

Pies, Tins, and Calculi

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Outline

This Talk

- A brief introduction to the π calculus

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- Milner's translation of λ into π

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- A translation from Tin to our enriched π

A Universal Model of Computation

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- Turing complete
- λ calculus
- Real world has concurrency
- Turing machines and λ calculi don't

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- Calculus of communicating systems
- Processes and channels, not functions and arguments
- Simple syntax

$$\begin{aligned} P := & \\ & (\nu x.P) \\ & P|Q \\ & x(y).P \\ & \bar{x}(y).P \\ & \perp \end{aligned}$$

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- Only data are channels
- Synchronous communication
- A universal model of computation?
- Milner provided an embedding of λ

The basic idea is that

- Variables send the result along the channel named by the variable
- Abstractions
 - receive the argument
 - receive the destination to send the result
 - run the body of the function with those connections
- Application
 - Runs the argument and function in parallel
 - Connects them with fresh channels
 - Replicates the argument in case it's used in multiple places

$$[x]u = x(u). \perp$$

$$[\lambda x.M]u = u(x).u(f).[M](f)$$

$$[MN](u) = \nu c.\nu d.([M](c)|c(d).c(u)|!d(v).[N](v))$$

In This Project

- `PureLam.hs`
- `PurePi.hs`
- `LamToPi.hs`

Lambda Calculus

```
data Lam = Abs Name Lam
  | App Lam Lam
  | Var Name
  | Print Name
```

```
data Val = VUnit
  | VAbs Name Lam
```

```
type Name = String
```

```
type Value = Name
```

```
data Proc = Receive Name Name Proc  
  | Send Name Name Proc  
  | Par Proc Proc  
  | Nu Name Proc  
  | Serv Proc  
  | Print Name  
  | Terminate
```

```
interpProc :: Proc -> Interp ()
interpProc (Print n) = putText n
interpProc Terminate = return ()
interpProc (Par p1 p2) = do
    forkM $ interpProc p1
    interpProc p2
interpProc (Serv p) = do
    forkM $ interpProc p
    interpProc (Serv p)
interpProc (Nu n p) = do
    m <- liftIO newEmptyMVar
    withChan n m $ interpProc p
```

```
interpProc (Send x y p) = do
  env <- asks fst
  case lookup x env of
    Nothing -> error "channel doesn't exist"
    Just m -> (liftIO $ putMVar m y) >> interpProc p
interpProc (Receive x y p) = do
  ec <- asks fst
  case lookup x ec of
    Nothing -> error "channel doesn't exist"
    Just m -> do
      v <- liftIO $ takeMVar m
      interpProc $ substName y v p
```

Lambda To Pi

```
transLam (L.Print n) _ = return $ P.Print n
transLam (L.Var x) n = return $ P.Send x n P.Terminate
transLam (L.Abs x b) n = do
  u <- fresh
  b' <- transLam b u
  return $ P.Receive n x $ P.Receive n u $ b'
transLam (L.App f a) n = do
  c <- fresh
  d <- fresh
  v <- fresh
  f' <- transLam f c
  a' <- transLam a v
  return $ P.Nu c $ P.Nu d $ P.Par (P.Par f'
    (P.Send c d $ P.Send c n $ P.Terminate))
    (P.Serv $ P.Receive d v a'))
```

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- Wanted to write something higher level

Common Expression Language

```
data Exp = EBinOp Exp Op Exp
  | EUnOp Op Exp
  | EInt Int
  | EBool Bool
  | EString String
  | EVar Var
  | EUnit
  | EPrint Exp
  | EName Name
```

```
data Val = VInt Int
  | VString String
  | VBool Bool
  | VUnit
  | VName Name
```

- Ordinary π calculus is sparse

- Ordinary π calculus is sparse
- Enriched with expressions and data

```
data Proc = Receive Exp Name Proc
  | Send Exp Exp Proc
  | Par Proc Proc
  | Nu Name Proc
  | Serv Proc
  | If Exp Proc Proc
  | Terminate
```

Tin: An Imperative Concurrent Language

- Slightly higher level

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- Imperative language

Tin: An Imperative Concurrent Language

- Slightly higher level
- Imperative language
 - while loops
 - if statements
 - sequenced code

Tin: Concurrency Model

- Each declaration is a process

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Tin: Concurrency Model

- Each declaration is a process
- Each process runs in parallel
- Each process can receive or send messages
- Sending and receiving is a blocking action
- No primitive locking

Tin AST

```
data Stmt = SExp Exp
          | SReceive [Var]
          | SSend Exp [Exp]
          | SWhile Exp [Stmt]
          | SIf Exp [Stmt] [Stmt]
```

```
data Decl = Decl Name [Stmt]
```

```
type Inbox = Chan Val
```

```
data InterpEnv = IE { inboxes :: NEnv Inbox, -- inboxes
                     venv :: VEnv Val, -- value env
                     outc :: Chan String, -- output queue
                     self :: Name}
```

```
type Interp = StateT InterpEnv IO
```

Fibonacci in Tin

```
f := {  
    send @h (1)  
    while (true) do {  
        receive (x)  
        send @h (x)  
    }  
}  
  
g := {  
    send @h (1)  
    while (true) do {  
        receive (x)  
        send @h (x)  
    }  
}  
  
h := {  
    receive (x,y)  
    while ((x < 100000)) do  
        print ((x + y))  
        send @f (y)  
        send @g ((x + y))  
        receive (x,y)  
    }  
}
```

Tin to Enriched Pi

```
progToPi :: [T.Decl] -> P.Proc
```

```
blockToProc :: String -> [T.Stmt] -> P.Proc -> Fresher P.Proc
```

```
stmtToProc :: String -> T.Stmt -> P.Proc -> Fresher P.Proc
```

```
stmtToProc n (T.SWhile e ss) p = do
  sp <- blockToProc n ss P.Terminate
  conn <- fresh
  dummy <- fresh
  return $ P.Par (P.Serv $ P.If e sp
    (P.Send (EName conn) EUnit P.Terminate))
    (P.Receive (EName conn) dummy p)
stmtToProc n (T.SIf e sts sfs) p = do
  tp <- blockToProc n sts p
  fp <- blockToProc n sfs p
  return $ P.If e tp fp
```

Questions?