Worksheet - 18 Feb 2020 **STAT 50 - Prof. Fitzgerald**

- Law of Total Probability

$$P(A) = P(A \cap B) + P(A \cap B^C) = P(A|B)P(B) + P(A|B^C)P(B^C)$$

- Bayes Rule

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$

1. Chef Joe makes delicious i.i.d. pancakes that vary slightly in heights. The mean pancake height is 0.5 inch, and the standard deviation is 0.1 inch. He serves them on i.i.d. plates that stand, on average, 0.3 inches off the top of the table, with standard deviation 0.02 inches.

   (a) What is the mean height above the table for a stack of 5 pancakes?

   (b) What is the standard deviation of the height above the table for a stack of 5 pancakes?

   (c) Joe calculates the average height of 9 pancakes. What is the mean and standard deviation of the average height?

2. Students are taking a multiple choice exam. About 60% of the students know the material well. If the student knows the material well, then the probability that they answer correctly is 0.9. If they don't know the material well, then the probability that they answer correctly is 0.4.

   (a) If a student answers one question correctly, what's the probability that they know the material well?

   (b) If a student answers 2 of 3 questions correctly, what's the probability that they know the material well?

   (c) If a student answers 2 of 3 questions correctly, what's the probability that they do not know the material well?

   (d) Based on these numbers, how effective is a 3 question multiple choice exam in measuring individual student performance?

   (e) Let n be the number of questions on the exam. If a student answers c questions correctly, what's the probability that the student knows the material well?

   (f) How many questions need to be on an exam such that if a student answers all correctly, the probability that they know the material well is at least 0.99?