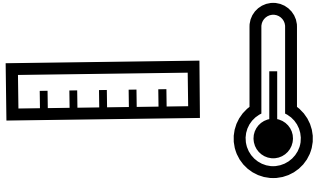
Basic statistical modelling in R
Linear Models (LMs)

Clarke van Steenderen
Department of Zoology and Entomology
Rhodes University
South Africa

Clarke.vansteenderen@ru.ac.za

Data types vary greatly:

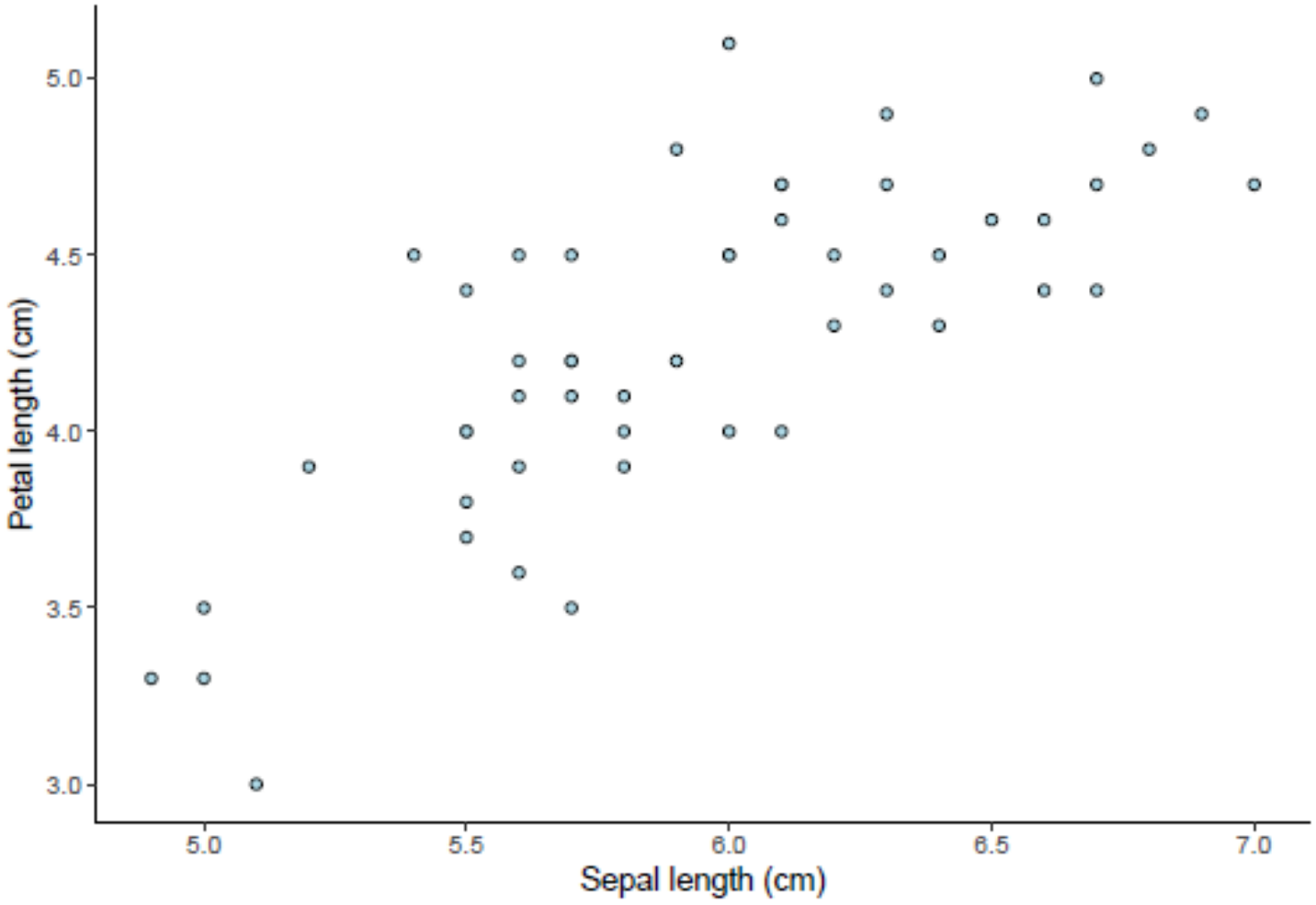
- Counts → how many species/individuals
- Binary → one of two states → male/female, dead/alive
- Continuous → measurements such as height or mass
- Proportions → proportion of a population with a disease
- Categorical → flowers classed into species or colours

