A Renormalized Polyakov Loop Susceptibility

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Strong-interaction matter under extreme conditions

Lattice renormalization, generally speaking

- The lattice is non-perturbative with $a^{-1} \sim$ UV and $(L\,a)^{-1} \sim$ IR.
- Eventually one takes $a \to 0$ limit, and like in perturbation theory,
  - one may encounter divergences $1/a$
  - that can be removed through operator and parameter renormalization.
- Such operators must be renormalized before interpreting physically.
- In particular needed before continuum limit extrapolation.

At $m = \infty$ with $N_c = 3$, deconfinement order parameter is Polyakov loop

$$P_{\vec{x}} \equiv \frac{1}{3}\,\mathrm{tr}\prod_{\tau} U_4\left(\vec{x},\tau\right), \qquad P \equiv \frac{1}{N_\sigma^3}\sum_{\vec{x}} P_{\vec{x}}.$$

Relates to static quark-antiquark free energy

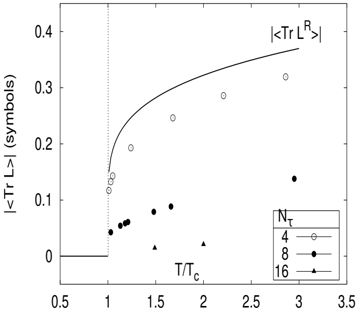
$$\exp\left[-\frac{F_{q\bar{q}}(r,T)}{T}\right] = \langle P_{\vec{x}} P^\dagger_{\vec{y}}\rangle \approx \langle P\rangle^2 \quad \text{(at large } r\text{)}.$$

Hence $\langle P\rangle = 0$ in the confined phase. $\langle P\rangle$ is of interest to e.g. studies at large quark mass, and sometimes used to construct effective actions.

A UV divergence

$\langle P \rangle$ contains a UV-divergent contribution

$$\left\langle P^{\text{bare}} \right\rangle \sim \exp\left[-kN_\tau g^2 + \mathcal{O}\left(g^4\right)\right] \qquad N_\tau = \frac{1}{aT}$$



- One might want to use $\langle P \rangle$ to figure out the phase in the continuum limit.
- But $\left\langle P^{\text{bare}} \right\rangle$ vanishes as $N_\tau \to \infty$!
- Reflected in lattice calculations[1].
- Needs to be renormalized!

[1] F. Zantow "Lattice renormalization of the Polyakov loop" en PhD thesis Germany: Bielefeld University, 2003.

Possible renormalizations of the Polyakov loop

$$P \sim \prod_{\tau=0}^{N_\tau-1} U_4(\tau) \quad \Rightarrow \quad P^{\text{ren}} = Z(g^2)^{N_\tau} P^{\text{bare}}$$

Two commonly used schemes include:

$\bar{q}q$-scheme[2]

- $F_{\bar{q}q}$ with add. renorm. $c(g^2)$
- F_1 has same $c(g^2)$
- $c(g^2) = a\,V_{T=0}(r_s) - a\,F_1(r_s, T)$
- Get $Z = \exp(-c/2)$

Gradient flow[3]

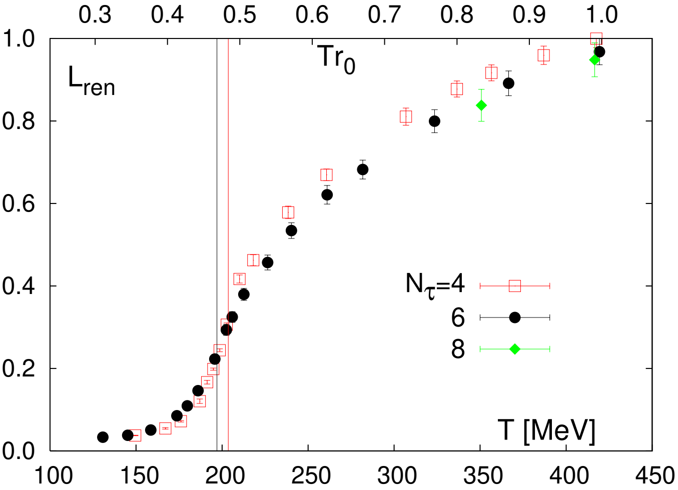
- Flow equation

$$\dot{V}_\mu(t) = -g^2 V_\mu(t)\,\partial_\mu S[V(t)]$$
$$V_\mu(0) = U_\mu$$

- Smearing scale $\sqrt{8t}$
- Maps gauge field to family of renorm. fields

[2] O. Kaczmarek et al., Phys. Lett. B, 543, 41–47 (2002).
[3] M. Lüscher, J. High Energy Phys. 2010.8, 071 (2010).

