

Lab #5

Fall 2013

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DUE 12 Nov 2013 in Discussion. See www.stat.ucdavis.edu/~beran/s232a/dental.dat for the data. Use class techniques to analyze the data. Attach your computer code to your report.

Seheult and Tukey (2001) analyzed a three-factor layout in which the response measures the hardness of dental fillings obtained by 5 Dentists (D) using 8 Gold alloys (G) and 3 Condensation methods (C). The objective of the experiment was to find a dental gold filling with greater hardness. Condensation, properly carried out, was known to increase the hardness of a filling. The three condensation techniques used in the experiment were: (1) electromalleting, in which blows are delivered mechanically at a steady frequency; (2) hand malleting, in which a small mallet is used to deliver blows; and (3) hand condensation. The reported hardness observations are each averages of ten measurements that are not available. It was reported anecdotally that dentist 5 appeared to be physically tired before the experiment.

As general model for the three-way layout of hardness measurements, consider

$$y_{ijk} = m_{ijk} + e_{ijk}, \quad 1 \leq i \leq p_G = 8, \quad 1 \leq j \leq p_C = 3, \quad 1 \leq k \leq p_D = 5.$$

The errors $\{e_{ijk}\}$ are taken to be i.i.d. $N(0, \sigma^2)$ with σ^2 unknown. Let y be the column vector of the hardness measurements $\{y_{ijk}\}$, ordered in mirror dictionary order. The dimension of y is $p \times 1$ with $p = p_G p_C p_D$. The command `y <- scan("dental.dat")` computes the vector y . The least squares estimator of m under the general model is just y . Analysis of the data is difficult because there is only one observation per factor-level triple, because possibly all of the main effects and interactions matter, and because some outliers exist.

Consider the candidate shrinkage estimators

$$\hat{m}(a) = (a_0 P_0 + a_G P_G + a_C P_C + a_D P_D + a_{GC} P_{GC} + a_{GD} P_{GD} + a_{CD} P_{CD} + a_{GCD} P_{GCD})y.$$

The projections are those of three-way ANOVA. The subscripts indicate the factors involved. The shrinkage factor multiplying each projection lies in $[0, 1]$.

- Find the expected value, under the general model, of $\hat{\sigma}^2 = |P_{GD}y|^2 / \text{tr}(P_{GD})$ and report its numerical value for the dental hardness data. Why does $\hat{\sigma}^2$ seem preferable to the analogous variance estimators based on the GC or CD or GCD interactions?
- Using the $\hat{\sigma}^2$ from part a, compute the adaptive shrinkage estimator \hat{m}_{shr} developed in class. Report its estimated risk $\hat{R}(\hat{m}_{shr})$ and the value of each estimated shrinkage factor.
- Explain mathematically why the estimator $\hat{\sigma}^2$ used forces $\hat{a}_{GD} = 0$.
- Make a normal quantile plot of the residuals $y - \hat{m}_{shr}$, adding the straight line that passes through the first and third quartile of the residuals. Comment on what the plot tells you.

- e) Compute, as in part b, the adaptive projection estimator \hat{m}_{pro} developed in class. Report its estimated risk $\hat{R}(\hat{m}_{pro})$ and the value of each estimated shrinkage factor.
- f) Averaging a three-factor fit over all levels of one factor yields a two-way marginal fit that estimates the corresponding two-way array of averaged means. For both the adaptive shrinkage estimator and the adaptive projection estimator: Compute the two-way CD marginal fit by averaging over all levels of G; the two-way marginal GC fit by averaging over all levels of D; and the two-way marginal GD fit by averaging over all levels of C. Display these six marginal fits as perspective plots on one sheet of paper. Label the plots to indicate adaptive shrinkage or adaptive projection; and label the axes to indicate the dentist, condensation method, and gold alloy.
- g) Study the plotted marginal fits in part f. Is hand condensation by dentists 4 and 5 less successful in making a filling hard than hand condensation by the other dentists? Does electromalleting or hand malleting harden a filling more than hand condensation, regardless of the gold alloy used? Do some gold alloys produce harder fillings than others, regardless of dentist?