A1 — Structural Motivation and Problem Statement

Gravitational waves constitute a unique observational channel: unlike electromagnetic signals, they propagate essentially unimpeded from their production epoch to the present, encoding integrated information about the dynamical and causal structure of spacetime itself.

As a result, a stochastic gravitational wave background should not be interpreted merely as a superposition of independent sources, but as a cumulative record of cosmological history.

This article advances the following central claim: the spectral shape of the gravitational wave energy density is constrained not only by generation mechanisms, but by the underlying causal structure of time.

Limitations of model-centric interpretations Standard analyses of gravitational wave backgrounds focus on specific production scenarios, such as phase transitions, compact object populations, or inflationary tensor modes. While valuable, such approaches implicitly assume that the spectral tilt of the background is a dynamical output of particular models.

This assumption overlooks a deeper question: whether the causal structure of cosmological time itself imposes model-independent constraints on the allowed form of the gravitational wave spectrum.

Reframing the question The question addressed in this work is therefore not which mechanism generates a given spectrum, but:

> Does a causal cosmological spacetime endowed with a rigid structural notion of time necessarily constrain the sign of the logarithmic slope of the gravitational wave energy spectrum?

We show that the answer is affirmative.

Gravitational wave spectra as causal records The present-day gravitational wave spectrum at frequency f receives contributions from a range of emission epochs, each mapped to f through cosmological redshift. This mapping is monotonic and causal: higher observed frequencies correspond, under minimal assumptions, to earlier emission times.

Consequently, the accumulated energy density per logarithmic frequency interval encodes information about the temporal ordering and accumulation of gravitational wave production across cosmic history.

A2 — Definition of the Gravitational Wave Energy Spectrum

We now introduce a rigorous and model-independent definition of the gravitational wave (GW) energy spectrum. This definition is required before any statement concerning spectral slopes or causal constraints can be made.

Energy density of gravitational waves Gravitational waves are treated here as perturbations h_{ij} of a background metric, propagating on cosmological scales. Their contribution to the total energy density of the Universe can be defined, in an averaged sense, through an effective stress-energy tensor.

Let ρ_{GW} denote the present-day energy density carried by gravitational waves, averaged over several wavelengths and periods so that a coarse-grained description is valid.

Spectral energy density The gravitational wave energy spectrum is defined as the energy density per logarithmic frequency interval,

$$\Omega_{\mathrm{GW}}(f) \equiv \frac{1}{\rho_c}\frac{d\rho_{\mathrm{GW}}}{d\ln f},$$

where $\rho_c = 3H_0^2/(8\pi G)$ is the critical density of the Universe and f is the frequency observed today.

This definition is purely kinematical and does not depend on the mechanism by which the gravitational waves were generated.

Observer independence The quantity $\Omega_{\mathrm{GW}}(f)$ is invariant under changes of observer within the cosmological rest frame. It is defined with respect to comoving observers and depends only on the global spacetime geometry and the history of gravitational wave production.

This invariance ensures that any constraint derived on the functional form of $\Omega_{\mathrm{GW}}(f)$ has physical, not merely coordinate, significance.

Separation between generation and propagation It is crucial to separate:

- the generation of gravitational waves at some emission epoch;
- their propagation through an expanding spacetime;
- the accumulation of contributions from different epochs.

The present analysis does not assume any specific generation mechanism. All generation details are encoded in a source term that is subsequently mapped to the observed spectrum through causal propagation.

Spectrum as a functional of spacetime history Formally, the observed spectrum can be written as a functional

$$\Omega_{\mathrm{GW}}(f) = \mathcal{F}[S(t), a(t)],$$

where $S(t)$ represents the cumulative gravitational wave source activity as a function of cosmic time and $a(t)$ is the cosmological scale factor.

The precise form of $\mathcal{F}$ is model-dependent, but its causal structure is not.

Minimal assumptions The results derived in this article rely only on the following assumptions:

- gravitational waves propagate causally at the speed of light;
- cosmological expansion is described by a monotonic scale factor $a(t)$;
- the stochastic background is formed by accumulation over cosmic time.

No assumption is made regarding inflation, phase transitions, or astrophysical sources.

Conclusion of the section We have defined the gravitational wave energy spectrum in a form that is independent of specific cosmological models. This definition isolates the structural role of causality and temporal ordering.

In the next section, we establish the causal mapping between observed frequency and emission epoch, which is the key ingredient for deriving structural constraints on the spectral slope.

A3 — Causality, Horizon Crossing, and Frequency Mapping

We now establish the causal mapping between the observed gravitational wave frequency and the cosmological time at which the corresponding modes were generated or last processed by the background spacetime. This mapping constitutes the key structural ingredient underlying all subsequent spectral constraints.

Redshift of gravitational wave frequencies Gravitational waves propagate freely once produced, and their physical wavelength λ_{phys} scales with the cosmological scale factor $a(t)$,

$$\lambda_{\text{phys}}(t) = a(t)\,\lambda_{\text{com}},$$

where λ_{com} is the comoving wavelength.

Accordingly, the frequency observed today is related to the frequency at an earlier cosmic time t by

$$f_0 = \frac{a(t)}{a_0}\,f(t),$$

with a_0 denoting the present-day scale factor.

This relation is purely kinematical and follows directly from causal propagation in an expanding spacetime.

Frequency as a temporal label Under minimal assumptions, the mapping between the observed frequency f_0 and the emission or processing time t is monotonic. Higher observed frequencies correspond to modes that were generated or last affected at earlier cosmic times.

This monotonicity is a direct consequence of:

- causal propagation at finite speed;
- monotonic expansion of the Universe;
- absence of frequency mixing during propagation.

As a result, frequency can be treated as a proxy for cosmological temporal ordering.

Horizon crossing and causal activation In many cosmological settings, gravitational wave modes are generated or become dynamically relevant when their physical wavelength is comparable to the Hubble radius,

$$\lambda_{\text{phys}}(t_*) \sim H^{-1}(t_*).$$

While the present analysis does not assume any specific horizon-crossing mechanism, the existence of a causal scale implies that modes of different frequencies are associated with different epochs of causal activation.

This association is structural and does not depend on the detailed source dynamics.

Causal ordering of spectral contributions Let $\Omega_{\rm GW}(f)$ be written schematically as an integral over cosmic time,

$$\Omega_{\rm GW}(f) = \int dt\, K(f,t)\, S(t),$$

where $S(t)$ represents the cumulative source activity and $K(f,t)$ is a causal kernel encoding redshift and propagation effects.

Causality requires that $K(f,t)$ have support only when the mode associated with frequency f is causally connected to the background at time t. This enforces an ordering: contributions to a given f originate from a restricted and ordered temporal domain.

Absence of temporal reordering Because gravitational waves propagate freely and do not interact significantly after production, there is no mechanism by which later events can contribute to higher frequencies than earlier events in a manner that violates causal ordering.

Therefore, the mapping between frequency and cosmic time preserves the temporal order of contributions.

Structural consequence The monotonic relation between f and t implies that the logarithmic derivative

$$\frac{d\ln \Omega_{\rm GW}}{d\ln f}$$

encodes information about how gravitational wave energy accumulates over cosmological time.

Any constraint on the sign of this derivative therefore constitutes a constraint on the temporal structure of the underlying cosmology.

Conclusion of the section We have shown that, under minimal and model-independent assumptions, the observed gravitational wave frequency acts as a causal label of emission or processing time. This establishes the foundation for translating statements about temporal structure into constraints on the gravitational wave spectrum.

B1 — Rigid Structural Time: Definition and Properties

We now introduce the concept of rigid structural time, which plays a central role in the derivation of constraints on the gravitational wave spectrum. This notion concerns the ordering and accumulation of physical processes in cosmology, independently of any specific dynamical realization.

Time as structural ordering In relativistic cosmology, time is not merely a coordinate parameter but an ordering structure that defines causal precedence between events. Any physical process must respect this ordering: causes precede effects, and information propagates forward along causal curves.

A structural notion of time refers to this ordering property abstracted from the detailed dynamics of fields or matter content.

Definition of rigid structural time A cosmological framework is said to possess rigid structural time if the causal ordering of events is fixed and cannot be dynamically rescaled, stretched, or reparameterized by the evolution of the background itself.

Formally, this means that there exists a preferred causal ordering such that:

- the temporal sequence of events is invariant under changes of physical scale;
- equal intervals of structural time correspond to equal causal weight in the accumulation of physical processes;
- no epoch contributes disproportionately due to dynamical time dilation.

This definition does not require a preferred clock, but it excludes dynamical mechanisms that distort the causal weight of different epochs.

Contrast with dynamically dilated time In some cosmological models, the background evolution induces an effective stretching of time, such that large intervals of structural ordering are mapped into narrow ranges of physical evolution. In such cases, early epochs dominate the cumulative effect of processes that are otherwise uniformly distributed in time.

Rigid structural time explicitly excludes this behavior. Each causal interval contributes comparably to cumulative observables.

Consequences for causal accumulation Consider an observable O that accumulates contributions over cosmic time, such as the energy density of a stochastic gravitational wave background. In a rigid structural time framework, the incremental contribution dO associated with a causal interval dt satisfies

$$dO \propto dt,$$

up to slowly varying factors that do not induce exponential or power-law distortions.

This proportionality expresses the absence of temporal bias in the accumulation process.

Monotonicity of cumulative contributions Because causal ordering is rigid, later epochs cannot suppress or outweigh the contributions of earlier epochs in a way that reverses monotonic trends. Cumulative quantities therefore evolve monotonically with structural time.

This monotonicity is a direct consequence of rigidity and does not depend on the nature of the sources.

Structural stability Rigid structural time is compatible with dynamical stability. Because no epoch dominates disproportionately, small perturbations in source activity do not lead to runaway amplification in cumulative observables.

This stability property will be essential in establishing bounds on spectral slopes.

Conclusion of the section We have defined rigid structural time as a property of cosmological frameworks in which causal ordering is fixed and free from dynamical temporal distortion. This property enforces uniform causal accumulation and monotonic behavior of integrated observables.

B2 — Structural Theorem for the Gravitational Wave Spectrum

We now prove the central structural result of this article. The proof relies exclusively on causal ordering, rigid structural time, and stability, and does not assume any specific gravitational wave generation mechanism.

Statement of the theorem In any cosmological framework endowed with rigid structural time and causal propagation of gravitational waves, the logarithmic slope of the gravitational wave energy spectrum satisfies

$$\frac{d \ln \Omega_{\mathrm{GW}}}{d \ln f} \geq 0.$$

Hypotheses The proof rests on the following minimal assumptions:

- gravitational waves propagate causally at the speed of light;
- the cosmological expansion is monotonic;
- the mapping between observed frequency f and emission or processing time t is monotonic;
- time possesses rigid structural ordering;
- the accumulation of gravitational wave energy is dynamically stable.

No assumptions are made regarding inflation, phase transitions, or astrophysical sources.

Spectral energy as a cumulative functional From the causal mapping established previously, the gravitational wave spectrum can be expressed as a cumulative integral over cosmic time,

$$\Omega_{\mathrm{GW}}(f) = \int_{t_{\min}(f)}^{t_{\max}(f)} A(t)\, dt,$$

where $A(t)$ denotes the effective rate of gravitational wave energy contribution per unit structural time, and the limits of integration are fixed by causal accessibility.

Rigid structural time implies that $A(t)$ does not contain exponential or power-law distortions that privilege specific epochs.

Frequency–time monotonicity Because the mapping between f and t is monotonic, increasing the observed frequency corresponds to restricting the integral to earlier or equal structural times,

$$f_2 > f_1 \;\Rightarrow\; [t_{\min}(f_2), t_{\max}(f_2)] \subseteq [t_{\min}(f_1), t_{\max}(f_1)].$$

Thus, higher frequencies sample subsets of the causal history contributing to lower frequencies.

Monotonic accumulation In a rigid structural time framework, the integrand $A(t)$ is non-negative and does not exhibit pathological temporal weighting. Therefore, restricting the integration domain cannot increase the cumulative contribution relative to a less restricted domain in a way that reverses monotonic trends.

Formally, this implies

$$\frac{d\Omega_{\mathrm{GW}}}{df} \geq 0$$

up to logarithmic normalization.

Logarithmic derivative Taking the logarithmic derivative with respect to frequency yields

$$\frac{d \ln \Omega_{\mathrm{GW}}}{d \ln f} = \frac{f}{\Omega_{\mathrm{GW}}} \frac{d\Omega_{\mathrm{GW}}}{df} \geq 0,$$

since both Ω_{GW} and $d\Omega_{\mathrm{GW}}/df$ are non-negative.

This establishes the claimed inequality.

Role of stability The stability assumption ensures that small fluctuations in $A(t)$ do not introduce oscillatory or singular behavior capable of invalidating the monotonicity argument. Without stability, pathological counterexamples could be constructed, but such models would be physically inconsistent.

Independence from generation mechanisms The proof makes no reference to how gravitational waves are generated. It applies equally to primordial, astrophysical, or exotic sources, provided they respect causality and rigid structural time.

Conclusion of the section We have shown that rigid structural time enforces a monotonic accumulation of gravitational wave energy with frequency, leading inevitably to the inequality

$$\frac{d \ln \Omega_{\mathrm{GW}}}{d \ln f} \geq 0.$$

Any violation of this inequality signals a departure from rigid structural time and therefore implies a fundamentally different temporal structure.

B3 — Robustness of the Spectral Inequality

We now demonstrate that the inequality

$$\frac{d \ln \Omega_{\mathrm{GW}}}{d \ln f} \geq 0$$

is robust under broad changes in cosmological context, source composition, and background evolution, provided the defining properties of rigid structural time and causal propagation are preserved.

Independence from source composition The derivation of the inequality makes no reference to the physical nature of gravitational wave sources. In particular, it does not assume:

- primordial vacuum fluctuations;
- phase transitions;
- compact object populations;
- astrophysical foregrounds.

All source details are encoded in the non-negative accumulation rate $A(t)$. As long as $A(t) \geq 0$ and respects causal ordering, the monotonicity argument remains valid.

Equation-of-state insensitivity The expansion history of the Universe enters the analysis only through the monotonicity of the scale factor $a(t)$. The specific equation of state of the dominant energy component does not alter the causal mapping between frequency and time.

Consequently, the inequality holds during radiation domination, matter domination, or mixed eras, provided no epoch introduces dynamical temporal distortion.

Invariance under time reparameterization One might attempt to evade the inequality by redefining the time parameter. However, rigid structural time is defined precisely to be invariant under such reparameterizations.

Any time transformation that preserves causal ordering and equal causal weight per interval leaves the accumulation structure unchanged. Transformations that violate this invariance correspond to abandoning rigid structural time and thus fall outside the theorem's domain of applicability.

Absence of fine-tuning loopholes Violations of the inequality would require finely tuned cancellations between contributions from different epochs. Such cancellations would necessitate negative contributions to $A(t)$ or oscillatory temporal kernels, both of which violate stability and physical consistency.

Therefore, the inequality is not sensitive to fine-tuning of source histories.

Stability against perturbations Consider small perturbations $\delta A(t)$ to the accumulation rate. Stability requires that these perturbations do not qualitatively alter the monotonic behavior of $\Omega_{\mathrm{GW}}(f)$.

Because the inequality is derived from ordering and positivity rather than precise functional form, it is preserved under arbitrarily small perturbations.

No dependence on amplitude normalization The proof constrains only the sign of the spectral slope. It is completely independent of the overall amplitude of the gravitational wave background.

Thus, uncertainties in normalization, efficiency, or transfer functions cannot invalidate the result.

Conclusion of the section We conclude that the inequality

$$\frac{d \ln \Omega_{\mathrm{GW}}}{d \ln f} \geq 0$$

is a robust structural consequence of rigid causal time and does not depend on cosmological details, source modeling, or amplitude normalization.

Any observed violation of this inequality would therefore imply a breakdown of rigid structural time rather than a failure of a specific generation model.

C1 — Slow-Roll Inflation and the Origin of Red Tilt

We now examine slow-roll inflationary cosmologies through the same structural lens developed in the previous sections. The purpose is not to assess their phenomenological success, but to classify their gravitational wave spectra in terms of temporal structure.

Inflation as dynamical time dilation In slow-roll inflation, the cosmological background undergoes quasi-exponential expansion. While often described in terms of accelerated scale-factor growth, this process has a direct and unavoidable consequence for temporal structure: large intervals of causal ordering are compressed into narrow ranges of physical time.

This compression constitutes a form of dynamical time dilation, in which early epochs acquire disproportionate causal weight relative to later ones.

Breakdown of rigid structural time Because slow-roll inflation dynamically stretches the scale factor, equal intervals of structural ordering do not correspond to equal contributions to cumulative observables.

Early horizon-exit events dominate the accumulation of gravitational wave energy, while later contributions are exponentially suppressed. This behavior directly violates the defining property of rigid structural time introduced in Section B1.

Spectral consequence of temporal stretching The dominance of early epochs implies that lower-frequency modes, which correspond to later horizon exit, receive systematically less cumulative contribution than higher-frequency modes.

As a result, the gravitational wave energy spectrum acquires a negative logarithmic slope,

$$\frac{d \ln \Omega_{\mathrm{GW}}}{d \ln f} < 0,$$

commonly referred to as a red tilt.

Model independence of the red tilt The argument presented here does not rely on:

- specific slow-roll parameters;
- the shape of the inflaton potential;
- reheating details;
- normalization of perturbations.

Any cosmological framework in which slow-roll conditions induce temporal compression necessarily leads to a red-tilted gravitational wave spectrum.

Structural classification From the perspective developed in this article, slow-roll inflationary cosmologies belong to the class of models with dynamically distorted time. Their gravitational wave spectra reflect this distortion through a negative spectral slope.

This classification is structural rather than phenomenological.

Conclusion of the section We conclude that the red tilt of the gravitational wave spectrum in slow-roll inflation is not an incidental dynamical feature, but a direct consequence of temporal stretching intrinsic to the inflationary framework.

This places slow-roll inflation in a class of cosmologies fundamentally distinct from those endowed with rigid structural time.

C2 — Structural-Time Cosmologies: Flat or Blue Spectra

We now analyze cosmological frameworks endowed with rigid structural time and derive the corresponding implications for the gravitational wave energy spectrum. This section completes the structural contrast initiated in Section C1.

Absence of temporal distortion In cosmologies with rigid structural time, the expansion history does not induce dynamical time dilation. Equal intervals of causal ordering correspond to equal causal weight in the accumulation of physical processes.

As a consequence, no epoch is exponentially privileged or suppressed in its contribution to cumulative observables such as the gravitational wave background.

Uniform causal accumulation Let $A(t)$ denote the effective rate of gravitational wave energy contribution per unit structural time, as defined in Section B2. In rigid-time cosmologies, $A(t)$ may vary slowly but does not exhibit exponential or power-law distortions associated with background-induced temporal stretching.

The cumulative spectrum therefore reflects a uniform or increasing accumulation of contributions as one moves toward earlier causal times.

Spectral implications Because higher observed frequencies correspond to earlier emission or processing times (Section A3), uniform causal accumulation implies that restricting the integration domain to higher frequencies does not reduce the cumulative energy density.

Consequently, the gravitational wave spectrum satisfies

$$\frac{d \ln \Omega_{\rm GW}}{d \ln f} \geq 0.$$

Equality corresponds to a flat spectrum, while strict inequality corresponds to a blue-tilted spectrum.

Model independence This result does not depend on:

- the detailed expansion law of the scale factor;
- the equation of state of the dominant energy component;
- specific gravitational wave generation mechanisms;
- normalization or amplitude of the spectrum.

The sign of the spectral slope is fixed by temporal structure alone.

Structural interpretation Flat or blue gravitational wave spectra arise naturally in cosmologies where time functions as a rigid ordering parameter rather than a dynamically distorted quantity.

In this sense, the spectral slope serves as a direct signature of the underlying temporal structure of the cosmological background.

Absence of intermediate behavior It is important to emphasize that intermediate behavior — a red tilt emerging without temporal distortion — is not structurally admissible within this framework.

Any sustained negative slope necessarily signals the presence of dynamical time dilation, while non-negative slopes indicate rigid structural time.

Conclusion of the section We conclude that cosmologies endowed with rigid structural time generically produce flat or blue-tilted gravitational wave spectra.

This result follows directly from uniform causal accumulation and stands in structural contrast to the red tilt characteristic of slow-roll inflation.

C3 — Mutual Exclusivity of Temporal Structures

We now demonstrate that cosmological frameworks endowed with rigid structural time and those exhibiting dynamical temporal distortion constitute mutually exclusive classes. This exclusivity is structural and does not depend on phenomenological details or model-specific assumptions.

Definition of the two temporal classes From the preceding sections, two distinct temporal structures have been identified:

- rigid structural time, characterized by uniform causal ordering and absence of dynamical time dilation;
- dynamically distorted time, characterized by background-induced temporal stretching that privileges specific epochs.

Each class imposes distinct and incompatible constraints on the accumulation of gravitational wave energy.

Incompatibility of accumulation laws In rigid structural time, cumulative observables grow monotonically with causal ordering, with no epoch dominating due to background evolution. In dynamically distorted time, early epochs dominate accumulation due to temporal stretching.

These two accumulation laws cannot be simultaneously satisfied. Any attempt to interpolate between them necessarily introduces:

- temporal distortion sufficient to induce a red tilt, or
- restoration of rigidity sufficient to eliminate it.

No intermediate accumulation behavior is structurally stable.

Spectral consequences Because the sign of the logarithmic slope of the gravitational wave spectrum is directly tied to the accumulation law, it follows that

$$\frac{d\ln\Omega_{\mathrm{GW}}}{d\ln f} < 0 \iff \text{dynamically distorted time,}$$

and

$$\frac{d\ln\Omega_{\mathrm{GW}}}{d\ln f} \geq 0 \iff \text{rigid structural time.}$$

These conditions are mutually exclusive.

Absence of continuous deformation One might attempt to evade this exclusivity by proposing a continuous deformation between the two temporal structures. However, any deformation that preserves causal ordering while introducing temporal bias necessarily breaks rigidity and induces a red tilt.

Conversely, any deformation that restores rigidity removes the red tilt.

Thus, no continuous path exists that preserves both properties simultaneously.

Structural, not phenomenological, dichotomy The dichotomy identified here is not a statement about specific models, but about classes of cosmological temporal structure. It persists independently of:

- detailed inflationary dynamics;
- reheating scenarios;
- late-time cosmological evolution.

It is therefore a structural classification, not a phenomenological one.

Conclusion of the section We conclude that rigid structural time and dynamically distorted time define two mutually exclusive cosmological classes. Their gravitational wave spectra encode this exclusivity through the sign of the logarithmic spectral slope.

This result establishes a sharp theoretical bifurcation that can be probed observationally.

D1 — Gravitational Wave Observatories as Structural Tests

We now examine the role of gravitational wave observatories in light of the structural results derived in this article. The central point is a change in logical status: such observatories do not merely constrain parameters within a given theory, but can adjudicate between fundamentally different temporal structures.

From amplitude measurements to structural diagnostics Traditional analyses of stochastic gravitational wave backgrounds emphasize amplitude sensitivity and parameter estimation. In this paradigm, observational data are used to infer the values of free parameters within a preselected theoretical framework.

The results of this article imply a different role. The sign of the logarithmic spectral slope,

$$\frac{d \ln \Omega_{\rm GW}}{d \ln f},$$

is not a continuous parameter to be fitted, but a discrete structural indicator.

Frequency windows as temporal windows Because observed frequency maps monotonically to emission or processing time, each frequency band probed by an observatory corresponds to a specific interval of cosmological history.

Measuring the spectral slope over a finite frequency window therefore amounts to sampling the causal accumulation of gravitational wave energy over a defined temporal interval.

Minimal observational requirement Importantly, the structural test proposed here does not require:

- precise amplitude normalization;
- detailed source modeling;
- full spectral reconstruction.

Determining the sign of the slope over a sufficiently broad frequency interval is sufficient to discriminate between the two temporal classes identified in Section C3.

Conclusion of the section We conclude that gravitational wave observatories can function as structural tests of cosmological time, rather than merely as instruments for parameter estimation.

By determining the sign of the gravitational wave spectral slope, such observatories can adjudicate between mutually exclusive classes of cosmological frameworks.

D2 — Theoretical Adjudication, Not Parameter Estimation

The structural results derived in this work imply a fundamental shift in the logical role of gravitational wave observations. Rather than serving solely as tools for parameter estimation within preselected cosmological models, such observations acquire the capacity to adjudicate between mutually exclusive theoretical frameworks.

Limits of parameter-centric methodology Parameter estimation presupposes that competing theories differ only by the values of continuous parameters within a shared structural framework. In such cases, observational data refine constraints without challenging the underlying theoretical architecture.

The results of this article demonstrate that this presupposition does not hold for cosmological temporal structure. Rigid structural time and dynamically distorted time represent distinct and incompatible architectures, not adjacent regions of a single parameter space.

Discrete structural outcomes The sign of the logarithmic slope of the gravitational wave spectrum constitutes a discrete structural outcome. It does not interpolate smoothly between positive and negative values without crossing a qualitative boundary associated with temporal distortion.

As such, the measurement of this sign functions as a binary test rather than a continuous constraint.

Implications for observational strategy From a structural perspective, the primary observational objective is not the precise reconstruction of $\Omega_{\mathrm{GW}}(f)$, but the robust determination of the sign of its slope over a sufficiently broad frequency interval.

This reframes the design and interpretation of observational programs: breadth of frequency coverage becomes more critical than fine-grained amplitude sensitivity.

Conclusion of the section We conclude that gravitational wave observations enable a form of theoretical adjudication that transcends parameter estimation. By determining the sign of the gravitational wave spectral slope, such observations can decisively discriminate between cosmological frameworks characterized by fundamentally different temporal structures.

D3 — Scientific Scope and Limitations

This article is deliberately restricted in scope. Its purpose is to establish structural constraints on the gravitational wave spectrum arising from the causal properties of cosmological time, rather than to provide phenomenological models or observational predictions.

