Using spline interpolation to extract the pure SU(3) deconfinement temperature

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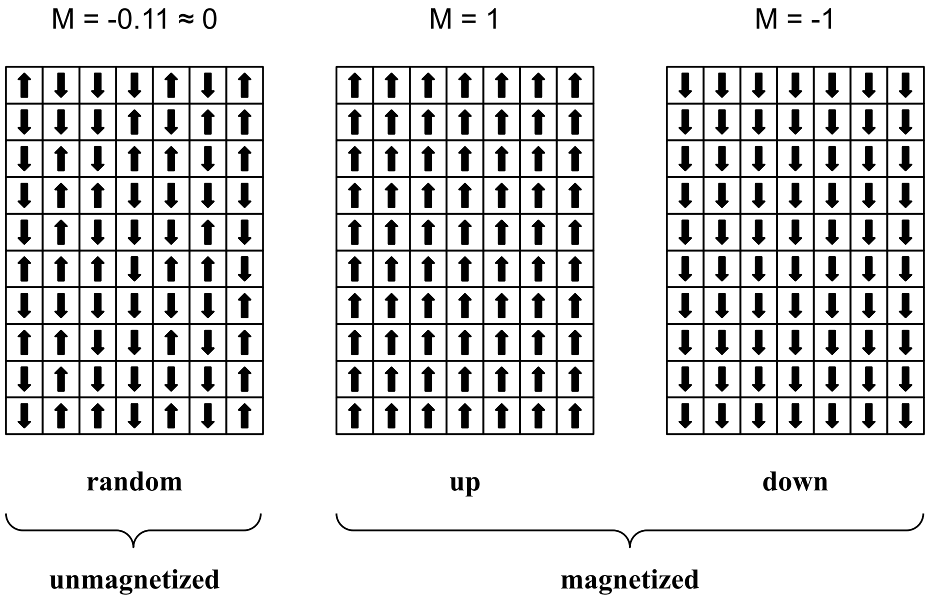
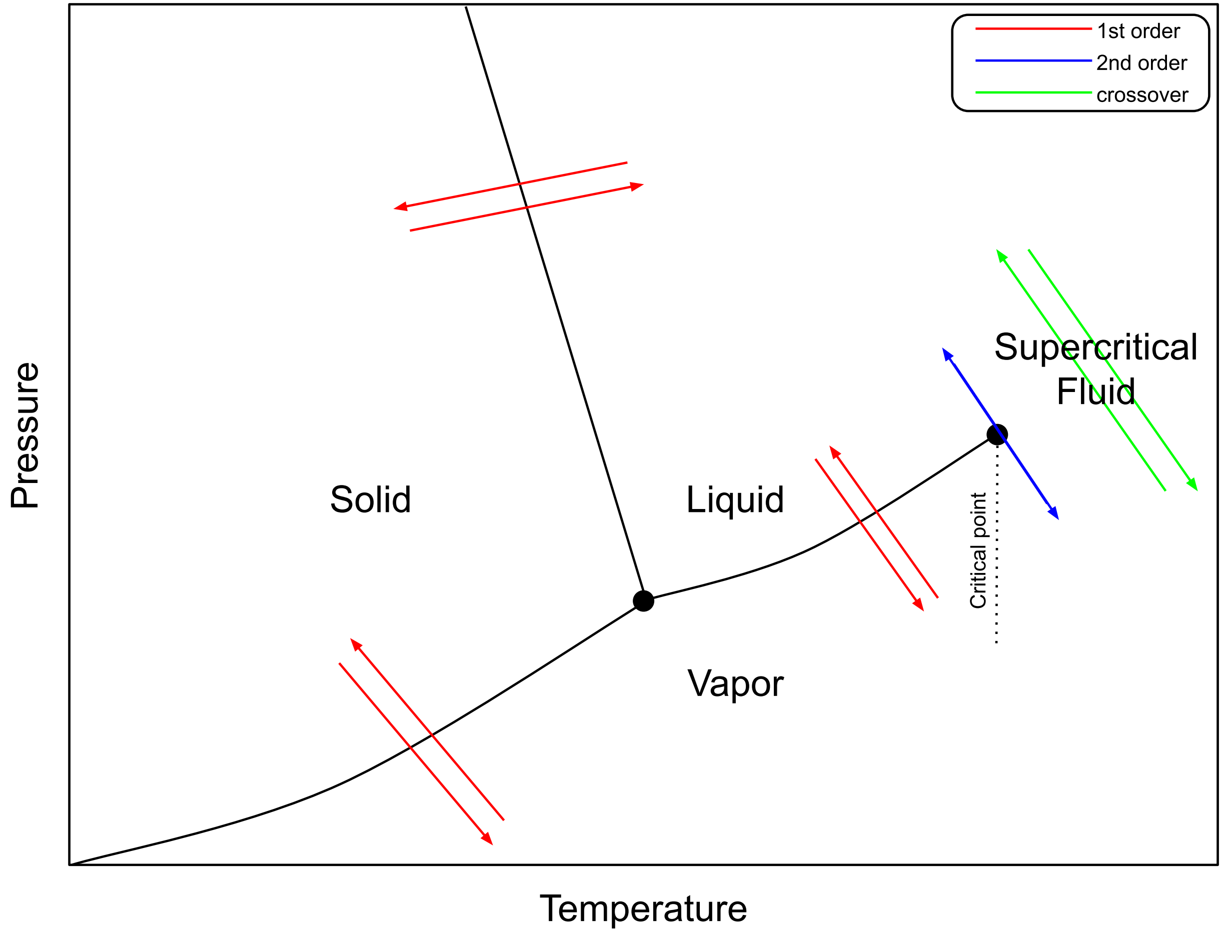
Phase Transitions