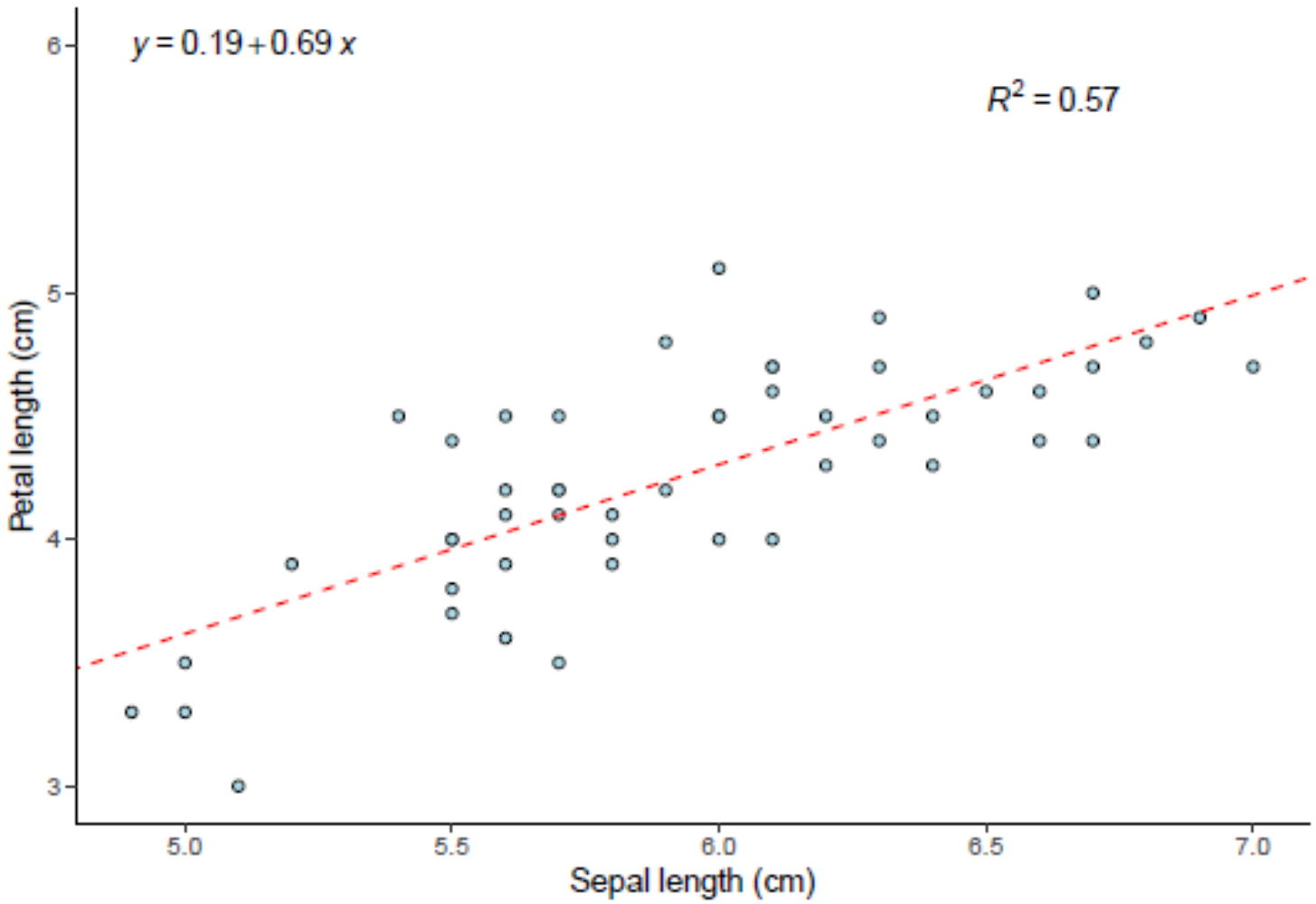


The type of data you have will determine the best choice of statistical test

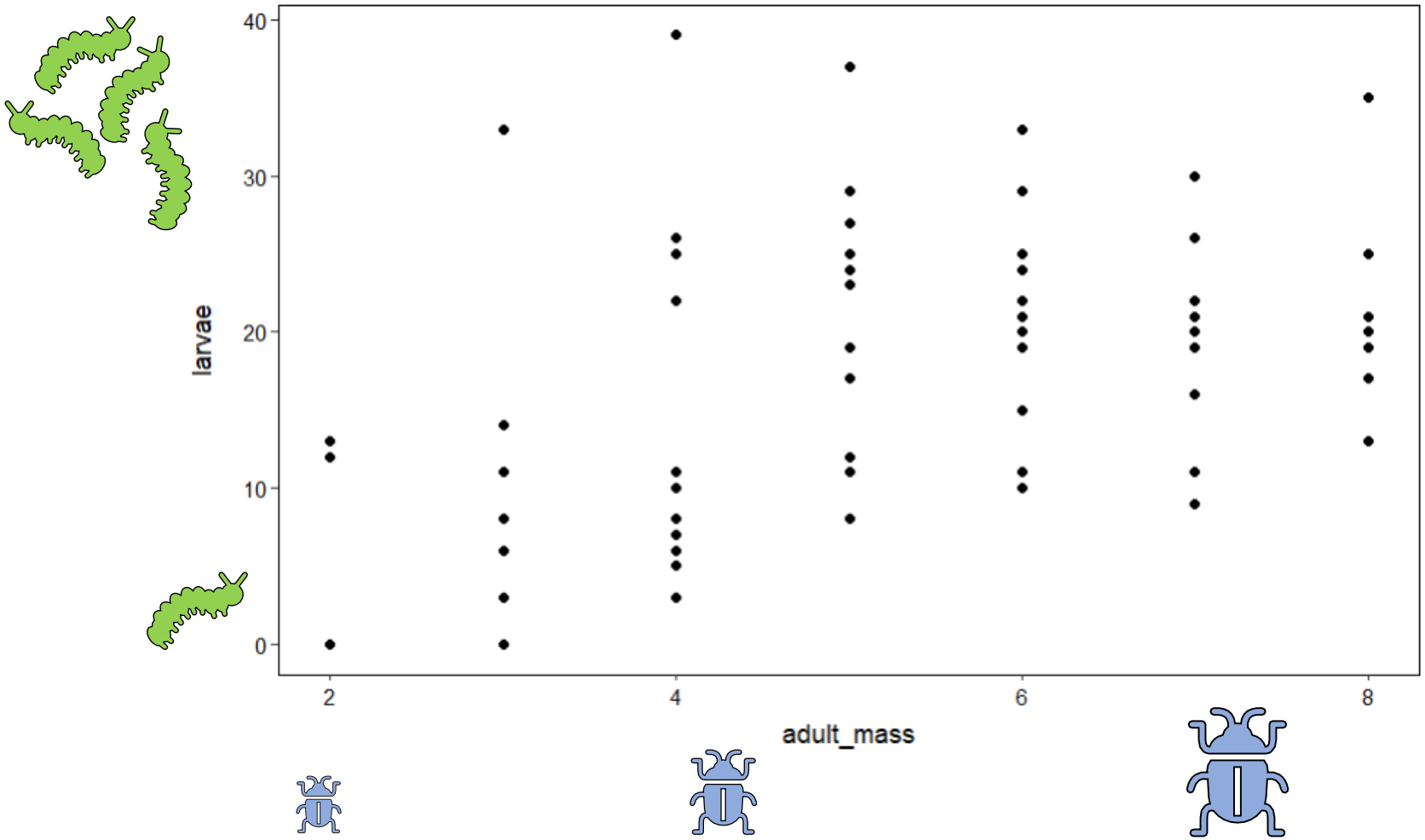
Linear (straight line!) regression is useful to find whether there is:

- **A relationship between X (a predictor variable) and Y (a response variable) → e.g. does body weight (X) affect longevity (Y)? Is there a + or – relationship?**
- **Does a change in X lead to a significant difference in Y?**
- **Can X be used to accurately predict new values of Y?**

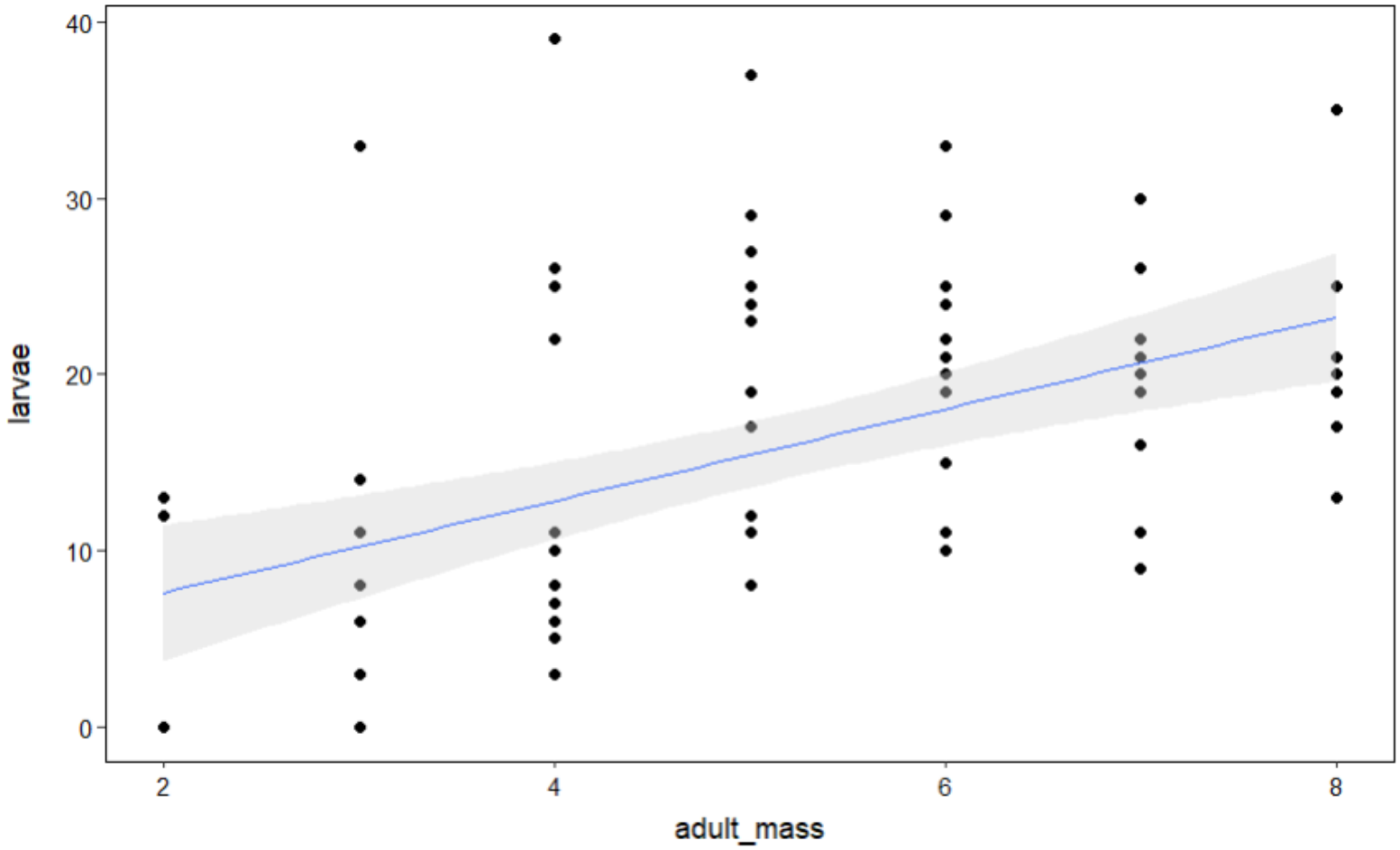

Petal length (cm)
5.0
4.5
4.0
3.5
3.0
5.0
5.5
6.0
6.5
7.0
Sepal length (cm)


y = 0.19 + 0.69 x
R² = 0.57
Petal length (cm)
Sepal length (cm)
3
4
5
6
5.0
5.5
6.0
6.5
7.0

What is the relationship between adult mass in an insect, and its reproductive output (number of larvae)? Positive, negative, none?



If we fit a straight line through these points, how well does it capture the trend? What is the variation in the data?



A linear regression line takes the form:

$$y = \beta_0 + \beta_1(x) + \varepsilon$$

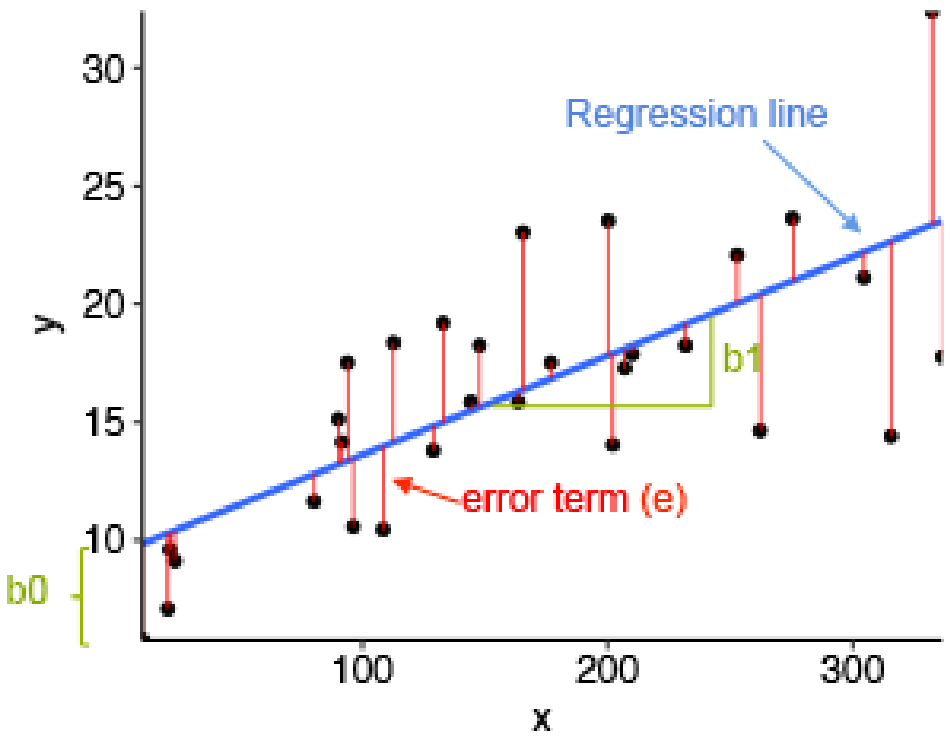
This should remind you of the equation of a straight line:

$$y = c + mx$$

- β_0 is the y-intercept
- β_1 is the gradient of the line
- ε is the error term → what is the difference between the actual measured y-values and the fitted line? This gives an indication of how much of the variation in the y-values is not captured by the model
- The β values are termed "Beta coefficients"

