Continuum-Limit Extrapolation of the Pure SU(3) Deconfinement Temperature Using Bayesian Model Averaging

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Lattice Field Theory: The Basics

Definition: LFT is a non-perturbative approach to quantum field theory on a discretized spacetime lattice

- Replaces continuous spacetime with a grid of points
- Can be simulated on a computer

Enables numerical calculations of quantum field theories, especially useful for **strong interactions**

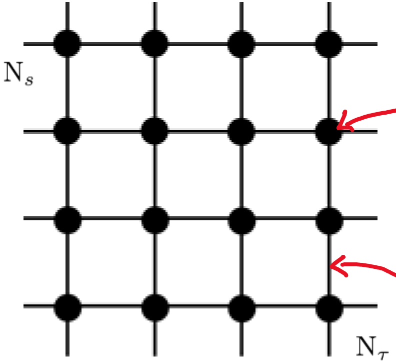


Image credit: Wikipedia

Quark fields ψ $\rightarrow$ defined on lattice sites

Gluon fields A_μ $\rightarrow$ defined on links between sites

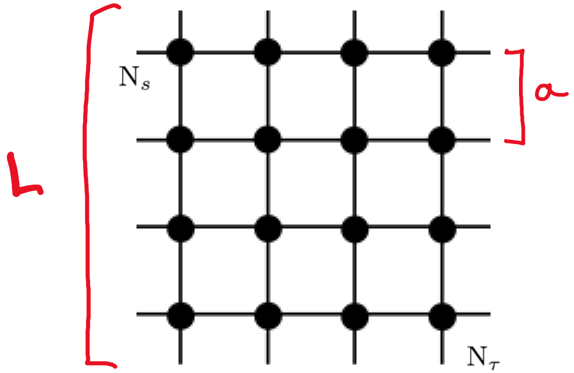
Lattice Parameters and Their Implications

Lattice spacing a : Distance between adjacent points on the lattice

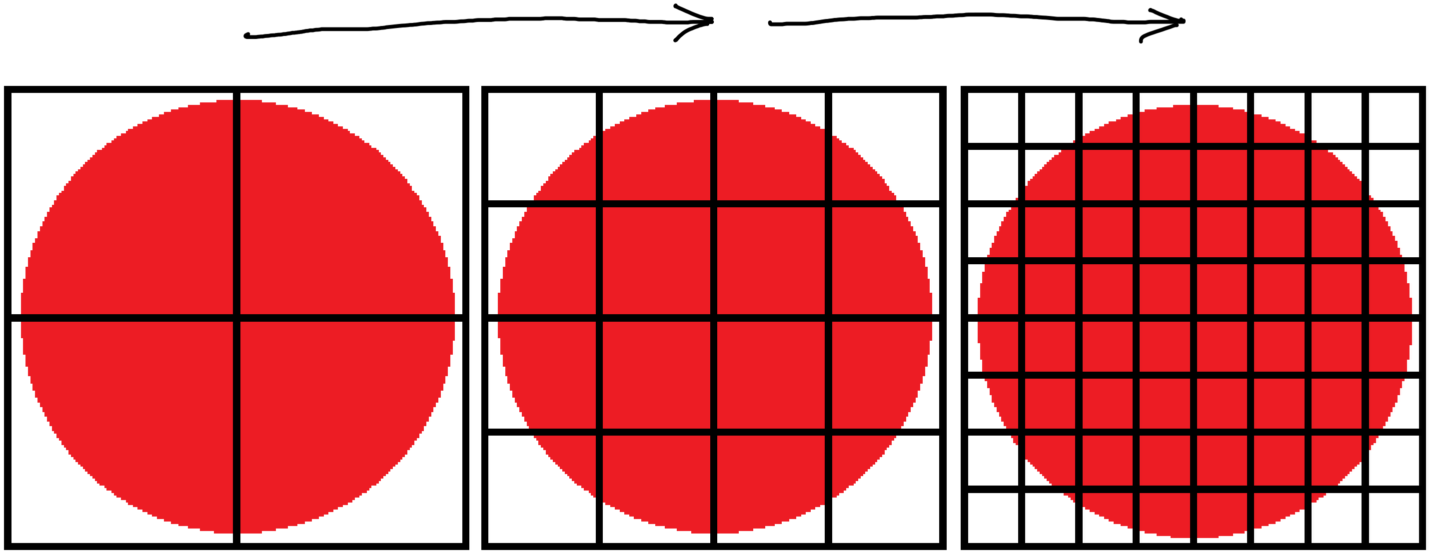
- Smaller $\mathrm{a} \rightarrow$ closer approximation to continuum

