Catalyzing Change in School Mathematics

Creating the Opportunities Our Students Deserve

This synthesis of the key messages and recommendations from the NCTM Catalyzing Change series suggests ways to initiate needed conversations within schools and districts on removing inequitable structures and broadening the purposes of learning mathematics.

DeAnn Huinker, Sarah B. Bush, and Karen J. Graham

Consider the full potential of a student's mathematical journey from prekindergarten through high school if you could dream it from start to finish. Imagine that the young child is curious about the world and loves to learn through play and exploration. This child is creative and clever in thinking about simple patterns they notice through music, books, toys, and nature. The child loves to count the number of birds they see at the feeder and may recognize when more or fewer birds appear than the last time they counted. As the child begins kindergarten and progresses through the elementary grades, this curiosity and creativity is nurtured and fostered, rather than stifled. The child develops a foundation of mathematical understanding through conceptually grounded and meaningful experiences and grows as a confident and capable thinker and doer of mathematics.

As this child transitions into middle school, they are now a young adolescent searching for their place in the world. Their middle school experience is a rich

place and has space to further develop deep mathematical understanding in ways that rightfully build on and extend previous knowledge and also empower them to understand and critique their world. These experiences provide opportunities to recognize injustices in the world and position one as part of the solution. The young adolescent leaves middle school recognizing the importance of mathematics and showing appreciation for the subject, both its usefulness and its beauty.

In high school, this student as an emerging adult is able to understand mathematical ideas in even more complex ways supported by the strong foundation developed in elementary and middle schools. The student further develops their mathematical literacy and considers mathematics-focused careers as both exciting and fully obtainable choices. The student knows that the mathematics they have learned will be critically important for their future success and that other advanced mathematics course options are available even if they do not pursue a STEM career. No matter what college or career path the student takes after graduation, they leave high school with a deep mathematical understanding, positive memories, and a great relationship with mathematics.

As you read this scenario, what mental image did you form of this student? What was their gender? What was their socioeconomic status? What was their background? The National Council of Teachers of Mathematics (NCTM) advocates for all students to have inspiring and positive mathematics experiences. Our mental image should include each and every student, no matter their gender, race, class, language, or ability status.

This article surveys four key recommendations that can serve as catalysts for achieving an equitable mathematics education for each and every student. The recommendations are official positions of NCTM and are discussed in detail through the NCTM Catalyzing Change series (see figure 1), which consists of early childhood and elementary, middle school, and high school grade band books (NCTM 2018, 2020a, 2020b). The purpose of this article is to provide a synthesis of the key messages from the Catalyzing Change series across grade bands and to recommend ways to use the series within schools and districts to initiate critical and needed conversations. We begin by first discussing the need to catalyze changes in school mathematics. Next, we share the four key recommendations and then discuss each recommendation in more depth. We close with suggested next steps for initiating needed and critical conversations. We hope that this article contributes to an intentional journey toward a more equitable mathematics education for each and every student.

WHY WE NEED TO CATALYZE CHANGE

The Catalyzing Change series is grounded in the belief that each and every student is capable of engaging in rigorous and challenging mathematics if provided access to a high-quality mathematics program. Yet, we are confronted by the reality that students' mathematics experiences are of uneven quality at every level. Programs often overproceduralize mathematics at the expense of developing deep conceptual understanding. Instruction often lacks authentic mathematical connections to real-world experiences, including students' interests, cultures, and communities. Disparities exist within individual classrooms, across grade levels within schools, and across schools within districts. The evidence is compelling that students who are identified as Black, Latinx, Indigenous, language learners, poor, and

DeAnn Huinker, huinker@uwm.edu, is a professor at the University of Wisconsin–Milwaukee. She is a past member of the NCTM Board of Directors and served as the task force and writing team chair for *Catalyzing Change in Early Childhood and Elementary Mathematics*.