$$y = \beta_0 + \beta_1(x) + \varepsilon$$

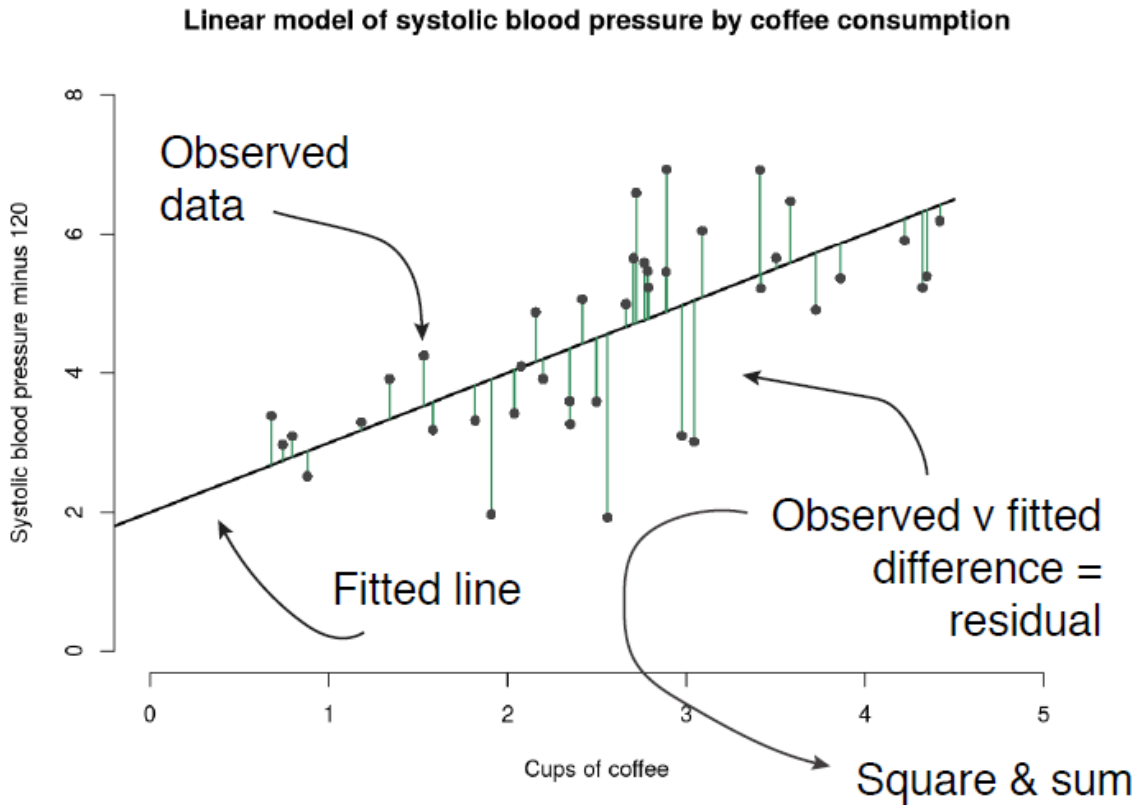
$$larvae = \beta_0 + \beta_1(adult\_mass) + \varepsilon$$



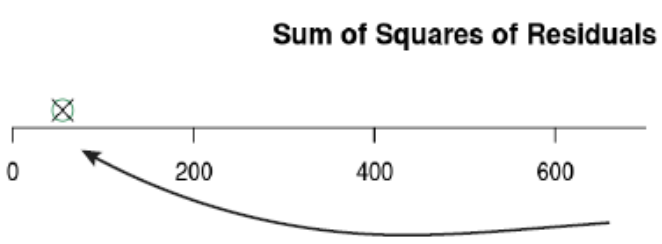
The average variation of points around the regression line is the Residual Standard Error (RSE). The lower the RSE, the better the fit (ideal value is close to 0). Measured in the units of the dependent variable (e.g. # of larvae)

The R^2 value (between 0 and 1) indicates how well the model fits the data. 1 = excellent, 0 = no fit at all (measured as a proportion)

A

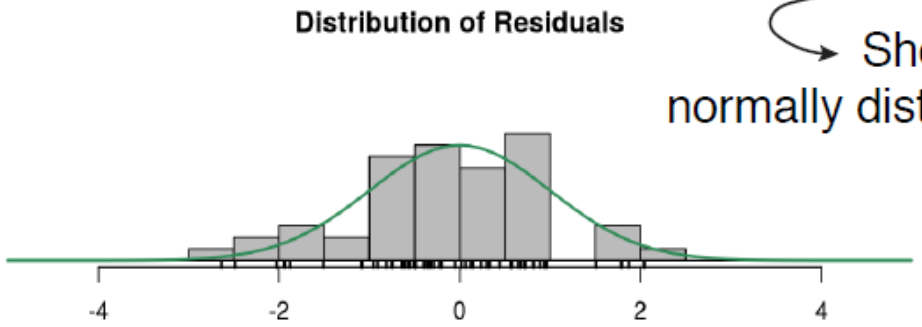


B



This is minimised for best fitting line

C



Should be normally distributed

Model fitting in R

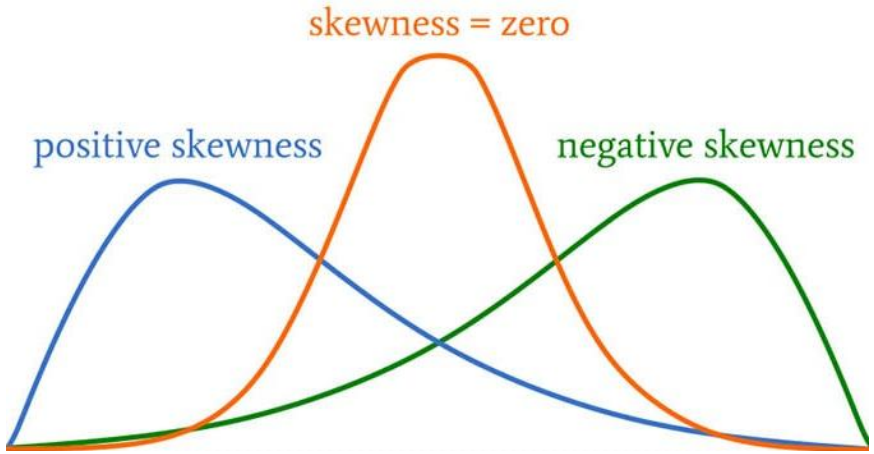
```
linear.mod.1 = lm(y ~ x, data = data)
linear.mod.1 = lm(larvae ~ adult_mass, data = in.data)
```

- `linear.mod.1`: the name in which we are storing the model → this can be any name you choose, but make it informative
- `lm`: the linear model function in R
- `Y~X` means Y as a function of X → how does Y change with a change in X? Note the use of the tilde sign (~)
- `in.data` is the input data that is read into R (e.g. from an Excel sheet)

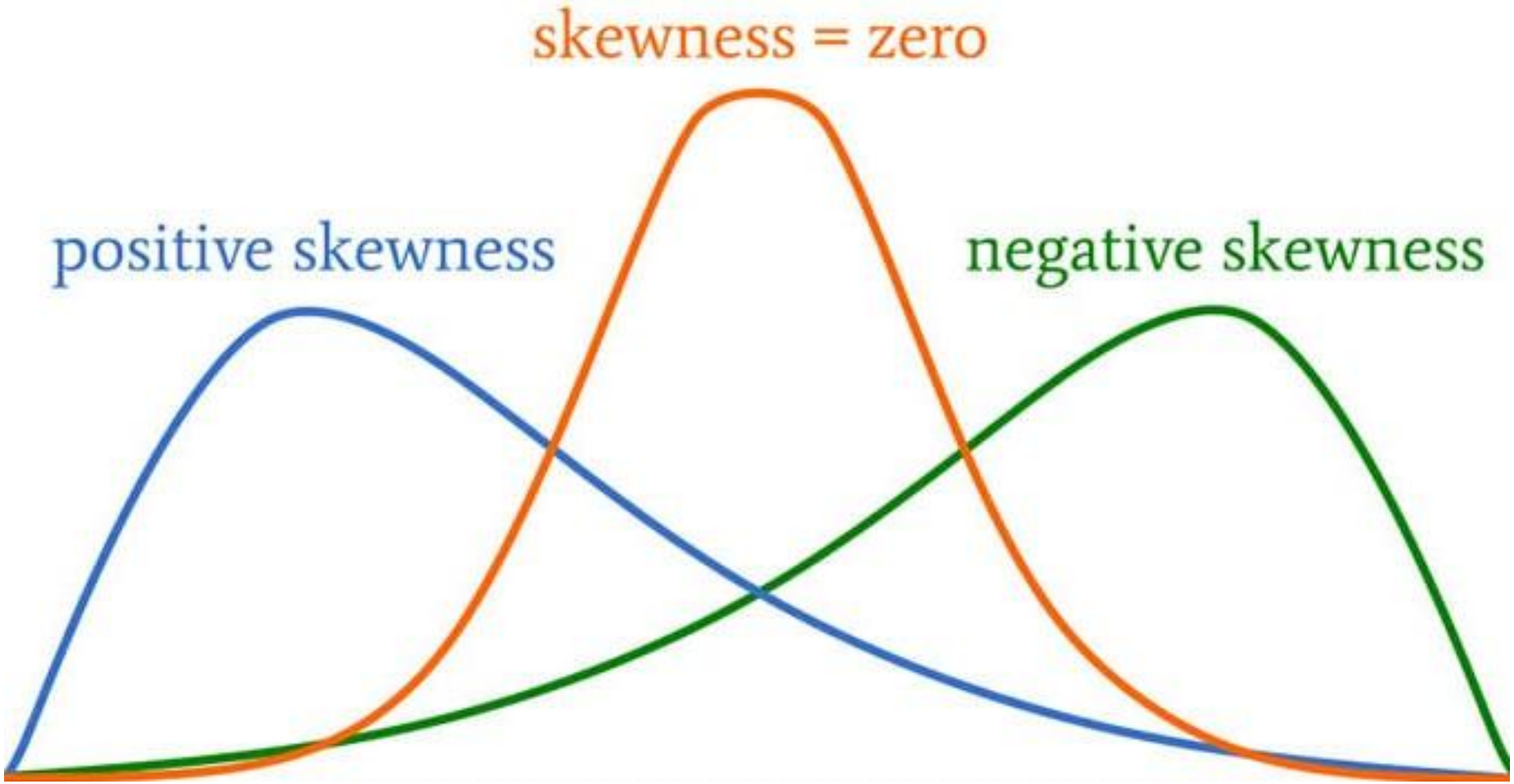
Check how well the model fits

For a statistical model to be reliable, there are a few assumptions that the data need to meet:

