Sensitivity of the Polyakov Loop to Chiral Symmetry Restoration

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Strong-interaction matter under extreme conditions

Global symmetry breaking

At $m = \infty$ with $N_c = 3$, the deconfinement order parameter is the Polyakov loop

$$P_{\vec{x}} \equiv \frac{1}{3} \operatorname{tr} \prod_{\tau} U_4\left(\vec{x}, \tau\right), \qquad P \equiv \frac{1}{N_\sigma^3} \sum_{\vec{x}} P_{\vec{x}},$$

which relates to color averaged quark-antiquark free energy

$$\exp\left[-\frac{F_{q\bar{q}}(r, T)}{T}\right] = \langle P_{\vec{x}} P_{\vec{y}}^\dagger \rangle \approx \langle \operatorname{Re} P \rangle^2 \quad \text{(at large } r\text{)}.$$

Hence $\langle \operatorname{Re} P \rangle = 0$ in the confined phase. In this phase $\langle \operatorname{Re} P \rangle$ is invariant under global $\mathbb{Z}_3$, which otherwise transforms non-trivially as $P \to z\,P$. Spontaneous breaking above T_d.

At $m = 0$ the chiral condensate $\langle \bar{\psi}\psi \rangle$ transforms non-trivially under $SU(2)_A$. Hence $\langle \bar{\psi}\psi \rangle > 0$ signals chiral symmetry breaking. Spontaneous breaking below T_c.

The original question

What are good observables that indicate deconfinement in QCD?

In pure SU(3) gauge theory, inflection points found at similar locations[1].

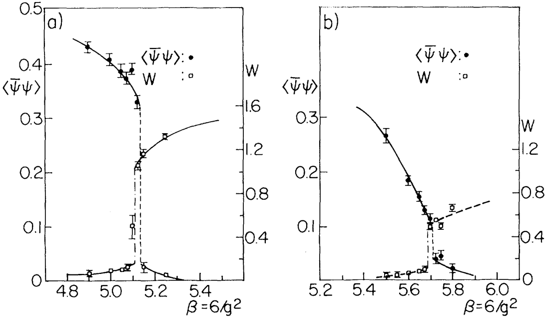


FIG. 2. $\langle\bar\psi\psi\rangle$ and W vs $\beta=6/g^2$ for SU(3) gauge theory on (a) 2×8^3 and (b) 4×8^3 lattices.

[1] J. Kogut et al., Phys. Rev. Lett. 50.6, 393–396 (1983).

At larger-than-physical quark mass

Similar locations[2] also for $N_f = 2 + 1$, physical m_s, and $m_\pi \approx 220$ [MeV].

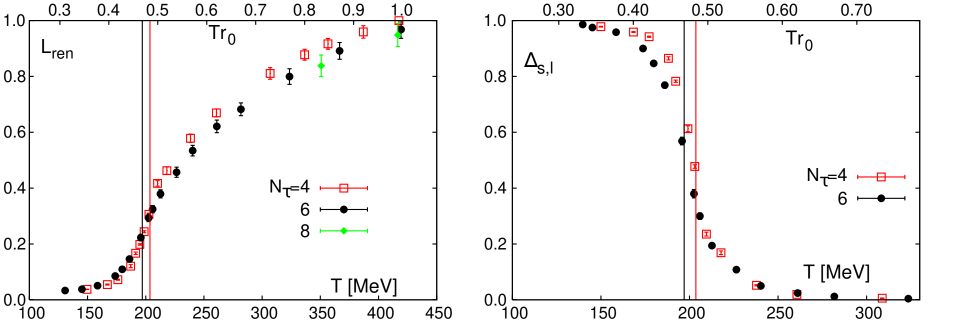
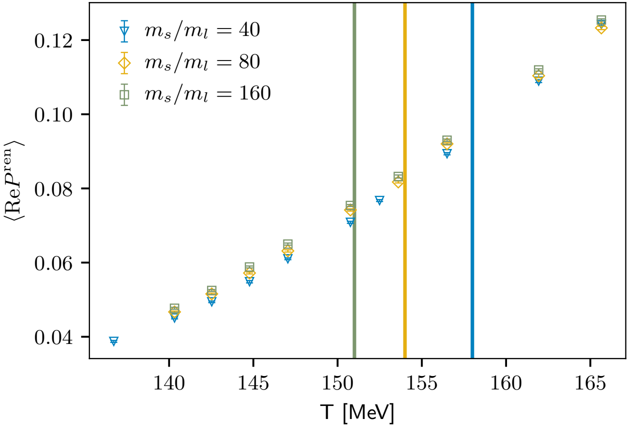


FIG. 11 (color online). Renormalized Polyakov loop on lattices with temporal extent N 4, 6 and 8 (left) and the normalized difference of light and strange quark chiral condensates def ned in Eq. (36). The vertical lines show the location of the transition temperature determined in [18] on lattices with temporal extent N 4 (right line) and in this analysis for N 6 (left line).

