, stir thoroughly, throw bay leaves on top.

Cook for 40 minutes

Turn off heat, wait til it stops bubbling and blend thoroughly.

Cook for 5-10 minutes after blending

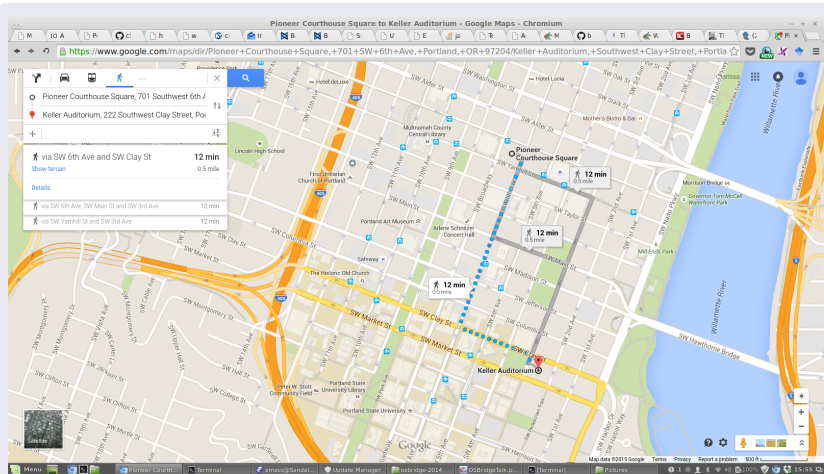
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Another finite process



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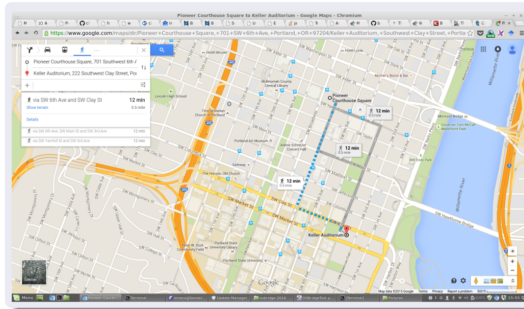
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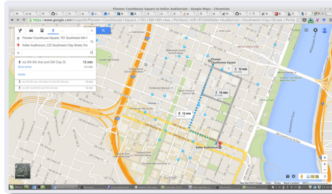
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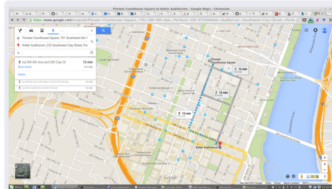
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I'm done with this joke now, I promise.

# Examples of Finite Processes

## Adding

$$\begin{aligned} 5 + 10 &= 4 + 11 \\ &= 3 + 12 \\ &= 2 + 13 \\ &= 1 + 14 \\ &= 0 + 15 \\ &= 15 \end{aligned}$$

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## Another Way

$$\begin{aligned}5 + 10 &= 6 + 9 \\&= 7 + 8 \\&= 8 + 7 \\&= 9 + 6 \\&= 10 + 5 \\&= 11 + 4 \\&= 12 + 3 \\&= 13 + 2 \\&= 14 + 1 \\&= 15\end{aligned}$$

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The following Haskell snippet that evaluates the sum of the integers from 1 to 10 is **also** a finite process

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```
let f x = sum [1..x] in f 10
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## Finite Description

- Finite length
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- number of other talks you'd rather be at

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## Why Finite Resources

No computer and no **physical process** that we know of can use an infinite quantity, thus infinite resources shouldn't be allowed in computation.



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## Why a Finite Alphabet?

An infinite alphabet can't have an implementation that is, itself, a finite process.

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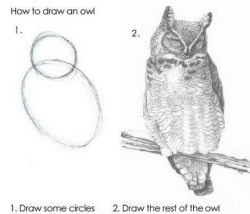
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- a way to **perform** the descriptions



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- Turing (basically) won the race!