- Order Parameter (ψ): zero in one phase and nonzero in another
  - ψ is magnetization in Ising model

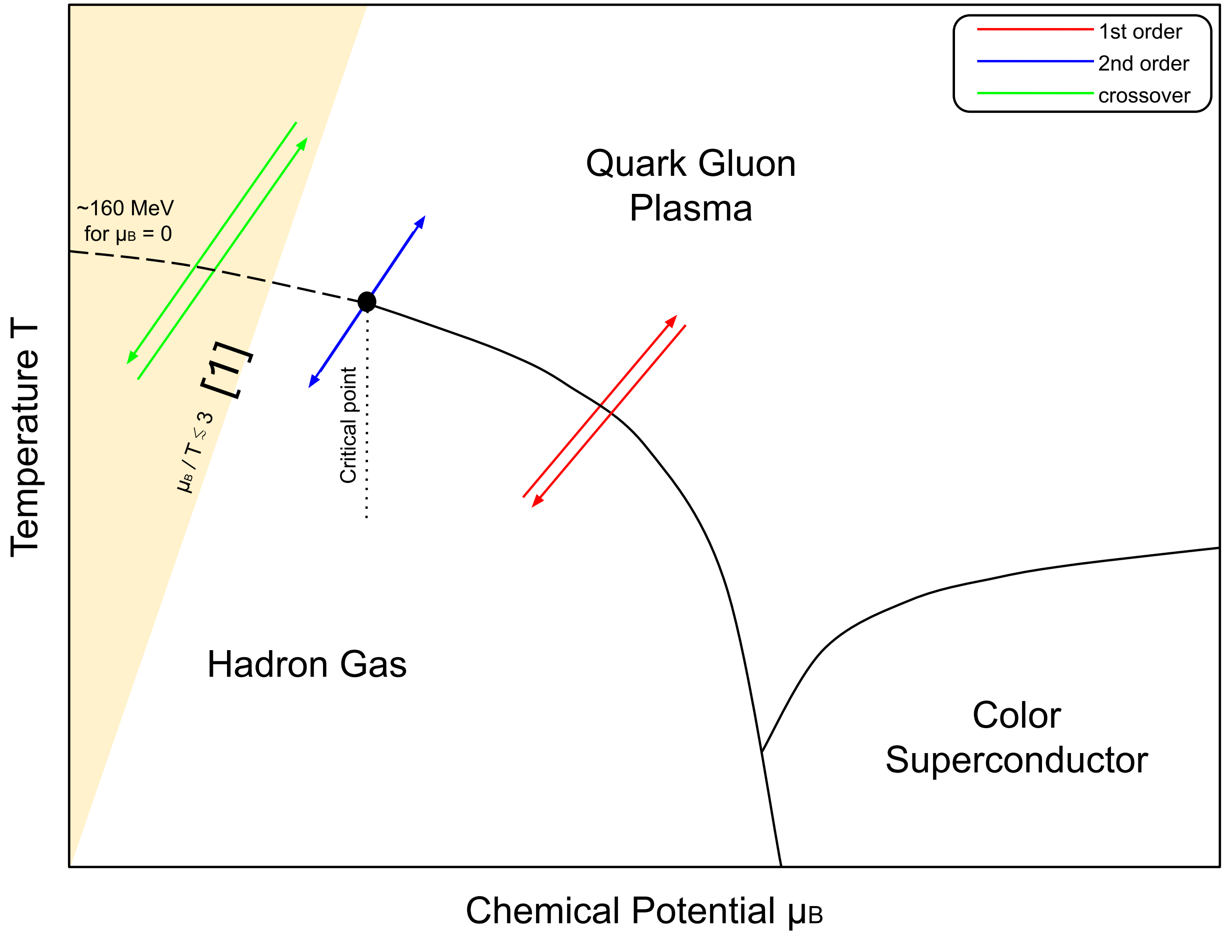


- First order transitions are discontinuous in order parameter and associated with a latent heat
- Second order transitions are discontinuous in the second derivative of free energy
- Crossovers are smooth