[2] M. Cheng et al., Phys. Rev. D, 77.1, 014511 (2008).

At smaller-than-physical quark mass

No longer the case for highly improved fermion actions, and at lower quark masses[3].



[3] D. A. Clarke et al., arXiv:1911.07668 [hep-lat], (2019).

How can we understand this behavior?

- At $m < \infty$, P still has inflection point somewhere.
- This is (often?) interpreted as some remnant of the $m = \infty$ critical behavior.
- But in the chiral limit, there is no clear global symmetry for deconfinement.
- Moreover P is purely gluonic, which means it's trivially invariant under chiral rotations.
- Therefore, from the perspective of some $\mathcal{L}_{\text{eff}}$ written in the chiral limit, it should rather be an energy-like operator, and we might expect it to inherit its behavior from the chiral transition in the same way any other energy-like operator would.

Let's explore this idea analytically and numerically.

Some useful quantities

$$H \equiv m_l / m_s$$ symmetry breaking parameter

$$t = (T - T_c)/T_c$$ reduced temperature

$$z \equiv z_0 t H^{-1/\beta\delta}$$ scaling variable

$$\chi_{mP} \equiv \frac{\partial \langle \mathrm{Re}\, P \rangle}{\partial H} = \langle \mathrm{Re}\, P \cdot \Psi \rangle - \langle \mathrm{Re}\, P \rangle \langle \Psi \rangle$$ mixed susceptibility

$$\Psi \equiv \frac{1}{2} \hat{m}_s \,\mathrm{tr}\, M_l^{-1}$$

$$F_q(T) = \lim_{r\to\infty} \frac{1}{2} F_{q\bar{q}}(r, T) = -T \log \langle \mathrm{Re}\, P \rangle$$ heavy quark free energy

$$\frac{1}{T} \frac{\partial F_q(T,H)}{\partial H} = -\frac{\chi_{mP}}{\langle \mathrm{Re}\, P \rangle}$$

P dependence on H

Being energy-like, P inherits singular behavior from 3d O(N) universality class[4]:

$$\langle P \rangle = \underbrace{AH^{(1-\alpha)/\beta\delta} f_f'(z)}_{\text{singular part}} + \underbrace{f_{\text{reg}}(T,H)}_{\text{regular part}}$$

with universal free energy scaling function f_f and critical exponents α, β, and δ. Prime indicates derivative w.r.t. z. We use O(2) since we will work at fixed $N_\tau = 8$, so

$$\beta = 0.3490, \qquad \delta = 4.780, \qquad \alpha = -0.017.$$

In $H \to 0$ limit, keeping only leading terms:

$$\langle P \rangle \sim a(T) - \begin{cases} b_-(T)\; H & T < T_c \\ b_0\; H^{(1-\alpha)/\beta\delta} & T = T_c \\ b_+(T)\; H^2 & T > T_c \end{cases} \qquad \frac{1-\alpha}{\beta\delta} = 0.61$$

[4] J. Engels and F. Karsch, Phys. Rev. D, 85, 094506 (2012).

F_q dependence on H

Using $F_q = -T \log \langle P \rangle$ and

$$\langle P \rangle = AH^{(1-\alpha)/\beta\delta} f_f'(z) + f_{\mathrm{reg}}(T, H)$$

one obtains after expanding the logarithm

$$\frac{F_q(T,H)}{T} = -\tilde{A}H^{(1-\alpha)/\beta\delta} f_f'(z) - \tilde{f}_{\mathrm{reg}}(T, H)$$

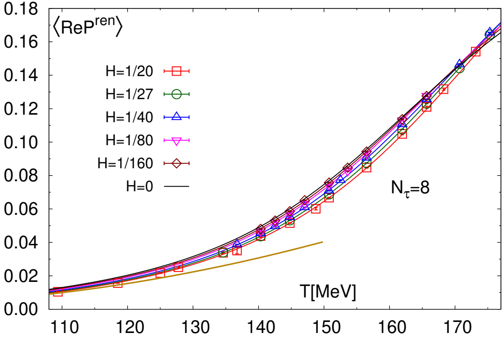
Same form as $\langle P \rangle$, so we can expect similar small H behavior (same leading powers) as before.