Sarah B. Bush, sarah.bush@ucf.edu, is an associate professor of K–12 STEM Education at the University of Central Florida. She is a member of the NCTM Board of Directors and was the task force and writing team chair for *Catalyzing Change in Middle School Mathematics*.

Karen J. Graham, karen.graham@unh.edu, is a professor in the Department of Mathematics and Statistics at the University of New Hampshire. She is a past member of the NCTM Board of Directors and served as the task force and writing team chair for *Catalyzing Change in High School Mathematics*.

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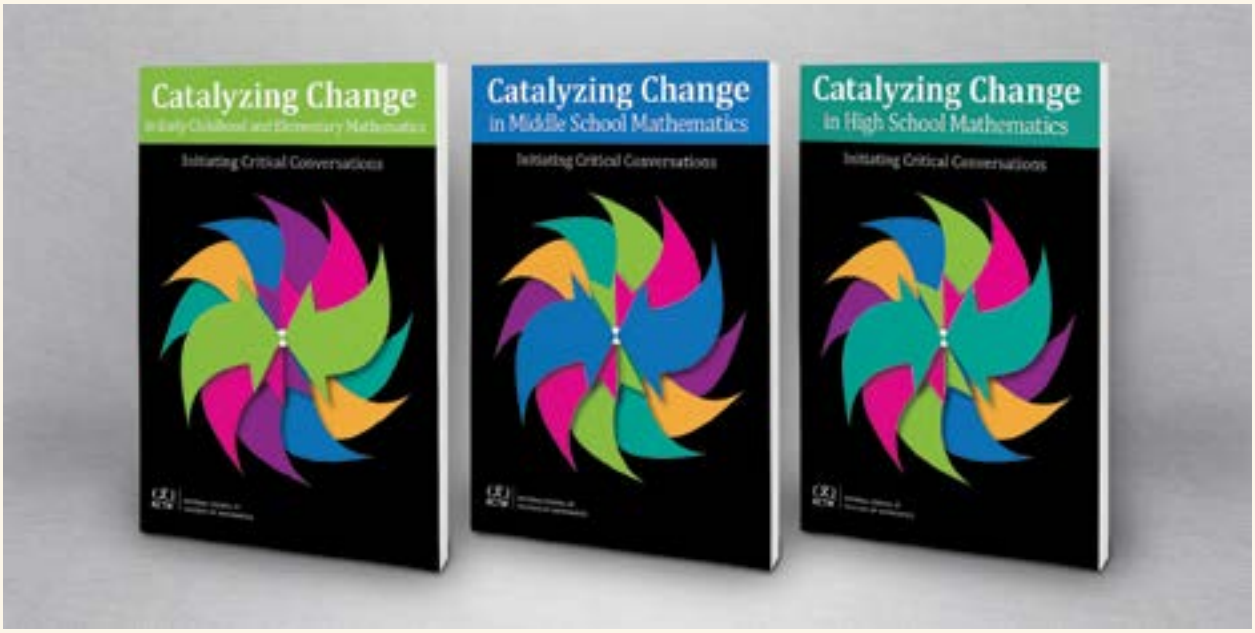
Fig. 1



The Catalyzing Change Series (NCTM 2018, 2020a, 2020b) includes books that support PK–12 teachers.

with disabilities, along with other marginalized learners, do not have the same opportunities as their peers to access and learn in mathematically powerful spaces.

The traditions and structures that embody PK–12 mathematics are long-standing and deeply rooted and require critical examination and transformation. Existing challenges in mathematics education point to serious reasons for why we must, collaboratively as a field, commit to catalyzing change:

- Too many students form negative views of mathematics and of themselves in relation to mathematics.
- Students who do not have regular opportunities to collaborate on challenging tasks, use various strategies, or focus on sense making have lower mathematics achievement than those afforded such opportunities.
- Societal perceptions of mathematics perpetuate the myth that only a selected few are destined to be good in mathematics.
- For far too long, mathematics education has left many students unprepared for college, a career, or life. (List adapted from NCTM 2018, 2020a, 2020b.)