Quantum Chromodynamics phase diagram

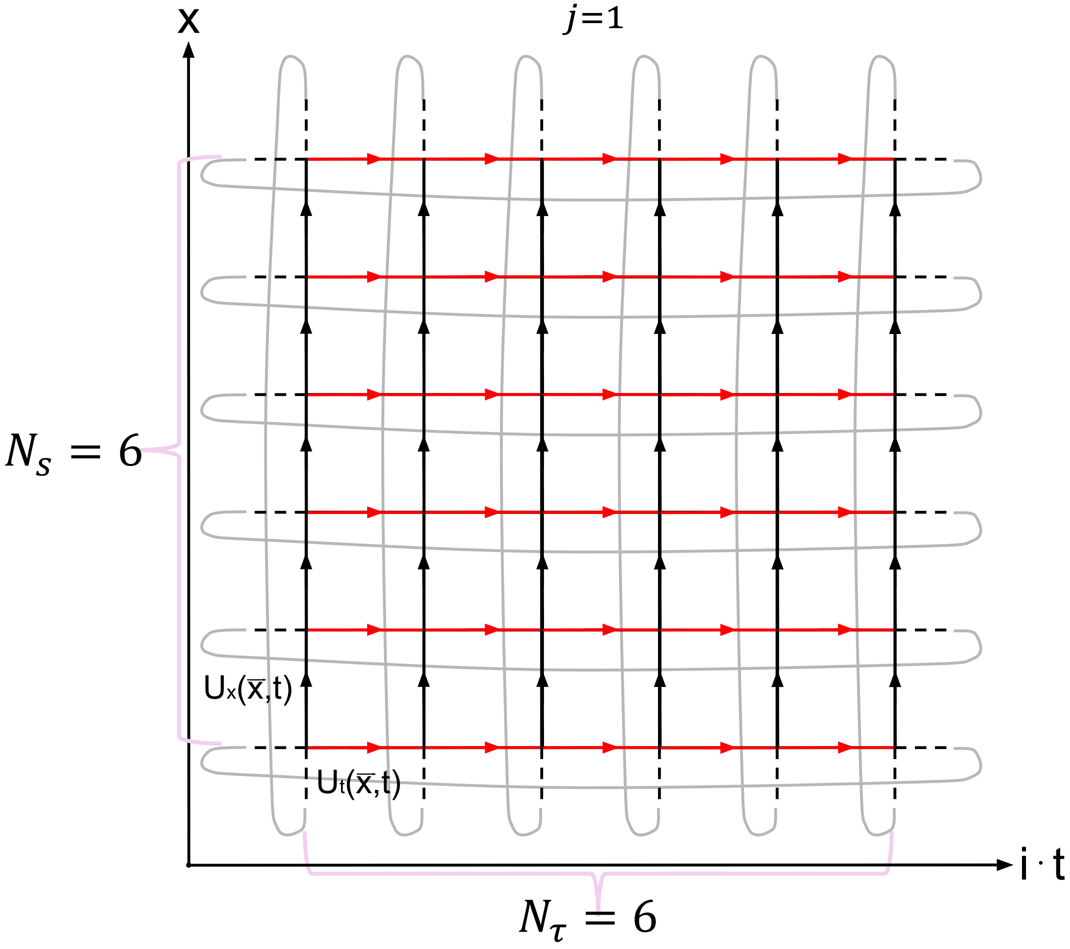
- Theory of interaction between quarks and gluons that comprise nuclear matter
- For zero baryon chemical potential μ_B there is a crossover transition
  - Quarks confined in hadrons to a sea of quarks and gluons
- Chiral Condensate and the Polyakov identify the transition (not strict order parameters)
- Quarks are computationally expensive
  - What about an approximation that does not consider them?



[1] Bollweg et al. Phys. Rev. D. **105**, (2022).