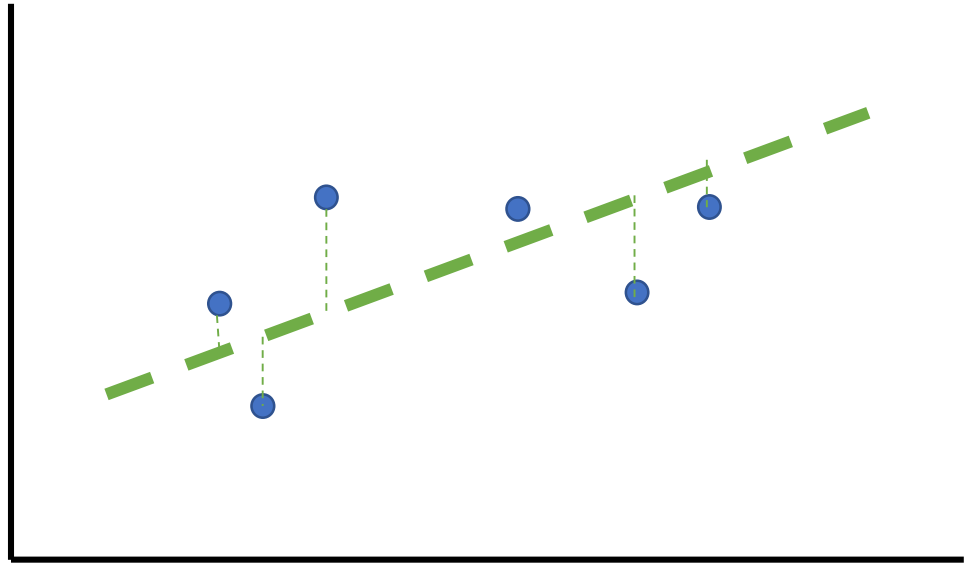
1. **Linearity**
2. **Independence → data collected from one site/individual does not affect data collected in others (e.g. plants in the same plot)**
3. **Normality of residuals (difference between observed and predicted values → normal bell curve) with a mean close to zero**
4. **Equal variance of residuals**

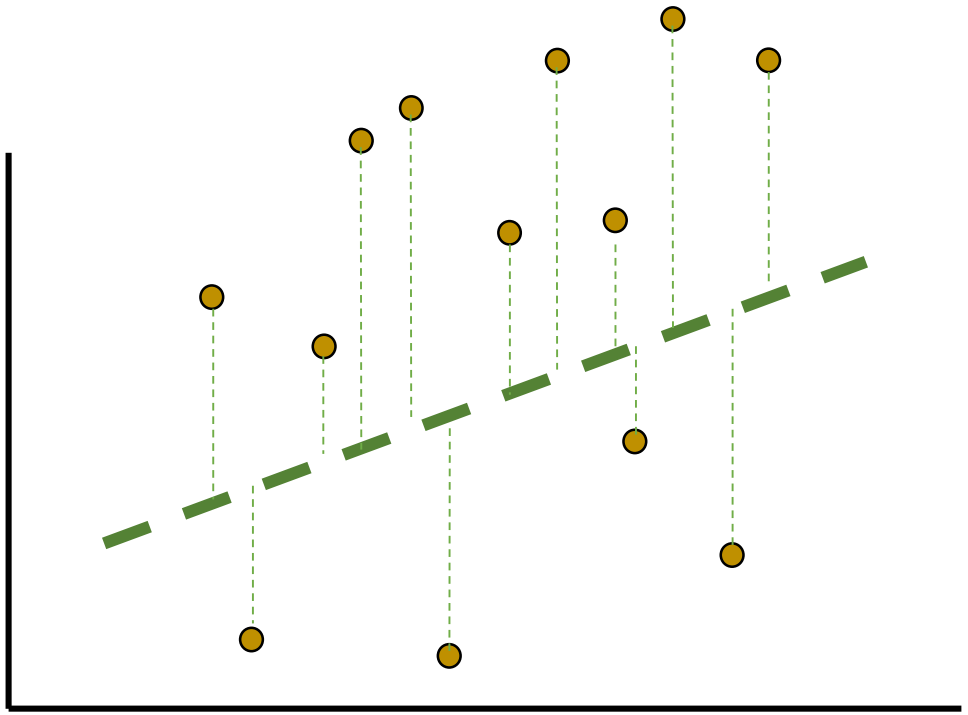


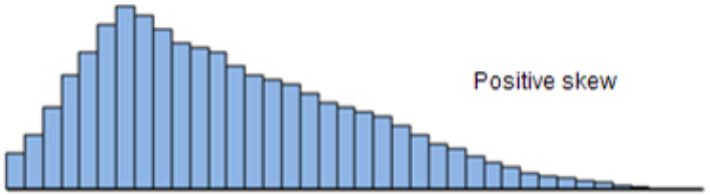
We need to run model diagnostic tests to check these. In R, the `DHARMa` package is very useful

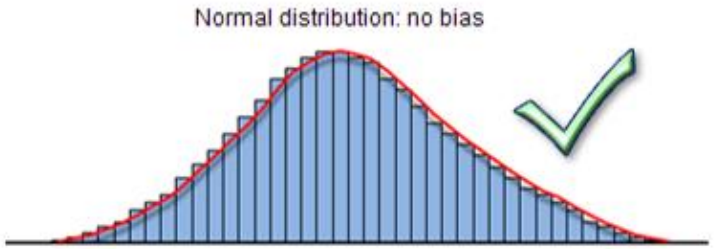

skewness = zero
positive skewness
negative skewness

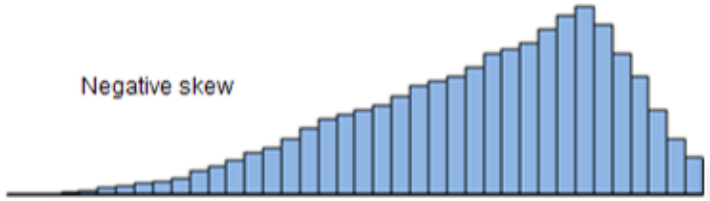
Raw residuals (observed – fitted)