Structural level of the analysis All results derived in this work operate at the level of causal and temporal structure. They follow from minimal assumptions about relativistic propagation, monotonic cosmological expansion, and the ordering of events in time.

No specific assumptions are made regarding:

- gravitational wave source populations;
- detailed inflationary dynamics;
- reheating or late-time astrophysics;
- detector noise or sensitivity curves.

No claims of detection or prediction This article does not claim that any particular gravitational wave spectrum has been observed, nor does it predict the outcome of future observations.

The results are conditional statements: they specify what must hold if a cosmological framework satisfies the defining properties of rigid structural time.

No quantitative forecasts Because the structural test proposed here depends only on the sign of the spectral slope, no quantitative forecast of amplitudes, signal-to-noise ratios, or confidence levels is provided.

Such forecasts require additional assumptions beyond the scope of this work.

Relation to model building The conclusions of this article constrain model building but do not replace it. Any phenomenological or dynamical model must be assessed for compatibility with the structural constraints identified here.

Models that violate these constraints are structurally inconsistent, regardless of their phenomenological appeal.

Limitations The limitations of the present analysis are explicit:

- it does not address mixed or transitional temporal structures;
- it does not consider non-causal or acausal models;
- it does not explore stochastic time reparameterizations.

These limitations reflect the intention to focus on clear, well-defined structural classes.

Conclusion of the section The results of this work should be interpreted strictly within their intended scope: as structural constraints on admissible gravitational wave spectra in causal cosmologies.

Any extension beyond this scope requires separate analysis and additional assumptions.

D4 — Final Conclusion

In this work, we have demonstrated that the energy spectrum of a stochastic gravitational wave background carries direct structural information about the nature of cosmological time. In particular, we have shown that the sign of the logarithmic spectral slope is not an adjustable phenomenological detail, but a necessary consequence of the underlying temporal structure.

Central result The central result can be summarized as follows: in cosmologies endowed with rigid structural time, the causal accumulation of gravitational wave energy necessarily enforces

$$\frac{d \ln \Omega_{\mathrm{GW}}}{d \ln f} \geq 0,$$

whereas cosmologies that exhibit dynamical time dilation produce negative slopes.

This dichotomy is structural and does not depend on specific models or particular generation mechanisms.

Cosmological classification On the basis of this result, the gravitational wave spectrum acquires the status of a fundamental cosmological classifier. Measuring the sign of its slope is equivalent to diagnosing the temporal structure class of the Universe.

This perspective shifts the focus from parametric fitting to structural discrimination.

Conceptual consequences The conceptual implications are broad. Time ceases to be merely an evolution parameter or a convenient coordinate and instead assumes an observable structural role.

In this sense, gravitational waves act as direct probes of the temporal architecture of cosmological spacetime.

Observational outlook Present and future gravitational wave observatories, in principle, possess the capability to perform this structural test. Robust determination of the sign of the spectral slope over an appropriate frequency range is sufficient to discriminate between the identified temporal classes.

Such a test does not require full spectral reconstruction or fine parameter tuning.

Closing remarks We conclude that the gravitational wave spectrum offers a privileged window into the structure of cosmological time. By revealing whether the Universe's time is rigidly structural or dynamically distorted, it provides a deep and observable criterion for the classification of cosmological frameworks.

This result establishes a direct connection between causality, temporal structure, and observational data, opening a new avenue for the fundamental investigation of time in cosmology.

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Structural Time Theory as an Eliminative Program in Fundamental Physics

A1 — Introduction: What This Article Is (and Is Not)

This article is not a proposal of a new fundamental interaction, nor the announcement of a unified theory of nature. It does not introduce new particles, fields, symmetries, or dynamical equations. It does not claim explanatory completeness, predictive closure, or empirical finality.

Instead, this work has a strictly epistemological function.

Purpose of the Article

The purpose of this article is to classify the Structural Time Theory (TFT) as a specific type of theoretical enterprise in fundamental physics: an *eliminative program.* Its role is not to explain physical phenomena, but to constrain, select, and exclude entire classes of theoretical interpretations based on structural consistency.

In this sense, TFT operates at a level logically prior to model construction, parameter fitting, and phenomenological explanation.

What This Article Is Not

To avoid category errors, we state explicitly what this article does not claim:

- It is not a Theory of Everything.
- It is not a complete description of all physical interactions.
- It is not a replacement for dynamical model-building.
- It does not aim to explain all observed phenomena.

Criticisms formulated in these terms therefore miss the intended domain of application.

Structural Versus Explanatory Theories

Fundamental physics hosts different kinds of theoretical frameworks. Some are explanatory and dynamical: they posit entities and laws to account for observed phenomena. Others are structural: they impose constraints on what kinds of descriptions are admissible, independently of specific dynamics.

Structural theories do not compete with explanatory theories; they delimit the space within which explanation is meaningful.

The Structural Time Theory belongs to the latter category.

The Core Thesis

The core thesis defended in this article is the following:

> *The Structural Time Theory functions as a theory of limits, selection, and exclusion. Its scientific role is to eliminate structurally inconsistent interpretations of physical phenomena, rather than to provide a complete dynamical account of nature.*

This thesis places TFT in a distinct epistemological class, with its own criteria of validity and evaluation.

Motivation: Recurrent Misreadings

Many objections raised against TFT — such as claims that it is incomplete, not explanatory enough, or not unified — arise from applying evaluation criteria appropriate to dynamical theories to a structural framework.

This article addresses this mismatch directly by clarifying the proper epistemological status of TFT.

Organization of the Article

Section A2 analyzes the problem of misclassification in fundamental physics and the recurrent category errors it generates. Block II positions TFT as a limit theory. Block III develops its role as a selection theory. Block IV formalizes its function as an exclusion theory. Block V addresses standard criticisms explicitly. Block VI situates TFT as an eliminative research program within the broader landscape of fundamental physics.

Scope

This article does not engage with new empirical data, experimental proposals, or model-specific predictions. Its contribution is conceptual and structural, and its success should be evaluated accordingly.

A2 — The Problem of Misclassification in Fundamental Physics

A significant fraction of conceptual disputes in fundamental physics arise not from disagreement over empirical facts or mathematical results, but from systematic misclassification of theories according to inappropriate epistemological criteria.

This section analyzes the nature of such misclassifications and clarifies why they recur, particularly in discussions involving structural frameworks such as the Structural Time Theory.

Category Errors in Theory Assessment

A category error occurs when an object is evaluated according to criteria that apply to a fundamentally different class of objects. In the context of physical theories, this typically manifests when a structural or limiting framework is judged using standards appropriate to dynamical, explanatory, or model-building theories.

Examples include demanding local mechanisms from global constraints, or requiring predictive completeness from theories whose function is to delimit admissible descriptions.

Historical Precedents

The history of physics provides numerous examples of such misclassification. General relativity was initially criticized for failing to explain the internal structure of matter. Quantum mechanics was faulted for not incorporating gravitation. Symmetry principles have been dismissed as "non-explanatory" because they do not generate dynamics on their own.

In each case, the criticism failed because it targeted a theory for not doing what it was never intended to do.

Structural Theories and Explanatory Demands

Structural theories impose constraints on possible descriptions rather than generating concrete models. Their success is measured by internal consistency, logical necessity, and eliminative power, not by the breadth of phenomena they directly explain.

Demanding that a structural theory "explain everything" conflates two distinct epistemic roles: explanation and admissibility.

The Cost of Misclassification

When structural theories are misclassified as incomplete explanatory theories, two pathologies arise. First, legitimate structural results are dismissed as "vague" or "non-predictive." Second, inconsistent models survive scrutiny because they are evaluated only on phenomenological grounds, rather than on structural coherence.

Misclassification thus distorts the theoretical landscape.

The Case of Time in Fundamental Physics

Time-related frameworks are especially prone to misclassification. Because time enters both as a coordinate and as a structural ordering principle, theories addressing its role are often expected to behave like dynamical field theories.

This expectation is misplaced. Constraints on temporal structure are logically prior to, and independent of, specific dynamical realizations.

Structural Evaluation Criteria

Structural theories must be evaluated according to criteria appropriate to their epistemological role, including:

- internal logical consistency;
- necessity of conclusions given stated assumptions;
- ability to eliminate inconsistent interpretations;
- independence from model-specific tuning.

Failure to apply these criteria leads to systematic misunderstanding.

Conclusion of the Section

The recurring misclassification of structural theories as deficient explanatory models reflects a broader epistemological confusion in fundamental physics. Clarifying this distinction is essential before meaningful assessment of the Structural Time Theory can take place.

In the next section, we position TFT explicitly within the class of structural limit theories and clarify the sense in which its conclusions function as boundaries rather than explanations.

B1 — Structural Theories Versus Dynamical Theories

To correctly situate the Structural Time Theory within fundamental physics, it is necessary to draw a clear distinction between structural theories and dynamical theories. These two categories serve different epistemological functions and must not be conflated.

Dynamical Theories

Dynamical theories are characterized by the introduction of degrees of freedom and equations of motion governing their evolution. Their primary aim is to explain how physical systems change in time and to generate quantitative predictions for observable phenomena.

Typical features of dynamical theories include:

- specification of fields, particles, or variables;
- governing equations or action principles;
- initial or boundary conditions;
- calculable time evolution.

Examples include quantum field theories, classical mechanics, and specific cosmological models.

Structural Theories

Structural theories, by contrast, do not introduce new dynamical variables or evolution equations. Instead, they impose constraints on the admissible form of dynamical descriptions by identifying necessary conditions for consistency, coherence, or interpretability.

Structural theories operate at a meta-level: they delimit the space within which dynamical theories may legitimately operate.

Time as Structure Versus Time as Dynamics

In dynamical theories, time typically appears as a parameter with respect to which change is described, or as a variable associated with a specific degree of freedom. In structural theories, time is treated as an ordering principle that constrains causal relations and accumulation processes.

The Structural Time Theory belongs explicitly to the latter category. It does not endow time with dynamics; it restricts how time may consistently function within physical descriptions.

Non-Competition Between the Two Classes

Structural and dynamical theories do not compete in the same explanatory space. A structural constraint cannot be falsified by the success of a particular dynamical model, just as a dynamical model cannot override a structural inconsistency.

This non-competition is often overlooked, leading to misplaced demands that structural theories generate dynamical predictions.

Evaluation Criteria

Because of their different roles, the two classes of theories must be evaluated according to different criteria. Dynamical theories are assessed by predictive accuracy, empirical adequacy, and computational power. Structural theories are assessed by necessity, generality, and eliminative strength.

Applying the wrong criteria leads to systematic misunderstanding.

Implications for TFT

Understanding TFT as a structural theory immediately resolves many standard objections. TFT is not deficient for failing to produce detailed dynamics; it would be incoherent if it attempted to do so.

Its scientific value lies in restricting the set of admissible temporal structures underlying any dynamical model.

Conclusion of the Section

The distinction between structural and dynamical theories is essential for properly situating the Structural Time Theory. TFT does not compete with dynamical models but constrains the conceptual space in which such models may be constructed.

In the next section, we formalize this role by defining TFT explicitly as a limit theory.

B2 — TFT as a Limit Theory

We now formalize the status of the Structural Time Theory as a *limit theory.* This classification is essential for understanding both its scientific function and the appropriate criteria for its evaluation.

Definition of a Limit Theory

A limit theory is not defined by the introduction of new entities or dynamics, but by the identification of boundary conditions that any admissible theoretical description must satisfy.

Limit theories specify:

- consistency boundaries;
- admissible structural regimes;
- forbidden conceptual configurations.

They operate by exclusion rather than construction.

Examples of Limit Theories in Physics

Physics already relies on several limit theories whose role is widely accepted. Causality in relativity restricts signal propagation. Symmetry principles constrain admissible interactions. Renormalizability limits acceptable quantum field theories.

None of these frameworks explain phenomena directly; they define the conditions under which explanation is possible.

Temporal Structure as a Limiting Principle

The Structural Time Theory identifies the temporal structure of physical descriptions as a limiting principle. It constrains how time may consistently enter the formulation of dynamical models, particularly with respect to causal ordering, accumulation, and global coherence.

TFT does not propose a specific temporal dynamics. It restricts the set of temporal assumptions that can be coherently maintained.

What TFT Limits

Specifically, TFT imposes limits on:

- the admissibility of isolated subsystems in cosmology;
- the treatment of time as a freely adjustable parameter;
- the interpretation of local dynamics without global causal accounting;
- the attribution of observed effects to purely local mechanisms when structural constraints are violated.

These limits are structural, not empirical.

Incompleteness as a Category Error

Because limit theories do not aim to describe the full content of physical reality, the notion of incompleteness does not apply to them in the same sense as it does to dynamical theories.

A limit theory is complete if it fully characterizes the boundaries it is meant to define. By this criterion, TFT is to be assessed by the clarity, necessity, and consistency of its eliminative constraints.

Relation to Dynamical Completion

Limit theories are compatible with multiple dynamical completions. TFT does not privilege any specific model; it constrains all of them equally.

Any dynamical theory that violates TFT constraints is structurally inconsistent, regardless of its phenomenological success.

Conclusion of the Section

We conclude that the Structural Time Theory is properly classified as a limit theory. Its scientific role is to define structural boundaries on admissible temporal descriptions in fundamental physics.

This classification resolves objections based on demands for completeness or explanatory closure, which are inappropriate for theories of this kind.

C1 — Selection Without Construction

A central feature of the Structural Time Theory is that it performs selection without construction. This characteristic is often misunderstood and misinterpreted as a weakness, when in fact it reflects a distinct and legitimate scientific function.

Construction Is Not a Prerequisite for Selection

In fundamental physics, selection does not require the explicit construction of models. A framework may eliminate or admit entire classes of theories based on structural consistency alone, without specifying the internal dynamics of the remaining candidates.

Selection operates at the level of admissibility, not realization.

Structural Selection Criteria

The Structural Time Theory selects admissible theoretical frameworks according to criteria that are independent of specific dynamical content, including:

- global causal coherence;
- consistency of temporal ordering;
- compatibility between local dynamics and global accumulation;
- stability under cosmological embedding.

Frameworks that violate these criteria are excluded, regardless of their phenomenological detail.

Non-Local Selection

Selection performed by TFT is inherently non-local. It evaluates whether local assumptions remain coherent when embedded in a global cosmological structure. This distinguishes structural selection from empirical falsification, which typically targets local predictions.

Non-local selection is especially relevant in cosmology, where isolation assumptions frequently fail.

Analogy with Established Practices

The legitimacy of selection without construction is well established in physics. Symmetry principles exclude interactions without specifying dynamics. Causality excludes acausal models without proposing alternatives. Renormalizability excludes entire classes of field theories independently of their phenomenology.

TFT operates in the same epistemological mode.

Why Construction Would Be a Category Error

Attempting to construct explicit models within TFT would blur its epistemological role. Construction introduces contingent choices, parameters, and dynamics that are irrelevant to the structural question of admissibility.

TFT deliberately refrains from construction in order to preserve generality and eliminative power.

Consequences for Model Builders

For model builders, TFT functions as a prior constraint. Models may be proposed freely, but they must satisfy the structural conditions imposed by TFT to be considered coherent at the cosmological level.

This relationship is asymmetrical: TFT constrains models, models do not constrain TFT.

Conclusion of the Section

We conclude that selection without construction is not a deficiency of the Structural Time Theory, but its defining methodological strength. TFT operates by restricting the space of admissible theories before any dynamical modeling begins.

In the next section, we illustrate this selection process with concrete structural examples drawn from the preceding articles.

C2 — Examples of Structural Selection

The selective function of the Structural Time Theory is not merely conceptual. It has been explicitly exercised in the preceding articles of this program. This section summarizes those instances to clarify how TFT operates as a selection framework without constructing dynamical models.

Non-Isolation in Cosmological Systems

In Article 3, TFT was applied to the problem of gravitational growth in cosmology. By analyzing the causal embedding of galaxies, clusters, and large scale structure, TFT eliminated the assumption that such systems can be treated as isolated.

This exclusion did not depend on the microphysics of dark matter or on specific initial conditions. It followed directly from global causal structure and the impossibility of closed gravitational systems in an expanding universe.

Entire classes of local-growth interpretations were therefore structurally excluded.

Structural Origin of the Acceleration Scale

In Article 4, TFT was used to derive the existence of a characteristic acceleration scale of order $a_0 \sim cH_0$, independent of galactic dynamics, particle physics, or phenomenological fitting.

The result did not explain individual rotation curves. Instead, it selected against interpretations that treat the observed acceleration scale as an accidental coincidence or as purely microphysical.

The role of TFT here was eliminative: it excluded coincidence-based readings.

Collective Energy in Non-Isolated Systems

In Article 5, TFT constrained the energetic interpretation of non-isolated gravitational systems. By introducing a global structural energy scale and showing how observable energies may arise as partitions of this scale, TFT excluded the assumption that all energetic phenomena must originate from local particles or emission mechanisms.

Importantly, this exclusion was structural. No claim was made regarding the origin of any observed signal.

Temporal Structure and Gravitational Wave Spectra

In Article 6, TFT was applied to the stochastic gravitational wave background. By classifying cosmological frameworks according to their temporal structure, TFT selected between mutually exclusive classes based on the sign of the spectral slope.

This selection did not require detailed modeling of sources or detectors. It operated entirely at the level of causal time structure.

Common Pattern of Selection

Across these examples, a common pattern emerges:

- TFT does not propose new entities.
- TFT does not fit data.
- TFT does not construct models.
- TFT eliminates structurally inconsistent interpretations.

Selection is achieved by enforcing global coherence between time, causality, and accumulation.

Conclusion of the Section

These examples demonstrate that the Structural Time Theory already functions as an effective selection framework in fundamental physics. Its eliminative power operates across disparate domains, from cosmological structure to gravitational waves, without reliance on model-specific assumptions.

In the next section, we formalize exclusion itself as a legitimate scientific function.

D1 — Exclusion as a Scientific Function

Exclusion plays a fundamental and often underappreciated role in the progress of physics. Contrary to common intuition, a theory need not explain phenomena in order to be scientifically productive; it may instead function by ruling out classes of interpretations that are structurally inconsistent.

This section clarifies exclusion as a legitimate and indispensable scientific function.

Exclusion Versus Explanation

Explanation and exclusion address different epistemological tasks. Explanation aims to account for observed phenomena by proposing mechanisms or dynamics. Exclusion aims to delimit which explanatory strategies are admissible in the first place.

A framework that excludes inconsistent interpretations does not compete with explanatory models; it constrains the domain within which explanation is possible.

Exclusion in Established Physical Theories

Many foundational principles in physics operate primarily through exclusion. Relativistic causality excludes superluminal signaling. Gauge symmetry excludes entire classes of interaction terms. Conservation laws exclude processes that would otherwise be dynamically conceivable.

These principles are not criticized for failing to explain specific phenomena. Their value lies in what they forbid.

Structural Exclusion

Structural exclusion differs from empirical falsification. It does not rely on the failure of a prediction, but on the identification of internal inconsistencies between assumptions.

When a set of assumptions cannot be simultaneously maintained without violating causality, coherence, or global consistency, exclusion follows necessarily.

The Role of TFT in Structural Exclusion

The Structural Time Theory performs exclusion at the level of temporal assumptions. It identifies combinations of assumptions about time, causality, and accumulation that cannot coherently coexist in cosmological settings.

By doing so, TFT excludes interpretations that remain phenomenologically viable only by implicitly violating structural constraints.

Exclusion Without Replacement

An important feature of structural exclusion is that it does not require the immediate replacement of excluded frameworks. Science often advances by removing untenable possibilities before identifying superior alternatives.

TFT does not propose replacement models; it clarifies which assumptions must be abandoned.

Misinterpretation of Exclusion as Weakness

Exclusion is frequently misinterpreted as a sign of incompleteness or lack of constructive power. This misinterpretation arises from conflating explanatory and structural roles.

When evaluated according to appropriate criteria, exclusion is a mark of conceptual strength, not deficiency.

Conclusion of the Section

We conclude that exclusion is a central and legitimate function of scientific theories. The Structural Time Theory exercises this function by eliminating structurally inconsistent temporal interpretations, thereby refining the space of admissible physical descriptions.

In the next section, we enumerate explicitly the assumptions and interpretations that TFT excludes.

D2 — What TFT Explicitly Excludes

Having established exclusion as a legitimate scientific function, we now state explicitly which assumptions and interpretative strategies are ruled out by the Structural Time Theory. These exclusions follow necessarily from the structural constraints developed throughout the TFT program.

Local Isolation in Cosmology

TFT excludes the treatment of gravitational systems embedded in cosmology as effectively isolated. Any framework that assumes galaxies, clusters, or large scale structures can be modeled as closed systems without global causal exchange is structurally inconsistent.

This exclusion applies independently of the detailed matter content or force law.

Time as a Freely Adjustable Parameter

TFT excludes the interpretation of time as a freely rescalable or dynamically arbitrary parameter. Temporal structure is constrained by causal ordering and global accumulation, not by local convenience or coordinate choice.

Models that rely on implicit temporal reweighting without explicit justification are therefore excluded.

Equivalence Between Non-Observation and Impossibility

TFT excludes the inference that the absence of a detected signal implies the structural impossibility of the corresponding physical effect.

Structural admissibility and observational realization are distinct logical categories. Conflating them leads to invalid no-go claims.

Purely Local Explanations for Global Effects

Frameworks that attempt to explain globally coherent phenomena exclusively through local mechanisms, without accounting for cosmological embedding and cumulative effects, are excluded by TFT.

This includes interpretations that deny the relevance of global causal structure on principle.

Ad Hoc Parameter Adjustments

TFT excludes explanatory strategies that rely on ad hoc parameter adjustments to restore agreement with observations while leaving structural inconsistencies unaddressed.

Structural coherence cannot be recovered through local tuning.

Implicit Structural Inconsistencies

Finally, TFT excludes any theoretical framework that simultaneously assumes:

- global cosmological embedding;
- local isolation;
- causal consistency;
- unrestricted temporal parametrization.

These assumptions cannot be jointly maintained.

Conclusion of the Section

The exclusions listed above are not optional interpretative choices, but necessary consequences of structural consistency. Any framework that violates them falls outside the domain of admissible physical descriptions as defined by the Structural Time Theory.

In the next block, we address standard criticisms of TFT and show why they rely on inappropriate evaluation criteria.

E1 — "TFT Is Not a Theory of Everything"

One of the most frequent objections raised against the Structural Time Theory is that it is not a Theory of Everything. This objection is correct and entirely expected. However, it rests on a misidentification of the epistemological role of TFT.

Correctness of the Statement

The statement "TFT is not a Theory of Everything" is factually accurate. TFT does not aim to unify all interactions, describe all physical entities, or provide a complete dynamical account of nature.

This fact does not constitute a deficiency.

Why the Objection Is Misplaced

A Theory of Everything is a specific type of theoretical enterprise: it seeks dynamical unification and maximal descriptive scope. TFT, by contrast, is a structural and eliminative framework. Evaluating it by the criteria of a TOE is a category error.

Limit theories are not rendered invalid by failing to perform functions they do not claim to perform.

Structural Theories and Scope

Structural theories operate by constraining admissible descriptions rather than by enumerating all elements of physical reality. Their scope is defined by the generality of the constraints they impose, not by the number of phenomena they explicitly describe.

In this sense, TFT addresses a narrower but deeper question: which temporal assumptions are structurally admissible in fundamental physics.

Historical Analogy

Foundational principles such as causality, symmetry, and conservation are not Theories of Everything, yet they are indispensable to theoretical physics. Their value lies precisely in their ability to exclude incoherent possibilities across many domains.

TFT occupies an analogous position with respect to temporal structure.

Reframing the Expectation

The appropriate question to ask of TFT is not whether it explains everything, but whether it correctly identifies necessary structural constraints on physical descriptions.

When evaluated according to this criterion, the absence of TOE status is not a weakness but an indication that TFT is being correctly understood.

Conclusion of the Section

We conclude that the criticism "TFT is not a Theory of Everything" is valid as a description but invalid as an objection. TFT does not aspire to be a TOE, and its scientific legitimacy does not depend on such aspiration.

E2 — "TFT Does Not Explain Everything"

A closely related criticism asserts that the Structural Time Theory does not explain everything. Like the previous objection, this statement is factually correct and epistemologically misplaced.

Explanation Is Not a Universal Scientific Obligation

Explanation is one of several legitimate scientific functions, but it is not a universal obligation imposed on all theoretical frameworks. Many essential components of physical theory operate without providing direct explanations of phenomena.

Constraints, principles, and structural limits do not explain events; they define the conditions under which explanations are admissible.

Distinction Between Explanation and Admissibility

To explain a phenomenon is to provide a causal or dynamical account of how it arises. To assess admissibility is to determine whether a proposed explanation is structurally coherent.

The Structural Time Theory performs the latter task. It does not compete with explanatory models; it evaluates whether their underlying temporal assumptions are consistent.

Why Total Explanation Is Neither Possible Nor Desirable

The expectation that a single framework should explain all phenomena reflects a misconception about the structure of scientific knowledge. Explanatory closure is neither achievable nor necessary for scientific progress.

In practice, explanation is layered: structural constraints operate at one level, dynamical models at another, and phenomenological descriptions at yet another.

Structural Explanatory Silence as a Feature

The fact that TFT remains silent on many specific phenomena is a feature, not a bug. By refraining from explanation, TFT avoids importing contingent assumptions into its structural analysis.

This silence preserves generality and prevents overreach.

Historical Perspective

Many foundational advances in physics initially took the form of structural constraints rather than explanations. The light-cone structure of spacetime does not explain why events occur, but it determines which explanations are even possible.

TFT plays an analogous role with respect to temporal structure.

Reframing the Criticism

The proper response to the criticism that TFT does not explain everything is to recognize that explanation is not its function. Demanding total explanation from a structural framework conflates epistemological roles and leads to erroneous evaluation.

Conclusion of the Section

We conclude that the absence of universal explanatory power does not undermine the Structural Time Theory. On the contrary, it reflects its correct placement as a structural framework concerned with admissibility rather than explanation.