Pure SU(3) vs QCD

$$P(\vec{x}) = tr\left(\prod_{j=1}^{N_\tau - 1} U_t(\vec{x}, j)\right)$$



- Pure SU(3) calculations consider only gluons and their interactions and ignore the existence of quarks
  - Formally equivalent to infinitely massive quarks
  - Gluons are represented as SU(3) matrix links between sites

- The transition, known as deconfinement, is of first order

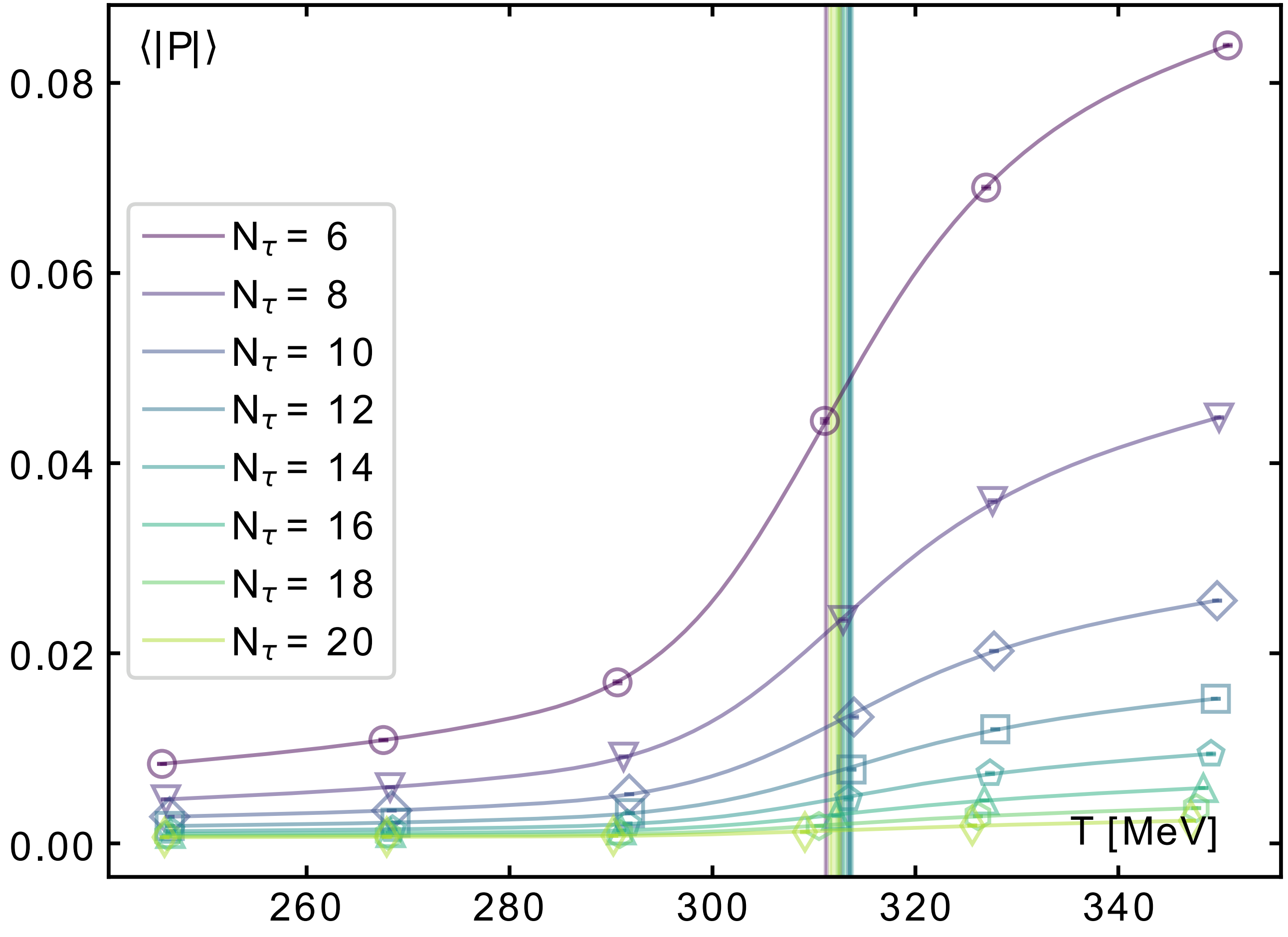
- The expectation value of the spatial average of the Polyakov loop magnitude is the order parameter of the deconfinement transition
  - Correlator of the Polyakov loop related to the static quark potential ($\propto \exp(-aN_t V(r))$)

Spline Interpolation

- By fixing temperature and changing N_τ we can change the lattice spacing

$$T = \frac{1}{a(\beta)N_\tau}$$

- Finite size changes a discontinuity in the order parameter to an inflection point
  - Pseudo critical temperature
- Functional form of the Polyakov loop unknown
- Each point is computationally expensive
- Splines allow us to interpolate from a limited number of points
  - The inflection point of the spline locates the critical temperature