NCTM challenges us to initiate critical conversations on policies, practices, and issues influencing mathematics education. In 2018, NCTM published *Catalyzing Change in High School Mathematics*: *Initiating Critical Conversations* and provided a framework for the transformation of high school mathematics. The initial discussions indicated widespread agreement that the recommendations for high school had implications for early childhood, elementary, and middle school, leading to the publication of *Catalyzing Change in Early Childhood and Elementary Mathematics: Initiating Critical Conversations* (NCTM 2020a) and *Catalyzing Change in Middle School Mathematics: Initiating Critical Conversations* (NCTM 2020b).

The Catalyzing Change series is unique from previous documents because it urges stakeholders to critically examine their mathematics programs in a holistic manner in order to take the necessary steps to uproot inequitable structures and ineffective practices. The status quo is simply unacceptable. Even though the mathematics education community has made progress in improving mathematics teaching and learning, this work remains far from complete.

KEY RECOMMENDATIONS

NCTM offers four key recommendations for strengthening school mathematics (see table 1) that are aligned across early childhood, elementary school, middle school, and high school (NCTM 2018, 2020a, 2020b). Achieving these recommendations will not be easy. Stakeholders need to engage in serious conversations that question current structures, policies, and practices and then work alongside one another to plan a forward path. It is our collective responsibility to the students of today and tomorrow to begin these difficult conversations and work to ensure equitable mathematics learning opportunities and outcomes for every student.

Recommendation 1. Broaden the Purposes of Learning Mathematics

NCTM calls for a unified and broader vision of learning mathematics while recognizing and appreciating the

Table 1 NCTM Key Recommendations for Catalyzing Change in School Mathematics (NCTM 2018, 2020a, 2020b)

	Early Childhood and Elementary	Middle School	High School
Broaden the Purposes of Learning Mathematics	Each and every child should develop deep mathematical understanding as confident and capable learners; understand and critique the world through mathematics; and experience the wonder, joy, and beauty of mathematics.	Each and every student should develop deep mathematical understanding; understand and critique the world through mathematics; and experience the wonder, joy, and beauty of mathematics, which all contribute to a positive mathematical identity.	Each and every student should learn the Essential Concepts in order to expand professional opportunities; understand and critique the world; and experience the wonder, joy, and beauty of mathematics.
Create Equitable Structures in Mathematics	Early childhood and elementary mathematics should dismantle inequitable structures, including ability grouping and tracking, and challenge spaces of marginality and privilege.	Middle school mathematics should dismantle inequitable structures, including tracking teachers as well as the practice of ability grouping and tracking students into qualitatively different courses.	High school mathematics should discontinue the practice of tracking teachers as well as the practice of tracking students into qualitatively different or dead-end course pathways.
Implement Equitable Mathematics Instruction	Mathematics instruction should be consistent with research-informed and equitable teaching practices that nurture children's positive mathematical identities and strong sense of agency.	Mathematics instruction should be consistent with research-informed and equitable teaching practices that foster students' positive mathematical identities and strong sense of agency.	Classroom instruction should be consistent with research-informed and equitable teaching practices.
Develop Deep Mathematical Understanding	Early childhood settings and elementary schools should build a strong foundation of deep mathematical understanding, emphasize reasoning and sense making, and ensure the highest-quality mathematics education for each and every child.	Middle schools should offer a common shared pathway grounded in the use of mathematical practices and processes to coherently develop deep mathematical understanding, ensuring the highest-quality mathematics education for each and every student.	High schools should offer continuous four-year mathematics pathways with all students studying mathematics each year, including two to three years of mathematics in a common shared pathway focusing on the Essential Concepts, to ensure the highest-quality mathematics education for all students.

unique needs of students at each grade band. The three purposes include—

- developing deep mathematical understanding;
- understanding and critiquing the world through mathematics; and
- experiencing the wonder, joy, and beauty of mathematics.

