Chi square Special case: $X \sim \chi^2_n \equiv \text{Gamma}(\frac{n}{2}, \frac{1}{2})$

$f(x) \propto x^{\frac{n}{2}-1} e^{\frac{-x}{2}}, \quad x > 0 \qquad \mathrm{E}\, X = n, \quad \mathrm{Var}\, X = 2n$

Let Z_i be iid $N(0,1)$. $\sum_{i=1}^n Z_i^2 \sim \chi^2_n$

Noncentral χ^2. Let $Y \sim N(\mu, I)$ be an n vector. Then

$$\|Y\|^2 \sim \chi^2_n(\|\mu\|^2)$$

F if num. and den. independent then

$$F(m,n) \equiv \frac{\frac{\chi^2_m}{m}}{\frac{\chi^2_n}{n}}$$

T if num. and den. independent then

$$t(n) = \frac{N(0,1)}{\sqrt{\frac{\chi^2_n}{n}}}$$

Conditional pdf:

$$f_{X|Y}(x|y) \equiv \frac{f_{X,Y}(x,y)}{f_Y(y)}$$

Iterated expectation:

$$E(Y) = E(E(Y|X))$$

Conditional variance formula:

$$\mathrm{Var}(Y) = \mathrm{Var}(E(Y|X)) + E(\mathrm{Var}(Y|X))$$

Singular Value Decompostion (SVD) Any matrix X can be written

$$X = UDV^T$$

with U, V orthogonal, and D diagonal.

Moore Penrose Psuedoinverse A^+ exists uniquely for every matrix A. Properties: $AA^+A = A, \quad A^+AA^+ = A^+$ and AA^+, A^+A are symmetric.

Projection matrix P are symmetric and idempotent. They have eigenvalues either 0 or 1.

$$P = P^T \qquad P^2 = P$$

Covariance of linear transformations

$$Cov(Ay, Bx) = ACov(y,x)B^T$$

Invert 2×2 matrix: $A = \left[\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right]$

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Sum identities:

$$\sum_{k=0}^{\infty} p^k = \frac{p}{1-p} \qquad \sum_{k=0}^{\infty} kp^k = \frac{p}{(1-p)^2} \qquad |p| < 1$$

Integration by parts:

$$\int uv' = uv - \int u'v$$

Matrix / Vector differentiation

$\frac{\partial A^T \beta}{\partial \beta} = A$, $\frac{\partial \beta^T A \beta}{\partial \beta} = (A + A^t)\beta = 2A\beta$ for A symmetric.

$\frac{\partial}{\partial \theta_i} \log(|A|) = tr(A^{-1} \frac{\partial A}{\partial \theta_i})$

General linear tests under H_0:

$$\frac{\frac{SSE_r - SSE_f}{p-r}}{\frac{SSE_f}{n-p}} \sim F_{p-r, n-p}$$

Three principles of experimental design: 1) Replication 2) Randomization 3) Blocking

One way ANOVA with n total observations, K groups:

SS		DF
SSTR	$\sum_{j=1}^K n_j(\bar{y_{j\cdot}} - \bar{y_{\cdot\cdot}})^2$	K - 1
SSE	$\sum_{i=1}^n (y_{ij} - \bar{y_{j\cdot}})^2$	n - K
SSTO	$\sum_{i=1}^n (y_{ij} - \bar{y_{\cdot\cdot}})^2$	n - 1

Multivariate Normal

$X \sim N(\mu, \Sigma)$, Σ positive definite

$$f(x) = \frac{\exp\{-\frac{1}{2}(x-\mu)^T \Sigma^{-1} (x-\mu)\}}{(2\pi)^{\frac{k}{2}} \sqrt{\det(\Sigma)}} = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

mgf: $M_X(t) = \exp(\mu' t + \frac{1}{2} t' \Sigma t)$

log likelihood for k vector $x \sim N(\mu, \Sigma)$

$$l_x = -\frac{k}{2} \log 2\pi - \frac{1}{2}\{\log\det\Sigma + (x-\mu)^T \Sigma^{-1}(x-\mu)\}$$

Stein's formula: $X \sim N(\mu, \sigma)$

$$\mathrm{E}(g(X)(X-\mu)) = \sigma^2\, \mathrm{E}(g'(X))$$

assuming these expectations are finite.

$X \sim N(\mu, \Sigma)$, A an $m \times n$ matrix, then

$$AX \sim N(A\mu, A\Sigma A^t)$$

For Σ full rank it's possible to transform between $Z \sim N(0, I)$ and X:

$$X = \Sigma^{1/2} Z + \mu \qquad Z = \Sigma^{-1/2}(X - \mu)$$

In block matrix form:

$$X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \sim N\left(\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}\right)$$

Assuming Σ_{11} is positive definite then the conditional distribution

$$X_2 | X_1 \sim N(\mu_2 + \Sigma_{21}\Sigma_{11}^{-1}(X_1 - \mu_1), \Sigma_{22} - \Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12})$$