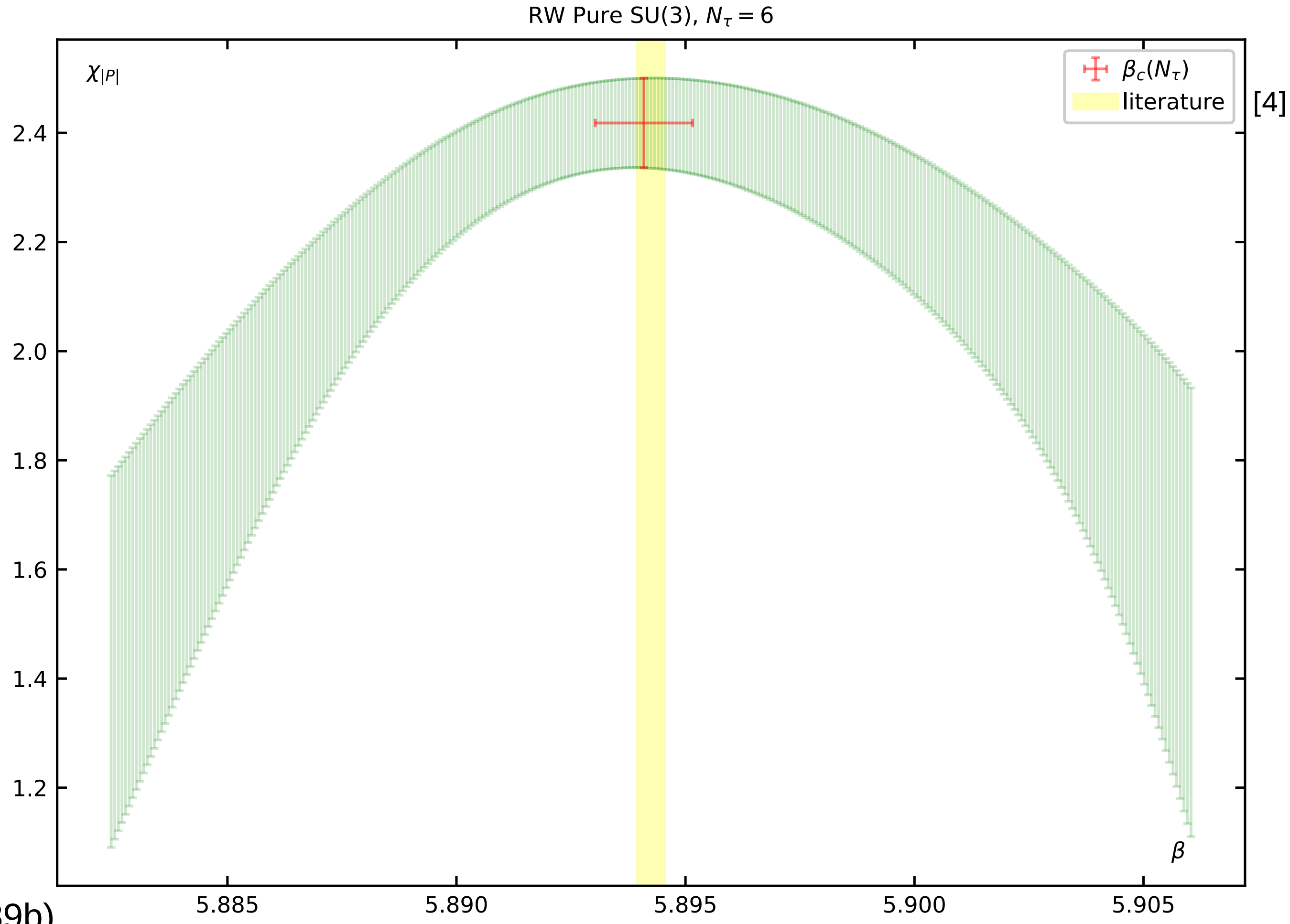


Reweighting

- Reweighting [2,3] makes direct use of the statistics of each simulation point
  - For an observable O
    $\langle O \rangle_{\beta'} = \left\langle \frac{Z_\beta}{Z_{\beta'}} e^{(\beta-\beta')S} O \right\rangle_\beta$
  - Need to compute the action of the configuration
  - Error grows with distance from simulation point

- At the transition the system is sensitive to an external field
  - Susceptibility diverges
  - For finite size susceptibility is maximized