Across PK–12, "the expanded purposes situate the learning of mathematics as important not only for college, career, and life but also as a human endeavor that values historical, cultural, and aesthetic perspectives of mathematics" (NCTM 2020a, p. 11). Importantly, the multiple purposes should be approached in ways that are reflected in daily mathematics instruction, not as separate lessons or projects.

Children naturally bring wonder and joy to their learning and are eager to "mathematize" their world—that is, to see and describe their environment with a mathematical lens. For example, children notice many symmetries found in nature and marvel at the number of stars in the sky. Children are inquisitive, and teachers need to be equally as curious about children's mathematical observations, questions, and insights as well as marvel at their brilliance. Unfortunately, the legacy of structures and practices, such as timed tests, in school mathematics undermine children's learning and mathematical dispositions in ways that have long-term negative consequences. In *Catalyzing Change,* NCTM calls on schools and districts to ensure that their mathematics programs emphasize the importance of going deep with conceptual ideas and provide sufficient instructional time for teachers and children to engage in extended mathematical inquiries and discussions that allow children to experience the wonder, joy, and beauty of mathematics and to see their world as mathematical.

In the middle grades, students are experiencing a time of rapid change in ways that are physical, cognitive, moral, psychological, and socio-emotional (AMLE 2010; Eccles et al. 1993) and are working to forge their own identities as they explore *who they are, who they can be, what they should be,* and *what they should do* (Beane 1997). Engaging students in mathematics experiences that are meaningful and relevant to their current lives is essential, as are opportunities for students to question and challenge the status quo in our world. When we provide opportunities for young adolescents to pose questions relevant to their lives, contest injustices, and challenge how the world is shaped, the true utility of mathematics for social justice becomes visible to students as they are empowered to be part of the solution (Gutstein 2003). Although it will need to be done carefully and thoughtfully, middle school students should explore sensitive and controversial topics as they use mathematics to understand and critique the world. One such context that serves as a clear example is the mathematics of gerrymandering (Safi, Bush, and Desai 2018). Students examine the societal impacts of reconfiguring districts to increase the likelihood of one group winning an election. "This task brings to the forefront how mathematics, in this case equal versus equivalent, allows students to begin to confront a real social, political, economic, and educational concern" (NCTM 2020b, p. 14).

In high school, students begin to consider how mathematics fits within possible professional opportunities, including those in STEM and non-STEM fields. In fact, at the very core of the *Catalyzing Change* conversation is the belief that every single student belongs in mathematics whether or not they pursue a STEM career. High school should promote mathematics as a tool to empower students as members of a democratic society, ultimately encouraging them to be engaged in their communities and to contribute to society by challenging injustices and making the world a better place (Gutstein and Peterson 2013). For example, current news stories provide a rich context for engaging students. A news article indicated that the typical household net worth increased by 30 percent for Black households and by 46 percent for Latinx households from 2013 to 2016 (Long and Jan 2017; access the article at https://www.denverpost.com/2017/09/28/great-recession-minorities-college-education-gains-in-wealth). A teacher using this news story might pose these questions:

1. What questions should you ask to make sense of this information?
2. In 2013, a typical net worth was $13,600 for Black households and $14,200 for Latinx households. What was the typical household net worth for each of these two groups in 2016? What new information or insights do the numerical values provide?
3. What are advantages and disadvantages of using percentages versus numerical values in describing the increase or decrease of net worth in this situation?

This task provides both an opportunity for students to critically examine how numbers stated in different

forms (in this case, percentages) may understate or overstate the actual situation and the opportunity to examine the issue of income disparity among different populations. Students could be encouraged to explore similar data in their own communities and discuss its implications, opportunities, and challenges.