# Turing and His Automatic Machines

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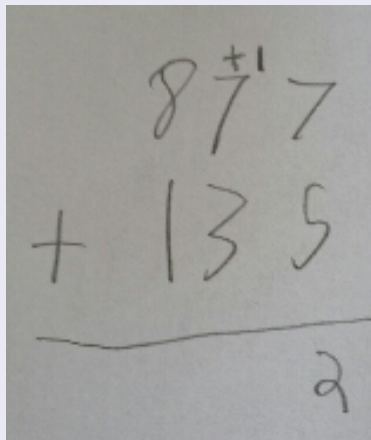
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# How Humans Compute

## An Example

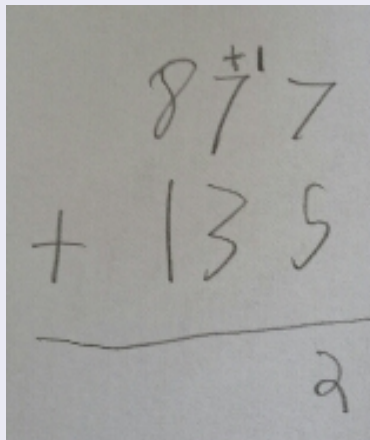


A photograph of a piece of paper with handwritten numbers. The top line shows '877' with a small '+' and '1' written above the second '7'. Below this is '+ 135'. A horizontal line is drawn under the numbers, and the number '2' is written below the line, aligned under the '5'.

$$\begin{array}{r} 877 \\ + 135 \\ \hline \end{array}$$

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A photograph of a piece of scratch paper with handwritten numbers. The top row shows '87' with a small '+' and '1' above the '7', followed by a '7'. The second row shows a '+' followed by '135'. A horizontal line is drawn below the second row, and the number '2' is written below the line.

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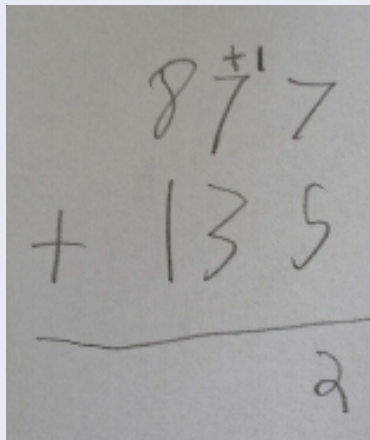
## How We Think

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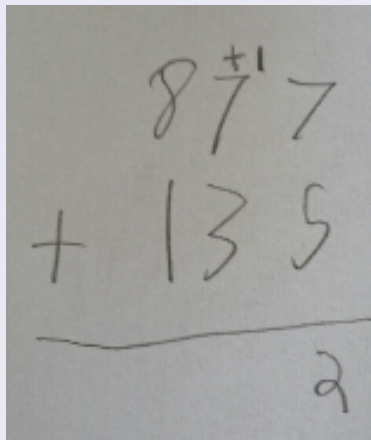
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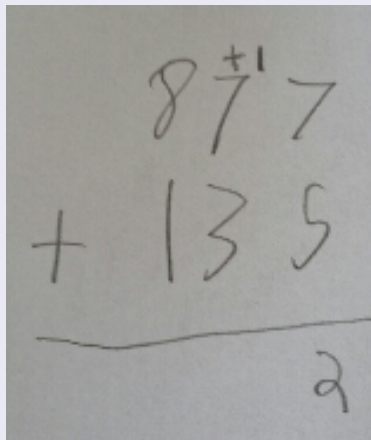
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## How We Think

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## How We Think

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- You can look at it and pick up where I left off
- Only requires a finite number of brain states to perform

# A Turing Machine

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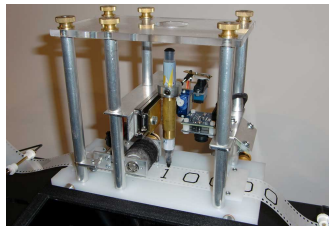
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Courtesy of  
<http://aturingmachine.com/hardware.php>



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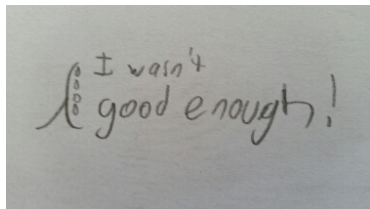
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# The Church-Turing Thesis

## Original Formulation

There is no model of computation more expressive than Turing machines/lambda calculus. [4]

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## Equivalent Formulation

No programming language can be more powerful than a Turing machine



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## Formal Specification

Is there a Turing machine that can tell if another Turing machine will halt on a given input?

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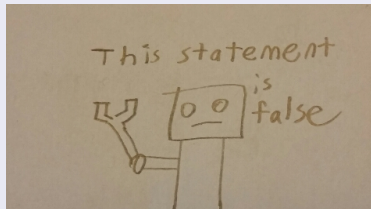
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## Informal Implication

Can you write a program that can detect if other programs have infinite loops?