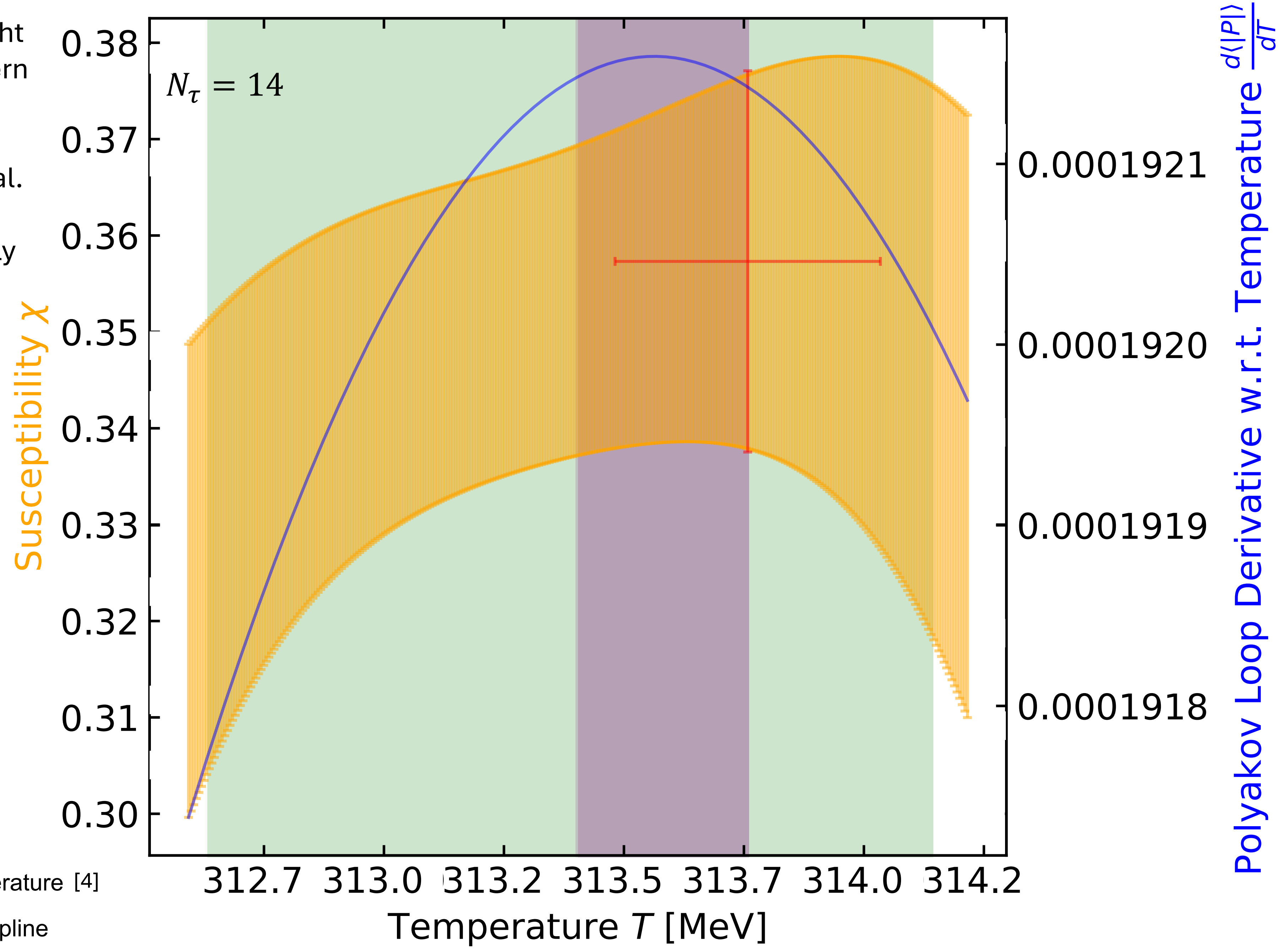
$$\chi = \frac{d\langle |P| \rangle}{dH} = N_s^3 \left(\langle |P|^2 \rangle - \langle |P| \rangle^2 \right)$$



[2] Ferrenberg & Swendsen Phys. Rev. Lett. **63**(12), 1195. (1989b).
[3] Ferrenberg & Swendsen Phys. Rev. Lett. **61**, 2365. (1989a).
[4] Francis et al. Phys. Rev. D. 91, (2015).

Results

- Shape of reweight curve is a concern
- Lower statistics than Francis et al.
- Spline error likely underestimated