Recommendation 2. Create Equitable Structures in Mathematics

System structures are defined as school or district policies, practices, or conditions that support or impede student learning of mathematics. In *Catalyzing Change*, NCTM specifically calls for the dismantling of ability grouping and the related structures of tracking students and teachers, structures that have been prominent in schools since the beginning of the 20th century. Starting in the elementary grades, students are already often ability grouped and sometimes in upper elementary are even tracked into qualitatively different courses. At the middle school level, tracking has traditionally been set fully into motion. The mathematics course a student takes as early as sixth or seventh grade ultimately determines their entire mathematics trajectory through high school and beyond. Ability grouping and tracking place students in qualitatively different experiences with many marginalized learners experiencing mathematics that is less engaging, is less rigorous, and has lower expectations for their mathematics learning. Often, these pathways are terminal in that they are not preparing students personally or professionally for their future.

Tracking is problematic for a multitude of reasons. It reinforces the notion that only some are capable of high mathematics achievement (Boaler 2011); placement is often on the basis of nonacademic factors (Stiff and Johnson 2011); and Black, Latinx, and students from low socioeconomic backgrounds are overrepresented in remedial intervention courses (Larnell 2016; Steele and Huhn 2018). Further, tracking students perpetuates the practice of also tracking teachers, in which the most experienced teachers, or the teachers perceived as the most effective, are unproportionally assigned to the higher-tracked courses. NCTM recommends balancing teaching assignments whenever possible in order to build a collective sense of responsibility for all students. Balanced teaching assignments might involve teachers assigned to both upper-level and entry-level mathematics courses and changing teaching assignments every two or three years (NCTM 2018).

Mathematics learning should be thought of as a journey and not a race; schools must guard against pressures to "race to algebra" or "race to calculus."

Acceleration in mathematics is also problematic. Too often critical concepts are rushed or skipped altogether (NCTM 2016). Mathematics learning should be thought of as a journey and not a race; schools must guard against pressures to "race to algebra" or "race to calculus." Evidence suggests that students who speed through content without developing a deep understanding are those who tend to disengage with mathematics when they are given the chance (Seeley 2009). However, in some limited situations, appropriate acceleration might be warranted for some students, provided that it takes into consideration students' professional aspirations and personal interests and allows for student choice and ownership in the decision making. NCTM is collecting case studies that document how schools and districts are dismantling inequitable structures, such as tracking, and moving toward mathematics programs that are equitable and inclusive and disrupt the use of deterministic labels used on both students and teachers. For example, San Francisco Unified School District began detracking in 2014. The district mathematics curriculum interweaves algebraic concepts throughout the elementary grades, leading to an eighth-grade course focused on linear equations and functions. The district "has decreased disparities in mathematics achievement and increased the number and diversity of students taking upper-level mathematics courses in high school" (NCTM 2020b, p. 33). To learn more about the San Francisco case or to read other case studies, visit https://www.nctm .org/Supporting-Resources-Catalyzing-Change-in -High-School-Mathematics.

Recommendation 3. Implement Equitable Mathematics Instruction

Equitable mathematics instruction is grounded in developing deep mathematical understanding while attending to fostering positive mathematics identities, a strong sense of mathematics agency, and shared mathematics authority in the classroom (see table 2). A student's mathematical identity is defined as the "dispositions and deeply held beliefs that students develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics in powerful ways" (Aguirre, Mayfield-Ingram, and Martin 2013, p. 14). Nasir (2002) contends that the relationship between learning and identity is bidirectional, with learning supporting a student's identity and a student's identity supporting their learning.

The ways in which students participate in mathematics and express their mathematical identities determine their level of agency (Murrell 2007). Students with a strong sense of agency are willing to engage in productive struggle, take risks in making their mathematical thinking visible, and realize that learning results when they successfully leverage an approach that works for them. Mathematical agency is cultivated through classroom environments in which students share in the mathematical authority. Teachers can assign competence to students in subtle ways that position them as mathematically competent, not only increasing the student's confidence in oneself but also raising the status of the student among peers. For example, a fourth-grade teacher might comment to a small group, "Tyrek, your tape diagram clearly shows how the quantities in the problem relate to each other; it would be valuable for your group to study it more closely and use it to help determine a next step in solving the problem." How students are positioned as learners, authors, and contributors to the collective and shared understanding of the class all have a profound impact on their mathematical identity and agency (Boaler 2016).