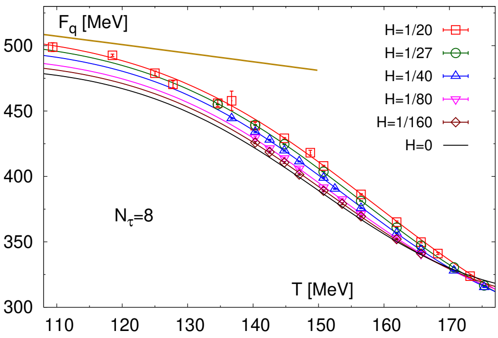
Set up

- $N_f = 2+1$ with HISQ action
- $N_\tau = 8$
- $N_s/N_\tau \geq 4$
- m_s fixed to its physical value
- m_s/m_ℓ varies from 20 to 160 (160 MeV $\gtrsim m_\pi \gtrsim$ 58 MeV)
- T in the vicinity of chiral crossover
- Renormalization constants, when needed, from TUMQCD[5]

[5] A. Bazavov et al., Phys. Rev. D, 93.11, 114502 (2016).

Results: $\langle \mathrm{Re}\, P \rangle$ and F_{qq}

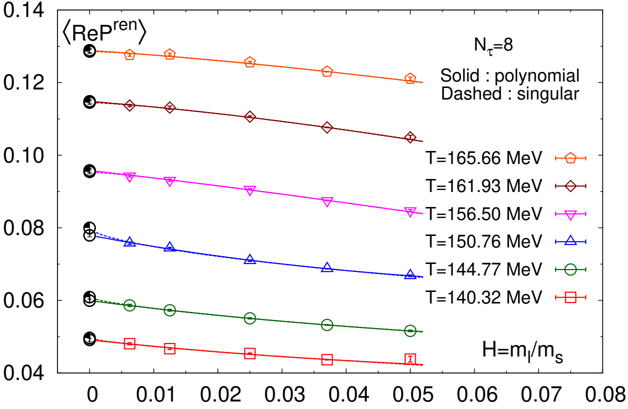




Gold line: static-light meson contribution computed in HRG[6].

[6] A. Bazavov and P. Petreczky, Phys. Rev. D, 87.9, 094505 (2013).

Results: P dependence on H



General form

$$F_q \sim a(T) - \begin{cases} b_-(T)\ H & T < T_c \\ b_0\ H^{0.61} & T = T_c \\ b_+(T)\ H^2 & T > T_c \end{cases}$$

suggests 3-parameter fits

$$P_{\sin}(H) = \exp\left[a + bH^c\right],$$
$$P_{\rm poly}(H) = \exp\left[a + bH + cH^2\right].$$

Former fit near $T_c^{N_\tau=8} \approx 144$ [MeV]

$$c = \begin{cases} 0.71(15) & T = 144.77\ [\mathrm{MeV}] \\ 0.63(11) & T = 150.76\ [\mathrm{MeV}]. \end{cases}$$

Derivatives with respect to H

Derivatives of observables w.r.t. H will be more sensitive to H. Hence we compute

$$\chi_{mP} = \frac{\partial \langle \mathrm{Re}\, P \rangle}{\partial H} = -AH^{(\beta-1)/\beta\delta} f_G'(z) + \frac{\partial}{\partial H} f_{\mathrm{reg}}(T,H),$$
$$\frac{1}{T}\frac{\partial F_q(T,H)}{\partial H} = -\tilde{A}H^{(\beta-1)/\beta\delta} f_G'(z) - \frac{\partial}{\partial H} \tilde{f}_{\mathrm{reg}}(T,H),$$

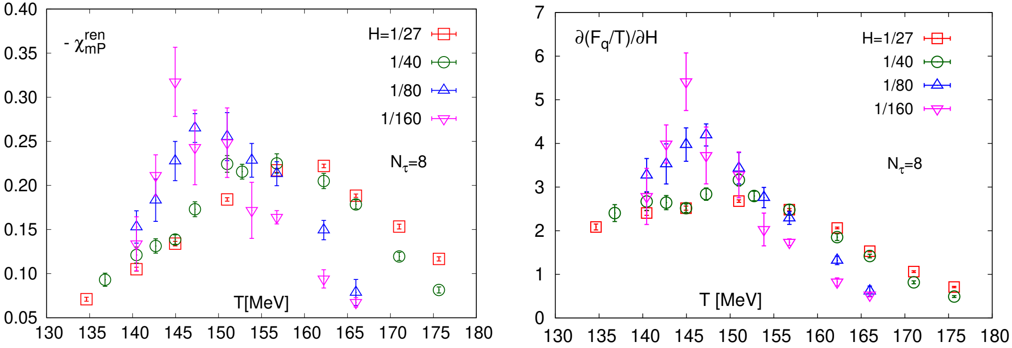
where the order parameter scaling function f_G is related to f_f by

$$f_G(z) = -\left(1+\frac{1}{\delta}\right) f_f(z) + \frac{z}{\beta\delta} f_f'(z).$$

and

$$\frac{\beta-1}{\beta\delta} = -0.39$$

Results: χ_{mP} and $\partial_H F_q$



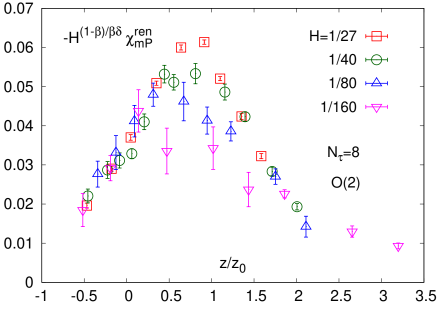
$$\chi_{mP} \sim -\frac{\partial F_q}{\partial H} \sim -H^{-0.39} + \text{regular}$$