# Why Can't We Solve The Halting Problem?

## Liar's Paradox

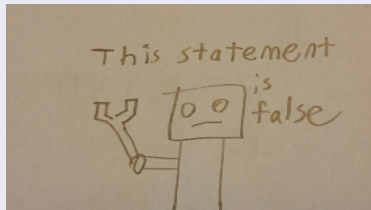


## Proof Idea

- Assume we have a program that can solve the halting problem

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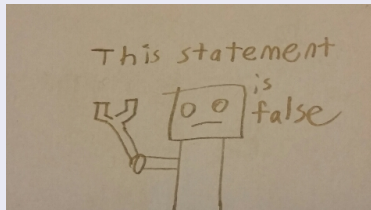
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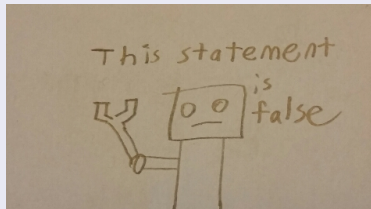


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- What does the liar say about itself?

## Full Employment Theorem

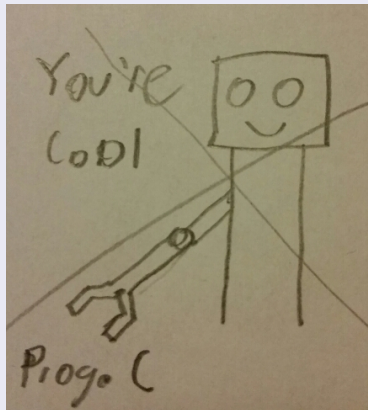
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# Virus Scanners

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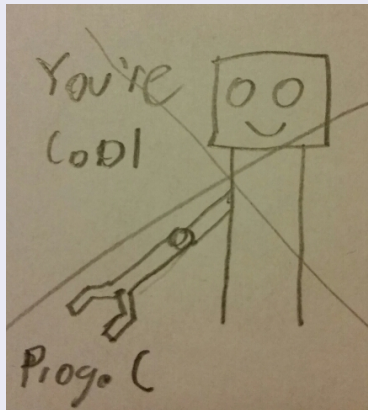


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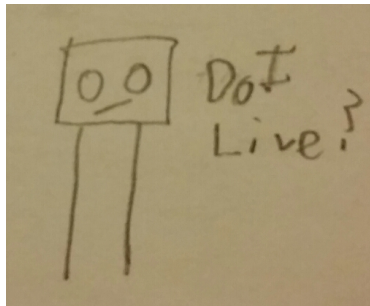
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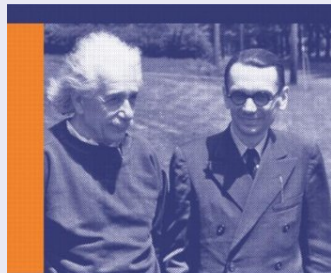
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## Goedel's Way



### Gödel's Way

Exploits into an  
undecidable world

Gregory Chaitin  
Newton da Costa  
Francisco Antonio Doria

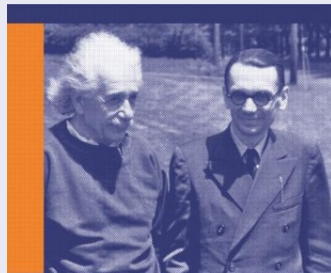


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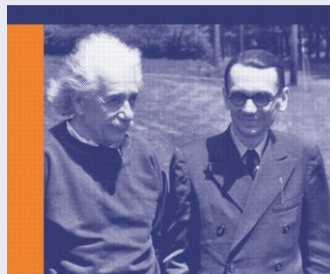


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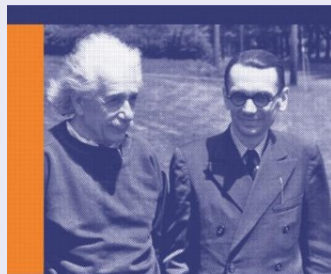


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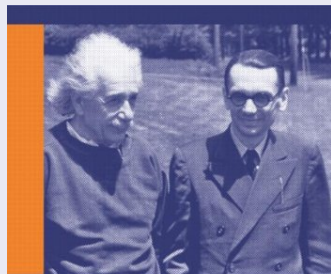


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- Computation has **limits**
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- How computation relates to the laws of the universe?
  - Much more unknown

# Any Questions?

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