Finite lattice size L : Total extent of the lattice

- Larger $\mathrm{L} \rightarrow$ better representation of infinite volume



Lattice Parameters and Their Implications



Reaching the Continuum Limit

Definition: Process of taking lattice spacing a → 0

Importance:

- Removes discretization artifacts

$$X_{Latt} = X_{Cont} + a^2 c_1 + \sum_{i=2}^{\infty} a^{2i} c_i$$

This talk: Focus on continuum-limit extrapolation of SU(3) deconfinement temperature

Gauge Observables

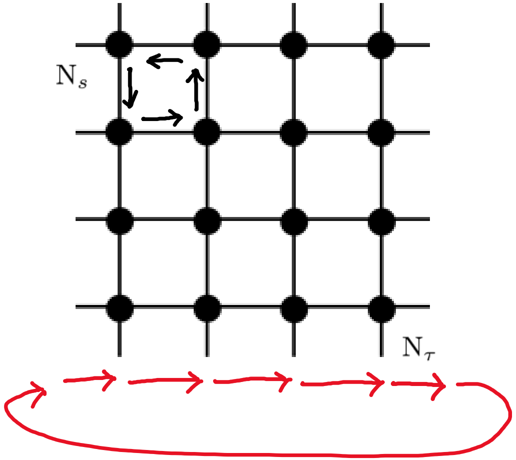
Plaquettes: Elementary closed loops on the lattice

- Used to construct the gauge action

Polyakov loops: Measurement of loops wrapping around the temporal direction

- Order parameter for **deconfinement transition**
- Used for: $\beta_c \rightarrow T_c$

$$P_{\vec{x}} \equiv \frac{1}{3}\mathrm{tr}\prod_{t=0}^{N_\tau-1} U_4(\vec{x},t)$$



Sampling Gauge Configurations

- Use Markov chains to sample SU(3) configurations

$$C_1 \to C_2 \to \ \ldots \ \to C_n$$

- Because $C_i \leftarrow C_{i-1}$, there is inherent correlation

Want: Effectively independent configurations, separate measurements with many intermediate configurations (500)

- Near the phase transition$\longrightarrow$even more intermediate configurations

Computational Challenges

Computational complexity:

- Scales with volume

$$Cost \propto N_s^3 \cdot N_\tau$$

- Scales with decreasing quark mass

- Requires repeating measurements at different spacings

$$X_{Latt} = X_{Cont} + a^2 c_1 + \sum_{i=2}^{\infty} a^{2i} c_i$$

	Pure SU(3) (infinite quark mass)	Nf 2+1 HISQ (physical quark mass)
Hardware	A100	A100
Dimensions	72^3 * 12	32^3 * 10
Time	19 Hrs	72 Hrs
Configurations	1000	48

Parallel Computing and Optimizations

Parallel Computing Strategies:

- Use of MPI (Message Passing Interface)
- Use of GPUs (massively parallel processors)

C++ Optimizations:

- Low-level control for performance optimization
- Custom memory allocators for lattice objects
- Cache-friendly data structures
- Vectorization of critical code sections

Image credit: NVIDIA

Pure SU(3)	**A100**	**RTX 3090**
Configurations	1000	1000
Dimensions	72^3 * 12	72^3 * 12
Time	19 Hrs	72 Hrs

Finite-Size Effects

Lattice spacing and extent introduce systematics:

- Minimum distance (lattice spacing)
- Maximum distance (lattice extent)

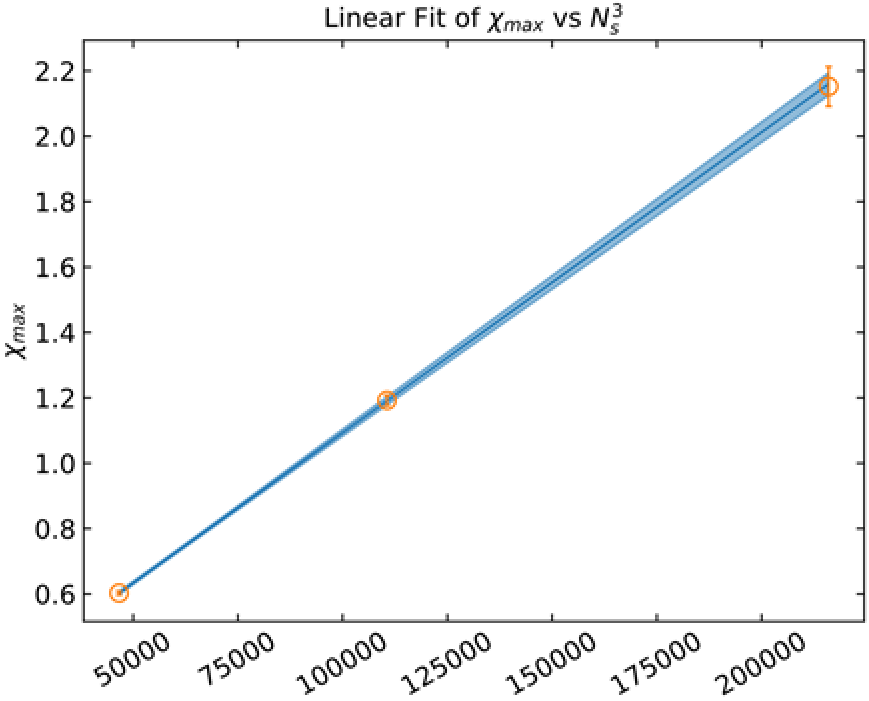
Aim to check if these systematics are more than our statistical error