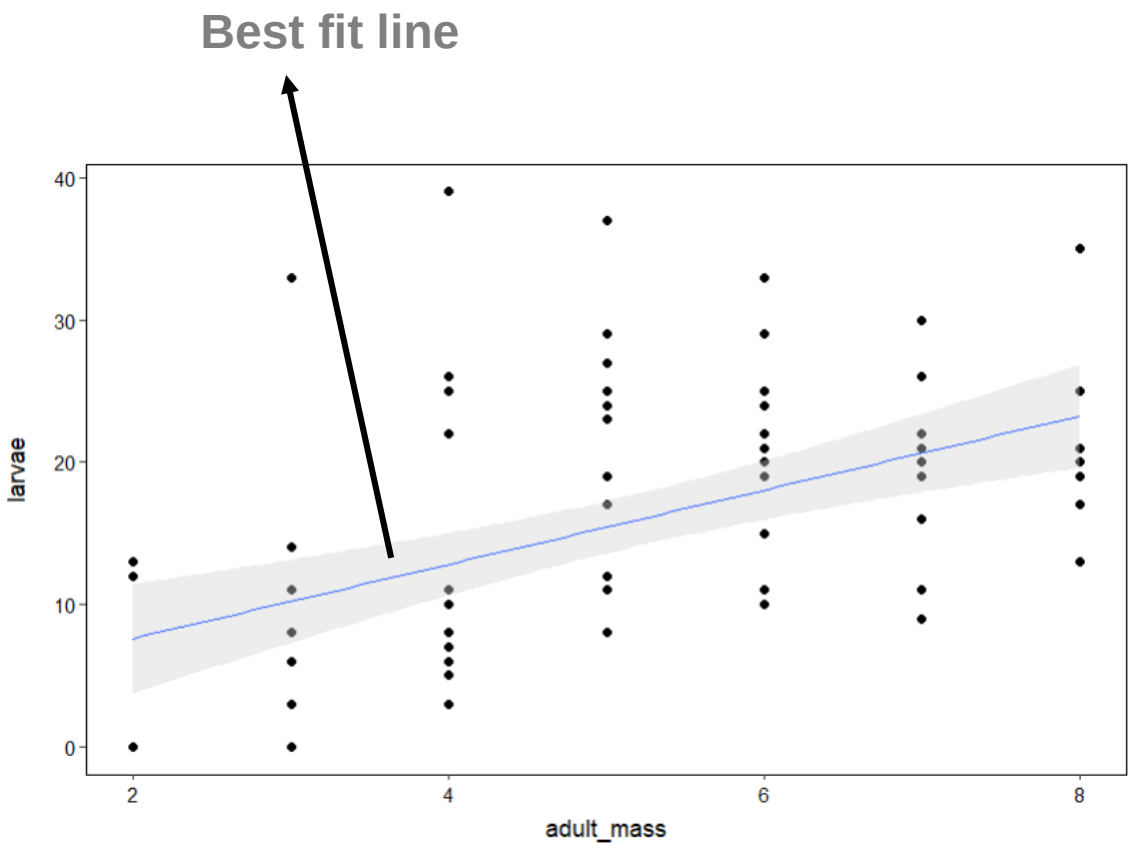


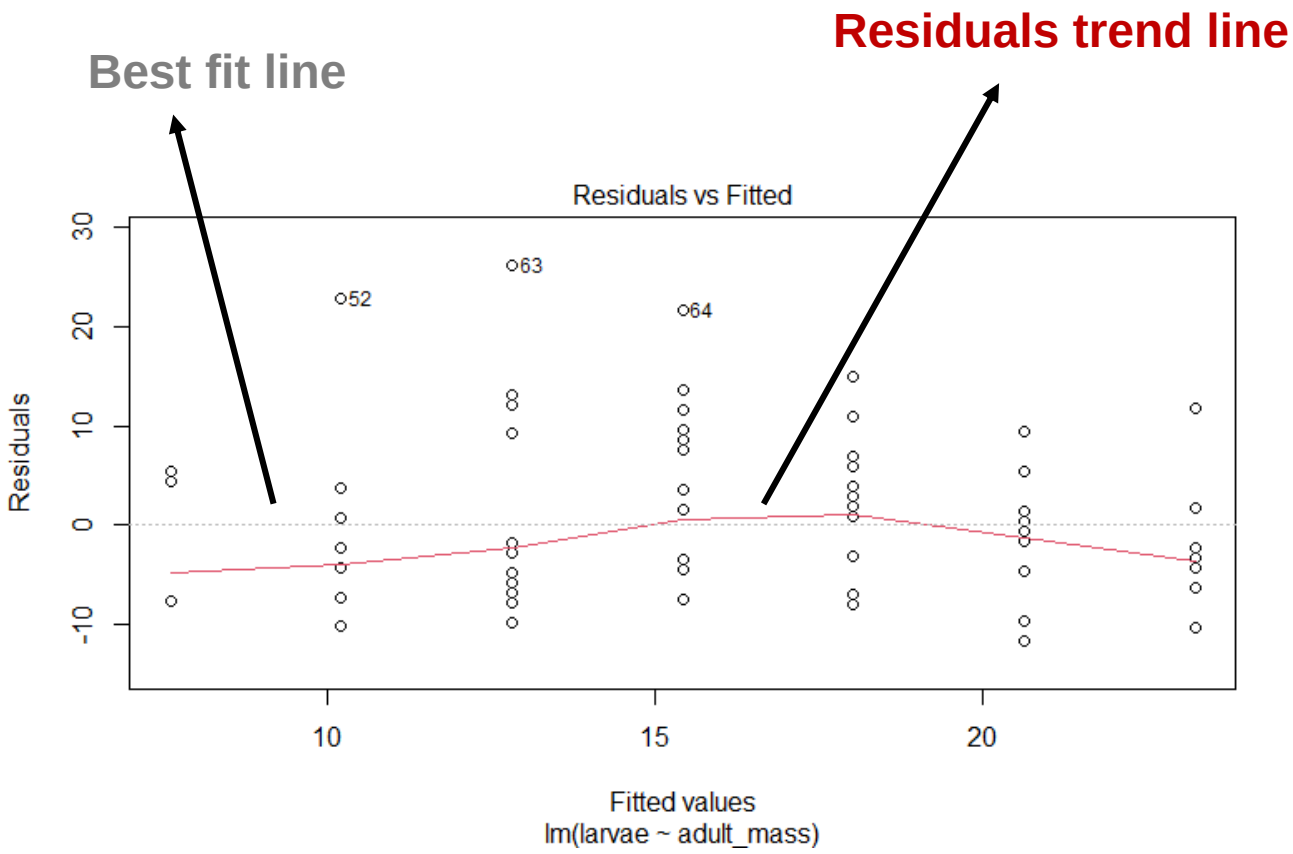




Linearity (and equal variance) → residual vs fitted plot

Residual values should cluster around the y = 0 line, with no clustering or patterning (constant spread, "shotgun pattern")