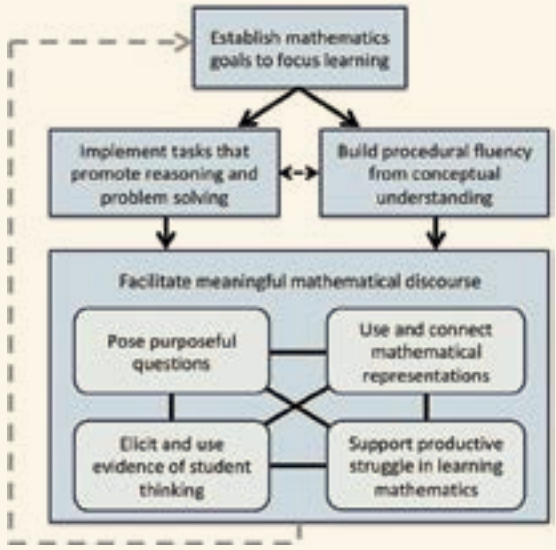
Students derive their mathematical identities and agency from their experiences in learning mathematics. In the Catalyzing Change series, NCTM highlights how equitable instruction is coupled with implementation of the eight research-informed Mathematics Teaching Practices, identified in *Principles to Actions: Ensuring Mathematical Success for All* (NCTM 2014). The Mathematics Teaching Framework, shown in figure 2, depicts the intersectionality of the teaching practices as a structure for high-quality mathematics instruction that is necessary to foster students' mathematical identities and agency (Boston et al. 2017; Huinker and Bill 2017; Smith, Steele, and Raith 2017). Consider this scenario:

> A teacher launches a lesson by eliciting student ideas and strategies about a mathematics problem. After some initial discussion, she invites her students to share their ideas with one another in small groups, collaboratively coming to consensus on a solution path as they offer, explain, critique, and justify a range of possibilities. The teacher then orchestrates a classroom-wide discussion, grounded in student work, as they reason together about the focal mathematics. (Langer-Osuna 2017, p. 237)

Table 2 Identity, Agency, and Authority in Mathematics (NCTM 2020a, 2020b)

Mathematics Identity Is . . .	Mathematics Agency Is . . .	Shared Mathematics Authority Is . . .
• A view of oneself as a doer, knower, and sense maker of mathematics.	• One's mathematical identity in action both inside and outside the classroom.	• Acknowledging students as authors of mathematical questions, ideas, strategies, and explanations.
• A deeply held belief about one's own ability to engage successfully with mathematics.	• Revealed in one's confidence, capacity, and willingness to engage mathematically.	• Centering classroom discourse on students' mathematical thinking and reasoning.
• Affected by how students are positioned in the mathematics classroom.	• Shaped by students' opportunities to choose, use, and discuss their own strategies.	• Positioning students as valuable contributors to the collective mathematical knowledge of the class.
• Affected by beliefs about the nature of mathematics and the learning of mathematics.	• Created in a space and place where students are empowered.	

Fig. 2



The Mathematics Teaching Framework can be implemented in daily instruction.

The brief scenario captures the essence of using the mathematics teaching framework as a guide for daily mathematics instruction. The task engaged students in problem solving and, thus, provided opportunities for deepening conceptual understanding. Students engaged in productive struggle, drawing on multiple representations, as they reached consensus on a solution path. The teacher needed a clear understanding of the mathematics learning goals to use them as a guide to facilitate discourse. Throughout the lesson, students had shared authority as the authors and owners of the mathematical solutions, explanations, and justifications.

Each and every classroom must become a powerful space for students to learn mathematics as doers, knowers, and sense makers. The ways in which students experience mathematics from one grade level to the next can be radically different and confusing to students as they are forming beliefs about what is mathematics, what it means to learn mathematics, and whether they have a place in mathematics. For far too many students, school mathematics leads to disengagement and disaffection. Equitable mathematics instruction not only supports deep mathematics learning but also fosters the development, within each student, of a mathematical identity that is positive and allows students to see themselves with agency to be successful in mathematics.