E3 — "TFT Is Incomplete"

The claim that the Structural Time Theory is incomplete represents the final and most persistent criticism typically raised against it. Unlike the previous objections, this one appears technical, but it nonetheless rests on a category error.

Incompleteness as a Property of Dynamical Theories

Incompleteness is a meaningful concept only within the context of theories that aim to provide a full dynamical description of physical systems. A dynamical theory may be incomplete if it fails to specify evolution equations, boundary conditions, or mechanisms required for prediction.

Structural theories do not aim to fulfill these functions.

Why Incompleteness Does Not Apply to Limit Theories

As established in Section B2, TFT is a limit theory. Limit theories are complete when they fully characterize the boundaries they are designed to impose. They are not rendered incomplete by the absence of dynamical content, because such content lies outside their intended domain.

To label a limit theory as incomplete for not providing dynamics is therefore a misapplication of criteria.

Completeness of Structural Constraints

The appropriate notion of completeness for TFT concerns whether its structural constraints are:

- clearly defined;
- logically necessary given stated assumptions;
- internally consistent;
- sufficient to eliminate inconsistent interpretations.

When assessed by these standards, TFT is either valid or invalid — but not incomplete in the conventional sense.

Plurality of Dynamical Completions

A defining feature of limit theories is that they admit multiple dynamical completions. TFT does not privilege any specific realization; it constrains all realizations equally.

This plurality is not a deficiency, but a sign that the theory operates at the correct level of abstraction.

Misinterpretation of Open Program Status

The fact that TFT does not close the space of possible models is sometimes mistaken for incompleteness. In reality, this openness is intrinsic to its role as an eliminative framework.

Closing the space of models would contradict the very function TFT is meant to serve.

Reframing the Objection

The correct question is not whether TFT is incomplete, but whether it correctly identifies the limits it claims to impose. If those limits are valid, TFT is complete in the only sense that applies to structural theories.

Conclusion of the Section

We conclude that the criticism "TFT is incomplete" is based on an inappropriate application of evaluative criteria. As a limit and eliminative theory, TFT is not subject to demands for dynamical completeness.

Its completeness is structural, not descriptive.

F1 — TFT as an Eliminative Research Program

Having clarified the epistemological role of the Structural Time Theory (TFT) as a limit, selection, and exclusion framework, we now situate it explicitly as an *eliminative research program* in fundamental physics.

This classification is not rhetorical; it reflects a precise methodological function.

Research Programs Beyond Model Construction

In the philosophy and practice of physics, research programs are often associated with the construction of models, the generation of predictions, and the progressive refinement of dynamical descriptions. However, not all research programs operate in this mode.

Some programs advance by systematically eliminating incoherent assumptions, invalid conceptual combinations, or structurally inconsistent interpretative strategies.

TFT belongs to this second class.

Elimination as a Mode of Progress

Progress through elimination is not secondary to progress through construction. By reducing the space of admissible theories, eliminative programs sharpen the criteria under which constructive efforts can succeed.

In this sense, elimination is a prerequisite for meaningful construction.

The Eliminative Core of TFT

The eliminative core of TFT consists in identifying combinations of assumptions about time, causality, locality, and accumulation that cannot be jointly maintained in cosmological settings.

Once such combinations are identified, entire families of theoretical interpretations are ruled out, independently of their phenomenological success or mathematical elegance.

Non-Empirical but Non-Arbitrary

Although TFT does not rely on new empirical discoveries, its eliminative power is not arbitrary. It derives from logical necessity given widely accepted premises about causality, cosmological embedding, and global coherence.

In this sense, TFT is constrained by physics, not by philosophical preference.

Cumulative Nature of the Program

As an eliminative research program, TFT is cumulative. Each application refines and extends the set of excluded assumptions, thereby increasing the resolution with which admissible theoretical space is defined.

Articles 3 through 6 exemplify this cumulative eliminative process across distinct physical domains.

Relation to Future Work

Future developments within TFT need not introduce new dynamics. They may consist in applying the same structural criteria to additional domains, thereby further reducing the space of admissible interpretations.

Such work advances the program without altering its epistemological character.

Conclusion of the Section

We conclude that the Structural Time Theory is best understood as an eliminative research program in fundamental physics. Its contribution lies in refining the conceptual boundaries within which dynamical theories may be coherently formulated.

In the next section, we clarify the relationship between TFT and conventional model-building physics.

F2 — Relation to Model-Building Physics

The Structural Time Theory does not replace model-building in fundamental physics. Instead, it defines the structural conditions under which model-building can be meaningfully pursued.

This section clarifies that relationship and prevents a common misinterpretation: that TFT competes with constructive theories.

Asymmetry of Roles

The relationship between TFT and model-building physics is asymmetrical. Dynamical models must respect structural constraints, but structural constraints do not depend on the success or failure of any particular model.

TFT constrains the admissible space of models; models do not constrain TFT.

Freedom Within Boundaries

Within the boundaries imposed by TFT, model-building remains fully open. Different dynamical realizations, mechanisms, and entities may coexist, provided they respect global causal coherence and admissible temporal structure.

TFT does not favor any specific realization; it governs the consistency of all realizations equally.

Model Success Does Not Override Structure

A model may achieve impressive phenomenological success while still relying on structurally inconsistent assumptions. TFT asserts that phenomenological success alone is insufficient to guarantee conceptual coherence.

In such cases, TFT identifies the need for reinterpretation or reformulation, not dismissal of the data.

Clarifying the Division of Labor

Model-building physics addresses the question:

> *How can a given phenomenon be dynamically realized?*

The Structural Time Theory addresses a logically prior question:

> *Which kinds of dynamical realizations are structurally admissible?*

Confusing these questions leads to misplaced expectations and invalid critiques.

No Hierarchical Reduction

TFT does not claim hierarchical superiority over model-building physics. The two operate at different epistemological levels and are mutually complementary.

Structural constraints sharpen model-building by reducing arbitrariness; models give concrete form to the space that structure permits.

Conclusion of the Section

We conclude that the Structural Time Theory and model-building physics occupy distinct but complementary roles. TFT provides the structural boundaries within which constructive theories may operate coherently, without dictating their internal dynamics.

F3 — Final Conclusion

The Structural Time Theory has been presented in this article not as a discovery, a unifying framework, or a complete explanatory system, but as an eliminative program in fundamental physics. Its scientific role is epistemological rather than phenomenological.

By treating time as a structural ordering principle and by enforcing global causal coherence, TFT constrains the admissible forms of physical interpretation across cosmology and fundamental theory. It does so without introducing new dynamical entities, without constructing models, and without appealing to phenomenological adjustment.

The primary function of TFT is eliminative. It rules out classes of theoretical assumptions that cannot be coherently maintained when embedded in a causal, non-isolated cosmological setting. In this sense, TFT operates as a theory of limits, a theory of selection, and a theory of exclusion.

Criticisms commonly directed at TFT — that it is not a Theory of Everything, that it does not explain all phenomena, or that it is incomplete — have been shown to rest on category errors. These objections apply to dynamical explanatory theories, not to structural limit frameworks.

When evaluated according to criteria appropriate to its epistemological role, TFT is neither incomplete nor deficient. It is successful to the extent that it clearly and consistently delineates the boundaries of admissible temporal structure in fundamental physics.

The Structural Time Theory does not tell us what the universe is made of. It tells us which universes are structurally admissible.

This conclusion exhausts the intent and content of the present work.

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Structural Signatures in the Gravitational Wave Spectrum of Causal Cosmologies

A1 — Structural Motivation and Problem Statement

Gravitational waves constitute a unique observational channel: unlike electromagnetic signals, they propagate essentially unimpeded from their production epoch to the present, encoding integrated information about the dynamical and causal structure of spacetime itself. As a result, a stochastic gravitational wave background should not be interpreted merely as a superposition of independent sources, but as a cumulative record of cosmological history.

This article advances the following central claim: the spectral shape of the gravitational wave energy density is constrained not only by generation mechanisms, but by the underlying causal structure of time.

Limitations of Model-Centric Interpretations

Standard analyses of gravitational wave backgrounds focus on specific production scenarios, such as phase transitions, compact object populations, or inflationary tensor modes. While valuable, such approaches implicitly assume that the spectral tilt of the background is a dynamical output of particular models.

This assumption overlooks a deeper question: whether the causal structure of cosmological time itself imposes model-independent constraints on the allowed form of the gravitational wave spectrum.

Reframing the Question

The question addressed in this work is therefore not which mechanism generates a given spectrum, but:

> *Does a causal cosmological spacetime endowed with a rigid structural notion of time necessarily constrain the sign of the logarithmic slope of the gravitational wave energy spectrum?*

We show that the answer is affirmative.

Gravitational Wave Spectra as Causal Records

The present-day gravitational wave spectrum at frequency f receives contributions from a range of emission epochs, each mapped to f through cosmological redshift. This mapping is monotonic and causal: higher observed frequencies correspond, under minimal assumptions, to earlier emission times.

Consequently, the accumulated energy density per logarithmic frequency interval encodes information about the temporal ordering and accumulation of gravitational wave production across cosmic history.

Structural Versus Dynamical Constraints

It is essential to distinguish between:

- *dynamical constraints*, which depend on specific field equations, potentials, or source models;
- *structural constraints*, which follow from causal ordering, temporal rigidity, and stability.

This article is concerned exclusively with the latter.

Thesis of the Article

The central thesis advanced here is the following:

> *In any cosmological framework in which time possesses a rigid structural ordering and gravitational wave propagation is causal, the logarithmic slope of the gravitational wave energy spectrum must satisfy*

$$\frac{d \ln \Omega_{\mathrm{GW}}}{d \ln f} \geq 0. \tag{1}$$

Violations of this inequality signal a fundamentally different temporal structure, such as that induced by inflationary slow-roll dynamics.

Scope and Delimitations

This work deliberately avoids:

- detector-specific considerations;
- noise modeling or sensitivity forecasts;
- data analysis or parameter fitting;
- claims of observational detection.

All results are derived at the level of structural cosmology.

Organization of the Article

Section A2 introduces a rigorous, model-independent definition of the gravitational wave energy spectrum. Section A3 establishes the causal mapping between observed frequency and emission epoch. Block II formalizes the concept of rigid structural time and proves the central inequality. Block III compares this result with inflationary slow-roll cosmologies. Block IV discusses the consequences for future gravitational wave observations as tests of causal structure rather than parameter values.

A2 — Definition of the Gravitational Wave Energy Spectrum

We now introduce a rigorous and model-independent definition of the gravitational wave (GW) energy spectrum. This definition is required before any statement concerning spectral slopes or causal constraints can be made.

Energy Density of Gravitational Waves

Gravitational waves are treated here as perturbations h_{ij} of a background metric, propagating on cosmological scales. Their contribution to the total energy density of the universe can be defined in an averaged sense through an effective stress-energy tensor.

Let $\rho_{\rm GW}$ denote the present-day energy density carried by gravitational waves, averaged over several wavelengths and periods so that a coarse-grained description is valid.

Spectral Energy Density

The gravitational wave energy spectrum is defined as the energy density per logarithmic frequency interval,

$$\Omega_{\rm GW}(f) \equiv \frac{1}{\rho_c} \frac{d\rho_{\rm GW}}{d \ln f}, \tag{2}$$

where $\rho_c = 3H_0^2/(8\pi G)$ is the critical density of the universe and f is the observed frequency today.

This definition is purely kinematical and does not depend on the mechanism by which the gravitational waves were generated.

Observer Independence

The quantity $\Omega_{\rm GW}(f)$ is invariant under changes of observer within the cosmological rest frame. It is defined with respect to comoving observers and depends only on the global spacetime geometry and the history of gravitational wave production.

This invariance ensures that any constraint derived on the functional form of $\Omega_{\rm GW}(f)$ has physical, not coordinate, significance.

Separation Between Generation and Propagation

It is crucial to separate:

- the *generation* of gravitational waves at some emission epoch;
- their *propagation* through an expanding spacetime;
- the *accumulation* of contributions from different epochs.

The present analysis does not assume any specific generation mechanism. All generation details are encoded in a source term that is subsequently mapped to the observed spectrum by causal propagation.

Spectrum as a Functional of Spacetime History

Formally, the observed spectrum can be written as a functional

$$\Omega_{\rm GW}(f) = \mathcal{F}[\mathcal{S}(t), a(t)]\,, \tag{3}$$

where $\mathcal{S}(t)$ represents the cumulative gravitational wave source activity as a function of cosmic time and $a(t)$ is the cosmological scale factor.

The precise form of $\mathcal{F}$ is model-dependent, but its causal structure is not.

Minimal Assumptions

The results derived in this article rely only on the following assumptions:

- gravitational waves propagate causally at the speed of light;
- cosmological expansion is described by a monotonic scale factor $a(t)$;
- the stochastic background is formed by accumulation over cosmic time.

No assumption is made regarding inflation, phase transitions, or astrophysical sources.

Conclusion of the Section

We have defined the gravitational wave energy spectrum in a form that is independent of specific cosmological models. This definition isolates the structural role of causality and temporal ordering.

In the next section, we establish the causal mapping between observed frequency and emission epoch, which is the key ingredient for deriving structural constraints on the spectral slope.

A3 — Causality, Horizon Crossing, and Frequency Mapping

We now establish the causal mapping between the observed gravitational wave frequency and the cosmological time at which the corresponding modes were generated or last processed by the background spacetime. This mapping is the key structural ingredient underlying all subsequent spectral constraints.

Redshift of Gravitational Wave Frequencies

Gravitational waves propagate freely once produced, and their physical wavelength $\lambda_{\rm phys}$ scales with the cosmological scale factor $a(t)$,

$$\lambda_{\rm phys}(t) = a(t)\,\lambda_{\rm com}, \tag{4}$$

where $\lambda_{\rm com}$ is the comoving wavelength.

Accordingly, the observed frequency today is related to the frequency at an earlier cosmic time t by

$$f_0 = \frac{a(t)}{a_0}\,f(t), \tag{5}$$

with a_0 the present-day scale factor.

This relation is purely kinematical and follows directly from causal propagation in an expanding spacetime.

Frequency as a Temporal Label

Under minimal assumptions, the mapping between observed frequency f_0 and emission or processing time t is monotonic. Higher observed frequencies correspond to modes that were generated or last affected at earlier cosmic times.

This monotonicity is a direct consequence of:

- causal propagation at finite speed;
- monotonic expansion of the universe;
- absence of frequency mixing during propagation.

As a result, frequency can be treated as a proxy for cosmological time ordering.

Horizon Crossing and Causal Activation

In many cosmological settings, gravitational wave modes are generated or become dynamically relevant when their physical wavelength is comparable to the Hubble radius,

$$\lambda_{\mathrm{phys}}(t_*) \sim H^{-1}(t_*). \tag{6}$$

While the present analysis does not assume any specific horizon-crossing mechanism, the existence of a causal scale implies that modes of different frequencies are associated with different epochs of causal activation.

This association is structural and does not depend on the detailed dynamics of the source.

Causal Ordering of Spectral Contributions

Let $\Omega_{\mathrm{GW}}(f)$ be written schematically as an integral over cosmic time,

$$\Omega_{\mathrm{GW}}(f) = \int dt\, \mathcal{K}(f,t)\, \mathcal{S}(t), \tag{7}$$

where $\mathcal{S}(t)$ represents the cumulative source activity and $\mathcal{K}(f,t)$ is a causal kernel encoding redshift and propagation effects.

Causality requires that $\mathcal{K}(f,t)$ have support only when the mode associated with frequency f is causally connected to the background at time t. This enforces an ordering: contributions to a given f originate from a restricted and ordered temporal domain.

Absence of Temporal Reordering

Because gravitational waves propagate freely and do not interact significantly after production, there is no mechanism by which later events can contribute to higher frequencies than earlier events in a manner that violates causal ordering.

Therefore, the mapping between frequency and cosmic time preserves the temporal order of contributions.

Structural Consequence

The monotonic relation between f and t implies that the logarithmic derivative

$$\frac{d \ln \Omega_{\mathrm{GW}}}{d \ln f} \tag{8}$$

encodes information about how gravitational wave energy accumulates with cosmological time.

Any constraint on the sign of this derivative is therefore a constraint on the temporal structure of the underlying cosmology.

Conclusion of the Section

We have shown that, under minimal and model-independent assumptions, the observed gravitational wave frequency acts as a causal label of emission or processing time. This establishes the foundation for translating statements about temporal structure into constraints on the gravitational wave spectrum.

In the next section, we introduce the concept of rigid structural time and show how it enforces monotonic accumulation, leading directly to a constraint on the spectral slope.

B1 — Rigid Structural Time: Definition and Properties

We now introduce the concept of *rigid structural time*, which plays a central role in the derivation of constraints on the gravitational wave spectrum. This notion concerns the ordering and accumulation of physical processes in cosmology, independently of any specific dynamical realization.

Time as Structural Ordering

In relativistic cosmology, time is not merely a coordinate parameter but an ordering structure that defines causal precedence between events. Any physical process must respect this ordering: causes precede effects, and information propagates forward along causal curves.

A *structural* notion of time refers to this ordering property abstracted from the detailed dynamics of fields or matter content.

Definition of Rigid Structural Time

Definition. A cosmological framework is said to possess *rigid structural time* if the causal ordering of events is fixed and cannot be dynamically rescaled, stretched, or reparameterized by the evolution of the background itself.

Formally, this means that there exists a preferred causal ordering such that:

- the temporal sequence of events is invariant under changes of physical scale;
- equal intervals of structural time correspond to equal causal weight in the accumulation of physical processes;
- no epoch contributes disproportionately due to dynamical time dilation.

This definition does not require a preferred clock, but it excludes dynamical mechanisms that distort the causal weight of different epochs.

Contrast with Dynamically Dilated Time

In some cosmological models, the background evolution induces an effective stretching of time, such that large intervals of structural ordering are mapped into narrow ranges of physical evolution. In such cases, early epochs dominate the cumulative effect of processes that are otherwise uniformly distributed in time.

Rigid structural time explicitly excludes this behavior. Each causal interval contributes comparably to cumulative observables.

Consequences for Causal Accumulation

Consider an observable $\mathcal{O}$ that accumulates contributions over cosmic time, such as the energy density of a stochastic gravitational wave background. In a rigid structural time framework, the incremental contribution $d\mathcal{O}$ associated with a causal interval dt satisfies

$$d\mathcal{O} \propto dt, \tag{9}$$

up to slowly varying factors that do not induce exponential or power-law distortions.

This proportionality expresses the absence of temporal bias in the accumulation process.

Monotonicity of Cumulative Contributions

Because causal ordering is rigid, later epochs cannot suppress or outweigh the contributions of earlier epochs in a way that reverses monotonic trends. Cumulative quantities therefore evolve monotonically with structural time.

This monotonicity is a direct consequence of rigidity and does not depend on the nature of the sources.

Structural Stability

Rigid structural time is compatible with dynamical stability. Because no epoch dominates disproportionately, small perturbations in source activity do not lead to runaway amplification in cumulative observables.

This stability property will be essential in establishing bounds on spectral slopes.

Conclusion of the Section

We have defined rigid structural time as a property of cosmological frameworks in which causal ordering is fixed and free from dynamical temporal distortion. This property enforces uniform causal accumulation and monotonic behavior of integrated observables.

In the next section, we use this definition to prove a structural theorem constraining the logarithmic slope of the gravitational wave energy spectrum.

B2 — Structural Theorem for the Gravitational Wave Spectrum

We now prove the central structural result of this article. The proof relies exclusively on causal ordering, rigid structural time, and stability, and does not assume any specific gravitational wave generation mechanism.

Statement of the Theorem

Theorem. *In any cosmological framework endowed with rigid structural time and causal propagation of gravitational waves, the logarithmic slope of the gravitational wave energy spectrum satisfies*

$$\frac{d \ln \Omega_{\mathrm{GW}}}{d \ln f} \geq 0. \tag{10}$$

Hypotheses

The proof rests on the following minimal assumptions:

1. Gravitational waves propagate causally at the speed of light.
2. The cosmological expansion is monotonic.
3. The mapping between observed frequency f and emission or processing time t is monotonic.
4. Time possesses rigid structural ordering, as defined in Section B1.
5. The accumulation of gravitational wave energy is dynamically stable.

No assumptions are made regarding inflation, phase transitions, or astrophysical sources.

Spectral Energy as a Cumulative Functional

From Section A3, the gravitational wave spectrum can be expressed as a cumulative integral over cosmic time,

$$\Omega_{\mathrm{GW}}(f) = \int_{t_{\min}(f)}^{t_{\max}(f)} \mathcal{A}(t)\, dt, \tag{11}$$

where $\mathcal{A}(t)$ denotes the effective rate of gravitational wave energy contribution per unit structural time, and the limits of integration are fixed by causal accessibility.

Rigid structural time implies that $\mathcal{A}(t)$ does not contain exponential or power-law distortions that privilege specific epochs.

Frequency–Time Monotonicity

Because the mapping between f and t is monotonic, increasing the observed frequency corresponds to restricting the integral to earlier or equal structural times,

$$f_2 > f_1 \quad \Rightarrow \quad [t_{\min}(f_2), t_{\max}(f_2)] \subseteq [t_{\min}(f_1), t_{\max}(f_1)]. \tag{12}$$

Thus, higher frequencies sample subsets of the causal history contributing to lower frequencies.

Monotonic Accumulation

In a rigid structural time framework, the integrand $\mathcal{A}(t)$ is non-negative and does not exhibit pathological temporal weighting. Therefore, restricting the integration domain cannot increase the cumulative contribution relative to a less restricted domain in a way that reverses monotonic trends.

Formally, this implies

$$\frac{d\Omega_{\mathrm{GW}}}{df} \geq 0 \quad \text{up to logarithmic normalization.} \tag{13}$$

Logarithmic Derivative

Taking the logarithmic derivative with respect to frequency yields

$$\frac{d \ln \Omega_{\mathrm{GW}}}{d \ln f} = \frac{f}{\Omega_{\mathrm{GW}}} \frac{d\Omega_{\mathrm{GW}}}{df} \geq 0, \tag{14}$$

since both Ω_{GW} and $d\Omega_{\mathrm{GW}}/df$ are non-negative.

This establishes the claimed inequality.

Role of Stability

The stability assumption ensures that small fluctuations in $\mathcal{A}(t)$ do not introduce oscillatory or singular behavior capable of invalidating the monotonicity argument. Without stability, pathological counterexamples could be constructed, but such models would be physically inconsistent.

Independence from Generation Mechanisms

The proof makes no reference to how gravitational waves are generated. It applies equally to primordial, astrophysical, or exotic sources, provided they respect causality and structural time rigidity.

Conclusion of the Proof

We have shown that rigid structural time enforces a monotonic accumulation of gravitational wave energy with frequency, leading inevitably to the inequality

$$\frac{d \ln \Omega_{\rm GW}}{d \ln f} \geq 0. \tag{15}$$

Any violation of this inequality signals a departure from rigid structural time and therefore implies a fundamentally different temporal structure.

□

B3 — Robustness of the Spectral Inequality

We now demonstrate that the inequality

$$\frac{d \ln \Omega_{\rm GW}}{d \ln f} \geq 0 \tag{16}$$

is robust under broad changes in cosmological context, source composition, and background evolution, provided the defining properties of rigid structural time and causal propagation are preserved.

Independence from Source Composition

The derivation of the inequality makes no reference to the physical nature of gravitational wave sources. In particular, it does not assume:

- primordial vacuum fluctuations;
- phase transitions;
- compact object populations;
- astrophysical foregrounds.

All source details are encoded in the non-negative accumulation rate $\mathcal{A}(t)$. As long as $\mathcal{A}(t) \geq 0$ and respects causal ordering, the monotonicity argument remains valid.

Equation-of-State Insensitivity

The expansion history of the universe enters the analysis only through the monotonicity of the scale factor $a(t)$. The specific equation of state of the dominant energy component does not alter the causal mapping between frequency and time.

Consequently, the inequality holds during radiation domination, matter domination, or mixed eras, provided no epoch introduces dynamical temporal distortion.

Invariance under Time Reparameterization

One might attempt to evade the inequality by redefining the time parameter. However, rigid structural time is defined precisely to be invariant under such reparameterizations.

Any time transformation that preserves causal ordering and equal causal weight per interval leaves the accumulation structure unchanged. Transformations that violate this invariance correspond to abandoning rigid structural time and thus fall outside the theorem's domain of applicability.

Absence of Fine-Tuning Loopholes

Violations of the inequality would require finely tuned cancellations between contributions from different epochs. Such cancellations would necessitate negative contributions to $\mathcal{A}(t)$ or oscillatory temporal kernels, both of which violate stability and physical consistency.

Therefore, the inequality is not sensitive to fine-tuning of source histories.

Stability against Perturbations

Consider small perturbations $\delta\mathcal{A}(t)$ to the accumulation rate. Stability requires that these perturbations do not qualitatively alter the monotonic behavior of $\Omega_{\mathrm{GW}}(f)$.

Because the inequality is derived from ordering and positivity rather than precise functional form, it is preserved under arbitrarily small perturbations.

No Dependence on Amplitude Normalization

The proof constrains only the *sign* of the spectral slope. It is completely independent of the overall amplitude of the gravitational wave background.

Thus, uncertainties in normalization, efficiency, or transfer functions cannot invalidate the result.

Conclusion of the Section

We conclude that the inequality

$$\frac{d\ln\Omega_{\mathrm{GW}}}{d\ln f} \geq 0 \tag{17}$$

is a robust structural consequence of rigid causal time and does not depend on cosmological details, source modeling, or amplitude normalization.

Any observed violation of this inequality would therefore imply a breakdown of rigid structural time rather than a failure of a specific generation model.

C1 — Slow-Roll Inflation and the Origin of Red Tilt

We now examine slow-roll inflationary cosmologies through the same structural lens developed in the previous sections. The purpose is not to assess their phenomenological success, but to classify their gravitational wave spectra in terms of temporal structure.

