Risk

Human beings have provided some form of insurance to one another for at least as long as recorded time (e.g., Bogucki (2011), Clare (2010), Trenerry (1926)), and the modern world is full of institutional arrangements that protect individuals against risk. One way way to derive demand for these institutions from an economic model is to assume that individuals are inherently *risk averse* in the sense that they are willing to pay something now in return for a more certain outcome in the future. The *expected utility* model is a simple representation for this sort of preference. In it, we assume that individuals can rank choices that influence the probability distribution over uncertain future payoffs π by comparing the expected value of an increasing state-independent concave utility function $u(\pi)$ across choices.

Let's think about this with a few examples. To keep things simple, let's assume initially that a farmer's preferences can be represented by a linear function of the mean and variance of π. In particular, for $\alpha > 0$ and $\beta > 0$, let

$$\mathbb{E}u(\pi) = \alpha\mathbb{E}\pi - \beta\text{var}(\pi).$$

Further, suppose that $\pi = \overline{\pi}$ or $\pi = 0$ with probabilities p and $1 - p$.

Exercises

(1) Demonstrate that a farmer with this utility function is willing to purchase actuarially fair insurance (premium = expected indemnity).

(2) Suppose there are two farmers, A and B, with $\beta_A > \beta_B$. Characterize optimal risk sharing between these two farmers.

Here's another exercise that relates to a pair of papers (Chibwana, Fisher, and Shively (2012) and Dercon (1996)) that you will read next week.

(3) A farmer has L total acres of land, and must choose the acreage l_i to plant this year to each crop. Suppose that there is no risk in the first crop; it has a certain return π_1. Crop 2 returns $\pi_2 > \pi_1$ with probability p and zero otherwise. How much acreage should the farmer plant to each crop?

(4) Write a Python class called `LandAllocation()` that has "attributes" for model parameters and "methods" for a) farmer utility (for now you can stick with mean-variance preferences), b) the probability distribution of each crop (you may assume there are just two crops as in the previous exercise.), c) a farmer's optimal acreage allocation, and e) a plot that summarizes some comparative static that you find interesting. Ideally your methods will accommodate arbitrary probability distributions.

(5) Suppose the farmer is offered insurance that costs P up front (the premium) and reimburses up to 75% of the expected output for each crop (assume that the farmer and the insurance company agree on the relevant distribution for computing this expectation, and that it's the one from your model). Exercise: What is the farmer's willingness to pay for this insurance? What is the welfare gain from insurance? Is there a profit maximizing premium and reimbursement rate for the insurance company?

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