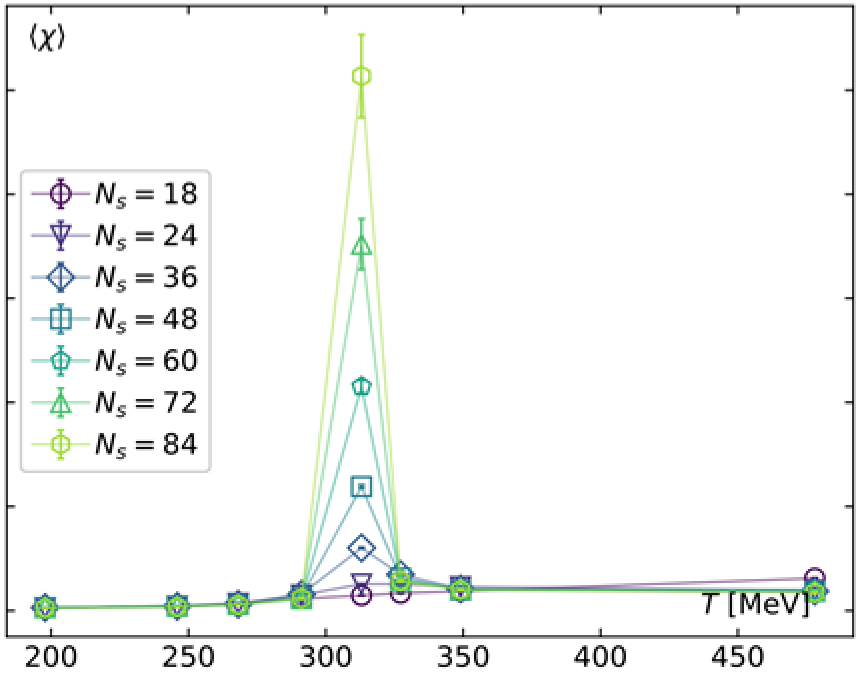
Check our measurements over increasing N_s with fixed N_τ to determine this

We find that finite-size effects shifts our central value by about 0.1%

Finite-Size Scaling in SU(3)

$$first\ order \Rightarrow \chi_{max} \sim N_s^3$$





Fukugita, M., Okawa, M., & Ukawa, A. (1989). Order of the deconfining phase transition in SU(3) lattice gauge theory. *Phys. Rev. Lett.*, *63*(17), 1768–1771. doi:10.1103/PhysRevLett.63.1768

Bayesian Model Averaging (BMA)

$$\langle a \rangle = \sum_M \langle a \rangle_M \underbrace{P(M|D)}_{\text{"model weight"}}$$

$$\sigma_a^2 = \langle a^2 \rangle - \langle a \rangle^2$$

$$= \underbrace{\sum_M \sigma_M^2 P(M|D)}_{\text{"statistical noise"}} + \underbrace{\sum_M \langle a \rangle_M^2 P(M|D) - \langle a \rangle^2}_{\text{"model spread"}}$$

BMA is a statistical technique that aims to account for model uncertainty by combining predictions from multiple models, weighted by their estimated likelihoods

Jay, W. I., & Neil, E. T. (2021). Bayesian model averaging for analysis of lattice field theory results. Physical Review D, 103(11), 114502. doi:10.1103/PhysRevD.103.114502