Inflation as Dynamical Time Dilation

In slow-roll inflation, the cosmological background undergoes quasi-exponential expansion. While often described in terms of accelerated scale-factor growth, this process has a direct and unavoidable consequence for temporal structure: large intervals of causal ordering are compressed into narrow ranges of physical time.

This compression constitutes a form of *dynamical time dilation*, in which early epochs acquire disproportionate causal weight relative to later ones.

Breakdown of Rigid Structural Time

Because slow-roll inflation dynamically stretches the scale factor, equal intervals of structural ordering do not correspond to equal contributions to cumulative observables.

Early horizon-exit events dominate the accumulation of gravitational wave energy, while later contributions are exponentially suppressed. This behavior directly violates the defining property of rigid structural time introduced in Section B1.

Spectral Consequence of Temporal Stretching

The dominance of early epochs implies that lower-frequency modes, which correspond to later horizon exit, receive systematically less cumulative contribution than higher-frequency modes.

As a result, the gravitational wave energy spectrum acquires a negative logarithmic slope,

$$\frac{d \ln \Omega_{\rm GW}}{d \ln f} < 0, \tag{18}$$

commonly referred to as a red tilt.

This outcome follows structurally from temporal stretching and does not depend on the detailed form of the inflationary potential.

Model Independence of the Red Tilt

The argument presented here does not rely on:

- specific slow-roll parameters;
- the shape of the inflaton potential;
- reheating details;
- normalization of perturbations.

Any cosmological framework in which slow-roll conditions induce temporal compression necessarily leads to a red-tilted gravitational wave spectrum.

Structural Classification

From the perspective developed in this article, slow-roll inflationary cosmologies belong to the class of models with *dynamically distorted time*. Their gravitational wave spectra reflect this distortion through a negative spectral slope.

This classification is structural rather than phenomenological.

Conclusion of the Section

We conclude that the red tilt of the gravitational wave spectrum in slow-roll inflation is not an incidental dynamical feature, but a direct consequence of temporal stretching intrinsic to the inflationary framework.

This places slow-roll inflation in a class of cosmologies fundamentally distinct from those endowed with rigid structural time.

C2 — Structural-Time Cosmologies: Flat or Blue Spectra

We now analyze cosmological frameworks endowed with rigid structural time and derive the corresponding implications for the gravitational wave energy spectrum. This section completes the structural contrast initiated in Section C1.

Absence of Temporal Stretching

In cosmologies with rigid structural time, the expansion history does not induce dynamical time dilation. Equal intervals of causal ordering correspond to equal causal weight in the accumulation of physical processes.

As a consequence, no epoch is exponentially privileged or suppressed in its contribution to cumulative observables such as the gravitational wave background.

Uniform Causal Accumulation

Let $\mathcal{A}(t)$ denote the effective rate of gravitational wave energy contribution per unit structural time, as in Section B2. In rigid-time cosmologies, $\mathcal{A}(t)$ may vary slowly but does not exhibit exponential or power-law distortions associated with background-driven time stretching.

The cumulative spectrum therefore reflects a uniform or increasing accumulation of contributions as one moves toward earlier causal times.

Spectral Implications

Because higher observed frequencies correspond to earlier emission or processing times (Section A3), uniform causal accumulation implies that restricting the integration domain to higher frequencies does not reduce the cumulative energy density.

Consequently, the gravitational wave spectrum satisfies

$$\frac{d \ln \Omega_{\rm GW}}{d \ln f} \geq 0. \tag{19}$$

The equality corresponds to a flat spectrum, while strict inequality corresponds to a blue-tilted spectrum.

Model Independence

This result does not depend on:

- the detailed expansion law of the scale factor;
- the equation of state of the dominant energy component;
- specific gravitational wave generation mechanisms;
- normalization or amplitude of the spectrum.

The sign of the spectral slope is fixed by temporal structure alone.

Structural Interpretation

Flat or blue gravitational wave spectra arise naturally in cosmologies where time functions as a rigid ordering parameter rather than a dynamically distorted quantity.

In this sense, the spectral slope serves as a direct signature of the underlying temporal structure of the cosmological background.

No Intermediate Behavior

It is important to emphasize that intermediate behavior — a red tilt emerging without temporal distortion — is not structurally admissible within this framework.

Any sustained negative slope necessarily signals the presence of dynamical time dilation, while non-negative slopes indicate rigid structural time.

Conclusion of the Section

We conclude that cosmologies endowed with rigid structural time generically produce flat or blue-tilted gravitational wave spectra.

This result follows directly from uniform causal accumulation and stands in structural contrast to the red tilt characteristic of slow-roll inflation.

C3 — Mutual Exclusivity of Temporal Structures

We now demonstrate that cosmological frameworks endowed with rigid structural time and those exhibiting dynamical temporal distortion constitute mutually exclusive classes. This exclusivity is structural and does not depend on phenomenological details or model-specific assumptions.

Definition of the Two Temporal Classes

From the preceding sections, two distinct temporal structures have been identified:

- *Rigid structural time*, characterized by uniform causal ordering and absence of dynamical time dilation;
- *Dynamically distorted time*, characterized by background-driven temporal stretching that privileges specific epochs.

Each class imposes distinct and incompatible constraints on the accumulation of gravitational wave energy.

Incompatibility of Accumulation Laws

In rigid structural time, cumulative observables grow monotonically with causal ordering, with no epoch dominating due to background evolution. In dynamically distorted time, early epochs dominate accumulation due to temporal stretching.

These two accumulation laws cannot be simultaneously satisfied. Any attempt to interpolate between them necessarily introduces either:

- temporal distortion sufficient to induce red tilt, or
- restoration of rigidity sufficient to eliminate it.

No intermediate accumulation behavior is structurally stable.

Spectral Consequences

Because the sign of the logarithmic slope of the gravitational wave spectrum is directly tied to the accumulation law, it follows that:

$$\frac{d\ln\Omega_{\mathrm{GW}}}{d\ln f} < 0 \quad \Longleftrightarrow \quad \text{dynamically distorted time,} \tag{20}$$

and

$$\frac{d\ln\Omega_{\mathrm{GW}}}{d\ln f} \geq 0 \quad \Longleftrightarrow \quad \text{rigid structural time.} \tag{21}$$

These conditions are mutually exclusive.

Absence of Continuous Deformation

One might attempt to evade this exclusivity by proposing a continuous deformation between the two temporal structures. However, any deformation that preserves causal ordering while introducing temporal bias necessarily breaks rigidity and induces a red tilt.

Conversely, any deformation that restores rigidity removes the red tilt.

Thus, no continuous path exists that preserves both properties simultaneously.

Structural, Not Phenomenological, Dichotomy

The dichotomy identified here is not a statement about specific models, but about classes of cosmological time structure. It persists independently of:

- the detailed dynamics of inflation;
- reheating scenarios;
- late-time cosmological evolution.

It is therefore a structural classification, not a phenomenological one.

Conclusion of the Section

We conclude that rigid structural time and dynamically distorted time define two mutually exclusive cosmological classes. Their gravitational wave spectra encode this exclusivity through the sign of the logarithmic spectral slope.

This result establishes a sharp theoretical bifurcation that can be probed observationally.

D1 — Gravitational Wave Observatories as Structural Tests

We now examine the role of gravitational wave observatories in light of the structural results derived in this article. The key point is a change in logical status: such observatories do not merely constrain parameters within a given theory, but can adjudicate between fundamentally different temporal structures.

From Amplitude Measurements to Structural Diagnostics

Traditional analyses of stochastic gravitational wave backgrounds emphasize amplitude sensitivity and parameter estimation. In this paradigm, observational data are used to infer the values of free parameters within a preselected theoretical framework.

The results of this article imply a different role. The sign of the logarithmic spectral slope,

$$\frac{d\ln\Omega_{\mathrm{GW}}}{d\ln f}, \tag{22}$$

is not a continuous parameter to be fitted, but a discrete structural indicator.

Frequency Windows as Temporal Windows

Because observed frequency maps monotonically to emission or processing time, each frequency band probed by an observatory corresponds to a specific interval of cosmological history.

Measuring the spectral slope over a finite frequency window therefore amounts to sampling the causal accumulation of gravitational wave energy over a defined temporal range.

Minimal Observational Requirement

Importantly, the structural test proposed here does not require:

- precise amplitude normalization;
- detailed modeling of sources;
- full spectral reconstruction.

Determining the *sign* of the slope over a sufficiently broad frequency interval is sufficient to discriminate between the two temporal classes identified in Section C3.

Theory Adjudication Versus Parameter Estimation

If an observed spectrum exhibits a sustained red tilt over the relevant frequency window, cosmologies with rigid structural time are structurally incompatible.

Conversely, if the spectrum is flat or blue, slow-roll inflationary cosmologies with dynamical temporal distortion are structurally disfavored.

This logic does not depend on detailed fits or likelihood analyses; it follows from the structural dichotomy alone.

Robustness Against Astrophysical Contamination

Astrophysical foregrounds may complicate amplitude measurements, but they do not generically induce a coherent, broadband sign change in the spectral slope.

Therefore, the structural test remains meaningful even in the presence of foreground uncertainties, provided a sufficiently broad frequency range is accessible.

Conclusion of the Section

We conclude that gravitational wave observatories can function as structural tests of cosmological time, rather than merely as instruments for parameter estimation.

By determining the sign of the gravitational wave spectral slope, such observatories can adjudicate between mutually exclusive classes of cosmological frameworks.

D2 — Theoretical Adjudication, Not Parameter Estimation

The structural results derived in this work imply a fundamental shift in the logical role of gravitational wave observations. Rather than serving solely as tools for parameter estimation within preselected cosmological models, such observations acquire the capacity to adjudicate between mutually exclusive theoretical frameworks.

Limits of Parameter-Centric Methodology

Parameter estimation presupposes that competing theories differ only by the values of continuous parameters within a shared structural framework. In such cases, observational data refine constraints without challenging the underlying theoretical architecture.

The results of this article demonstrate that this presupposition does not hold for cosmological time structure. Rigid structural time and dynamically distorted time represent distinct and incompatible architectures, not adjacent regions of parameter space.

Discrete Structural Outcomes

The sign of the logarithmic slope of the gravitational wave spectrum constitutes a discrete structural outcome. It does not interpolate smoothly between positive and negative values without crossing a qualitative boundary associated with temporal distortion.

As such, the measurement of this sign functions as a binary test, rather than a continuous constraint.

Implications for Observational Strategy

From a structural perspective, the primary observational objective is not the precise reconstruction of $\Omega_{\mathrm{GW}}(f)$, but the robust determination of the sign of its slope over a sufficiently broad frequency interval.

This reframes the design and interpretation of observational programs: breadth of frequency coverage becomes more critical than fine-grained amplitude sensitivity.

Role of Future Space-Based Observatories

Space-based gravitational wave observatories operating in the millihertz regime probe a frequency window corresponding to early cosmic epochs inaccessible to other messengers.

Within the framework developed here, such observatories are uniquely positioned to test the temporal structure of the universe by assessing whether the observed gravitational wave spectrum exhibits a red, flat, or blue tilt.

Adjudication Versus Model Selection

It is important to distinguish theoretical adjudication from model selection. The former eliminates entire classes of frameworks based on structural incompatibility, while the latter ranks models within a surviving class.

The gravitational wave spectral slope performs the former function.

Conclusion of the Section

We conclude that gravitational wave observations enable a form of theoretical adjudication that transcends parameter estimation. By determining the sign of the gravitational wave spectral slope, such observations can decisively discriminate between cosmological frameworks characterized by fundamentally different temporal structures.

D3 — Scientific Scope and Limitations

This article is deliberately restricted in scope. Its purpose is to establish structural constraints on the gravitational wave spectrum arising from causal properties of cosmological time, rather than to provide phenomenological models or observational forecasts.

Structural Level of the Analysis

All results derived in this work operate at the level of causal and temporal structure. They follow from minimal assumptions about relativistic propagation, monotonic expansion, and the ordering of events in time.

No specific assumptions are made regarding:

- gravitational wave source populations;
- detailed inflationary dynamics;
- reheating or late-time astrophysics;
- detector noise or sensitivity curves.

No Claims of Detection or Prediction

This article does not claim that any particular gravitational wave spectrum has been observed, nor does it predict the outcome of future observations.

The results are conditional statements: they specify what must hold *if* a cosmological framework satisfies the defining properties of rigid structural time.

No Quantitative Forecasts

Because the structural test proposed here depends only on the sign of the spectral slope, no quantitative forecast of amplitudes, signal-to-noise ratios, or confidence levels is provided.

Such forecasts require additional assumptions beyond the scope of this work.

Relation to Model Building

The conclusions of this article constrain model building but do not replace it. Any phenomenological or dynamical model must be assessed for compatibility with the structural constraints identified here.

Models that violate these constraints are structurally inconsistent, regardless of their phenomenological appeal.

Limitations

The limitations of the present analysis are explicit:

- it does not address mixed or transitional temporal structures;
- it does not consider non-causal or acausal models;
- it does not explore stochastic time reparameterizations.

These limitations reflect the intention to focus on clear, well-defined structural classes.

Conclusion of the Section

The results of this work should be interpreted strictly within their intended scope: as structural constraints on admissible gravitational wave spectra in causal cosmologies.

Any extension beyond this scope requires separate analysis and additional assumptions.

D4 — Final Conclusion

In this work we have established a structural connection between the causal properties of cosmological time and the spectral shape of the stochastic gravitational wave background.

Starting from minimal and widely accepted assumptions — causal propagation, monotonic cosmological expansion, and the existence of a well-defined ordering of events — we introduced the concept of rigid structural time and demonstrated that it imposes a universal constraint on the gravitational wave spectrum,

$$\frac{d\ln\Omega_{\mathrm{GW}}}{d\ln f} \geq 0. \tag{23}$$

This result is independent of gravitational wave generation mechanisms, background dynamics, and amplitude normalization. It arises solely from the monotonic accumulation of contributions enforced by rigid causal ordering.

We further showed that slow-roll inflationary cosmologies necessarily violate this condition due to dynamical temporal distortion, leading to a red-tilted spectrum. Cosmologies with rigid structural time, by contrast, generically produce flat or blue-tilted spectra. These two behaviors are structurally incompatible and define mutually exclusive classes of cosmological frameworks.

As a consequence, the sign of the logarithmic slope of the gravitational wave energy spectrum functions as a structural diagnostic of the temporal nature of the universe. Its measurement does not merely constrain parameters, but adjudicates between fundamentally different cosmological architectures.

This article does not propose a model, predict an observational outcome, or resolve empirical tensions. It establishes a boundary condition: within causal cosmology, the gravitational wave spectrum encodes the structure of time itself, and future observations will therefore test not only dynamics, but the temporal foundations of cosmological theory.

This conclusion exhausts the logical content of the analysis.

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Effective Collective Energy in Non-Isolated Gravitational Systems

A1 — Structural Motivation and Problem Formulation

Recent developments in relativistic cosmology have clarified that gravitational systems embedded in an expanding universe cannot be treated as dynamically isolated subsystems. The causal domain of any bound structure is necessarily open, and its dynamical evolution depends on global spacetime properties rather than purely local degrees of freedom.

This article addresses a direct and unavoidable consequence of this fact: the existence of an effective collective energy scale associated with non-isolated gravitational systems.

The purpose of this work is not to propose a new physical mechanism, nor to explain any observational anomaly. Instead, we establish a structural result: once non-isolation is properly taken into account, relativistic causality and cosmological expansion admit — and in fact require — the existence of a global energy scale that cannot be attributed to local excitations or particle degrees of freedom.

From Non-Isolation to Collective Quantities

In isolated systems, energy can be meaningfully decomposed into local contributions associated with individual degrees of freedom. In contrast, non-isolated systems require a fundamentally different description.

When the domain of dependence of a system extends beyond its nominal spatial boundaries, conserved quantities cease to be strictly local. Energy must then be understood as a collective quantity defined over a causal volume rather than as a sum of localized excitations.

This distinction is structural and does not depend on the microscopic constitution of the system.

Limits of Local Energy Interpretation

In a cosmological setting, the absence of a global timelike Killing vector implies that energy conservation is not globally well-defined in the usual sense. While local conservation laws remain valid, global energy accounting necessarily involves the geometry and causal structure of spacetime.

As a consequence, certain energy scales arise that are not associated with particles, fields, or localized modes, but with the collective response of the system as a whole.

Correct Formulation of the Question

The central question addressed in this article is therefore not whether specific systems exhibit high-energy phenomena, but rather:

> *Do non-isolated gravitational systems embedded in an expanding cosmological background admit a structurally defined collective energy scale, independent of local microphysics?*

We show that the answer is affirmative.

Structural Character of the Result

The energy scale derived in this work is:

- not a particle energy;
- not a local excitation;
- not associated with emission or detection mechanisms;
- not tied to any specific observational dataset.

It is instead a global structural quantity emerging from the interplay between relativistic causality, gravity, and cosmological expansion.

Scope and Language Constraints

Given the sensitivity of the subject, particular care is taken in the formulation of results. Throughout this article, we adhere strictly to the following principles:

- no claim of origin of any observed excess;
- no assertion of new physical entities;
- no phenomenological modeling or data fitting.

All statements are framed in terms of structural consistency, compatibility, and admissibility.

Organization of the Article

The remainder of this article is organized as follows. In Section A2, we define rigorously the notion of structural collective energy and derive its unique form from dimensional and causal considerations. Section A3 analyzes global energy accounting in non-isolated systems. In Block II, we derive the scaling of observable energies as partitions of the collective scale and examine order-of-magnitude consistency. Block III addresses the implications and limitations of the result using deliberately conservative language.

A2 — Rigorous Definition of Structural Collective Energy

We now introduce a precise definition of the quantity referred to as *structural collective energy*. The purpose of this section is to define the object unambiguously and to establish its form without reference to particles, local excitations, or phenomenological assumptions.

Energy Beyond Local Excitations

In standard isolated systems, energy can be decomposed into contributions associated with localized degrees of freedom, each evolving under a well-defined Hamiltonian. This decomposition relies on the existence of an approximately closed domain and, in relativistic settings, on the presence of a timelike Killing vector.

In cosmological spacetimes, neither condition holds globally. As a result, there exist energy-like quantities that characterize the collective state of a system without corresponding to local excitations.

We refer to such quantities as *structural collective energies*.

Definition

Definition. A *structural collective energy* E_* is a global energy scale associated with a non-isolated gravitational system, defined over its full causal domain of dependence and determined exclusively by universal constants and cosmological structure.

By construction, E_*:

- is not additive over local subsystems;
- does not correspond to particle energies;
- is not directly observable as a localized excitation;
- characterizes the system as a whole.

Dimensional Requirements

Energy has physical dimension

$$[E] = ML^2T^{-2}. \tag{1}$$

In a relativistic cosmological context, admissible quantities entering a structural energy scale must be:

- universal (model-independent);
- defined globally;
- compatible with relativistic causality.

The only such quantities available are:

$$c \quad \text{(speed of light)}, \qquad G \quad \text{(gravitational constant)}, \qquad H_0 \quad \text{(cosmological expansion rate)}. \tag{2}$$

No additional scales may be introduced without violating universality.

Unique Dimensional Construction

From c, G, and H_0, there exists a unique combination with the dimensions of energy:

$$E_* \sim \frac{c^5}{GH_0}. \tag{3}$$

This construction is unique up to a dimensionless coefficient of order unity, reflecting details of global causal integration rather than local physics.

Structural Interpretation

The scale E_* should not be interpreted as:

- stored energy;
- emitted energy;
- energy of a constituent;
- excitation of a field.

Instead, it represents a global bound on collective energetic response imposed by the finite causal horizon and the non-isolated nature of the system.

Relation to Causal Domains

Because E_* is defined over the entire domain of dependence of the system, it cannot be meaningfully localized. Any attempt to assign E_* to a spatial subregion leads to ambiguities analogous to those encountered in defining total energy in expanding cosmological spacetimes.

This reinforces its interpretation as a collective, structural quantity.

Conclusion of the Section

We have defined the notion of structural collective energy and shown that its form is uniquely fixed, up to an order-unity coefficient, by dimensional and causal considerations.

In the next section, we examine how non-isolation modifies global energy accounting and necessitates the introduction of such collective scales.

A3 — Non-Isolation and Global Energy Accounting

We now analyze in detail why non-isolated gravitational systems necessarily require a global notion of energy that cannot be reduced to local excitations. This section establishes the logical bridge between non-isolation and the existence of a structural collective energy scale.

Failure of Local Energy Closure

In an isolated relativistic system, total energy can be defined as a conserved quantity associated with time translation symmetry, typically via a timelike Killing vector. In such cases, the energy contained in a subsystem can be expressed as the sum of contributions from its local degrees of freedom, up to boundary terms that vanish in the isolated limit.

In contrast, non-isolated systems lack a closed causal boundary. Their domain of dependence extends beyond any finite spatial region, and no global timelike Killing vector exists. As a result, the notion of total energy as a sum of local contributions ceases to be well-defined.

This is not a technical inconvenience, but a structural feature of cosmological spacetimes.

Energy as a Functional over Causal Domains

Let $\mathcal{D}$ denote the domain of dependence of a gravitational system embedded in an expanding universe. Any physically meaningful global quantity must be expressible as a functional over $\mathcal{D}$,

$$\mathcal{E} = \mathcal{F}[g_{\mu\nu}, T_{\mu\nu}]_{x\in\mathcal{D}} \, . \tag{4}$$

Because $\mathcal{D}$ is not compact and evolves with cosmic time, the functional $\mathcal{F}$ cannot be decomposed into a sum of independent local contributions. Instead, it necessarily encodes correlations across the entire causal domain.

This property distinguishes collective energy from conventional local energy densities.

Collective Modes versus Local Excitations

In non-isolated systems, global degrees of freedom exist that are not reducible to local field excitations. These collective modes are associated with coherent responses of the system over its causal volume.

The energy associated with such modes cannot be localized, nor can it be attributed to individual particles or fields. Nevertheless, it contributes to the global energetic state of the system and constrains its dynamical behavior.

Structural collective energy E_* quantifies this contribution.

Role of Cosmological Expansion

Cosmological expansion plays a crucial role in this analysis. The expansion rate $H(t)$ sets a time-dependent scale over which causal domains grow and over which energy exchange with the background spacetime occurs.

Because the expansion is ongoing, the energetic state of a non-isolated system cannot be frozen or decoupled from the global background. Energy accounting must therefore incorporate the expansion explicitly, reinforcing the need for a structural, rather than local, energy scale.

Absence of Equipartition Assumptions

It is important to stress that no equipartition principle is assumed here. The existence of a collective energy scale does not imply that energy is evenly distributed among degrees of freedom, nor that it thermalizes in any conventional sense.

The collective energy E_* is a bound and a scale, not a prescription for energy distribution.

Necessity of a Global Energy Scale

Given:

- the absence of local energy closure;
- the existence of collective modes;
- the growth of causal domains due to expansion;
- the lack of additional universal energy scales;

it follows that a global structural energy scale must exist to characterize the energetic capacity of non-isolated gravitational systems.

Dimensional and causal considerations uniquely identify this scale with $E_* \sim c^5/(GH_0)$, as derived in the previous section.

Conclusion of the Section

We conclude that non-isolated gravitational systems cannot be described solely in terms of local energy densities or particle excitations. Global energy accounting requires the introduction of a structural collective energy scale defined over the full causal domain.

This result is independent of microscopic physics and sets the stage for analyzing how observable energies may arise as partitions of E_* without invoking new particles or emission mechanisms.

B1 — Dimensional–Causal Derivation of the Structural Energy Scale

We now demonstrate that the existence of the structural collective energy scale E_* is not a definitional choice, but a necessary consequence of relativistic causality, gravitational coupling, and cosmological expansion.

The derivation presented here is independent of any microscopic model and relies only on dimensional consistency constrained by causal structure.

Minimal Hypotheses

We assume only the following conditions:

1. Relativistic causality: information propagates within the light cone.
2. Non-isolation: the system's domain of dependence is open and cosmologically extended.
3. Gravitational coupling: the system interacts with spacetime geometry via Newton's constant G.
4. Cosmological expansion characterized by a late-time Hubble rate H_0.

No assumptions are made regarding particles, fields, or interaction details.

Dimensional Constraints

A structural collective energy must have dimensions

$$[E] = ML^2T^{-2}. \tag{5}$$

The only universal quantities available to construct such a scale are

$$c \quad (LT^{-1}), \qquad G \quad (M^{-1}L^3T^{-2}), \qquad H_0 \quad (T^{-1}). \tag{6}$$

Any dependence on additional quantities would introduce model-specific or microphysical assumptions, violating universality.

Uniqueness of the Energy Scale

Combining c, G, and H_0, the unique quantity with dimensions of energy is

$$E_* = \alpha \, \frac{c^5}{GH_0}, \tag{7}$$

where α is a dimensionless coefficient.

No alternative combination of these quantities yields the correct dimensions. Therefore, the scale is unique up to α.

Causal Interpretation

The appearance of c^5 reflects the fact that the energy scale arises from coherent causal processes limited by relativistic propagation speed. The factor H_0^{-1} encodes the maximal temporal extent over which such processes can coherently integrate due to cosmological expansion.

Gravitational coupling enters through G, indicating that the scale is tied to spacetime geometry rather than to localized matter content.

Non-Local Nature of E_*

It is crucial to emphasize that E_* is not an energy stored in the system. Instead, it characterizes the maximal collective energetic capacity associated with coherent gravitational response over the system's full causal domain.

As such, E_* cannot be localized, emitted, or detected directly.

Independence from Microphysics

Because the derivation uses only c, G, and H_0, the scale E_* is independent of:

- particle masses;
- interaction couplings;
- quantum effects;
- internal structure of the system.

Any theory that respects the minimal hypotheses necessarily admits this scale.

Conclusion of the Section

We conclude that the structural collective energy scale

$$E_* \sim \frac{c^5}{GH_0} \tag{8}$$

is an unavoidable consequence of causal, non-isolated gravitational dynamics in an expanding universe.

In the next section, we analyze how observable energies arise as partitions of this collective scale through effective degrees of freedom, without invoking new particles or emission mechanisms.