Recommendation 4. Develop Deep Mathematical Understanding

The fourth recommendation prioritizes the building of a strong foundation of deep mathematical understanding. Students with richly connected knowledge are empowered with a rich set of tools to apply mathematics meaningfully to critique the world, to make sound decisions, and to solve problems in their personal and future professional lives. In the following sections, we will highlight the implications of Recommendation 4 for each of the grade bands.

Early childhood and elementary. Disrupting the cycle of rote and disconnected learning of mathematics is necessary for children to achieve deep understanding as confident and capable learners. The way mathematics is taught matters because "when mathematics instruction is shallow, learning stays at the surface. When mathematics instruction goes deep, children are empowered to explore the richness of the mathematical landscape" (NCTM 2020a, p. 77).

Mathematical practices and processes define how children engage as *doers of mathematics*. Four pairs of interrelated practices and processes are identified in *Catalyzing Change in Early Childhood and Elementary Mathematics,* which are particularly important for our youngest learners:

1. Representing and connecting
2. Explaining and justifying
3. Contextualizing and decontextualizing
4. Noticing and using mathematical structures

As children make connections among multiple representations (e.g., draw an array for 4 × 7), they not only deepen their mathematical understanding but also increase their resources to explain and justify their reasoning. Children are doers of mathematics when they contextualize mathematical statements with real-world situations (e.g., What might be a story for 47 ÷ 3?) or decontextualize by extracting the quantities and operation relationships from a problem situation. Another key aspect of doing is noticing and using mathematical structures (e.g., place value, number relationships, and operation properties) to support reasoning and problem solving.

Mathematics content learning must focus on building a strong foundation so that children develop as *knowers and sense makers of mathematics*. The five content domains for early childhood and elementary

school include (1) whole number concepts and operations, (2) fraction concepts and operations, (3) early algebraic concepts and reasoning, (4) data concepts and statistical thinking, and (5) geometric and measurement concepts and spatial reasoning.

For example, a hallmark of number and operation sense is computational flexibility. How might a child approach solving 1005 − 998 = ? Does the child immediately line up the digits by place values and begin regrouping, or does the child pause to consider the meaning of subtraction as finding a difference; notice that the numbers are very close together, and then reason, "I can start at 998, add 2 more to get to 1000, and add 5 more to get to 1005, so the difference is 7." Number and operation sense must be intentionally developed through experiences that value multiple strategies and diverse ways of solving problems, as well as be woven into the culture of the mathematics classroom. The ultimate success of learning mathematics in early childhood and elementary school is students who are confident in themselves as they transition into the demands of middle school mathematics.

Middle school. In middle school, students should have consistent opportunities to engage in mathematics in ways that mirror how they will engage in mathematics as adults, which inherently includes problem solving through messy problems. With many of the ideas in middle school mathematics and statistics being some of the most common and important students will use as adults, we must ensure that mathematics is learned well and with depth. Such deep learning is accomplished through rich contextual and noncontextual tasks in order to transform and empower students as their mathematical, statistical, and, more broadly, STEM literacy is developed. Focusing on only content standards is not enough to lead to the transformative changes needed. It is through embracing the mathematical practices and processes that we truly meet the potential of the college and career readiness standards, where students are active participants in their own learning. Regardless of which content standards your setting has adopted, it is important to consider how they are being implemented. For example, overproceduralizing content standards will lose the intent to promote a balance of procedural fluency and conceptual understanding. A sampling of topics commonly overproceduralized in the middle grades includes an overly algorithmic focus on ratios and unit rates; computation with fractions (continued from the elementary grades); rote memorization of steps to solve multistep equations without connecting to properties; a focus on statistics that is primarily on the calculation of numerical and statistical summaries rather than on developing statistical thinking to build understanding of variability and the ability to interpret data; and finding the area, surface area, and volume of two- and three-dimensional shapes using formulas void of opportunities to develop conceptual understanding.