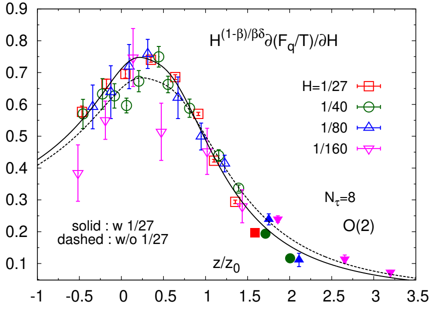
Results: Rescaled susceptibilities

Singular part suggests 3-parameter fits, e.g.

$$H^{(1-\beta)/\beta\delta}\partial_H F_q = -\tilde{A}\, f_G'\left(z_0\frac{T-T_c}{T_c}H^{-1/\beta\delta}\right)$$

They deliver $T_c = 145.5(5),\ 144.3(6)$ [MeV] (compare $T_c^{N_\tau=8} = 144(2)$ [MeV])





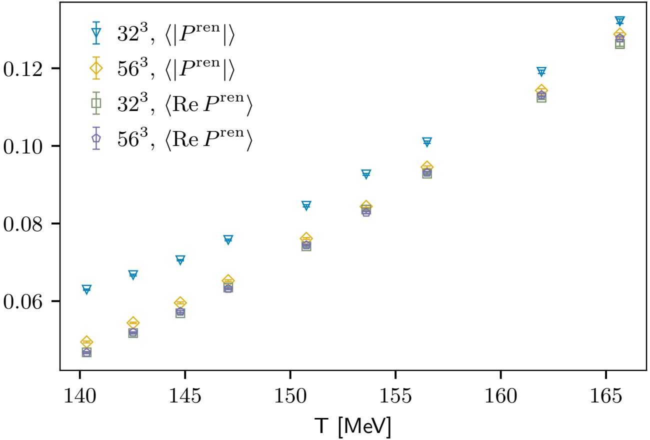
Summary, work in progress

- The Polyakov loop is sensitive to the chiral phase transition near the chiral limit.
- In particular χ_{mP} and $\partial_H F_q$ diverge as $H \to 0$ according to the O(2) universality class.
- Would like to quantify the contribution of regular terms to χ_{mP}.
- Would like to understand temperature derivatives.

Thanks for your attention.

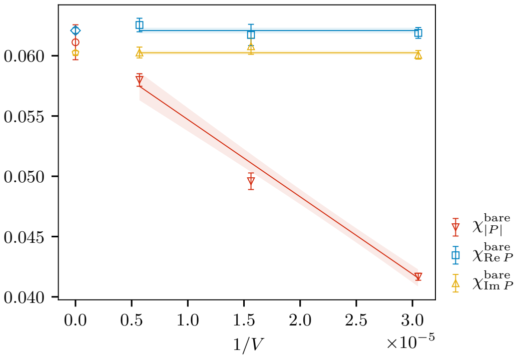
Additional details: P dependence on V

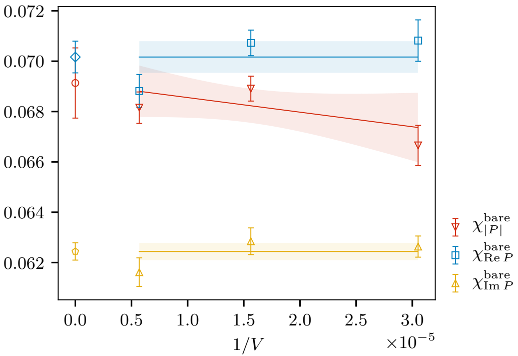
N_σ dependence of P. $N_\tau = 8$ and $m_s/m_l = 80$ for these.



Additional details: χ_P finite size scaling

Finite size scaling of various susceptibilities for $T \approx 140$ [MeV] (left) and $T \approx 165$ [MeV] (right). $N_\tau = 8$ and $m_s/m_l = 80$ for these.





Additional details: current statistics

m_s/m_l	N_σ	avg. # TU
20	32	99 000
27	32	1 500 000
40	40	110 000
80	56	35 000
	40	33 000
	32	73 000
160	56	17 000