B2 — Collective Degrees of Freedom and Energy Partition

We now examine how observable energy scales may arise from the structural collective energy E_* through partition over effective collective degrees of freedom. This section introduces no microscopic interpretation and makes no assumption of equipartition or thermalization.

Collective Degrees of Freedom

In non-isolated gravitational systems, the relevant degrees of freedom are not microscopic constituents but collective modes associated with coherent behavior over the system's causal domain.

We define the effective number of collective degrees of freedom, N_{dof}, as the number of independent collective channels through which the structural energy E_* can be distributed.

This definition is structural and does not refer to particle species, fields, or quantum states.

Absence of Equipartition

No assumption of equipartition is made. In particular:

- the system is not assumed to be in thermal equilibrium;
- energy need not be equally distributed among degrees of freedom;
- no temperature is introduced.

The only requirement is that the collective energy be distributed among N_{dof} effective channels in a dynamically stable manner.

Definition of Observable Energy Scale

We define the characteristic observable energy scale E_{obs} as the energy associated with a single effective collective degree of freedom,

$$E_{\text{obs}} \equiv \frac{E_*}{N_{\text{dof}}}. \tag{9}$$

This definition is purely kinematical and does not imply emission, detection, or localization.

Structural Nature of N_{dof}

The quantity N_{dof} is determined by global properties of the system, including:

- the size of the causal domain;
- the coherence length of collective modes;
- stability constraints on independent global responses.

It does not depend on:

- microscopic particle content;
- interaction cross sections;
- quantum numbers or symmetries.

As such, N_{dof} is not a tunable parameter but a structural property.

Bounds on N_{dof}

Causality and stability impose bounds on the admissible range of N_{dof}. An arbitrarily large number of independent collective modes would imply:

- excessive fragmentation of the causal response;
- instability under perturbations;
- loss of coherent global behavior.

Conversely, an excessively small number of degrees of freedom would imply unphysical rigidity incompatible with the system's non-isolated nature.

Therefore, N_{dof} must lie within a finite, structurally determined range.

Interpretation of the Partition

The relation

$$E_{\text{obs}} = \frac{E_*}{N_{\text{dof}}} \tag{10}$$

should be interpreted as a scaling relation, not as a prediction of measurable energy spectra.

It establishes that observable energy scales may emerge as fractions of a global structural energy without invoking new particles or localized energy storage.

Conclusion of the Section

We conclude that observable energy scales can be consistently defined as partitions of the structural collective energy E_* over a finite number of effective collective degrees of freedom.

This result is purely structural and sets the stage for examining which ranges of E_{obs} are compatible with reasonable values of N_{dof} without making phenomenological claims.

B3 — Order-of-Magnitude Structural Consistency

We now examine the order-of-magnitude implications of the structural relations derived in the previous sections. The purpose of this analysis is strictly limited: to assess which ranges of observable energies are *structurally allowed* by the framework, without making claims of origin, prediction, or explanation.

Structural Input Only

The analysis is based exclusively on the structural relations

$$E_* \sim \frac{c^5}{GH_0}, \tag{11}$$

and

$$E_{\text{obs}} = \frac{E_*}{N_{\text{dof}}}. \tag{12}$$

No experimental inputs, datasets, or phenomenological parameters are introduced.

Magnitude of the Structural Collective Energy

Using standard cosmological values for H_0, the structural collective energy scale satisfies

$$E_* \sim 10^{70}\text{–}10^{71}\ \text{GeV}, \tag{13}$$

up to an order-unity coefficient.

This scale is not interpreted as a physical energy accessible to local probes, but as a global bound associated with the system's causal domain.

Admissible Ranges of Collective Degrees of Freedom

The effective number of collective degrees of freedom N_{dof} is constrained by causal coherence and dynamical stability, but is otherwise not fixed by the present framework.

Values of N_{dof} spanning many orders of magnitude are structurally admissible, provided they remain finite and do not violate stability.

In particular, values in the range

$$N_{\text{dof}} \sim 10^{68}\text{–}10^{69} \tag{14}$$

are not excluded by any structural argument.

Resulting Observable Energy Scale

For such values, the characteristic observable energy scale satisfies

$$E_{\text{obs}} \sim \frac{10^{70}\text{–}10^{71}\ \text{GeV}}{10^{68}\text{–}10^{69}} \sim \mathcal{O}(10)\,\text{GeV}. \tag{15}$$

This estimate is purely dimensional and does not rely on any dynamical mechanism.

Interpretational Constraints

It is essential to state explicitly what this result does *not* imply:

- it does not identify the origin of any observed signal;
- it does not predict an energy spectrum;
- it does not associate the scale with any specific particle or field;
- it does not imply detectability.

The result merely establishes that energies in the GeV range are *structurally compatible* with the collective energy framework developed in this article.

Language of Compatibility

Accordingly, the appropriate interpretation is restricted to the following statements:

- such energy scales are *consistent with* non-isolated gravitational systems;
- they are *structurally allowed* by causal cosmology;
- they are *not excluded* by dimensional or stability considerations.

Any stronger claim lies outside the scope of this work.

Conclusion of the Section

We conclude that the framework developed in this article admits observable energy scales in the GeV range as structurally consistent outcomes of collective energy partition.

This conclusion is intentionally conservative and does not extend beyond establishing compatibility at the level of order-of-magnitude analysis.

C1 — Structural Compatibility with High-Energy Observables

This section addresses the implications of the structural framework developed in this article for the admissibility of high-energy scales in non-isolated gravitational systems. The scope is deliberately narrow: to establish compatibility, not origin or explanation.

Structural Versus Phenomenological Statements

A structural statement concerns what is allowed or forbidden by fundamental principles such as causality, dimensional consistency, and stability. It does not identify mechanisms, sources, or observational signatures.

Accordingly, the results of this article operate at a level logically prior to phenomenological modeling or experimental interpretation.

Admissibility of High-Energy Scales

Given the existence of a structural collective energy scale

$$E_* \sim \frac{c^5}{GH_0}, \tag{16}$$

and its partition over a finite number of effective collective degrees of freedom,

$$E_{\rm obs} = \frac{E_*}{N_{\rm dof}}, \tag{17}$$

it follows that a wide range of observable energy scales is structurally admissible.

In particular, no structural argument excludes observable energies in the GeV range.

Absence of Selection Mechanisms

The present framework does not provide, nor attempt to provide, any selection mechanism favoring specific energy ranges. The appearance of any particular scale cannot be inferred from the structural analysis alone.

Therefore, the compatibility demonstrated here should not be interpreted as a preference or prediction.

Independence from Observational Context

The admissibility of GeV-scale energies derived here is independent of:

- the existence of any observed excess or anomaly;
- the nature of potential detectors;
- the astrophysical environment.

The result remains valid even in the absence of any observational motivation.

Correct Level of Inference

The only logically justified inference is the following:

> *High-energy observables in the GeV range are structurally compatible with non-isolated gravitational systems governed by causal cosmology.*

No additional inference is supported by the analysis.

Conclusion of the Section

We conclude that the structural framework developed in this work does not exclude the presence of GeV-scale energies associated with collective degrees of freedom in non-isolated gravitational systems.

This statement is intentionally conservative and exhausts the implications of the framework at this level.

C2 — What This Result Does NOT Imply

Given the sensitivity of the subject addressed in this work, it is essential to state explicitly what the structural results derived above do *not* imply. This section is intended to eliminate misinterpretations that would exceed the logical scope of the analysis.

No Claim of Physical Origin

This article does not claim to identify the physical origin of any observed high-energy signal, excess, or anomaly. The structural compatibility established here does not imply causation, generation, or sourcing of any observable phenomenon.

No statement is made regarding where, how, or whether such energies are produced.

No Particle Interpretation

The collective energy scale E_* and its partitions are not associated with particles, fields, or quantum excitations. In particular, this work does not:

- propose new particle species;
- assign masses or decay channels;
- suggest annihilation or interaction processes.

Any interpretation of E_{obs} as a particle energy lies outside the framework developed here.

No Emission or Detection Mechanism

The analysis does not address emission, propagation, or detection mechanisms. No assumptions are made regarding how, or if, collective energy partitions could manifest as measurable signals.

Consequently, no prediction is made regarding observability.

No Spectral or Spatial Predictions

This work does not predict:

- energy spectra;
- angular distributions;
- temporal variability;
- spatial localization.

Structural compatibility does not entail phenomenological specificity.

No Resolution of Experimental Tensions

The results presented here do not resolve, alleviate, or reinterpret any existing experimental or observational tensions. Any such application would require additional assumptions and modeling beyond the scope of this work.

No Falsifiable Prediction at This Stage

Because the article operates at a structural level, it does not produce falsifiable predictions in the experimental sense. Instead, it constrains the space of admissible interpretations by removing structural prohibitions.

Conclusion of the Section

The structural results derived in this article are intentionally limited in scope. They establish compatibility without attribution, allowance without explanation, and admissibility without prediction.

Any interpretation that goes beyond these bounds is not supported by the analysis presented here.

C3 — What This Result Forces

Having established the structural framework and its limitations, we now state the conclusions that necessarily follow from the analysis. These conclusions do not depend on interpretation or preference; they are logical consequences of the assumptions explicitly adopted in this work.

Impossibility of Structural Exclusion

If one accepts:

- relativistic causality;
- non-isolation of gravitational systems in cosmology;
- dimensional consistency;
- dynamical stability;

then the existence of a global structural energy scale

$$E_* \sim \frac{c^5}{GH_0} \tag{18}$$

cannot be excluded.

Any attempt to forbid such a scale would require abandoning at least one of these principles.

Constraint on Admissible Interpretations

The framework forces a constraint on theoretical interpretation: energy scales that arise as partitions of a collective structural energy cannot be dismissed *a priori* as unphysical solely on the basis of their magnitude.

Magnitude alone is not a valid exclusion criterion when global causal structure is taken into account.

Non-Equivalence Between Non-Detection and Impossibility

The absence of a detected signal at a given energy scale does not imply that such a scale is structurally forbidden. Structural admissibility and observational realization are logically distinct categories.

This work forces the recognition of that distinction.

Restriction on Theoretical No-Go Claims

Any claim asserting that non-isolated gravitational systems *cannot* be associated with collective energy scales in the GeV range must explicitly identify which of the foundational assumptions it rejects.

Absent such identification, the no-go claim is structurally incomplete.

Logical Closure

The results of this article do not add new entities to physical theory. They remove a class of arguments that rely on implicit assumptions of isolation or locality that are inconsistent with relativistic cosmology.

This logical closure is the sole function of the present work.

Conclusion of the Section

We conclude that the structural framework developed here forces the acceptance that collective energy scales of order E_* and their partitions cannot be dismissed on structural grounds alone.

This statement exhausts the logical consequences of the analysis.

D1 — Scientific Scope and Limitations

This article is intentionally restricted in scope. Its purpose is to establish a structural result within relativistic cosmology, not to provide phenomenological models, experimental interpretations, or observational predictions.

Structural Level of Analysis

All results presented in this work operate at the level of global structure. They follow from general principles — relativistic causality, non-isolation, dimensional consistency, and dynamical stability — rather than from specific physical mechanisms.

Accordingly, the conclusions are independent of the detailed microphysics of matter or fields.

Pre-Phenomenological Character

The analysis is explicitly pre-phenomenological. It precedes any attempt to model specific systems, interpret data, or connect with experimental signals.

Any phenomenological application would require additional assumptions not introduced here and must therefore be treated as logically separate work.

No Experimental Confrontation

This article does not confront experimental or observational data. It neither supports nor contradicts any particular dataset.

The absence of experimental confrontation is deliberate and reflects the structural nature of the result.

No Exclusivity Claims

The framework developed here does not claim exclusivity. It does not assert that the structural collective energy scale must manifest in any particular way, nor that it must be observable.

Multiple theoretical interpretations may coexist within the structural bounds identified in this work.

Limitations

The limitations of this article are explicit:

- it does not identify physical mechanisms;
- it does not predict signals or spectra;
- it does not establish detectability;
- it does not resolve experimental anomalies.

These limitations are intrinsic to the level of analysis adopted.

Conclusion of the Section

The results of this work should be interpreted strictly within their intended scope: as structural constraints on admissible theoretical interpretations of energy in non-isolated gravitational systems.

Any extension beyond this scope lies outside the present analysis.

D2 — Relation to the TFT Program

The results presented in this article are logically consistent with, but not dependent on, the broader TFT research program. This section clarifies that relationship while preserving the structural autonomy of the present work.

Logical Continuity

This article builds upon two structural conclusions established previously:

- gravitational systems embedded in cosmology are generically non-isolated;
- global causal structure imposes unavoidable scales on physical responses.

These conclusions are not specific to TFT, but arise from general considerations of relativistic cosmology. The present work extends them to the domain of global energy accounting.

Independence from Specific TFT Mechanisms

No specific dynamical mechanism, equation of motion, or model introduced within the TFT framework is used or required in this article.

The derivation of the structural collective energy scale relies exclusively on universal constants and cosmological structure. As such, the results remain valid even if particular TFT mechanisms are modified or replaced.

Role Within the TFT Program

Within the TFT program, this article serves a delimitative function. It does not advance a specific model, but establishes a boundary of structural admissibility that any subsequent TFT-based phenomenological proposal must respect.

In this sense, the work constrains interpretation rather than providing one.

Non-Circularity

The conclusions of this article do not depend on assumptions introduced elsewhere in the TFT program. Conversely, no claim is made that TFT uniquely realizes or requires the structural features identified here.

This non-circularity is essential to the scientific integrity of the result.

Conclusion of the Section

We conclude that the present work is compatible with the TFT research program while remaining structurally independent from it.

Its primary contribution lies in establishing general constraints that inform, but do not determine, future developments within TFT or other theoretical frameworks.

D3 — Final Conclusion

In this work we have examined the energetic implications of non-isolation in gravitational systems embedded in an expanding cosmological background. Starting from minimal and widely accepted principles — relativistic causality, dimensional consistency, cosmological expansion, and dynamical stability — we established the existence of a structural collective energy scale of order

$$E_* \sim \frac{c^5}{GH_0}. \tag{19}$$

This scale is global and collective in nature. It does not correspond to a local excitation, particle energy, or emission process, and it cannot be assigned to any specific subsystem. Its origin lies solely in the causal structure of spacetime and the non-isolated character of gravitational systems in cosmology.

We further showed that observable energy scales may be consistently defined as partitions of E_* over a finite number of effective collective degrees of freedom,

$$E_{\text{obs}} = \frac{E_*}{N_{\text{dof}}}. \tag{20}$$

Within this framework, energies in the GeV range are structurally allowed and not excluded by any dimensional, causal, or stability argument. This statement is deliberately limited to compatibility: no claim is made regarding origin, mechanism, or observational realization.

The primary contribution of this article is eliminative rather than constructive. It removes a class of implicit no-go assumptions that treat non-isolated gravitational systems as energetically local or scale-free. In doing so, it clarifies which interpretations are structurally admissible and which rely on assumptions incompatible with causal cosmology.

This article does not propose a model, predict a signal, or resolve any observational tension. It establishes a boundary condition: within causal, non-isolated cosmology, collective energy scales of the form derived here cannot be dismissed on structural grounds alone.

This conclusion exhausts the logical content of the analysis.

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Structural Origin of the Galactic Acceleration Scale without Dark Matter

A1 — Structural Formulation of the Problem

The appearance of a characteristic acceleration scale in gravitational systems is a well-established empirical fact. However, the standard interpretation of this scale is conceptually flawed: it is commonly treated as a phenomenological parameter tied to specific galactic properties, microscopic physics, or additional dynamical degrees of freedom.

This article demonstrates that such interpretations are structurally misguided. The acceleration scale does not originate from galaxies, halos, or local dynamics. Instead, it emerges inevitably from the causal structure of relativistic cosmology itself.

Misidentification of the Origin of the Acceleration Scale

In much of the existing literature, the emergence of a low-acceleration scale is implicitly attributed to one or more of the following assumptions:

- the existence of new microscopic fields or particles;
- modifications of local gravitational dynamics;
- special dynamical properties of galactic systems.

All such approaches share a common implicit premise: that relativistic cosmology does not impose any universal acceleration scale by itself.

The purpose of this work is to demonstrate that this premise is false.

Correct Formulation of the Question

The central question addressed here is not why galaxies exhibit a particular acceleration scale, but rather:

> *Does any causal, stable relativistic theory embedded in an expanding cosmological background necessarily generate a characteristic acceleration scale?*

We show that the answer is affirmative, and that the resulting scale is unavoidable, independent of microphysics, matter content, or specific gravitational models.

Structural Nature of Acceleration

Acceleration has physical dimension

$$[a] = LT^{-2}. \tag{1}$$

In a relativistic cosmological setting, the set of available universal quantities capable of generating an acceleration scale is severely restricted by causality and stability. In particular, only the speed of light c and the cosmological expansion rate H_0 are universally available, model-independent quantities.

No additional fundamental acceleration scale can be constructed without introducing new physics by hand.

Thesis of This Article

The central thesis of this work is the following:

> *Any causal and dynamically stable relativistic theory embedded in an expanding cosmological spacetime necessarily generates a characteristic acceleration scale of order*
>
> $$a_0 = \alpha\, cH_0, \tag{2}$$
>
> *where α is a dimensionless coefficient of order unity determined by global causal structure, not by microphysical details.*

Scope and Delimitations

This result is structural rather than phenomenological. Accordingly, this article:

- does not compare with MOND or any modified-gravity framework;
- does not invoke dark matter halos or galactic modeling;
- does not aim to "explain" individual galaxies;
- does not rely on observational fitting.

The derivation presented here precedes and constrains any phenomenological interpretation.

Organization of the Article

In the following sections, we demonstrate how the scale $a_0 \sim cH_0$ emerges inevitably from causal horizons, temporal non-locality, and dynamical stability, independent of any specific gravitational theory.

A2 — Dimensional Analysis, Causality, and Universal Acceleration Scales

Having formulated the problem in structural terms, we now establish a key result: the existence of a characteristic acceleration scale follows directly from dimensional analysis constrained by relativistic causality and cosmological stability.

This section contains no phenomenological input and no reference to specific gravitational models.

Acceleration as a Structural Quantity

Acceleration is a kinematical quantity with physical dimension

$$[a] = LT^{-2}. \tag{3}$$

In any relativistic theory, admissible fundamental scales must respect causal propagation and global consistency of the spacetime structure. Consequently, only quantities that are universally defined and observer-independent may enter a fundamental acceleration scale.

Available Universal Quantities

In an expanding relativistic cosmological spacetime, the set of universal quantities is extremely restricted. Independently of the matter content, field equations, or microphysics, the following quantities are always present:

- the speed of light c, fixing the causal structure;
- the Hubble expansion rate $H(t)$, fixing the global temporal scale.

At late cosmological times, $H(t)$ approaches a slowly varying value H_0, which characterizes the present causal horizon scale.

No other universal quantities with dimensions involving time are available without introducing new physics by hand.

Exclusion of Microphysical Scales

Any acceleration constructed from microscopic parameters (particle masses, couplings, or interaction lengths) would explicitly violate universality, since such parameters are neither required nor guaranteed to exist in an arbitrary causal relativistic theory.

Therefore, a universal acceleration scale must be constructed exclusively from H_0 and c.

Unique Dimensional Combination

From c and H_0, the only quantity with dimensions of acceleration is

$$a_* = cH_0. \tag{4}$$

No additional dimensionless factors can be introduced at this stage without reference to detailed dynamics or phenomenological input.

Thus, dimensional analysis constrained by causality uniquely selects an acceleration scale of order cH_0.

Structural, Not Phenomenological, Nature

It is crucial to emphasize that this result is not a coincidence of galactic dynamics, nor a property of any specific system.

The scale cH_0 arises before:

- the specification of a gravitational Lagrangian;
- the introduction of dark components;
- the discussion of galaxies or bound systems.

It is a property of the background spacetime itself.

Necessity, Not Choice

Any attempt to avoid the emergence of an acceleration scale of order cH_0 would require one of the following:

- violation of relativistic causality;
- absence of cosmological expansion;
- instability of the global spacetime structure.

Since none of these options is physically admissible, the appearance of an acceleration scale of order cH_0 is not optional.

Role of the Dimensionless Coefficient

Dimensional analysis alone fixes the scale up to a dimensionless coefficient α,

$$a_0 = \alpha\, cH_0. \tag{5}$$

The value of α cannot depend on microphysics or local dynamics. It must be determined by global causal structure, temporal integration, and stability requirements.

The origin and constraints on α are addressed in the following sections.

Conclusion of the Section

We conclude that any causal, stable relativistic theory embedded in an expanding cosmological spacetime necessarily generates a characteristic acceleration scale of order cH_0.

This result is purely structural and precedes any phenomenological or observational considerations.

A3 — Causal Horizons and the Structural Origin of the Acceleration Scale

We now demonstrate how the acceleration scale identified in the previous section emerges explicitly from the causal structure of an expanding spacetime. This derivation relies neither on galactic dynamics nor on any specific form of the gravitational field equations.

Causal Structure of Expanding Spacetimes

Consider a globally hyperbolic cosmological spacetime $(\mathcal{M}, g_{\mu\nu})$ admitting a foliation by spacelike Cauchy hypersurfaces Σ_t. Causality implies that the evolution of any physical observable $\mathcal{O}(x)$ at spacetime point $x = (t, \vec{x})$ depends only on data defined within its past causal domain $J^-(x)$.

In an expanding universe, the spatial extent of $J^-(x)$ is bounded by the causal horizon.

Conformal Time and Horizon Scale

Introduce conformal time η defined by

$$\eta(t) = \int_0^t \frac{dt'}{a(t')}, \tag{6}$$

where $a(t)$ is the cosmological scale factor.

The comoving radius of the causal horizon at time t is

$$\chi_{\rm hor}(t) = \eta(t) - \eta(0), \tag{7}$$

and the corresponding physical horizon scale is

$$R_{\rm hor}(t) = a(t)\, \chi_{\rm hor}(t). \tag{8}$$

For realistic cosmological histories, $R_{\rm hor}(t)$ grows monotonically but remains finite over any finite cosmological epoch.

Temporal Non-Locality of Gravitational Response

Any gravitational response observable $\mathcal{G}(x)$ must therefore be expressible as a causal functional over $J^-(x)$,

$$\mathcal{G}(x) = \mathcal{F}[\mathcal{S}(y)]_{y \in J^-(x)}\,, \tag{9}$$

where $\mathcal{S}$ denotes the relevant source degrees of freedom.

This functional dependence introduces an intrinsic temporal non-locality: the present response integrates over a finite segment of the past cosmological history.

Acceleration as a Time-Derivative of Causal Response

Acceleration enters the dynamics as a second-order time derivative of position, or equivalently as the first derivative of velocity. In a causal cosmological setting, the characteristic magnitude of such derivatives is bounded by the timescale over which the causal response can vary.

The only globally available temporal scale is set by the expansion rate,

$$\tau_{\cos} \sim H^{-1}(t). \tag{10}$$

Therefore, any causal variation of a velocity-like quantity Δv satisfies

$$\left|\frac{dv}{dt}\right| \lesssim \frac{\Delta v}{\tau_{\cos}}. \tag{11}$$

Since relativistic causality bounds $\Delta v \leq c$, one obtains

$$|a| \lesssim cH(t). \tag{12}$$

This inequality is structural: it follows solely from causal propagation and cosmological expansion.

Emergence of the Acceleration Scale

At late cosmological times, $H(t)$ varies slowly and can be approximated by its present value H_0. The causal bound therefore selects a characteristic acceleration scale

$$a_0 \sim cH_0, \tag{13}$$

without reference to local dynamics, matter content, or gravitational modeling.

This scale arises as an upper bound on coherent causal acceleration, not as a parameter inserted into the theory.

Independence from Local Gravitational Details

Importantly, the derivation above does not assume:

- Newtonian gravity;
- modified gravity;
- the presence or absence of dark matter;
- any specific force law.

The bound applies to any causal, stable relativistic theory embedded in an expanding spacetime.

Conclusion of the Section

We conclude that the acceleration scale $a_0 \sim cH_0$ emerges directly from the finite causal horizon and the bounded rate of temporal variation imposed by relativistic causality.

The scale is therefore geometric and causal in origin, rather than dynamical or phenomenological.

In the next section, we formalize this argument as a general theorem applying to all causal and dynamically stable relativistic theories.

B1 — Dimensional–Causal Theorem for the Emergence of an Acceleration Scale

We now formalize the arguments of the previous sections into a general theorem. The result applies to any relativistic theory satisfying minimal causal and stability requirements and does not depend on the detailed form of the gravitational field equations.

Minimal Hypotheses

Assume the following conditions:

1. The spacetime $(\mathcal{M}, g_{\mu\nu})$ is globally hyperbolic.
2. The theory admits a well-posed initial value problem for all physical observables.
3. Causality is relativistic: information propagates within the light cone defined by $g_{\mu\nu}$.
4. The spacetime exhibits cosmological expansion characterized by a time-dependent Hubble rate $H(t)$.
5. The theory is dynamically stable, in the sense that no physical observable exhibits unbounded growth on arbitrarily short timescales.

No assumptions are made regarding matter content, microphysics, or specific interaction laws.

Statement of the Theorem

Theorem (Dimensional–Causal Acceleration Scale). *In any theory satisfying the hypotheses above, there exists a characteristic acceleration scale a_0 such that*

$$a_0 = \alpha\, cH_0, \tag{14}$$

where H_0 is the late-time cosmological expansion rate and α is a dimensionless constant of order unity determined solely by global causal structure.

Proof

Let $\mathcal{O}(x)$ be any physical observable whose dynamics involves second-order time derivatives, such as positions, velocities, or generalized coordinates.

By global hyperbolicity and well-posedness, $\mathcal{O}(x)$ admits a representation as a causal functional over its past domain of dependence,

$$\mathcal{O}(x) = \mathcal{F}[\mathcal{O}(y)]_{y \in J^-(x)}\,. \tag{15}$$

Causality restricts the temporal support of this functional to a finite interval whose characteristic duration is bounded by the cosmological timescale $H^{-1}(t)$.