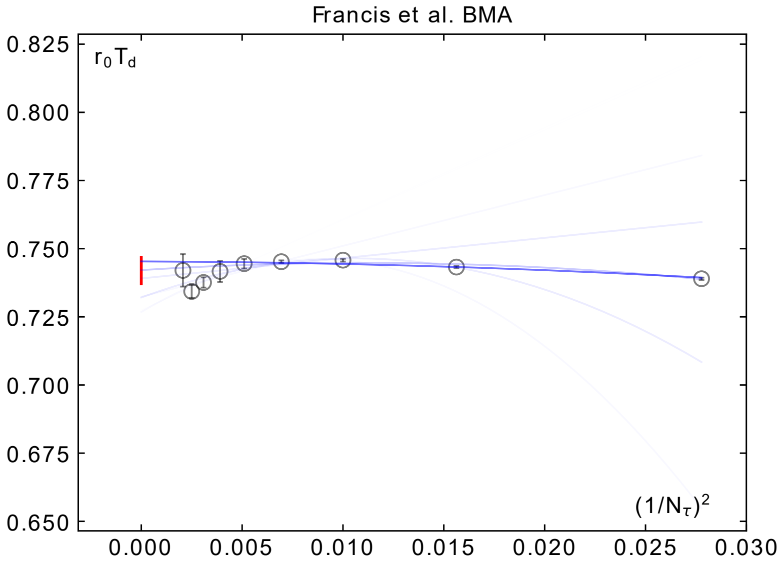
Summary of Statistics

N_tau	6	8	10	12	14	16	18	20
N_conf	1.0e4	1.0e4	1000	1.0e4	~860	8000	4000	1300

(One to two orders of magnitude fewer than literature)

Francis, A., Kaczmarek, O., Laine, M., Neuhaus, T., & Ohno, H. (2015). Critical point and scale setting in SU(3) plasma: An update. *Phys. Rev. D*, *91*(9), 096002. doi:10.1103/PhysRevD.91.096002

Continuum-Limit Extrapolation



$$X_{Latt} = X_{Cont} + a^2 c_1 + \sum_{i=2}^{\infty} a^{2i} c_i$$

$$T = \frac{1}{a(\beta) N_\tau}$$

Literature w/ BMA	0.7420(54)
Literature	0.7457(45)

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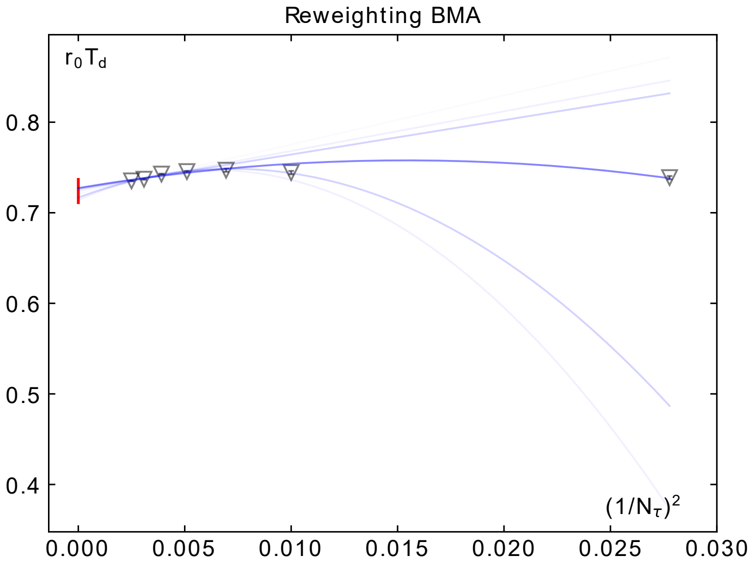

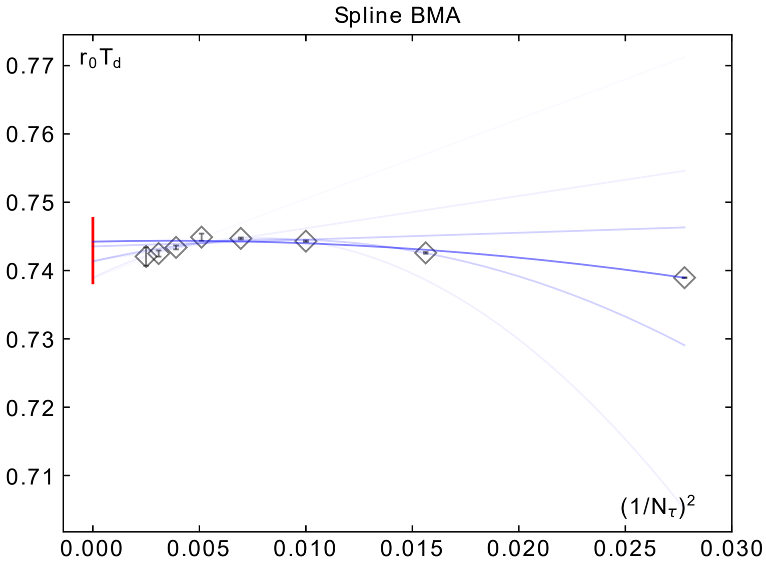

Literature	SRI w/ RW	SRI w/ Splines
0.7457(45)	0.724(15)	0.7429(50)

Summary

- Both RW and spline results agree with literature value of Td
- Found some modest finite-size effects
- Literature error is arguably slightly underestimated according to BMA

Thank you!