lit v spl p = 0.84
lit vs rw p = 0.63
rw vs spl p = 0.54

literature [4]
spline
reweight

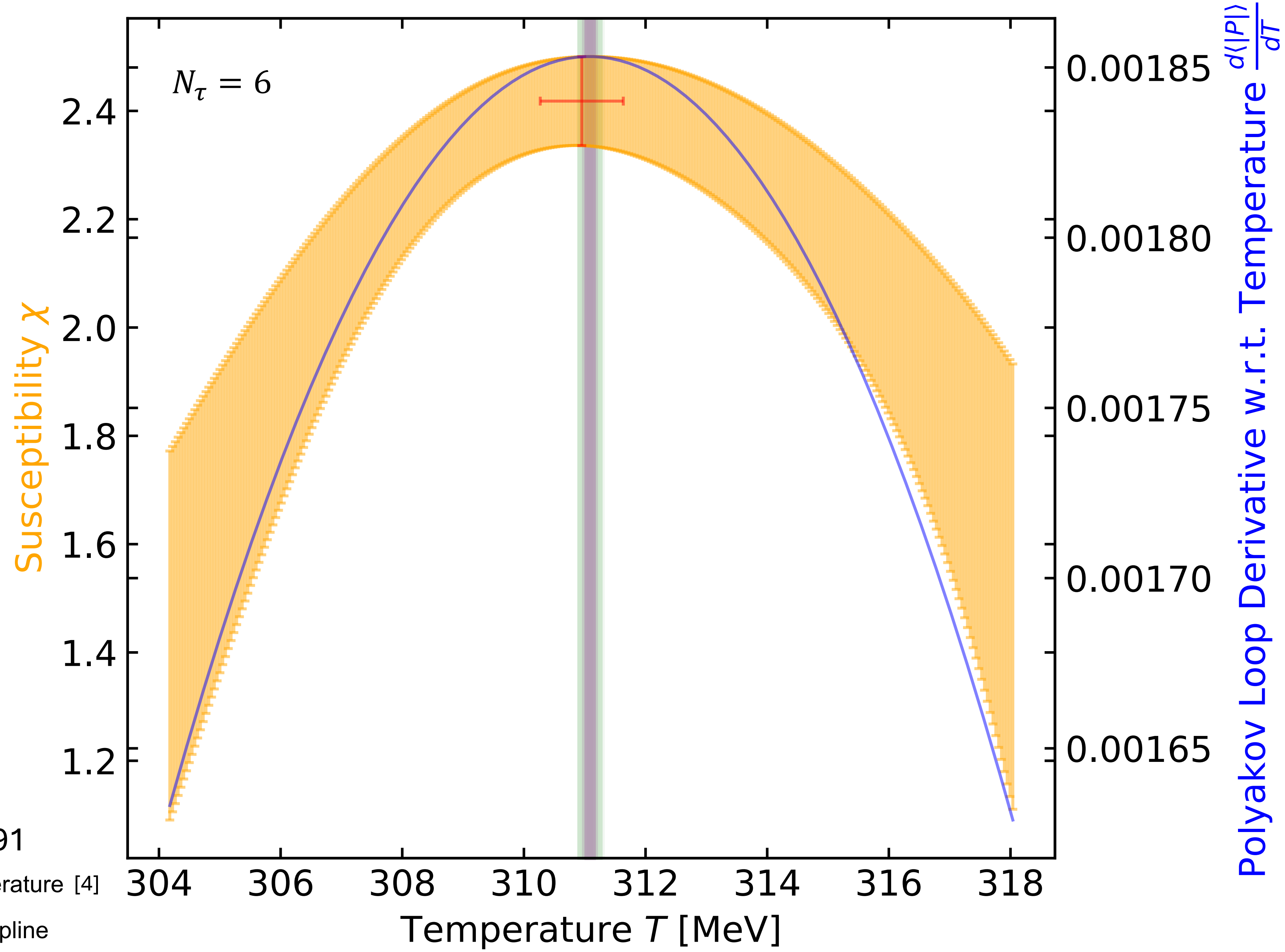
[4] Francis et al. Phys. Rev. D. 91, (2015).

Splines vs Reweighting: advantages and disadvantages

- Splines allow global extrapolation using a small amount of data
  - Locating the general area of a phase transition
- Introduces systematic error that is difficult to account for
  - Free to choose many spline parameters

- Reweighting errors are simple to propagate and reliably reproduced literature results
- Reweighting range becomes smaller close to the continuum
  - Higher number of statistics required
  - Must be near transition