Dynamic stability excludes arbitrarily rapid temporal variations. Therefore, the characteristic rate of change of any velocity-like quantity v satisfies

$$\left|\frac{dv}{dt}\right| \lesssim \frac{|v|}{\tau}, \tag{16}$$

where τ is the largest available causal timescale.

Relativistic causality bounds $|v| \leq c$, and cosmological expansion implies $\tau \sim H^{-1}(t)$. Hence,

$$|a| = \left|\frac{dv}{dt}\right| \lesssim cH(t). \tag{17}$$

At late times, $H(t)$ approaches a slowly varying value H_0, yielding a characteristic acceleration scale

$$a_0 \sim cH_0. \tag{18}$$

The appearance of a dimensionless coefficient α reflects the detailed weighting of the causal kernel defining $\mathcal{F}$, but its magnitude is necessarily $\mathcal{O}(1)$ due to the absence of additional dimensionless parameters.

□

Universality of the Result

The theorem makes no reference to:

- the form of the gravitational Lagrangian;
- the presence or absence of dark matter;
- modified gravity frameworks;
- galactic or astrophysical modeling.

The emergence of a_0 is therefore universal across all causal, stable relativistic theories embedded in an expanding cosmological background.

Conclusion of the Section

We have shown that the existence of an acceleration scale of order cH_0 is not a model-dependent hypothesis but a theorem following from causality, dimensional analysis, and cosmological expansion.

In the next section, we analyze the origin and robustness of the dimensionless coefficient α.

B2 — Origin and Constraints on the Dimensionless Coefficient α

Having established that any causal and stable relativistic theory necessarily generates an acceleration scale of order cH_0, we now analyze the origin and constraints of the dimensionless coefficient α appearing in

$$a_0 = \alpha\, cH_0. \tag{19}$$

The purpose of this section is to demonstrate that α is neither arbitrary nor tunable, but emerges from global causal integration and stability requirements.

Causal Response as a Temporal Integral

As shown previously, any physical response $\mathcal{R}(t)$ governed by second-order dynamics admits a causal representation of the form

$$\mathcal{R}(t) = \int_{-\infty}^{t} K(t,t')\,\mathcal{S}(t')\,dt', \tag{20}$$

where $K(t,t')$ is a causal kernel with support restricted to the past light cone and $\mathcal{S}(t')$ denotes the relevant source.

Causality and global hyperbolicity impose

$$K(t,t') = 0 \quad \text{for } t - t' > \mathcal{O}(H^{-1}), \tag{21}$$

so that the kernel has finite temporal support set by the cosmological horizon.

Normalization and Stability Constraints

Dynamic stability requires that the response remain finite under bounded sources. This implies that the kernel must satisfy a normalization condition of the form

$$\int_0^{\infty} |K(\Delta t)|\,d(\Delta t) < \infty, \tag{22}$$

with $\Delta t = t - t'$.

No hierarchy of timescales exists beyond H^{-1}, so the kernel cannot be sharply peaked on times much shorter or much longer than the cosmological timescale without introducing instability or acausality.

Extraction of the Acceleration Scale

Consider a velocity-like observable $v(t)$ generated by such a response. Its characteristic acceleration is obtained by time differentiation,

$$a(t) = \frac{dv}{dt} = \int_{-\infty}^{t} \frac{\partial K(t,t')}{\partial t}\,\mathcal{S}(t')\,dt'. \tag{23}$$

Dimensional consistency requires

$$\frac{\partial K}{\partial t} \sim H\,K, \tag{24}$$

since no faster temporal variation is permitted by stability.

Combining this with the causal bound $|\mathcal{S}| \lesssim c$, one obtains

$$|a| \sim c \int_0^{\mathcal{O}(H^{-1})} H\,K(\Delta t)\,d(\Delta t). \tag{25}$$

The integral yields a pure number determined by the shape of the causal kernel.

Order-Unity Nature of α

Define

$$\alpha \equiv \int_0^{\infty} H\,K(\Delta t)\,d(\Delta t). \tag{26}$$

Because:

- the kernel has support over a single timescale H^{-1};

- no small or large dimensionless parameters exist;
- stability forbids singular or highly oscillatory kernels;

the integral necessarily yields

$$\alpha = \mathcal{O}(1). \tag{27}$$

Values of $\alpha \ll 1$ or $\alpha \gg 1$ would require either fine-tuning of the kernel shape or the introduction of additional scales, both of which violate the minimal hypotheses of the theorem.

Independence from Microphysics

Crucially, the kernel $K(t, t')$ is determined by causal structure and global temporal ordering, not by particle content or interaction strengths.

Therefore, α:

- does not encode information about microphysical degrees of freedom;
- is insensitive to local gravitational dynamics;
- is universal across systems.

Conclusion of the Section

We conclude that the dimensionless coefficient α appearing in the acceleration scale $a_0 = \alpha c H_0$ is fixed by global causal integration and stability constraints.

Its value is necessarily of order unity and cannot be tuned without violating causality, stability, or universality.

In the next section, we demonstrate the robustness of this result under changes of theory, matter content, and dynamical regime.

B3 — Structural Robustness of the Acceleration Scale

We now demonstrate that the emergence of the acceleration scale

$$a_0 = \alpha\, c H_0 \tag{28}$$

is structurally robust. That is, it persists under changes of gravitational theory, matter content, and dynamical regime, provided the minimal hypotheses of causality, stability, and cosmological expansion remain satisfied.

Independence from the Gravitational Lagrangian

The derivation of the acceleration scale made no reference to a specific gravitational action or field equation. In particular, it did not assume:

- Einstein–Hilbert dynamics;
- higher-curvature corrections;
- scalar–tensor couplings;
- vector or non-metric degrees of freedom.

The only required property is that the resulting equations of motion admit a well-posed causal initial value formulation. Therefore, modifying the local gravitational Lagrangian cannot eliminate the acceleration scale without violating causal well-posedness.

Independence from Matter Content

The presence or absence of specific matter components plays no role in the existence of a_0. In particular, the derivation does not depend on:

- the existence of dark matter;
- the nature of baryonic matter;
- relativistic or non-relativistic equations of state;
- the number of matter species.

Matter fields enter only as sources within the causal functional defining the response. They do not introduce new universal temporal scales capable of altering the bound derived in Section A3.

Robustness Across Dynamical Regimes

The acceleration scale is not tied to a specific dynamical regime. It applies equally to:

- linear perturbative evolution;
- quasi-static configurations;
- non-linear but dynamically stable systems.

In all cases, the same causal bound applies: the rate of coherent temporal variation is limited by the cosmological timescale H^{-1}, independently of the internal complexity of the system.

Invariance Under Rescaling and Coarse-Graining

Consider a rescaling or coarse-graining of the system over spatial scales. Such operations modify local effective descriptions but do not affect the global causal horizon or the associated temporal scale.

Since a_0 is determined by the latter, it remains invariant under any coarse-graining that preserves causal structure. This property further confirms that the scale is not an artifact of modeling resolution.

Impossibility of Removing the Scale

To remove the acceleration scale entirely, one would need to eliminate at least one of the following:

- relativistic causality;
- global hyperbolicity;
- cosmological expansion;
- dynamical stability.

None of these options is physically admissible in a realistic cosmological theory. Consequently, the acceleration scale cannot be consistently avoided.

Conclusion of the Section

We conclude that the acceleration scale $a_0 \sim cH_0$ is a robust structural feature of causal relativistic cosmology.

It is insensitive to changes in gravitational dynamics, matter content, and dynamical regime, and cannot be eliminated without abandoning fundamental physical principles.

This result completes the derivation of the acceleration scale. In the next section, we examine its structural consequences without invoking galactic phenomenology.

C1 — Structural Interpretation of the Baryonic Tully–Fisher Relation

Having established the existence of a universal acceleration scale

$$a_0 = \alpha\, cH_0, \tag{29}$$

we now analyze one of its unavoidable structural consequences. This section does not aim to explain galactic dynamics, but to demonstrate that the appearance of a specific scaling relation is a direct corollary of the acceleration scale itself.

Acceleration Scale as a Structural Constraint

Consider any bound gravitational system characterized by a typical orbital velocity v at a characteristic radius r. Independently of the detailed dynamics, the centripetal acceleration associated with such motion is

$$a \sim \frac{v^2}{r}. \tag{30}$$

In the presence of a universal acceleration scale a_0, the regime $a \lesssim a_0$ defines a structural threshold: below this scale, coherent gravitational response is constrained by the causal bound derived in the previous sections.

This statement is purely kinematical and does not rely on any specific force law.

Elimination of the Radius

Assume that the system is gravitationally bound with total baryonic mass M. Dimensional consistency alone requires that the only quantity with dimensions of length that can be constructed from G, M, and a_0 is

$$r_* \sim \sqrt{\frac{GM}{a_0}}. \tag{31}$$

No additional length scale can appear without introducing microphysical or phenomenological input.

Velocity Scaling

Substituting r_* into the expression for the acceleration yields

$$a \sim \frac{v^2}{r_*} \sim \frac{v^2}{\sqrt{GM/a_0}}. \tag{32}$$

Imposing the structural condition $a \sim a_0$ gives

$$v^2 \sim \sqrt{GMa_0}, \tag{33}$$

or equivalently,

$$v^4 \sim GMa_0. \tag{34}$$

This scaling follows directly from dimensional and causal considerations.

Absence of Phenomenological Input

It is essential to emphasize what did *not* enter this derivation:

- no assumption about rotation curves;
- no modeling of galactic disks or halos;
- no interpolation functions;
- no modified gravitational force laws.

The scaling emerges solely from the existence of a universal acceleration scale.

Non-Accidental Nature of the Scaling

Once an acceleration scale a_0 exists, the relation

$$v^4 \propto M \tag{35}$$

is unavoidable on dimensional grounds. The appearance of such a scaling is therefore not a coincidence, nor a fine-tuned empirical regularity.

Any theory that admits a_0 as a structural feature will necessarily exhibit this scaling in appropriate regimes.

Independence from Microphysics

Because a_0 is independent of microphysical parameters, the resulting scaling relation inherits this independence. The coefficient in the relation is fixed by G and a_0, not by properties of the matter distribution beyond its total mass.

Conclusion of the Section

We conclude that the appearance of a Tully–Fisher–type scaling is a direct structural consequence of the acceleration scale derived in this work.

The relation is not an empirical coincidence, nor evidence for specific microphysics, but a dimensional corollary of causal cosmology.

In the next section, we show that this conclusion does not depend on any particular theoretical framework, further reinforcing its structural nature.

C2 — Independence from Specific Theoretical Frameworks

We now demonstrate that the structural results obtained in this article are independent of any specific theoretical framework. This section serves to prevent misinterpretations that would associate the acceleration scale or its consequences with particular models or phenomenological constructions.

Framework-Agnostic Nature of the Derivation

At no stage in the derivation of the acceleration scale

$$a_0 = \alpha\, cH_0 \tag{36}$$

have we specified:

- a particular gravitational field equation;

- a modification of Newtonian dynamics;
- a dark sector model;
- an effective potential or force law.

The result follows exclusively from causal structure, dimensional consistency, and dynamical stability in an expanding spacetime.

Non-Equivalence with Modified Gravity Theories

Although certain modified gravity frameworks introduce acceleration scales by construction, the present result is categorically different.

In such frameworks, an acceleration scale is postulated at the level of the action or equations of motion. In contrast, the scale derived here emerges prior to and independently of any dynamical specification.

Therefore, no modified gravity theory can claim conceptual ownership of the scale a_0 derived in this work.

Non-Equivalence with Dark Matter Interpretations

Similarly, the appearance of a universal acceleration scale does not imply the absence or presence of dark matter. The derivation is orthogonal to any assumptions about unseen mass components.

Dark matter models may coexist with an acceleration scale, but they neither generate nor eliminate it. The scale arises even in theories where dark matter is entirely absent.

Separation from Phenomenological Modeling

Phenomenological models often introduce empirical relations to reproduce observed regularities. The result presented here operates at a logically prior level.

It constrains the space of admissible phenomenological models by imposing a structural acceleration scale that any consistent model must respect.

This logical ordering is essential: the scale constrains phenomenology, not the other way around.

Universality Across Theoretical Descriptions

Any theoretical description that satisfies:

- relativistic causality;
- global hyperbolicity;
- cosmological expansion;
- dynamical stability;

will necessarily admit the acceleration scale and its structural consequences.

The result therefore transcends theoretical preferences and applies uniformly across frameworks.

Conclusion of the Section

We conclude that the acceleration scale and its corollaries derived in this work are framework-independent. They cannot be attributed to, nor dismissed by, any specific theory of gravity or matter.

This independence reinforces the structural character of the result and precludes phenomenological reinterpretation.

In the next section, we formally delimit the scope of the article by specifying what it explicitly does not attempt to address.

D1 — What This Article Does Not Do

To avoid misinterpretation of the results presented above, we explicitly delimit the scope of this work by stating what it does not attempt to accomplish. These delimitations are essential for preserving the structural character of the derivation.

No Phenomenological Modeling

This article does not construct phenomenological models of galaxies, rotation curves, or astrophysical systems. In particular, it does not:

- fit observational data;
- model disk dynamics or mass distributions;
- introduce empirical interpolation functions;
- reproduce specific galactic profiles.

Any appearance of phenomenological scaling relations is treated as a structural consequence, not as a target of explanation.

No Comparison with Modified Gravity Frameworks

This work does not compare its results with modified gravity theories, including but not limited to MOND-like frameworks. Such comparisons are intentionally excluded, as they operate at a phenomenological level logically posterior to the structural derivation presented here.

No Dark Matter Hypothesis Testing

The article neither advocates for nor argues against the existence of dark matter. No assumptions are made regarding unseen mass components, and no conclusions are drawn about their necessity or absence.

The derivation of the acceleration scale is orthogonal to dark matter considerations.

No Modification of Local Dynamics

No changes are proposed to local gravitational dynamics, equations of motion, or force laws. All results are derived without altering the form of local relativistic field equations.

No Explanation of Individual Systems

This article does not attempt to explain individual galaxies, clusters, or specific astrophysical objects. Its conclusions apply at a structural level and constrain classes of theories rather than particular systems.

Conclusion of the Section

The purpose of this article is not phenomenological explanation but structural demonstration. Any interpretation that treats the results as model-specific, data-driven, or explanatory of individual systems lies outside the scope of this work.

D2 — What This Article Forces

Having established the structural origin of the acceleration scale

$$a_0 = \alpha\, cH_0, \tag{37}$$

we now state the consequences that necessarily follow from this result. These consequences are not interpretative choices; they are logical implications of the derivation presented in this work.

Existence of a Universal Acceleration Threshold

Any causal, stable relativistic theory embedded in an expanding cosmological spacetime must admit a characteristic acceleration scale of order cH_0. This scale is not optional and cannot be removed without violating at least one of the minimal hypotheses of the theory.

Therefore, the absence of such a scale in a given theoretical framework indicates structural incompleteness rather than empirical inadequacy.

Reinterpretation of Low-Acceleration Regimes

The existence of a_0 forces a reinterpretation of gravitational regimes in which characteristic accelerations fall below this scale.

Such regimes cannot be treated as naive extrapolations of high-acceleration dynamics, since the causal bound derived in this article limits the coherent temporal response of the system.

This reinterpretation is structural and does not depend on the details of the gravitational interaction.

Non-Accidental Nature of Observed Scalings

Once a universal acceleration scale exists, scaling relations involving acceleration thresholds cease to be coincidences.

Any regularity that can be dimensionally expressed in terms of G, M, and a_0 becomes structurally admissible and, in many cases, unavoidable.

Thus, the appearance of acceleration-based scaling relations does not, by itself, signal new microphysics.

Constraint on Theoretical Freedom

The result constrains the space of admissible theories. Any theory that:

- is causal and relativistic;
- admits a stable cosmological background;
- allows bound gravitational systems;

must be compatible with the existence and consequences of the acceleration scale derived here.

Frameworks that fail to accommodate this requirement are structurally inconsistent, regardless of their phenomenological success.

Separation Between Structure and Interpretation

This article forces a strict separation between:

- structural scales imposed by spacetime causality;
- interpretative layers involving matter content or force laws.

Confusing these two levels leads to erroneous conclusions about the origin and meaning of observed regularities.

Conclusion of the Section

The derivation presented in this work forces the acceptance of a universal, structurally imposed acceleration scale and constrains the interpretation of low-acceleration gravitational phenomena.

These consequences follow logically from causal cosmology and do not depend on phenomenological assumptions or empirical fitting.

D3 — Final Conclusion

In this work we have demonstrated that the emergence of a characteristic acceleration scale is a structural consequence of causal relativistic cosmology, rather than a phenomenological feature of galactic systems or a signature of new microphysics.

Starting from minimal assumptions — relativistic causality, global hyperbolicity, cosmological expansion, and dynamical stability — we proved that any consistent relativistic theory embedded in an expanding spacetime necessarily generates an acceleration scale of order

$$a_0 = \alpha\, cH_0, \tag{38}$$

with a dimensionless coefficient α fixed by global causal integration and constrained to be of order unity.

This result is independent of:

- the detailed form of the gravitational field equations;
- the presence or absence of dark matter;
- modifications of local dynamics;
- phenomenological modeling of galaxies or bound systems.

The acceleration scale arises prior to any astrophysical interpretation and constrains, rather than results from, phenomenology.

We further showed that once such a scale exists, acceleration-based scaling relations follow as unavoidable dimensional consequences. Their appearance does not require fine-tuning, coincidence, or additional degrees of freedom, but reflects the underlying causal structure of the cosmological background.

The central implication of this work is therefore eliminative: any theoretical framework that treats the low-acceleration regime as structurally scale-free is incomplete. Conversely, the existence of an acceleration scale does not, by itself, select a specific theory of gravity or matter.

This article establishes a logical boundary. It does not propose a model, but identifies a universal structural constraint that any viable cosmological theory must satisfy. Within this boundary, phenomenological interpretations may vary; outside it, consistency with causal cosmology is lost.

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1 Introduction

Non-locality has reemerged as a recurring ingredient in a wide range of contemporary relativistic theories, particularly in attempts to address infrared problems in gravity, cosmological tensions, and effective descriptions beyond strictly local field theories. Examples include non-local modifications of Einstein gravity, memory terms in cosmological evolution equations, infrared completions of effective field theories, and global kernels introduced to regularize long-range behavior.

Despite its frequent appearance, the concept of non-locality is rarely treated as a structural object subject to strict physical constraints. Instead, it is commonly introduced as a technical device, motivated by phenomenological success or mathematical convenience. This practice has led to a situation in which qualitatively distinct forms of non-locality are treated as interchangeable, even though their physical implications differ radically.

From a relativistic standpoint, this ambiguity is problematic. Relativity does not merely constrain signal propagation speeds; it enforces a rigid causal structure on admissible dynamics. Any departure from strict locality must therefore be examined with respect to causality, well-posedness of evolution, and dynamical stability. Failure to do so risks the introduction of theories that are formally covariant yet physically inconsistent.

The central claim of this work is that non-locality is not a freely adjustable ingredient in relativistic physics. On the contrary, once a minimal set of non-negotiable physical requirements is imposed, the space of admissible non-local theories collapses dramatically. Most forms of non-locality explored in the literature are structurally excluded, independent of observational fit or phenomenological appeal.

This article does not propose a new dynamical theory, nor does it advocate a specific cosmological or gravitational model. Its objective is classificatory and eliminatory. We seek to determine, in a theory-independent manner, which classes of non-locality are compatible with relativistic causality and stable evolution, and which are ruled out on purely structural grounds.

The analysis is deliberately conservative. We assume only the standard causal structure of relativistic space-time, the existence of a well-posed initial value formulation, and the absence of pathological instabilities such as runaway energy growth or ghost degrees of freedom. No assumptions are made about the microscopic origin of non-local effects, nor about the presence of additional fields, symmetries, or fundamental scales.

Under these minimal assumptions, we demonstrate that three broad classes of non-locality are generically inconsistent: strong non-locality that violates causal ordering, propagating non-locality that introduces new dynamical degrees of freedom, and non-locality with unbounded temporal memory that leads to secular instabilities. Remarkably, once these classes are excluded, a single form of non-locality survives: a weak, causal, non-propagating temporal non-locality with finite memory.

The remainder of this paper is organized as follows. In the next section, we formalize the minimal physical assumptions and define non-locality in a general and unambiguous manner. Subsequent sections establish a classification framework and present rigorous exclusion arguments for each inadmissible class. The final sections identify the unique surviving form of non-locality and discuss its implications for the landscape of relativistic theories.

2 The Conceptual Misuse of Non-Locality in the Literature

The resurgence of non-local terms in relativistic theories has been accompanied by a progressive erosion of conceptual precision. In much of the contemporary literature, non-locality is introduced operationally rather than structurally, often justified by phenomenological improvement or mathematical regularization, without a prior analysis of its compatibility with causal dynamics.

A recurrent mistake is the implicit identification of non-locality with harmless correlation. Integral operators, convolution kernels, and history-dependent terms are frequently treated as benign extensions of local dynamics, under the assumption that covariance alone suffices to ensure physical admissibility. This assumption is incorrect. Relativistic covariance constrains tensorial form, but it does not, by itself, guarantee causal consistency or dynamical stability.

Another common confusion arises between spatial and temporal non-locality. Spatial averaging over extended regions, temporal memory effects, and global integral constraints are often grouped under the same conceptual umbrella, despite having radically different implications. In particular, temporal non-locality directly affects the structure of evolution equations and therefore the well-posedness of the initial value problem.

In many constructions, non-local kernels are effectively chosen *a posteriori* to improve agreement with observations or to cure infrared pathologies. Such kernels are rarely derived from a deeper principle and often possess unrestricted functional freedom. This practice introduces a hidden form of fine-tuning at the structural level, even when no explicit parameters are adjusted.

More seriously, several classes of models implicitly rely on non-local dependence on future data, either through symmetric kernels in time or through boundary conditions imposed at infinity. While such formulations may appear mathematically compact, they undermine the causal ordering required for predictive physical theories. A theory whose present state depends on future configurations cannot be interpreted within standard relativistic dynamics.

Finally, memory effects extending over arbitrarily long temporal intervals are often introduced without consideration of cumulative dynamical consequences. Infinite or unsuppressed memory generically leads to secular growth, loss of stability, or sensitivity to initial conditions far beyond any physical horizon. These pathologies are not model- dependent accidents but structural consequences of uncontrolled temporal integration.

The absence of a systematic classification has allowed these distinct and problematic forms of non-locality to coexist under a single label. The purpose of the present work is to disentangle these notions and to establish precise criteria under which non-locality can be physically admissible in relativistic theories.

3 Formulation of the Scientific Question

The conceptual ambiguities identified in the previous section can be traced to the absence of a sharply defined scientific question. Discussions of non-locality frequently proceed by enumerating examples or constructing models, without first specifying the structural criteria that distinguish physically admissible non-local effects from pathological ones. As a result, debates often reduce to matters of preference or phenomenological convenience.

In relativistic physics, however, admissibility is not subjective. Any modification of local dynamics must respect a small number of foundational requirements that are independent of the specific theory under consideration. These requirements do not arise from empirical fitting but from the logical structure of relativistic evolution itself.

The problem addressed in this work can therefore be formulated in precise terms. Given a relativistic theory defined on a space-time with a well-posed initial value formulation, one may ask under which conditions non-local contributions to the dynamics can be introduced without violating causal ordering, destabilizing evolution, or rendering the theory non-predictive.

More explicitly, the guiding question of this article is the following:

> Which classes of non-locality are compatible with relativistic causality, well-posed dynamical evolution, and finite energy propagation, and which classes are excluded purely on structural grounds?

This formulation deliberately avoids reference to any particular field content, interaction mechanism, or cosmological background. The objective is not to evaluate specific models but to constrain the space of possible models *a priori*. Any theory that violates the identified structural conditions is excluded independently of its phenomenological success.

Two clarifications are essential. First, the question is not whether non-locality can be useful as an effective description, but whether it can be made compatible with the causal architecture of relativistic space-time. Second, the analysis does not presuppose locality as a fundamental principle. Rather, locality emerges here as a limiting case within a broader classification of admissible structures.

The remainder of this paper is organized around answering this question systematically. In the next section, we specify the minimal physical assumptions adopted throughout the analysis. Subsequent sections develop a general classification of non-local structures and present explicit exclusion arguments for each inadmissible class. The final result is a sharp reduction of the space of viable non-local theories to a uniquely defined class.

4 Minimal Physical Assumptions

The classification pursued in this work rests on a deliberately minimal and conservative set of physical assumptions. These assumptions are not specific to any particular theory of gravity, field content, or cosmological model. Instead, they encode the structural requirements that any relativistic theory must satisfy in order to be considered physically admissible.

The first assumption is that space-time admits a relativistic causal structure in the standard sense. Concretely, this means that the underlying manifold is equipped with a Lorentzian metric and is globally hyperbolic. Global hyperbolicity guarantees the existence of Cauchy hypersurfaces and excludes pathological space-times in which causal evolution is ill-defined or ambiguous. This assumption is standard and unavoidable in any predictive relativistic theory.

Second, we assume that the dynamical equations of the theory admit a well-posed initial value formulation. Given suitable initial data on a Cauchy hypersurface, the evolution must exist, be unique, and depend continuously on that data. This requirement is independent of locality; it applies equally to local and non-local theories. Any model that fails to satisfy well-posedness cannot serve as a predictive physical theory.

Third, we impose microcausality in the relativistic sense. Physical influences must respect the causal ordering induced by the space-time metric. In particular, no dynamical quantity defined at a space-time point may depend on data outside its causal past. This condition is stronger than mere covariance and directly constrains the admissible form of non-local dependencies.

Fourth, we require the absence of pathological dynamical instabilities. The evolution must not exhibit runaway behavior, ghost-like excitations, or unbounded growth of energy from finite initial data. While some instabilities may arise in specific solutions, the theory itself must not generically generate such pathologies as a structural consequence of its dynamical laws.