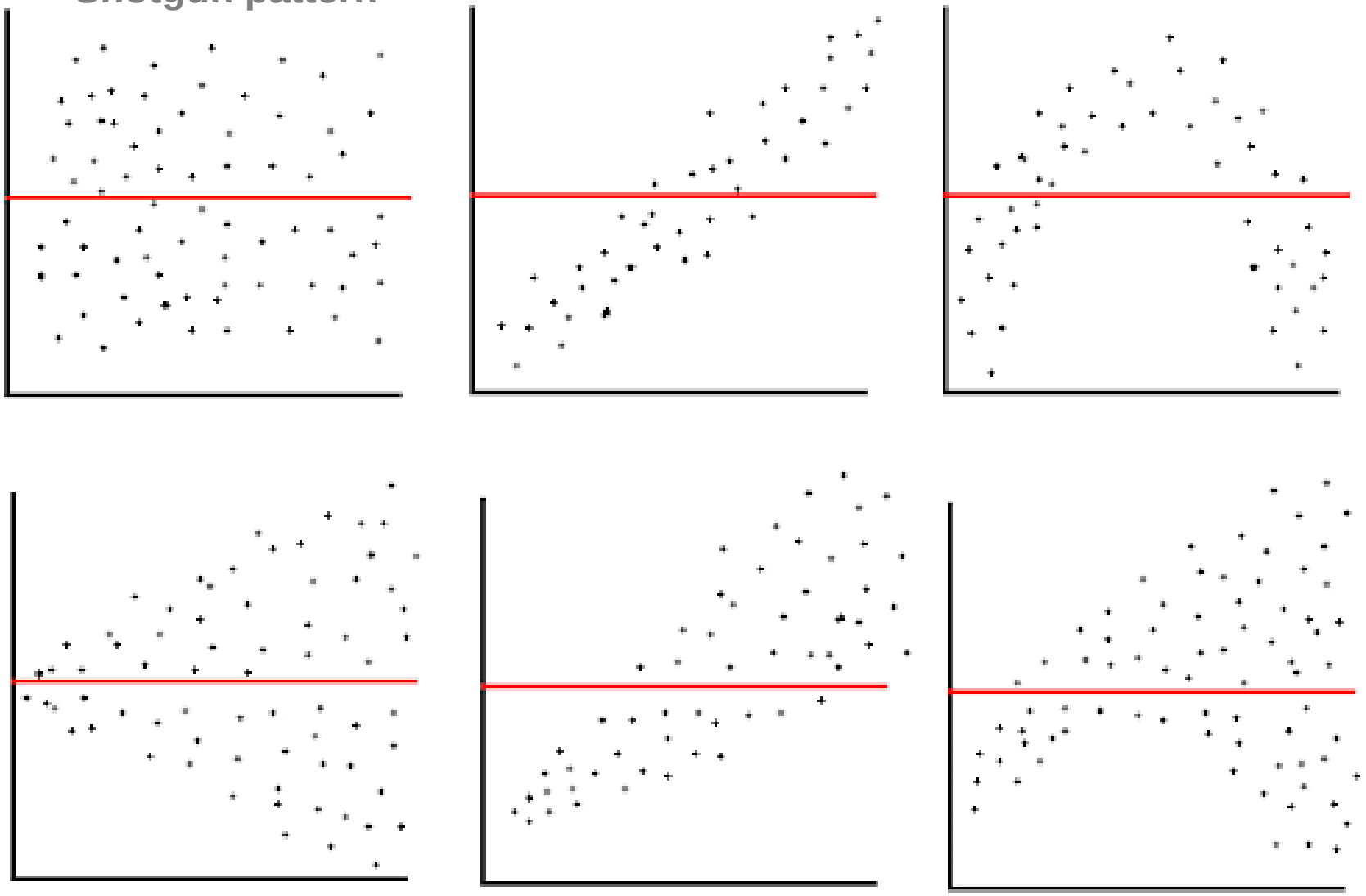




Residual plot examples

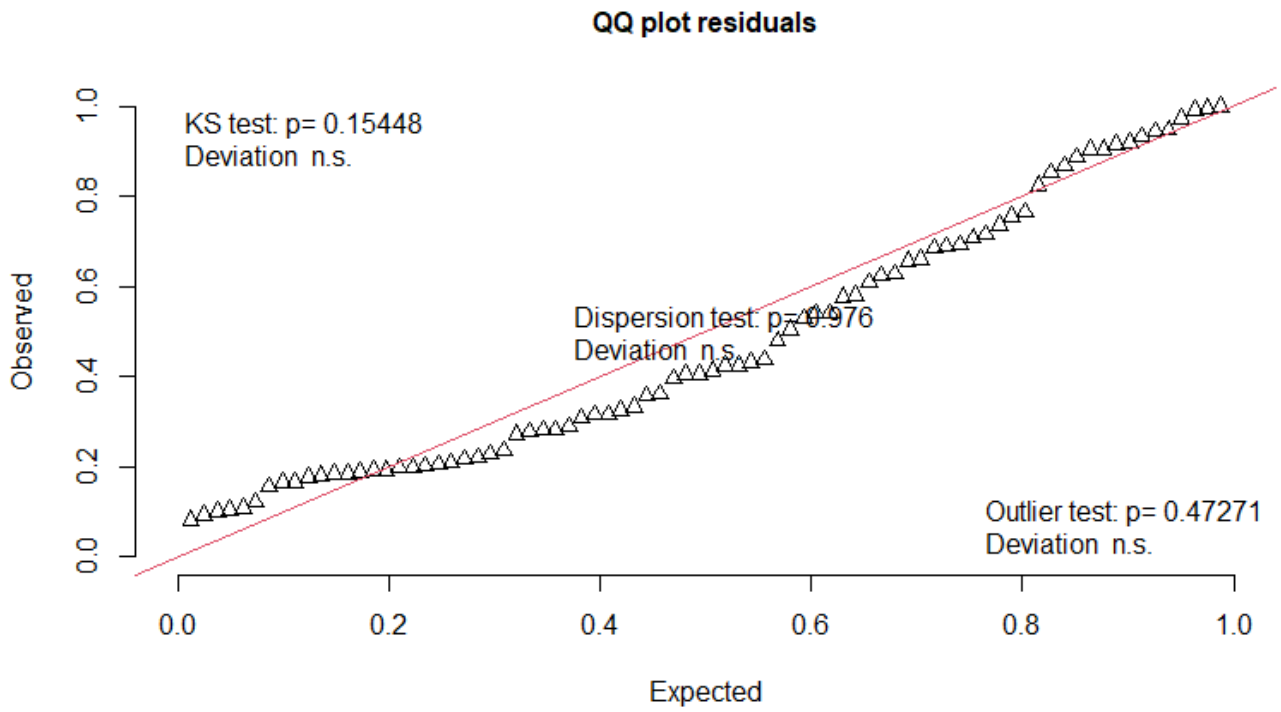
Shotgun pattern

Suitable for a LM

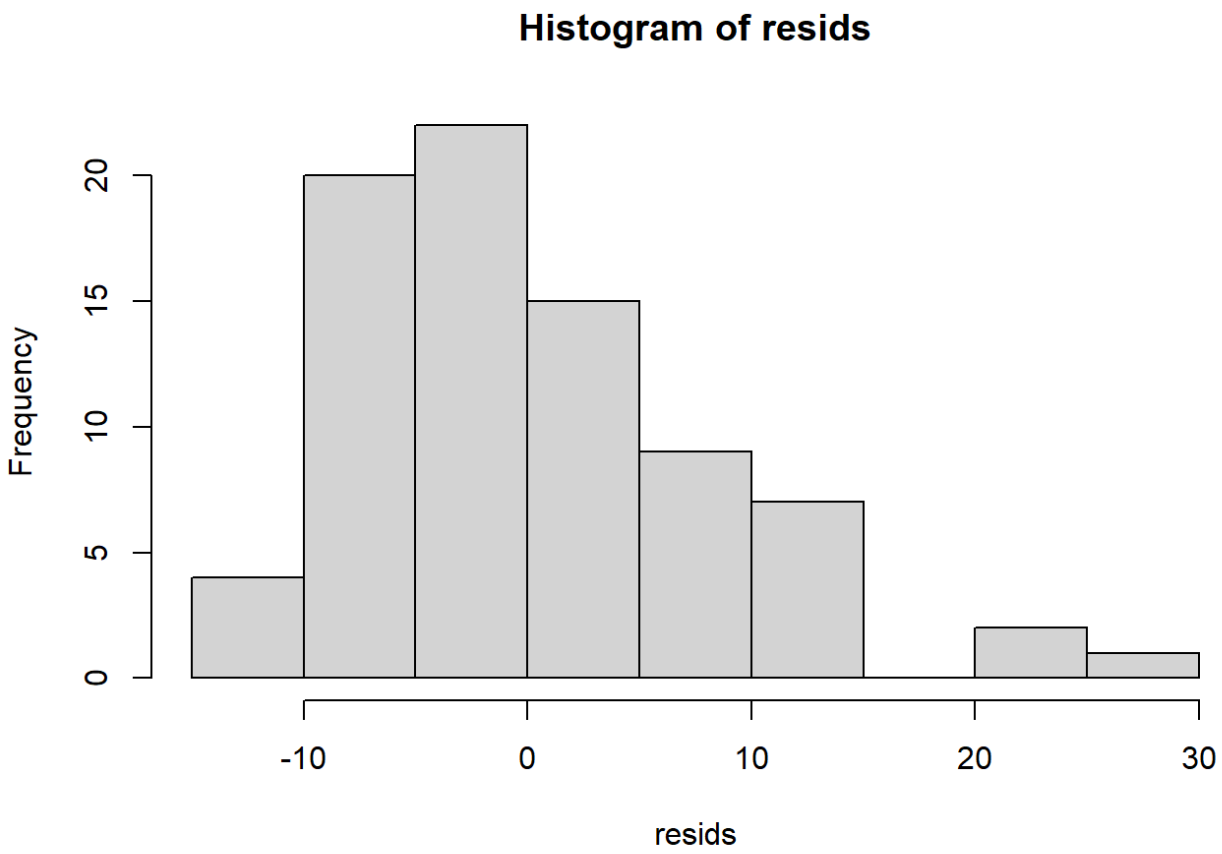


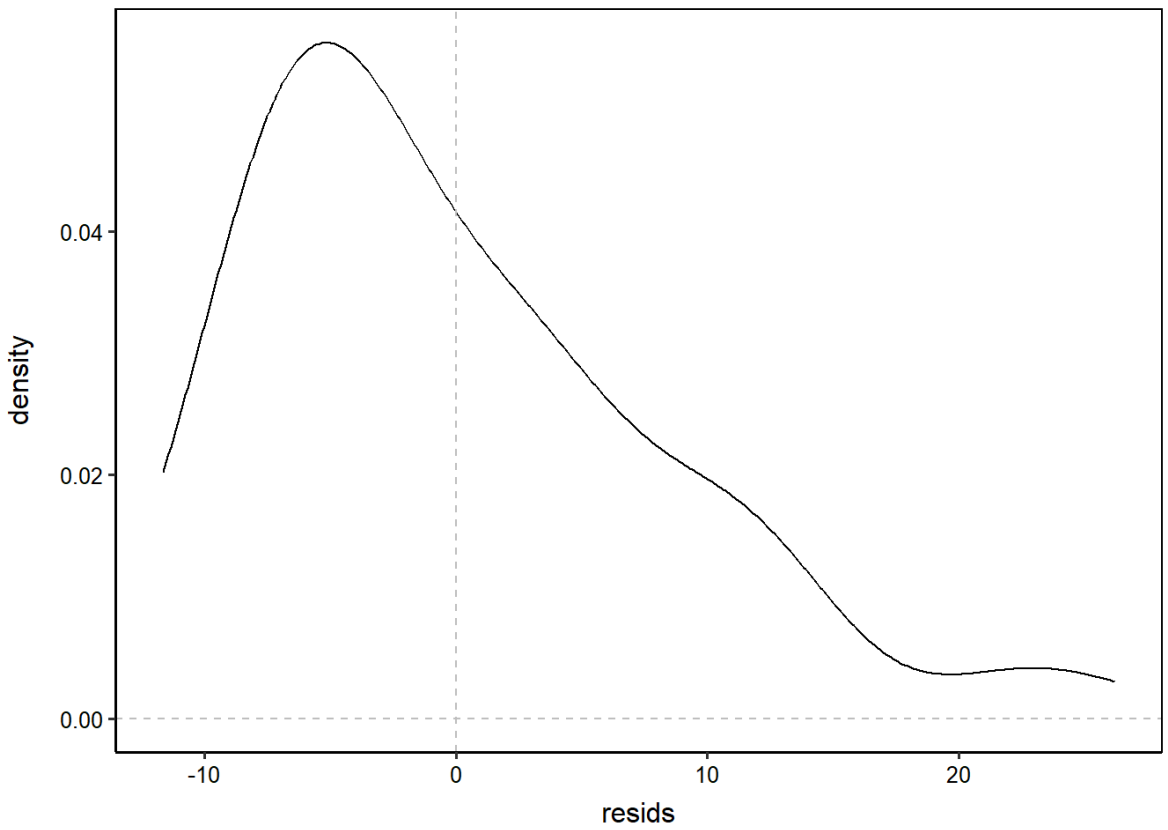
Normality of residuals → QQ plot

Residual values should cluster around the straight line with gradient of 1 (red line). Kolmogorov-Smirnov test shows whether residuals are normally distributed ($p > 0.05$) or not ($p < 0.05$). Dispersion and outlier tests should ideally be non-significant



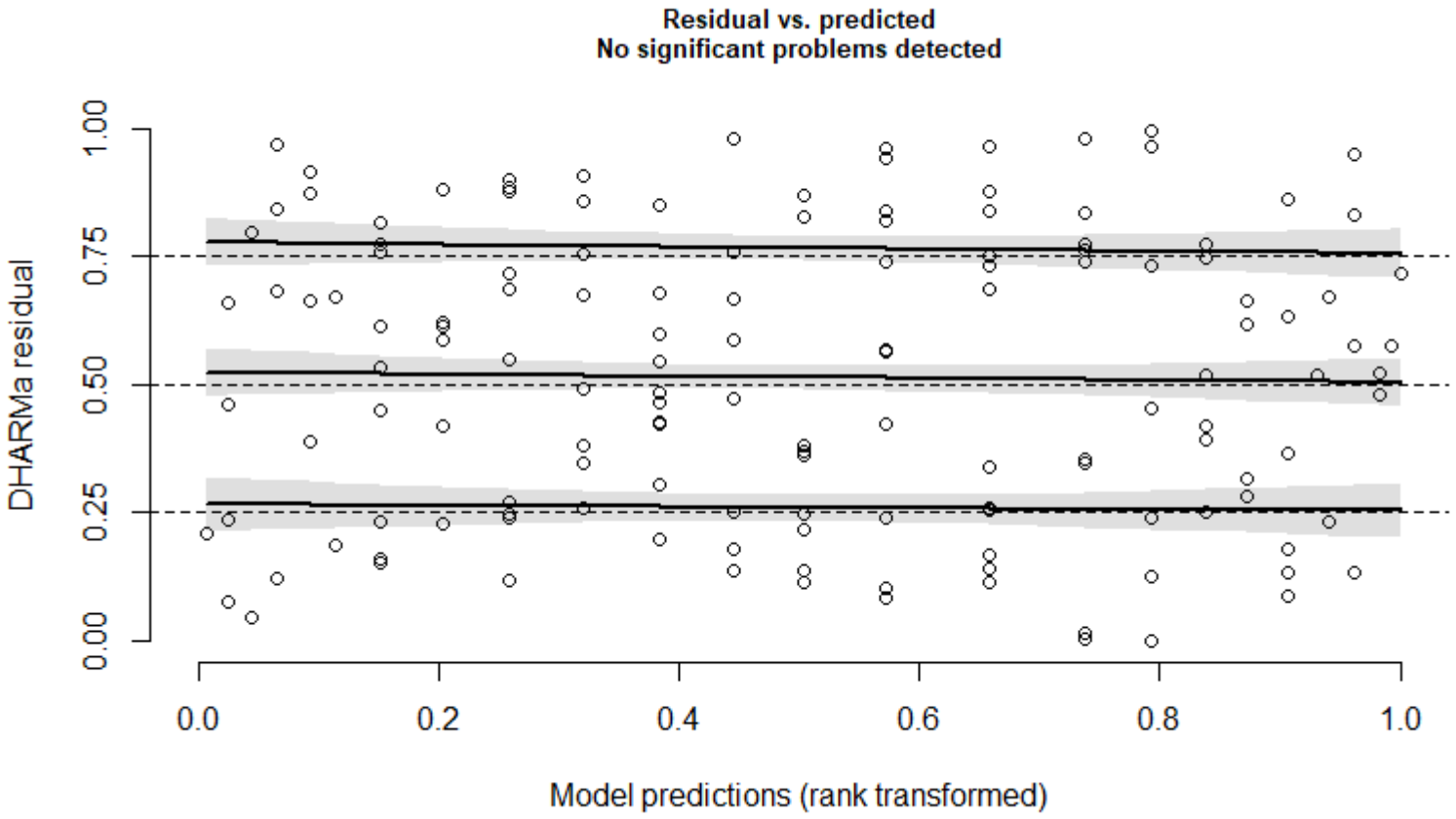
Despite a slight + skewness in residuals, they are normally-distributed (KS test)