Catalyzing Change in Middle School Mathematics identified five key content domains: (1) number, (2) ratio and proportion, (3) algebra and function, (4) statistics and probability, and (5) geometry and measurement. Importantly, each content domain should engage students in three key interwoven threads: (1) mathematical and statistical modeling, (2) mathematical reasoning, and (3) strategic use of technology. Through embracing these interwoven threads and the practices and processes while maintaining the integrity of the content standards, the true potential of middle school mathematics can be realized.

High school. At the high school level, NCTM recommends four years of mathematics instruction for every student, so they can be productive in both their personal and professional lives. A set of 41 Essential Concepts are described in *Catalyzing Change in High School Mathematics,* which include fundamental understandings in number, algebra and functions, statistics and probability, and geometry and measurement. These concepts are essential for expanding each student's professional and postsecondary educational opportunities as well as for understanding and critiquing processes in daily life (from banking and personal finance to climate change and political claims). The Essential Concepts are not a set of standards but instead represent the fundamental ideas that students should understand and remember long after their formal schooling is complete.

Essential Concepts should be organized within a common pathway for the first two to three years of high school mathematics. A common pathway supports a single curriculum and common rigorous and engaging instruction for all students in a school setting and avoids the creation of separate and unequal tracks. Once students have completed the common pathway, they should continue their study of mathematics directly related to their personal and career interests and aspirations. Possible advanced courses include Precalculus, AP Calculus, AP Statistics,

Quantitative Literacy, Data Science, Financial Mathematics, Mathematical Modeling, or Discrete Mathematics.

Next Steps for Catalyzing Change

This article has provided an overview of the four key recommendations for *Catalyzing Change* in PK–12 mathematics. These four recommendations represent research-informed actions to ensure that mathematics programs are powerful, equitable, just, and inclusive mathematics learning spaces. In summary, it is our collective responsibility TO ensure that each and every student experiences mathematics learning focused on deep and robust understanding that lasts for a lifetime (not until the next test). Students deserve experiences that enable them to use their mathematical knowledge to model and solve authentic problems, that foster positive mathematical identities and a strong sense of mathematical agency, and that generate competence and pride in students' mathematical abilities. Critical conversations are needed to pursue and create equitable structures that promote a just, equitable, and inclusive high-quality mathematics education for each and every student. In table 3, we share several conversation starters from the series organized by the four key recommendations as a resource to jump-start your work.

As a next step, we encourage you to read the Catalyzing Change series, focusing on the grade band book that most closely aligns to your role, and to use the available resources (see nctm.org/change for recorded webinars, executive summary, book study guides, and other resources) to start the *Catalyzing Change* conversation in your setting.

Table 3 Conversation Starters for the Key Recommendations

Broaden the Purposes of Learning Mathematics	**Create Equitable Structures in Mathematics**
• In what ways can opportunities to understand and critique the world through mathematics be integrated into the mathematics curriculum as part of what teachers bring each day to mathematics instruction?	• What steps can be taken to identify and eliminate biases that lead to differential learning opportunities in our school for mathematics?
• What supports are needed to ensure that teachers' ongoing professional learning, teamwork, and planning are focused on implementing our school mathematics curriculum with integrity?	• In what ways can individuals in our school communicate with educators, families, and students about the multiple purposes of learning mathematics and related shifts in instruction?
Implement Equitable Mathematics Instruction	**Develop Deep Mathematical Understanding**
• To what degree do teachers in our school or district embody equitable mathematics teaching practices?	• To what extent are the mathematical practices and processes prioritized in classrooms in our school or district? Why might this be the case?
• What is the quantity and quality of the mathematics-focused collaborative time for teachers and instructional leaders in our school?	• What specific school or district structures, policies, and practices support or undermine teaching for deep mathematical understanding?

Note: Conversation starters adapted from NCTM (2020a, 2020b).

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