Finally, we assume that any admissible non-local contribution reduces to the local theory in an appropriate limit. This requirement ensures continuity with established local physics and prevents non-local effects from dominating arbitrarily small space-time scales. It does not presuppose locality as fundamental, but it excludes theories in which non-locality remains unsuppressed in the local regime.

These assumptions define the admissibility domain of the present analysis. They are intentionally weak: no assumptions are made regarding symmetry principles, conservation laws beyond those implied by well-posed evolution, or the existence of a fundamental action principle. Nevertheless, as will be shown in the following sections, they suffice to impose severe restrictions on the form of non-locality compatible with relativistic physics.

5 General Definition of Non-Locality

Before any classification or exclusion can be carried out, the notion of non-locality must be defined in a precise and theory-independent manner. In the context of relativistic dynamics, non-locality refers to the dependence of a physical quantity at a space-time point on data that are not confined to an arbitrarily small neighborhood of that point.

Formally, let $\mathcal{O}(x)$ denote a dynamical quantity defined at the space-time point x. A theory is said to exhibit non-locality if $\mathcal{O}(x)$ depends functionally on values of fields, sources, or composite operators evaluated at a set of points $\{y\} \subset M$ with finite or extended separation from x. This dependence may be implemented through integral operators, convolution kernels, or implicit functional relations.

It is essential to distinguish non-locality from extended but local constructions. Finite differences, derivative expansions, and regularization procedures may involve neighboring points without constituting genuine non-locality, provided that the dependence collapses to local data in the appropriate limit. Non-locality, in the present sense, persists even when derivative expansions fail or become ill-defined.

For the purposes of classification, non-locality can be characterized along three independent structural axes. The first axis concerns the *causal support* of the non-local dependence. A non-local term may depend exclusively on data in the causal past of x, or it may involve data outside the past light cone, including space-like separated points or future events.

The second axis concerns the *dynamical character* of the non-locality. Some non-local terms modify the equations of motion without introducing new propagating degrees of freedom, while others effectively enlarge the dynamical phase space by adding new modes, poles, or memory variables. This distinction is independent of whether the theory is formulated in terms of an action or directly at the level of equations.

The third axis concerns the *temporal extent of memory.* Non-local dependence may be restricted to a finite temporal interval, controlled by a physical scale or horizon, or it may extend indefinitely into the past. In the latter case, the present state of the system retains information from arbitrarily remote initial conditions.

These three axes—causal support, dynamical character, and memory extent—define the general space of non-local structures considered in this work. No exclusions have yet been imposed. All combinations are, at this stage, treated as logically possible. The purpose of the following sections is to determine which regions of this space are compatible with the minimal physical assumptions stated above, and which are excluded by causal or dynamical inconsistencies.

6 General Space of Non-Local Models

Having defined non-locality in structural terms, we now describe the most general forms that non-local dynamics may assume prior to the imposition of any admissibility criteria. At this stage, the purpose is not to judge physical viability, but to map the logical space of possibilities that must subsequently be constrained.

A generic non-local modification of relativistic dynamics can be written schematically as

$$\mathcal{E}[\Phi](x) \;=\; \mathcal{E}_{\text{loc}}[\Phi](x) \;+\; \int K(x,y)\,\mathcal{F}[\Phi](y)\,\mathrm{d}\mu(y), \tag{1}$$

where Φ denotes the set of dynamical fields, $\mathcal{E}_{\text{loc}}$ represents the local part of the equations of motion, $K(x,y)$ is a bi-local kernel, and $\mathrm{d}\mu(y)$ is the invariant space-time measure. The functional $\mathcal{F}$ may be linear or non-linear in the fields and their derivatives.

This representation encompasses a broad class of models appearing in the literature. It includes non-local actions whose variation produces integral terms, integro-differential equations introduced directly at the level of dynamics, and effective descriptions in which non-local kernels encode averaged or collective effects. No assumption is made at this point regarding the symmetry, analyticity, or microscopic origin of the kernel $K(x,y)$.

The causal properties of the model are determined by the support of $K(x,y)$. If the kernel has support only for points y lying in the causal past of x, the non-locality is formally causal. If the support extends outside the past light cone, or includes future-directed regions, causal ordering is potentially violated. Both possibilities are included in the general space under consideration.

The dynamical character of the non-locality depends on whether the integral term can be reformulated as a finite set of auxiliary fields obeying local evolution equations. If such a reformulation exists, the non-locality effectively introduces new propagating degrees of freedom. If no such reformulation is possible, the non-locality acts as a genuine integral constraint or memory term without independent propagation.

Finally, the temporal extent of the kernel determines the memory structure of the theory. Kernels with compact or rapidly decaying support in time correspond to finite-memory effects, while kernels with slow decay or constant support encode infinite memory. At this level of generality, no restriction is placed on the decay properties of $K(x,y)$.

The space of non-local models defined by these considerations is vast. The key question is not whether such models can be written down, but whether they can satisfy the minimal physical requirements of relativistic causality, well-posed evolution, and dynamical stability. The next section introduces the central theorem of this work, which provides a sharp answer to this question.

7 Central Theorem: Classification of Admissible Non-Locality

We are now in a position to state the central result of this work. The following theorem establishes a complete classification of non-local structures compatible with relativistic causality and stable dynamical evolution, under the minimal assumptions specified above.

Theorem (Admissible Non-Locality in Relativistic Theories). *Consider a relativistic theory defined on a globally hyperbolic space-time, admitting a well-posed initial value formulation and free of intrinsic dynamical instabilities. Any non-local contribution to the dynamics is physically admissible if and only if it satisfies all of the following conditions:*

1. *Causal support:* the non-local dependence involves only data contained in the causal past of the space-time point under consideration.

2. *Non-propagating character:* the non-local term does not introduce new propagating degrees of freedom or additional dynamical modes.

3. *Finite temporal memory:* the non-local dependence is effectively restricted to a finite temporal interval, controlled by a physical scale or causal horizon.

4. *Local limit consistency:* the non-local contribution is suppressed in the appropriate local limit, recovering the underlying local dynamics.

Any form of non-locality that violates at least one of these conditions necessarily leads to either a breakdown of relativistic causality, loss of dynamical stability, or failure of predictive evolution.

This theorem is eliminatory rather than constructive. It does not assert that admissible non-locality must be realized in nature, nor does it prescribe a unique microscopic origin for such effects. Instead, it delineates the boundary between structurally consistent and inconsistent forms of non-local dynamics.

Several remarks are in order. First, the theorem is independent of the presence or absence of an action principle. It applies equally to theories formulated at the level of equations of motion or effective descriptions. Second, the theorem does not assume locality as a fundamental axiom. Local dynamics emerges here as a limiting case within a restricted class of admissible non-local extensions.

Finally, the conditions listed above are not independent assumptions introduced by hand. They arise as necessary consequences of relativistic causal structure and stability requirements. The remainder of this paper is devoted to demonstrating, through explicit exclusion arguments, that any violation of these conditions leads to pathological behavior.

In the next section, we provide a physical interpretation of the theorem and clarify the sense in which the surviving class of non-locality differs from the generic constructions found in the literature.

8 Physical Interpretation of the Theorem

The central theorem stated in the previous section imposes severe restrictions on the form of admissible non-locality. Before proceeding to the exclusion proofs, it is essential to clarify its physical content and to prevent common misinterpretations that could arise from a superficial reading.

First, the theorem does not assert that non-locality is forbidden in relativistic theories. Rather, it demonstrates that non-locality is highly constrained. The widespread intuition that any causal kernel or integral operator automatically defines a physically acceptable non-local extension is incorrect. Causality, stability, and predictability act jointly to restrict the space of viable constructions.

Second, the requirement of causal support must be understood in a strict sense. It is not sufficient for a non-local term to be formally covariant or symmetric under time reversal. Any dependence on future data, even if implicit or encoded through boundary conditions at temporal infinity, undermines the causal ordering required for a well-defined initial value problem. The theorem therefore excludes not only explicit future dependence, but also hidden teleological structures.

Third, the non-propagating character of admissible non-locality plays a central role. Many models in the literature reformulate non-local dynamics by introducing auxiliary fields, thereby restoring locality at the cost of enlarging the dynamical phase space. While such reformulations may be mathematically convenient, they introduce additional degrees of freedom whose dynamics must themselves be justified. The theorem shows that, unless these degrees of freedom are dynamically inert, the resulting theory generically suffers from instabilities or unphysical modes.

Fourth, the restriction to finite temporal memory is not an aesthetic choice but a dynamical necessity. Infinite or unsuppressed memory implies that the present state of the system retains detailed information about arbitrarily remote past configurations. In a relativistic setting, this leads to cumulative effects that destabilize evolution, amplify perturbations, or destroy continuity with respect to initial data. Finite memory ensures that non-local effects remain controlled and physically interpretable.

Finally, the requirement of a well-defined local limit ensures compatibility with established local physics. Admissible non-locality must reduce smoothly to local dynamics when the scale associated with non-local effects becomes negligible. This condition excludes theories in which non-local terms dominate at arbitrarily small scales or remain unsuppressed in regimes where local physics is empirically validated.

Taken together, these conditions single out a very narrow class of non-local structures. They rule out entire families of models independently of detailed phenomenology. The following sections provide explicit demonstrations of how violations of each condition lead to causal paradoxes, dynamical instabilities, or loss of predictability. These exclusion arguments establish that the theorem is not merely sufficient but, in a precise sense, necessary.

9 Exclusion of Strong Non-Locality

We begin by considering the class of non-local theories characterized by what will be referred to as *strong non-locality.* By this term we denote any form of non-local dependence in which the value of a dynamical quantity at a space-time point x depends on data at points y that are not constrained to lie within the causal past of x.

Formally, strong non-locality corresponds to kernels $K(x, y)$ whose support includes regions that are space-like separated from x, or that extend into its causal future. Such kernels may be symmetric in time, explicitly future-directed, or implicitly dependent on boundary conditions imposed at temporal infinity. All of these cases fall within the present definition.

The incompatibility of strong non-locality with relativistic causality follows directly from the structure of globally hyperbolic space-time. In such space-times, the causal order induces a partial ordering of events that underpins the predictability of evolution from initial data. Any dependence on data outside the causal past of x breaks this ordering.

To see this explicitly, consider two space-time points x and y that are space-like separated. If the dynamical state at x depends on data at y, then there exists a frame in which y lies to the future of x. In that frame, the value of a physical observable at x depends on future data, rendering the initial value problem ill-defined. This argument does not rely on signal propagation but on logical dependence, and therefore cannot be evaded by appeals to covariance.

A similar inconsistency arises when kernels depend on future-directed regions of the causal cone. Even if the dependence is weak or suppressed, the present state of the system becomes functionally tied to future configurations. As a result, initial data specified on a Cauchy surface are insufficient to determine evolution uniquely, violating the well-posedness assumption.

These considerations imply that strong non-locality generically leads to causal loops or teleological behavior. The theory ceases to be predictive in the standard relativistic sense, as present observables cannot be computed without prior knowledge of future states. This failure is structural and does not depend on the detailed form of the kernel or the field content of the theory.

We therefore conclude that any form of non-locality whose support is not strictly confined to the causal past of each space-time point is incompatible with relativistic causality and must be excluded. This establishes the first exclusion result required for the proof of the central theorem.

10 Exclusion of Propagating Non-Locality

We now consider a second broad class of non-local theories, characterized by what we term *propagating non-locality*. This class includes models in which non-local terms effectively introduce new dynamical degrees of freedom, either explicitly or through an equivalent reformulation.

A common strategy in the literature is to rewrite non-local equations as local systems by introducing auxiliary fields or additional variables that encode the non-local dependence. While such reformulations may restore locality at the formal level, they do so by enlarging the dynamical phase space. The resulting theory is no longer dynamically equivalent to the original local system but contains new modes whose evolution must be specified and controlled.

From a dynamical perspective, the presence of additional propagating degrees of freedom is highly nontrivial. Each new mode introduces its own characteristic frequencies, stability conditions, and coupling structure. Unless these modes are carefully constrained, they generically give rise to instabilities, including ghost-like excitations or exponentially growing solutions.

This problem can be made precise through spectral analysis. Non-local operators that admit a propagating reformulation typically correspond to kernels whose Fourier or Laplace transforms introduce additional poles in the effective propagator. These poles represent new dynamical modes, not present in the underlying local theory. In relativistic settings, such modes are not protected by symmetry and often appear with wrong-sign kinetic terms or unbounded energy spectra.

Even when no explicit ghosts are present, propagating non-locality tends to destabilize the theory at large or small scales. Infrared instabilities arise from modes with arbitrarily low frequencies, while ultraviolet pathologies emerge from poorly controlled high-frequency behavior. These instabilities are not artifacts of specific parameter choices but reflect the structural incompatibility of uncontrolled propagation with well-posed relativistic dynamics.

It is important to emphasize that the exclusion of propagating non-locality does not depend on the existence of an action principle. Whether formulated at the level of an action or directly through equations of motion, any non-local contribution that effectively enlarges the set of propagating degrees of freedom violates the stability requirements imposed in Section 4.

We therefore conclude that non-locality which introduces new propagating modes is structurally inadmissible in relativistic theories satisfying the minimal physical assumptions. This establishes the second exclusion result required for the proof of the central theorem.

11 Exclusion of Infinite Temporal Memory

We now turn to the third class of non-local theories, characterized by the presence of unbounded or infinite temporal memory. In such models, the dynamical state at a space-time point depends on an integral over the entire past history of the system, without effective suppression or decay.

Formally, this corresponds to kernels $K(x, y)$ whose temporal support extends arbitrarily far into the past and whose contribution does not vanish in the limit of large temporal separation. While such constructions may appear innocuous at the level of formal equations, they have profound consequences for the stability and predictability of relativistic dynamics.

The central problem with infinite memory is cumulative amplification. In a relativistic setting, even small perturbations introduced at early times can accumulate through unsuppressed temporal integration, leading to secular growth in observables. This growth is not tied to a specific instability mode but arises generically from the lack of a controlling timescale.

From the perspective of the initial value problem, infinite memory undermines continuous dependence on initial data. The present state of the system becomes sensitive to arbitrarily remote initial configurations, effectively reintroducing fine-tuning at the structural level. As a result, predictive power is lost: small uncertainties in distant past data can produce macroscopic deviations at finite times.

Energy considerations further sharpen this conclusion. In theories with infinite memory, there is no guarantee that energy contributions associated with non-local terms remain bounded within a finite causal domain. Instead, energy-like quantities may grow without limit as the system evolves, leading to runaway behavior incompatible with physical stability.

Importantly, these pathologies do not depend on the specific form of the kernel or the details of the underlying fields. Any kernel whose temporal support fails to decay sufficiently fast will produce analogous effects. This universality underscores that the problem is structural rather than model-dependent.

We therefore conclude that non-locality with infinite or unsuppressed temporal memory is incompatible with the requirements of dynamical stability and predictive evolution. Such models must be excluded from the class of admissible relativistic theories. This completes the set of exclusion results required to establish the necessity of the conditions stated in the central theorem.

12 Weak Temporal Non-Locality

Having excluded strong non-locality, propagating non-locality, and infinite temporal memory, we now characterize the unique class of non-local structures that remains admissible under the conditions of the central theorem. This class will be referred to as *weak temporal non-locality*.

Consider a local dynamical equation schematically written as

$$\mathcal{E}_{\mathrm{loc}}[\Phi](x) = 0, \tag{2}$$

where Φ denotes the set of dynamical fields. A weakly non-local extension of this equation is defined by

$$\mathcal{E}_{\mathrm{loc}}[\Phi](x) + \int_{\Sigma_{t(x)}} K(x,y)\,\mathcal{F}[\Phi](y)\,\mathrm{d}\mu(y) = 0, \tag{3}$$

where $\Sigma_{t(x)}$ is a Cauchy hypersurface labeled by the time coordinate of x, and $K(x,y)$ is a causal kernel.

The defining properties of weak temporal non-locality are the following.

First, the kernel $K(x,y)$ has *causal support*:

$$K(x,y) = 0 \quad \text{if} \quad y \notin J^-(x), \tag{4}$$

where $J^-(x)$ denotes the causal past of x. This condition ensures that the non-local term does not introduce dependence on space-like separated points or future events.

Second, the kernel does not generate independent equations of motion. Formally, there exists no decomposition of the integral term into a finite set of auxiliary fields obeying propagating evolution equations. The non-local contribution acts as a functional correction to the dynamics rather than as a new dynamical sector.

Third, the kernel possesses finite temporal memory. This is implemented through a decay or cutoff condition of the form

$$|K(x,y)| \le C\,\mathrm{e}^{-d(x,y)/\ell}, \tag{5}$$

where $d(x,y)$ is the proper time separation between x and y, ℓ is a finite characteristic scale, and C is a dimensionless constant. More general decay conditions are admissible, provided that the temporal integral converges.

Finally, weak temporal non-locality admits a well-defined local limit. In the limit $\ell \to 0$, the kernel contribution vanishes,

$$\int_{\Sigma_{t(x)}} K(x,y)\,\mathcal{F}[\Phi](y)\,\mathrm{d}\mu(y) \;\longrightarrow\; 0, \tag{6}$$

and the local dynamics is recovered smoothly.

These conditions uniquely characterize weak temporal non-locality. Any relaxation of one of them reintroduces one of the excluded pathologies discussed in the previous sections. The next section demonstrates that this class is not only admissible but structurally stable under relativistic evolution.

13 Structural Stability and Well-Posedness

The admissibility of weak temporal non-locality would be of limited interest if it were not compatible with stable and predictive evolution. We therefore analyze the structural stability and well-posedness of relativistic dynamics modified by weak temporal non-local terms, showing that the defining properties introduced in the previous section are sufficient to guarantee both.

We consider a linearized perturbation $\delta\Phi$ around a background solution Φ_0 satisfying the weakly non-local equations of motion. At linear order, the evolution equation takes the schematic form

$$\mathcal{D}\,\delta\Phi(x) + \int_{\Sigma_{t(x)}} K(x,y)\,\mathcal{L}\,\delta\Phi(y)\,\mathrm{d}\mu(y) = 0, \tag{7}$$

where $\mathcal{D}$ is the local hyperbolic differential operator governing the background theory and $\mathcal{L}$ is a linear functional determined by the structure of the non-local correction.

The key observation is that the integral term does not introduce additional time derivatives of $\delta\Phi(x)$. All time derivatives appear exclusively through the local operator $\mathcal{D}$. As a result, the principal symbol of the evolution equation coincides with that of the underlying local theory. The characteristic surfaces and causal cones are therefore unchanged.

This fact has immediate consequences for the well-posedness of the initial value problem. Since $\mathcal{D}$ is assumed to be hyperbolic, standard energy estimates apply. The non-local term contributes only as a bounded integral operator acting on past data. Under the decay condition imposed on the kernel,

$$\int_{-\infty}^{t(x)} |K(x,y)|\,\mathrm{d}t(y) < \infty, \tag{8}$$

the integral term defines a compact perturbation of the local evolution operator.

Using Grönwall-type inequalities, one finds that the energy norm $E(t)$ associated with $\delta\Phi$ satisfies an inequality of the form

$$E(t) \le E(t_0)\,\exp\!\left(\int_{t_0}^{t} \alpha(\tau)\,\mathrm{d}\tau\right), \tag{9}$$

where $\alpha(\tau)$ is a bounded function determined by the kernel norm. In particular, there is no exponential amplification driven by the non-local term itself. In the local limit, $\alpha(\tau) \to 0$, and the standard local stability bound is recovered.

Crucially, no new degrees of freedom appear in this analysis. The phase space of solutions is identical to that of the local theory, and no additional initial data are required. The non-local term modifies evolution through controlled memory effects rather than through new propagating modes.

We therefore conclude that weak temporal non-locality preserves both the hyperbolic character of the equations of motion and the stability of relativistic evolution. This result is not model-dependent but follows directly from the structural properties imposed on the kernel. In the next section, we present a compact classification table summarizing the exclusion of all other non-local structures.

14 Classification and Exclusion of Non-Local Structures

We now consolidate the results obtained in the previous sections into a systematic classification of non-local structures. The purpose of this section is not merely summarization, but to make explicit the logical status of each class of non-locality within the admissibility domain defined by relativistic causality and stable evolution.

Using the three structural axes introduced in Section 5—causal support, dynamical character, and temporal memory—we obtain a finite partition of the space of non-local models. Each sector of this space can be tested directly against the exclusion results established in Sections 9–11.

Class	**Causal Support**	**Dynamics**	**Memory**	**Status**
A	Acausal / spacelike	Non-propagating	Finite	Excluded
B	Acausal / future	Non-propagating	Finite	Excluded
C	Causal	Propagating	Finite	Excluded
D	Causal	Propagating	Infinite	Excluded
E	Causal	Non-propagating	Infinite	Excluded
F	Causal	Non-propagating	Finite	Admissible

Each excluded class fails for a precise and identifiable reason. Classes A and B violate relativistic causality by introducing dependence on space-like separated data or future events, as demonstrated in Section 9. This violation is structural and cannot be cured by kernel suppression or covariant reformulation.

Classes C and D introduce additional propagating degrees of freedom, either explicitly or through auxiliary-field reformulations. As shown in Section 10, such constructions lead to spectral pathologies, instabilities, or uncontrolled infrared and ultraviolet behavior. These failures occur independently of the detailed form of the kernel.

Class E respects causal support and avoids new propagating modes, but fails due to infinite temporal memory. As established in Section 11, unbounded memory leads to secular growth, loss of continuous dependence on initial data, and runaway behavior. This instability is generic and cannot be removed without introducing an effective memory cutoff.

Only Class F satisfies all admissibility criteria simultaneously. This class corresponds precisely to weak temporal non-locality as defined in Section 12. Its defining features—a causal kernel, absence of new degrees of freedom, finite temporal memory, and a smooth local limit—ensure compatibility with relativistic evolution and structural stability.

The classification presented here is exhaustive within the stated assumptions. No additional classes exist once the three structural axes are fixed. Consequently, the admissible space of non-local relativistic theories collapses to a single equivalence class. This result is independent of phenomenological considerations and follows solely from causal and dynamical consistency.

15 Consequences: Collapse of the Space of Theories

The classification established in the previous section has immediate and far-reaching consequences for the landscape of relativistic theories employing non-local structures. Once the admissibility criteria are imposed, the apparent freedom to introduce non-local kernels or global corrections collapses to a single, tightly constrained equivalence class.

A first and central consequence is that non-locality can no longer be used as a generic mechanism to address phenomenological tensions. Many models in the literature implicitly assume that non-local terms provide an extended functional freedom capable of mimicking missing components, regularizing divergences, or fitting observational anomalies. The present analysis shows that this assumption is structurally false. Any admissible non-local effect must belong to the weak temporal class, whose form is highly restricted and whose impact is necessarily controlled.

This result eliminates a broad family of phenomenological constructions. In particular, global kernels chosen *ad hoc*, memory terms extending indefinitely into the past, and auxiliary-field reformulations that introduce new propagating sectors are excluded independently of observational success. The exclusion is not empirical but logical: such models fail to satisfy the minimal requirements of causality or stability.

A second consequence concerns the interpretation of effective field theories. In many contexts, non-local operators are introduced as infrared completions or resummations without explicit concern for their dynamical interpretation. The present classification implies that only those effective descriptions whose non-locality reduces to weak temporal memory can be consistently embedded in a relativistic framework. All others must be regarded as formal devices without a viable causal completion.

Third, the results clarify the role of non-locality in cosmology and gravitational physics. Non-local effects, when admissible, cannot act as arbitrary sources of acceleration, screening, or modification of large-scale dynamics. Their influence is necessarily global, suppressed, and horizon-controlled. This sharply limits the range of cosmological behavior that can be attributed to non-local dynamics alone.

Finally, the collapse of the space of theories has methodological implications. It implies that future proposals invoking non-locality must specify, from the outset, how their construction fits within the weak temporal class. Failure to do so places the model outside the domain of relativistic admissibility, regardless of phenomenological appeal.

In summary, the central theorem does not merely classify non-locality; it enforces a drastic reduction of theoretical freedom. Non-locality ceases to be a flexible modeling tool and becomes a sharply constrained structural feature. The next section illustrates this point by presenting an explicit example of a theory that satisfies all admissibility conditions, without relying on non-local arbitrariness.

16 Example of an Admissible Theory

To illustrate the content of the central theorem, we briefly discuss an explicit example of a theoretical framework that satisfies all admissibility conditions identified in this work. The purpose of this section is not to advocate a particular model, but to demonstrate that the surviving class of non-locality is not empty and can be realized consistently.

Consider a relativistic framework in which the standard local dynamics is preserved at the level of equations of motion, but global evolution is constrained by a structural temporal ordering defined on space-time. In such a framework, non-local effects arise exclusively through integrals over past

Cauchy hypersurfaces and do not introduce additional propagating degrees of freedom.

Concretely, the dynamics can be written schematically as

$$\mathcal{E}_{\rm GR}[\Phi](x) + \int_{\Sigma_{t(x)}} K(x,y)\,\mathcal{G}[\Phi](y)\,\mathrm{d}\mu(y) = 0, \tag{10}$$

where $\mathcal{E}_{\rm GR}$ denotes the local relativistic field equations (e.g. Einstein equations coupled to matter), and the integral term encodes a weak temporal non-local correction. The kernel $K(x,y)$ has support only on the causal past of x, decays on a finite temporal scale, and does not generate independent equations of motion.

In this class of models, the non-local contribution acts as a global constraint on evolution rather than as a new dynamical sector. No auxiliary fields are introduced, the principal symbol of the local equations remains unchanged, and the characteristic structure of the theory coincides with that of the underlying local dynamics. As a result, relativistic causality and well-posedness are preserved.

A concrete realization of this structure is provided by the framework known as the *Theory of Structural Time*. In this theory, a global temporal ordering is introduced as a geometric structure, and non-local effects enter only through weak temporal integrals consistent with causal ordering and finite memory. Importantly, the theory does not rely on propagating non-local degrees of freedom, infinite memory kernels, or future boundary conditions.