Equal variance → residual vs predicted

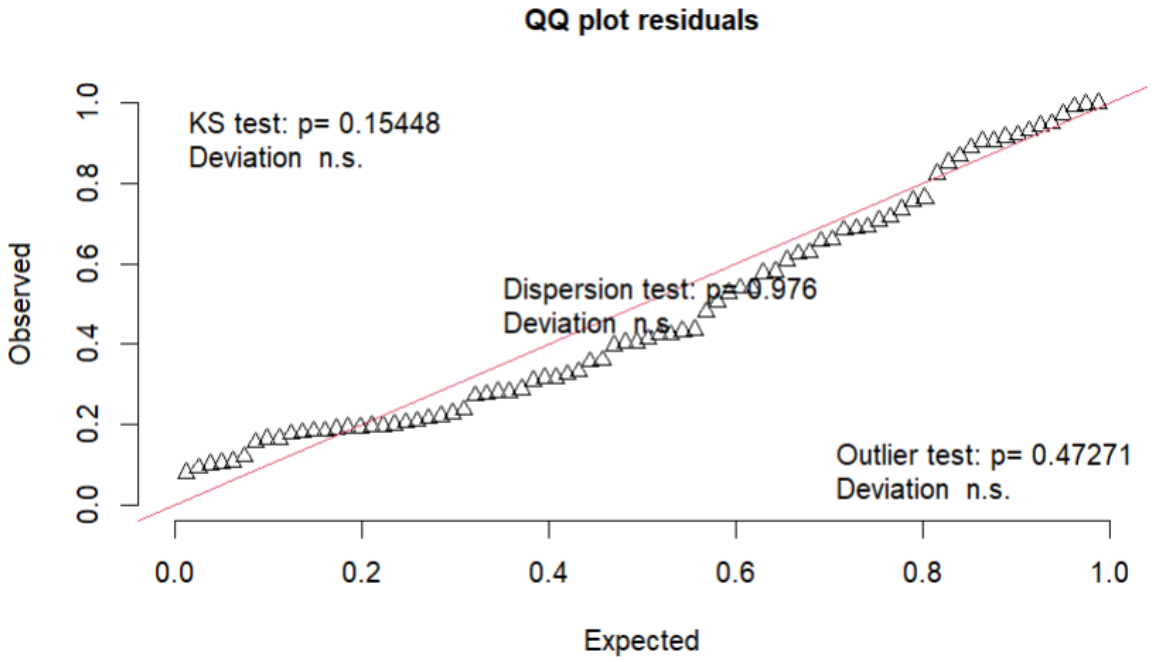
Bold lines should fall along the y = 0.25, 0.5, and 0.75 quartile range marks



Check how well the model fits

```
DHARMa::plotQQunif(linear.mod.1)
```

This QQ plot shows that residuals are normally distributed (KS test), that there is no overdispersion, and no outliers. We want to see the points roughly matching the straight red line (y = x)

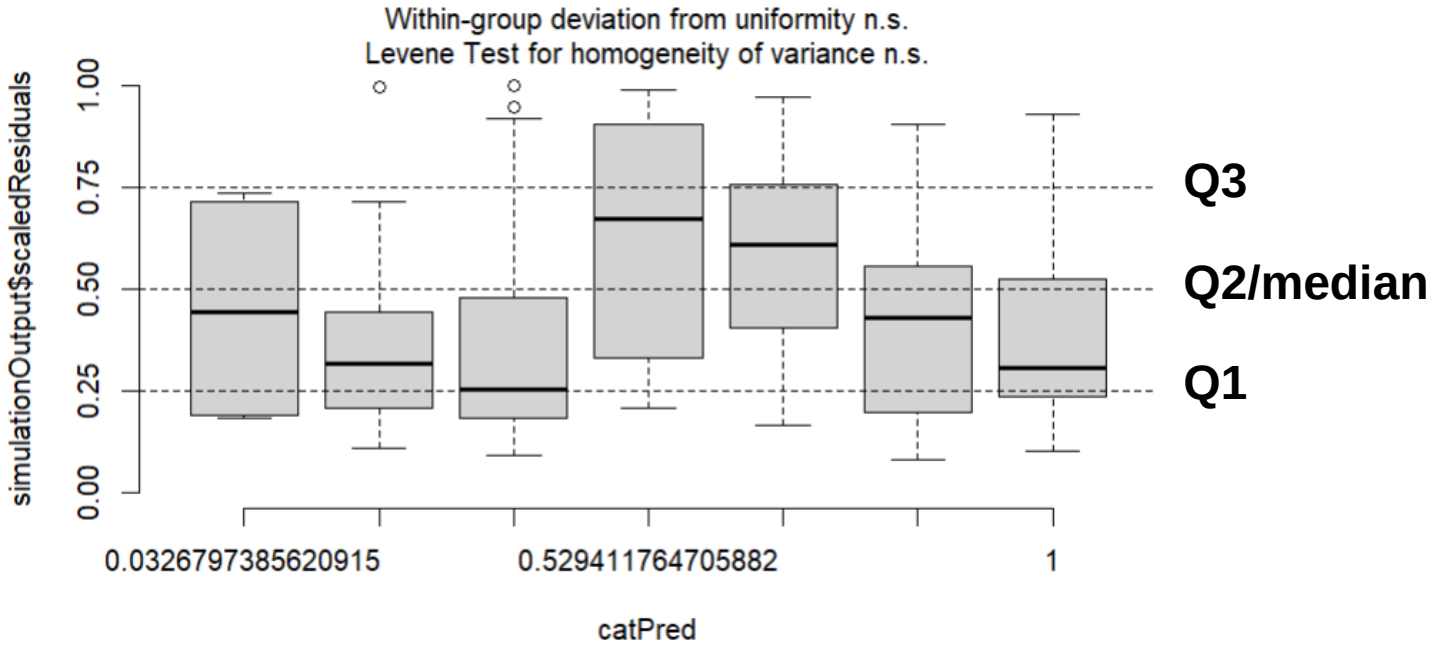


Check how well the model fits

```
DHARMa::plotResiduals(linear.mod.1)
```

Within-group deviation (Kolmogorov-Smirnov test (KS)) per mass category and Levene tests for homogeneity of variance show that there are no issues with the residuals in this linear model. We expect the three interquartile ranges of the boxplots to match the dotted horizontal lines

Boxplots are shown in the case of different categories or groups. Here, they are weight categories



Check how well the model fits

```
summary(linear.mod.1)
```

The summary output for our model shows that our intercept (β_0) = 2.399, and our gradient (β_1) is 2.6066. Our straight line is $y = 2.6x + 2.4$

```
Call:
lm(formula = larvae ~ adult_mass, data = in.data)

Residuals:
    Min      1Q  Median      3Q     Max 
-11.645  -6.825  -2.022   4.068  26.175 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) β0 2.3990     2.9254   0.820    0.415    
adult_mass  β1 2.6066     0.5431   4.799 7.52e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 8.236 on 78 degrees of freedom
Multiple R-squared:  0.228,	Adjusted R-squared:  0.2181 
F-statistic: 23.03 on 1 and 78 DF,  p-value: 7.521e-06
```

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```

- **β_1 suggests that for every unit increase in biomass (1 g), the number of larvae increase by 2.6**
- **Our p-value is < 0.001, which means that adult mass has a significant effect on larvae number**
- **RSE tells us that adult mass (x) predicts larvae numbers (y) with an average error of 8.2 larvae → average size of the residuals (smaller = better). Ranges from 0 to +∞**

```
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lm(formula = larvae ~ adult_mass, data = in.data)

Residuals:
    Min      1Q  Median      3Q     Max 
-11.645  -6.825  -2.022   4.068  26.175 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) β0 2.3990     2.9254   0.820    0.415    
adult_mass  β1 2.6066     0.5431   4.799 7.52e-06 ***
---
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Multiple R-squared:  0.228,	Adjusted R-squared:  0.2181 
F-statistic: 23.03 on 1 and 78 DF,  p-value: 7.521e-06
```

- **The R^2 value indicates that our linear model explains 0.22 (22%) of the variation in the data → how much of the variability in *y* is explained by *x*? Larger = better. Ranges from 0 to 1**
  What might this value (0.22) indicate? Is this a reliable model?
- **F-statistic → tells you whether the model is significant (larger = better). Is the model better than chance? Ranges from 0 to +∞**

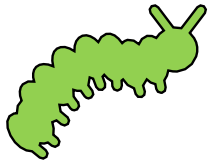
Check how well the model fits

```
confit(linear.mod.1)
```

```
              2.5 %   97.5 %
(Intercept)  -3.424936 8.222914
adult_mass    1.525312 3.687797
```

The 95% confidence interval (CI) suggests that a 1 g increase in adult mass will result in 1.5 – 3.7 more individual larvae. If we were to repeat this experiment 100 times, our larvae estimates will fall in this range 95 times out of the 100.

