1 Math

2 Applied

Topics covered up to 31 October

Matrix / Vector differentiation

2.1 Linear Models

Least Squares Principle

$$\arg\min_\beta \|Y - X\beta\|^2$$

Normal Model

$$Y = X\beta + \epsilon, \qquad \epsilon \sim N(0, \sigma^2 I)$$

Normal Equations - Any b satistfying this solves the least squares

$$X^T X b = X^T y$$

Gauss Markov Theorem - $\hat{\beta}(X^T X)^{-1} y$ is Best Linear Unbiased Estimator (BLUE) of β.

2.2 General Techniques

Singular Value Decompostion (SVD) Any matrix X can be written

$$X = UDV^T$$

with U, V orthogonal, and D diagonal.

Moore Penrose Psuedoinverse A^+ exists uniquely for every matrix A.

Projection matrix P are symmetric and idempotent. They have eigenvalues either 0 or 1.

$$P = P^T \qquad P^2 = P$$

Multivariate Normal $y \sim N(\mu, \Sigma)$, A an $m \times n$ matrix, then

$$Ay \sim N(A\mu, A\Sigma A^t)$$