Thanks for listening



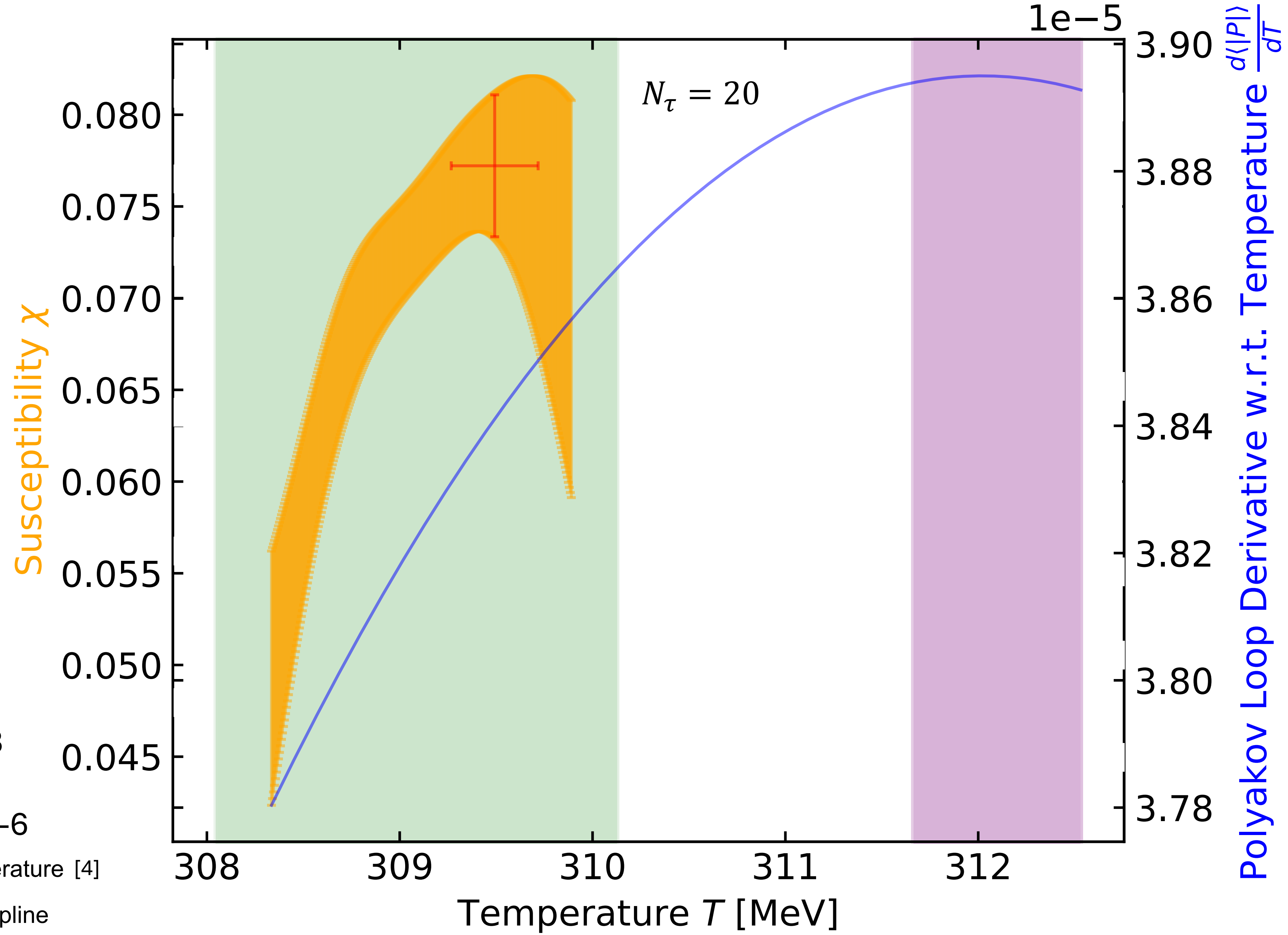
lit v spl p = 0.97
lit vs rw p = 0.91
rw vs spl p = 0.0.91

literature [4]

spline

reweight

[4] Francis et al. Phys. Rev. D. 91, (2015).



lit v spl p = 4.7e-3

lit vs rw p = 0.71

rw vs spl p = 1.3e-6

literature [4]

spline

reweight

[4] Francis et al. Phys. Rev. D. 91, (2015).

Bibliography

[1] D. Bollweg, J. Goswami, O. Kaczmarek, F. Karsch, S. Mukherjee, P. Petreczky, C. Schmidt, and P. Scior, *Taylor Expansions and Padé Approximants for Cumulants of Conserved Charge Fluctuations at Nonvanishing Chemical Potentials*, Physical Review D **105**, (2022).

[2] Ferrenberg, A. M., & Swendsen, R. H. (1989b). Optimized Monte Carlo data analysis. Phys. Rev. Lett., 63(12), 1195.

[3] Ferrenberg, A. M., & Swendsen, R. H. (1989a). New Monte Carlo Technique for Studying Phase Transitions. Phys. Rev. Lett., 61, 2365.

[4] A. Francis, O. Kaczmarek, M. Laine, T. Neuhaus, and H. Ohno, *Critical Point and Scale Setting in SU(3) Plasma: An Update*, Physical Review D **91**, (2015).

Further Reading

C. Gattringer and C. Lang, *Quantum Chromodynamics on the Lattice: An Introductory Presentation* (Springer Science & Business Media, 2009).