Example $\langle P^{\text{ren}} \rangle$ in action



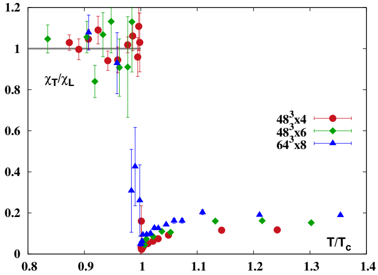
[4] M. Cheng et al., Phys. Rev. D, 77.1, 014511 (2008).

A naive "renormalized" susceptibility

The Polyakov loop susceptibility is

$$T^3\chi_{|P|} = \frac{N_\sigma^3}{N_\tau^3}\left(\langle |P|^2\rangle - \langle |P|\rangle^2\right).$$

One may consider replacing $P^{\text{bare}} \to P^{\text{ren}}$ (i.e. factor Z^{2N_τ}) but some problem remains. Can manifest e.g. in ratios, as shown below[5].



$$\frac{\chi_T}{\chi_L} \equiv \frac{\langle \mathrm{Re}\, P^2\rangle - \langle \mathrm{Re}\, P\rangle^2}{\langle \mathrm{Im}\, P^2\rangle - \langle \mathrm{Im}\, P\rangle^2}$$

Any multiplicative renorm. cancels in the ratio. Here we see some remaining a-dependence in the high-T region.

[5] P. M. Lo et al., Phys. Rev. D, 88.1, 014506 (2013).

A proper renormalized susceptibility

Now have a look at this:

$$\left\langle |P^{\text{ren}}|^2 \right\rangle = \left\langle P^{\text{ren}} P^{\text{ren}\dagger} \right\rangle = \frac{1}{N_\sigma^6} \left\langle \sum_{\vec{x},\vec{y}} P^{\text{ren}}_{\vec{x}} P^{\text{ren}\dagger}_{\vec{y}} \right\rangle$$

- Has issues with $\vec{x} = \vec{y}$ "contact term".
- Also will have short-distance problems since for $r = |\vec{x} - \vec{y}| \to 0$

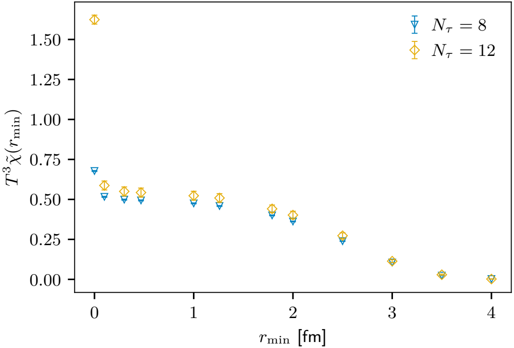
$$\ln \left\langle P^{\text{ren}}_{\vec{x}} P^{\text{ren}\dagger}_{\vec{y}} \right\rangle = -\frac{F^{\text{ren}}_{\bar{q}q}(r,T)}{T} \sim \frac{g^2}{rT} \sim g^2 N_\tau$$

i.e. this term should grow exponentially with N_τ.

SOLUTION: Remove all terms with $r < r_{\text{min}}$ (physical units)

Preliminary results

$$T^3\tilde{\chi}(r_{\text{min}}) \equiv \frac{1}{N_\sigma^3 N_\tau^3} \sum_{\substack{\vec{x},\vec{y} \\ a|\vec{x}-\vec{y}|>r_{\text{min}}}} \left\langle P_{\vec{x}}^{\text{ren}} P_{\vec{y}}^{\text{ren}\,\dagger} \right\rangle$$



- $T = 157$ [MeV]
- m_s/m_l physical
- $48^3 \times 12$ and $32^3 \times 8$

Summary

- $\langle P \rangle$ renormalizes multiplicatively.
- For χ_P some divergence remains, coming from the short-distance terms of a sum over correlations.
- Can remove these by hand by imposing some r_{min}.
- (Very) preliminary results seem at least promising!
- May allow for eventual $a \to 0$ extrapolation.

Thanks for your attention.