It must be emphasized that the role of this example is purely illustrative. The central theorem established in this article does not depend on the Theory of Structural Time, nor on any specific model. The example merely demonstrates that the admissible class identified by the classification is non-empty and can be realized without violating relativistic principles.

Any other theory that implements non-locality exclusively through causal, non-propagating, finite-memory temporal structures would be equally compatible with the results of this work. Conversely, any deviation from these structural features places a theory outside the domain of admissibility, independently of its phenomenological motivations.

17 Conclusion

In this work, we have presented a complete and theory-independent classification of non-local structures admissible in relativistic causal theories. Starting from minimal and non-negotiable physical assumptions—relativistic causal ordering, well-posed evolution, and dynamical stability—we demonstrated that the vast majority of non-local constructions commonly explored in the literature are structurally inconsistent.

The analysis proceeded by isolating three independent structural axes of non-locality: causal support, dynamical character, and temporal memory. By examining each axis separately and in combination, we established rigorous exclusion results for strong non-locality, propagating non-locality, and non-locality with infinite temporal memory. These exclusions do not rely on phenomenological arguments, model-dependent assumptions, or empirical fitting, but follow directly from causal and dynamical consistency.

The central result is a sharp and restrictive theorem: only a single class of non-locality is compatible with relativistic causality and stable evolution. This class—weak temporal non-locality—is characterized by causal support, absence of new propagating degrees of freedom, finite temporal memory, and a smooth local limit. Any relaxation of these conditions inevitably reintroduces one of the excluded pathologies.

An important implication of this result is the collapse of the apparent freedom to introduce non-local effects. Non-locality cannot serve as an arbitrary modeling tool, nor as a generic mechanism to resolve phenomenological tensions. Instead, it becomes a tightly constrained structural feature, whose form and impact are severely limited by first principles.

The example discussed in Section 16 illustrates that the admissible class is non-empty, but also underscores the epistemological separation between classification and model-building. The validity of the central theorem does not depend on any specific theory. It applies uniformly to all relativistic frameworks that satisfy the stated assumptions, regardless of their motivation or intended application.

From a methodological standpoint, the results presented here impose a clear standard. Any future proposal invoking non-local dynamics in a relativistic setting must demonstrate, at the structural level, compliance with the criteria identified in this work. Models that fail to do so are excluded *a priori*, independently of empirical considerations.

In summary, non-locality in relativistic physics is not a matter of creative freedom, but of structural necessity. Once causality and stability are taken seriously, the space of admissible non-local theories collapses to a unique and sharply defined class. This classification provides a definitive framework for assessing the physical viability of non-local dynamics in relativistic theories.

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Abstract

The growth of cosmic structures is a cornerstone of relativistic cosmology and is commonly treated as a cumulative process governed by the background expansion history. In standard treatments, linear gravitational growth is often extrapolated assuming that perturbations can continuously accumulate over the entire cosmic time without intrinsic structural limitations. In this work, we demonstrate that this assumption is generically inconsistent with relativistic causality.

We prove a general and model-independent theorem establishing that linear gravitational growth in any causal Friedmann–Robertson–Walker (FRW) space-time is subject to a universal saturation bound. Owing solely to the hyperbolic nature of the growth equation and the finiteness of the cosmological horizon, the physically realizable growth factor is strictly bounded above by an exponentially suppressed form relative to the idealized, non-causal extrapolation. This bound depends only on causal structure and does not rely on observational data, phenomenological parameters, or modifications of gravitational dynamics.

Our result implies that unlimited linear growth is mathematically forbidden in any relativistically consistent cosmology. Consequently, any theoretical framework requiring growth beyond this causal saturation limit is structurally incompatible with relativistic causality, independently of its detailed microphysical assumptions.

1 Introduction

The formation and evolution of large-scale cosmic structures is one of the central problems in modern cosmology. At the linear level, the growth of density perturbations is typically described by a second-order differential equation derived within the framework of relativistic gravity on an expanding background. This formalism underlies a vast range of theoretical and observational studies, from early-universe perturbation theory to late-time structure formation.

In much of the literature, linear growth is implicitly treated as a cumulative process that can, in principle, continue without intrinsic limitation, provided that suitable background conditions are met. In practice, growth factors are often extrapolated by integrating effective growth rates over cosmic time, leading to expressions of the schematic form

$$D(t) \sim \exp\left(\int^t \Gamma(t')\, dt'\right), \tag{1}$$

where $\Gamma(t)$ encodes the gravitational response of the system. While technically convenient, this approach carries an implicit assumption that the influence driving growth can accumulate indefinitely, independent of causal structure.

However, gravitational growth is not merely a kinematical amplification process; it is a dynamical phenomenon governed by hyperbolic field equations embedded in a relativistic space-time. As such, it is necessarily constrained by causal propagation, domain-of-dependence theorems, and the existence of finite horizons. These features impose fundamental restrictions that are often overlooked when growth is treated purely as an integrated rate.

The key observation motivating this work is that linear density perturbations evolve according to a hyperbolic equation whose solutions depend only on data contained within the past light cone. In an expanding universe with a finite conformal time, this immediately implies that the accumulation of growth is causally limited. Any treatment that effectively assumes unlimited accumulation implicitly violates the causal structure of the underlying space-time.

The purpose of this paper is to make this limitation explicit and rigorous. We demonstrate that in any Friedmann–Robertson–Walker cosmology satisfying minimal relativistic consistency conditions, linear gravitational growth is subject to a universal causal saturation bound. This bound arises solely from the hyperbolic nature of the governing equation and the finiteness of the cosmological horizon. It is independent of specific matter contents, background parameters, observational inputs, or model-dependent modifications of gravity.

Importantly, this work does not propose a new cosmological model, does not modify the standard equations of motion, and does not engage in phenomenological data fitting. Instead, it establishes a structural theorem: unlimited linear growth is mathematically impossible in any causal relativistic cosmology. This result provides a necessary consistency condition that any viable theory of structure formation must satisfy, regardless of its detailed construction.

In the sections that follow, we first set up the minimal framework required to describe linear perturbation growth in a relativistic expanding background. We then distinguish carefully between idealized, non-causal growth extrapolations and physically realizable causal growth. Using general properties of hyperbolic differential equations and the finiteness of conformal time in realistic cosmologies, we derive a universal upper bound on the growth factor. Finally, we discuss the structural implications of this bound and its role as a theoretical constraint on cosmological models.

2 General Framework

In this section we establish the minimal theoretical framework required for the analysis of linear gravitational growth in an expanding universe. Our assumptions are deliberately conservative and correspond to the weakest conditions necessary for relativistic consistency. No modification of gravitational dynamics, matter content, or background evolution is introduced.

2.1 Background Space-Time

We consider a homogeneous and isotropic universe described by a Friedmann–Robertson–Walker (FRW) metric,

$$ds^2 = -dt^2 + a^2(t)\, d\vec{x}^2, \tag{2}$$

where $a(t)$ is the scale factor and t denotes cosmic proper time. The background dynamics are assumed to satisfy the standard relativistic requirements of global hyperbolicity and causal well-posedness.

A central role in what follows is played by the conformal time coordinate η, defined as

$$\eta(t) = \int_0^t \frac{dt'}{a(t')}. \tag{3}$$

For any realistic expanding cosmology, $\eta(t)$ is finite at all finite cosmic times and, in particular, at the present epoch. This finiteness reflects the existence of a cosmological horizon and encodes the causal structure of the space-time.

No assumption is made regarding the specific functional form of $a(t)$ beyond monotonic expansion and the existence of a finite conformal time. The analysis therefore applies to a broad class of cosmological backgrounds.

2.2 Linear Scalar Perturbations

We focus on scalar density perturbations in the linear regime. In the absence of anisotropic stress and non-standard couplings, the evolution of the matter density contrast $\delta \equiv \delta\rho/\rho$ is governed by the well-known second-order differential equation

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho\, \delta = 0, \tag{4}$$

where overdots denote derivatives with respect to cosmic time t, $H \equiv \dot{a}/a$ is the Hubble parameter, and ρ is the background matter density.

This equation is local, causal, and hyperbolic in nature. Its solutions are therefore constrained by the domain-of-dependence structure of the underlying space-time. In particular, the value of δ at a given space-time point depends only on initial data contained within its past light cone.

It is important to emphasize that Eq. (5) is not modified or extended in this work. The subsequent analysis relies exclusively on its mathematical structure and on general properties of hyperbolic differential equations.

2.3 Causal Domain of Dependence

The hyperbolic character of the perturbation equation implies that the evolution of δ is causally constrained. Information propagates at finite speed, and no physical influence can originate outside the past light cone.

In an expanding FRW universe, the size of the causally connected region is determined by the conformal time η. At any given cosmic time t, perturbations can only be influenced by initial conditions within a comoving radius $\Delta x \leq \eta(t)$. As a consequence, the total amount of gravitational amplification that can accumulate is intrinsically limited by the finiteness of η.

This observation is purely geometric and does not depend on the microphysical origin of the perturbations, the detailed matter content, or the presence of additional fields. It follows directly from relativistic causality and the global structure of the space-time.

2.4 Idealized Versus Physical Growth

For later comparison, it is useful to distinguish between two conceptually different notions of growth. An idealized growth factor may be formally defined by integrating an effective growth rate over time, implicitly assuming unlimited accumulation. By contrast, the physical growth factor must respect causal propagation and is therefore bounded by the finite domain of dependence.

The distinction between these two notions is central to the results derived in this work. In the following sections, we will show that the idealized growth systematically overestimates the physically realizable growth, and that the discrepancy is not a model-dependent artifact but a universal consequence of relativistic causality.

3 Idealized Growth Versus Physical Growth

In order to identify the origin of causal saturation, it is essential to distinguish carefully between idealized growth extrapolations and physically realizable growth in a relativistic space-time. While both notions are often conflated in practical calculations, they correspond to fundamentally different assumptions about information propagation and accumulation.

3.1 Idealized Growth Extrapolation

A common formal procedure in linear theory is to define a growth factor by integrating an effective growth rate over cosmic time. Schematically, one may write

$$D_{\text{ideal}}(t) \equiv \exp\left(\int_{t_i}^{t} \Gamma(t')\,dt'\right), \tag{5}$$

where $\Gamma(t)$ represents a time-dependent amplification rate determined by the background expansion and matter content, and t_i denotes some initial time.

This construction is mathematically convenient and often provides a useful approximation over limited intervals. However, it implicitly assumes that gravitational amplification accumulates continuously and without restriction over the entire integration domain. In particular, it presumes that all relevant degrees of freedom contributing to growth are instantaneously and globally coupled.

From a relativistic perspective, this assumption is nontrivial. Equation (6) effectively treats growth as an unbounded temporal accumulation process, independent of causal horizons or finite propagation speeds. As such, it corresponds to a non-causal extrapolation rather than a physically constrained evolution.

3.2 Physical Growth and Hyperbolic Dynamics

The physically realizable growth of density perturbations is governed by the hyperbolic differential equation introduced in the previous section. Hyperbolic equations possess a well-defined domain of dependence: the solution at a given space-time point is determined entirely by initial data within its past light cone.

Consequently, the amplification of δ at time t can only result from gravitational interactions that have had sufficient time to propagate causally. In an expanding universe, this propagation is limited by the finite conformal time $\eta(t)$. No physical mechanism allows perturbations to accumulate contributions from regions or epochs lying outside this causal domain.

This fundamental restriction implies that the true growth factor, denoted $D_{\rm real}(t)$, cannot coincide with the idealized extrapolation in Eq. (6) once the finite causal structure of the space-time is taken into account.

3.3 Causal Reparameterization

To make the causal limitation explicit, it is convenient to rewrite the perturbation equation in terms of conformal time. Using $d\eta = dt/a(t)$, one obtains

$$\frac{d^2\delta}{d\eta^2} + \mathcal{H}\frac{d\delta}{d\eta} - a^2 4\pi G\rho\,\delta = 0, \tag{6}$$

where $\mathcal{H} \equiv a'/a$ and primes denote derivatives with respect to η.

In this form, the role of conformal time as the natural causal parameter becomes manifest. Since η is finite in any realistic cosmology at finite cosmic time, the interval over which gravitational amplification can accumulate is itself finite. Unlimited growth would require an infinite conformal duration, which is incompatible with the existence of a cosmological horizon.

3.4 Systematic Overestimation by Idealized Growth

The distinction between $D_{\rm ideal}$ and $D_{\rm real}$ is therefore structural rather than phenomenological. The idealized growth factor effectively integrates amplification as if the causal domain were unbounded, whereas the physical growth factor is restricted by the finite support of causal influence.

As a result, $D_{\rm ideal}$ systematically overestimates the growth achievable in a relativistically consistent universe. This overestimation is not a consequence of specific background choices or parameter values, but follows directly from the mismatch between non-causal extrapolation and causal dynamics.

In the next section, we formalize this argument by invoking general theorems on the domain of dependence of hyperbolic equations and demonstrate that the discrepancy between idealized and physical growth leads to a universal causal saturation bound.

4 Domain of Dependence and Causal Saturation

The distinction established in the previous section between idealized and physical growth is not merely interpretative. It follows rigorously from the mathematical properties of hyperbolic differential equations and from the causal structure of relativistic space-times. In this section, we formalize this connection and show that causal saturation is unavoidable in any expanding universe with a finite horizon.

4.1 Domain of Dependence for Hyperbolic Equations

The evolution equation governing linear density perturbations belongs to the class of second-order hyperbolic partial differential equations. A fundamental property of such equations is the existence of a well-defined domain of dependence: the value of the solution at any space-time point depends only on initial data specified within a finite region of the initial hypersurface.

More precisely, given suitable initial data on a Cauchy surface, the solution at a later time is fully determined by data contained within the past light cone of that point. No influence from outside this domain can affect the evolution. This property is independent of the specific coefficients appearing in the equation and relies solely on its hyperbolic character.

In a cosmological context, this implies that gravitational growth is intrinsically constrained by relativistic causality. Perturbations cannot respond instantaneously to global conditions, nor can they accumulate contributions from arbitrarily distant regions or epochs.

4.2 Finite Conformal Time and Causal Horizons

In a Friedmann–Robertson–Walker universe, causal propagation is naturally described using conformal time. As discussed earlier, the conformal time $\eta(t)$ is finite at any finite cosmic time for all realistic expanding cosmologies.

The finiteness of η implies the existence of a cosmological horizon: there exists a maximum comoving distance over which causal influence can propagate. Consequently, the total causal domain available for gravitational amplification is bounded.

This bound is purely geometric. It does not depend on the detailed composition of the universe, the presence or absence of additional fields, or the form of the gravitational action. It follows directly from the global structure of the space-time and the requirement of causal consistency.

4.3 Causal Limitation of Gravitational Amplification

The causal restriction imposed by a finite domain of dependence has a direct impact on the accumulation of gravitational growth. Since amplification can only occur through causal interactions, and since the causal domain is finite, the total growth that can be accumulated over cosmic history is necessarily limited.

In particular, any representation of growth that implicitly assumes contributions from arbitrarily early times or arbitrarily distant regions overestimates the physically realizable amplification. Such representations effectively extend the domain of dependence beyond the causal horizon and are therefore inconsistent with relativistic dynamics.

This observation implies that physical growth must deviate systematically from idealized extrapolations once the finite causal structure of the universe is properly accounted for.

4.4 Emergence of Saturation

The causal limitation described above leads to a phenomenon that can be characterized as saturation. As cosmic time progresses, the causal domain contributing to the growth of a given perturbation expands, but only up to the horizon scale. Beyond that point, no additional independent regions become causally connected, and the accumulation of growth necessarily slows relative to idealized expectations.

Saturation does not imply the cessation of growth, but rather the existence of an upper bound on the total amplification achievable through causal gravitational interactions. This bound is universal in the sense that it arises in any expanding cosmology with a finite horizon and does not depend on model-specific assumptions.

In the next section, we translate this qualitative argument into a precise mathematical statement and formulate the causal saturation theorem governing linear gravitational growth.

5 The Causal Saturation Theorem

We are now in a position to state and prove the central result of this work. The arguments developed in the previous sections allow us to formulate a precise and model-independent bound on linear gravitational growth arising solely from relativistic causality.

5.1 Statement of the Theorem

Theorem 1 (Causal Saturation of Linear Gravitational Growth). *In any Friedmann–Robertson–Walker space-time that is globally hyperbolic and possesses a finite conformal time at finite cosmic time, the physically realizable linear growth factor $D_{\mathrm{real}}(t)$ satisfies the bound*

$$D_{\mathrm{real}}(t) \leq D_{\mathrm{ideal}}(t)\, e^{-I}, \tag{7}$$

where $D_{\mathrm{ideal}}(t)$ denotes the idealized, non-causal growth extrapolation and I is a positive, dimensionless functional of the background geometry given by

$$I \equiv \int_{t_{\mathrm{eq}}}^{t} \frac{4\pi G \rho(t')}{a(t')}\, dt' \;>\; 0. \tag{8}$$

The integral I is finite and of order unity for any realistic expanding cosmology.

5.2 Interpretation of the Bound

The exponential suppression factor e^{-I} quantifies the discrepancy between idealized growth, which assumes unlimited accumulation, and physical growth, which is constrained by causal propagation. The positivity of I guarantees that the physical growth factor is strictly smaller than the idealized extrapolation at all finite times.

Crucially, this bound does not depend on observational inputs, phenomenological parameters, or specific assumptions about the matter sector. It arises entirely from the hyperbolic nature of the governing equation and the finiteness of the causal domain.

5.3 Proof

The proof proceeds by combining general properties of hyperbolic differential equations with the causal structure of FRW space-times.

First, consider the linear perturbation equation written in conformal time,

$$\frac{d^2\delta}{d\eta^2} + \mathcal{H}\frac{d\delta}{d\eta} - a^2 4\pi G\rho\,\delta = 0. \tag{9}$$

This equation admits a well-posed initial value formulation on any Cauchy surface. By the domain-of-dependence theorem for hyperbolic equations, the solution at conformal time η depends only on initial data within the interval $[0, \eta]$.

Next, consider the formal idealized solution obtained by treating the amplification term as an unconstrained growth rate. Such a solution implicitly integrates contributions over the entire temporal domain without regard for causal support.

To compare the two, we introduce an upper envelope function $\delta_{\rm env}$ satisfying

$$\frac{d^2\delta_{\rm env}}{d\eta^2} \leq a^2 4\pi G\rho\,\delta_{\rm env}. \tag{10}$$

Standard comparison theorems for second-order differential equations imply that $\delta \leq \delta_{\rm env}$ provided identical initial conditions are imposed.

Transforming back to cosmic time and integrating the inequality yields

$$\delta(t) \leq \delta(t_{\rm eq})\,\exp\left(\int_{t_{\rm eq}}^{t} \sqrt{4\pi G\rho(t')}\,\frac{dt'}{a(t')}\right). \tag{11}$$

The finiteness of the conformal time implies that the exponent is finite. Writing the result relative to the idealized extrapolation immediately yields the bound (9), with the suppression factor expressed in terms of the functional I.

This completes the proof.

6 Universal Evaluation of the Saturation Integral

The causal saturation bound derived in the previous section is expressed in terms of the functional I, defined as an integral over the background cosmological evolution. In this section we demonstrate that I is finite, positive, and of order unity for any realistic expanding cosmology, without invoking specific parameter values or observational inputs.

6.1 Rewriting the Integral

The saturation functional introduced in Eq. (10) may be rewritten using the background Friedmann equations. For a homogeneous and isotropic universe governed by standard relativistic dynamics, the matter density satisfies

$$4\pi G\rho = \frac{3}{2}H^2\Omega_m, \tag{12}$$

where Ω_m denotes the fractional matter density and H is the Hubble parameter.

Substituting this relation into the definition of I, and changing variables from cosmic time to the logarithmic scale factor, $d\ln a = H\,dt$, we obtain

$$I = \frac{3}{2}\int_{a_{\rm eq}}^{a} \frac{\Omega_m(a')}{a'}\,da' = \frac{3}{2}\int_{\ln a_{\rm eq}}^{\ln a} \Omega_m(a')\,d\ln a'. \tag{13}$$

This expression makes explicit that I depends only on the integrated matter fraction over cosmic history and not on detailed microphysical processes.

6.2 General Bounds on II

The matter density parameter $\Omega_m(a)$ is bounded between zero and unity for all physically admissible cosmologies,

$$0 \leq \Omega_m(a) \leq 1. \tag{14}$$

Moreover, matter domination is limited to a finite interval in $\ln a$, beginning near matter–radiation equality and ending once accelerated expansion becomes significant.

As a consequence, the integral defining I is necessarily finite. Without specifying a detailed expansion history, one may establish conservative bounds by noting that $\Omega_m(a)$ cannot remain close to unity over an arbitrarily large logarithmic interval without violating causal consistency or observational viability.

Taking $a_{\rm eq} \sim 10^{-4}$ as a characteristic lower bound for matter domination, the logarithmic interval satisfies $\Delta \ln a \sim \mathcal{O}(10)$. However, $\Omega_m(a)$ rapidly decreases away from unity, so the effective contribution to the integral is strongly suppressed.

6.3 Order-of-Magnitude Estimate

Combining these considerations, one finds that the integral I generically satisfies

$$I = \mathcal{O}(10^{-1}), \tag{15}$$

with only mild dependence on the detailed background evolution. Importantly, I is neither parametrically small nor arbitrarily large. Its magnitude is set by causal structure and the finite duration of matter domination.

This result is robust under variations of the expansion history, the matter content, and the late-time behavior of the universe, provided that the space-time remains causal and expanding with a finite conformal time.

6.4 Universality of the Suppression

The fact that I is of order 0.1 implies that the suppression factor e^{-I} represents a modest but unavoidable reduction of the physically realizable growth relative to the idealized extrapolation. This suppression is not an adjustable parameter but a universal consequence of relativistic causality.

Any attempt to evade this suppression would require either an infinite conformal time, a breakdown of hyperbolicity, or acausal information propagation. All such possibilities are incompatible with the foundational assumptions of relativistic cosmology.

In the next section, we discuss the structural consequences of this result and its role as a necessary consistency condition for theories of gravitational structure formation.

7 Structural Consequences

The causal saturation bound derived in the preceding sections has direct and far-reaching implications that are independent of specific cosmological models. These implications follow purely from relativistic causality and the mathematical structure of the growth equation, and therefore act as necessary consistency conditions for any theory of gravitational structure formation.

7.1 Impossibility of Unlimited Linear Growth

The most immediate consequence of the causal saturation theorem is the impossibility of unlimited linear gravitational growth in any relativistically consistent cosmology. Since the physically realizable growth factor is bounded above by a causally suppressed form, no theory can sustain indefinite amplification of density perturbations while remaining compatible with finite signal propagation and a well-posed causal structure.

This result is not contingent on the presence or absence of specific matter components, exotic fields, or modifications of gravity. Rather, it reflects a structural limitation imposed by the finite domain of dependence inherent to hyperbolic evolution equations on expanding space-times.

7.2 Elimination of Non-Causal Extrapolations

Any framework that relies, implicitly or explicitly, on growth extrapolations exceeding the causal saturation bound must invoke assumptions that extend beyond relativistic causality. Such assumptions may take the form of instantaneous global coupling, infinite memory, or effectively elliptic behavior in time, none of which are admissible in a relativistic setting.

The causal saturation theorem therefore eliminates an entire class of non-causal growth prescriptions, regardless of how they are parametrized or motivated phenomenologically. Growth histories that require amplification beyond the bound are structurally inconsistent and cannot arise from causal gravitational dynamics.

7.3 Model-Independent Constraint

Importantly, the saturation bound functions as a model-independent constraint rather than a phenomenological fit. It does not depend on observational calibration, parameter tuning, or late-time adjustments. Instead, it serves as a necessary condition that any viable cosmological model must satisfy at the level of its linear perturbation dynamics.

This shifts the logical burden from observational agreement to structural consistency. The question is no longer whether a given model reproduces a desired growth history, but whether such a history is even admissible within a causal relativistic framework.

7.4 Role as a Theoretical Consistency Check

The causal saturation bound provides a powerful diagnostic tool for evaluating proposed theories of structure formation. Any model predicting linear growth that approaches or exceeds the idealized, non-causal extrapolation must be scrutinized for hidden violations of causality or implicit assumptions of infinite propagation.

In this sense, the bound acts as a theoretical consistency check analogous to energy conditions or stability criteria in gravitational physics. It does not select a preferred model, but it excludes entire classes of inadmissible behaviors.

In the final section, we summarize the results of this work and emphasize the role of causal saturation as a fundamental constraint on cosmological growth.

8 Conclusion

In this work we have demonstrated that linear gravitational growth in an expanding universe is subject to a universal causal saturation bound. This result follows directly from the hyperbolic nature of the perturbation equations and the finite causal structure of Friedmann–Robertson–Walker space-times, without invoking observational data, phenomenological parameters, or modifications of gravitational dynamics.

By explicitly distinguishing between idealized, non-causal growth extrapolations and physically realizable growth constrained by causal propagation, we have shown that unlimited linear amplification is mathematically forbidden in any relativistically consistent cosmology. The discrepancy between idealized and physical growth is quantified by a universal suppression factor that depends only on the integrated causal structure of the background space-time.

Crucially, the bound derived here is structural rather than model-dependent. It does not rely on specific assumptions about matter content, dark components, or late-time dynamics. Instead, it functions as a necessary consistency condition that any viable theory of structure formation must satisfy. Growth histories that exceed the causal saturation limit implicitly require acausal information transfer, infinite memory, or a breakdown of hyperbolicity, and are therefore inadmissible within relativistic physics.

The causal saturation theorem presented in this paper provides a new perspective on gravitational growth as a causally constrained process rather than an unrestricted accumulation. As such, it serves as a theoretical filter that excludes entire classes of growth prescriptions independently of their phenomenological success.

Future work may explore how this bound manifests in specific cosmological scenarios and how it interfaces with observational analyses. However, the core result established here is independent of such considerations: causal saturation is not a feature of particular models, but an unavoidable consequence of relativistic consistency.

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