Reminder: our β_1 was 2.6 → this should fall within the CI range



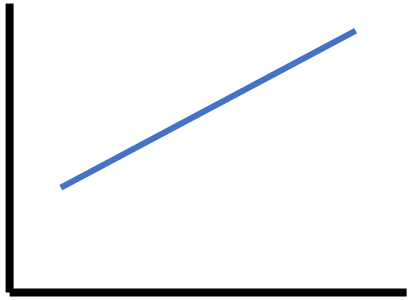
Hypothesis testing

- **Null (H_0)**

Body mass does not have a significant effect on reproductive output (number of larvae)

- **Alternative (H_1)**

Body mass significantly influences reproductive output



Hypothesis testing

Instead of lm(), we will run a glm() → more on that later

```
H0.model = glm(larvae ~ 1, data = in.data,
family = gaussian)

H1.model = glm(larvae ~ 1 + adult_mass,
data = in.data, family = gaussian)
```

A Gaussian distribution is specified for normally-distributed residuals

Notice how the `H0.model` excludes adult mass, and only runs the model with an intercept term (~1). We are not including any predictor variables here, and are assuming that β_1 (gradient) is not significantly different from zero (i.e. a horizontal line)

The ~1 tells R: run this model without any predictor variables, and estimate the intercept (which would also be the mean # of larvae)

```
H0.model = glm(larvae ~ 1, data = in.data,
family = gaussian)

H1.model = glm(larvae ~ 1 + adult_mass,
data = in.data, family = gaussian)
```

summary(H0.model)

```
> summary(H0.model)

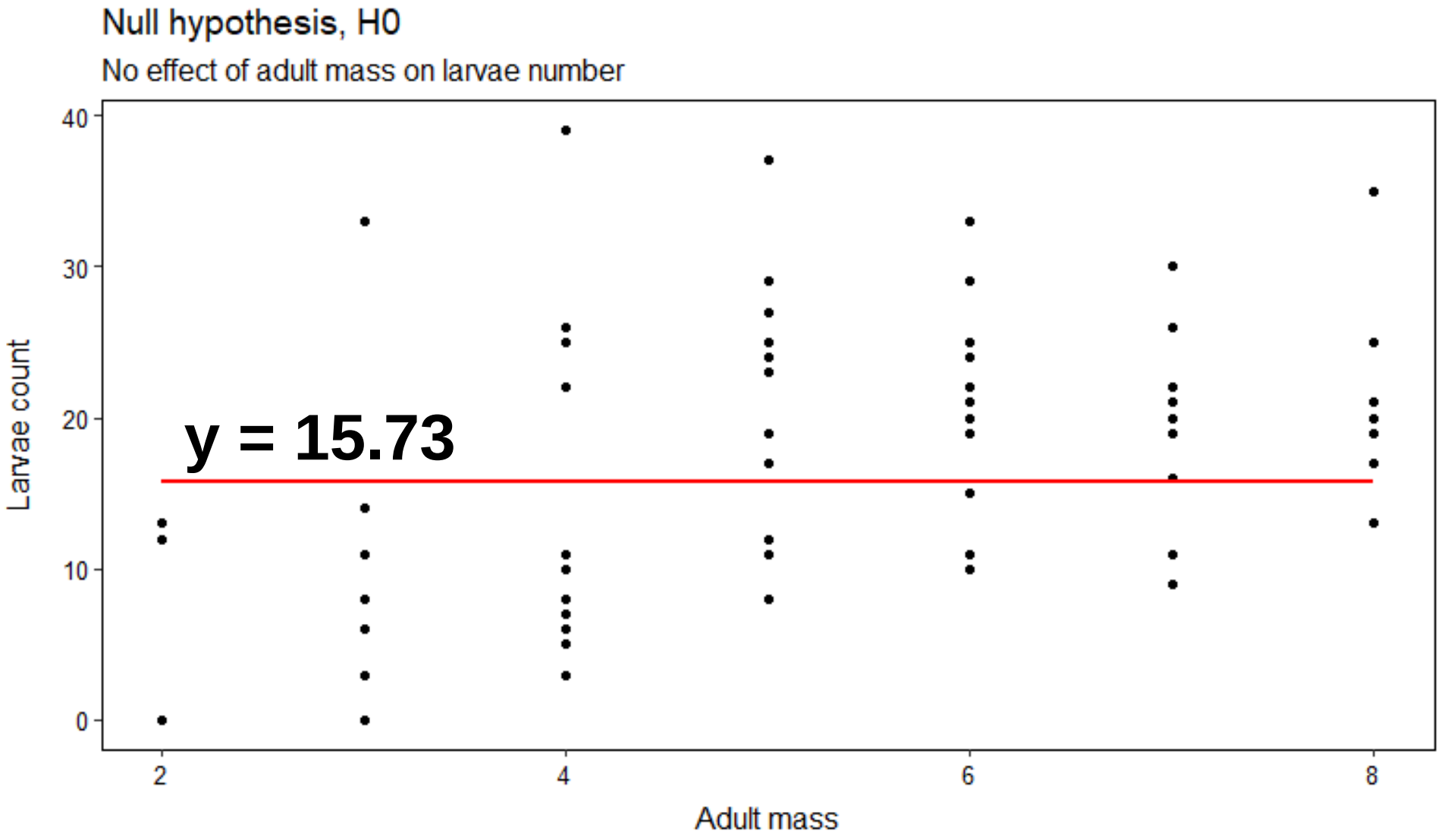
Call:
glm(formula = larvae ~ 1, family = gaussian, data = in.data)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   15.725      1.041    15.1   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for gaussian family taken to be 86.75886)

    Null deviance: 6853.9  on 79  degrees of freedom
Residual deviance: 6853.9  on 79  degrees of freedom
AIC: 587.07

Number of Fisher Scoring iterations: 2
```


Null hypothesis, H0
No effect of adult mass on larvae number
y = 15.73
Larvae count
Adult mass
0
10
20
30
40
2
4
6
8

```
summary(H1.model)

Call:
glm(formula = larvae ~ 1 + adult_mass, family = gaussian, data =
in.data)

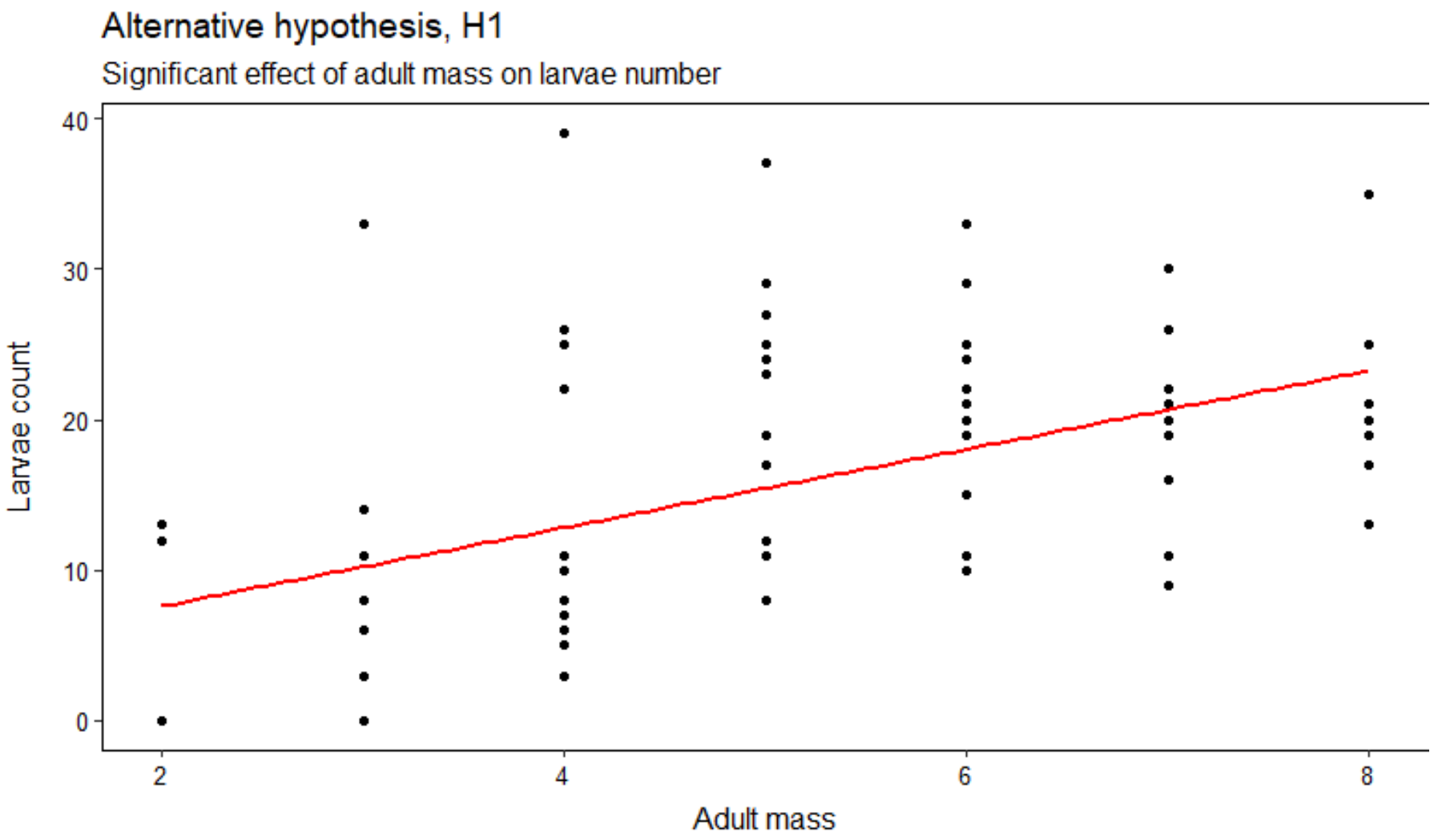
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   2.3990     2.9254   0.820    0.415
adult_mass    2.6066     0.5431   4.799 7.52e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for gaussian family taken to be 67.83828)

    Null deviance: 6853.9  on 79  degrees of freedom
Residual deviance: 5291.4  on 78  degrees of freedom
AIC: 568.37

Number of Fisher Scoring iterations: 2
```

$$y = 2.6x + 2.4$$


Alternative hypothesis, H1
Significant effect of adult mass on larvae number
Larvae count
Adult mass
0
10
20
30
40
2
4
6
8

Lower Residual deviance and AIC values are better

```
> summary(H0.model)

Call:
glm(formula = larvae ~ 1, family = gaussian, data = in.data)

Null deviance: 6853.9  on 79  degrees of freedom
Residual deviance: 6853.9  on 79  degrees of freedom
AIC: 587.07
```

```
> summary(H1.model)

Call:
glm(formula = larvae ~ 1 + adult_mass, family = gaussian, data = in.data)

Null deviance: 6853.9  on 79  degrees of freedom
Residual deviance: 5291.4  on 78  degrees of freedom
AIC: 568.37
```