Counting, Permutations, and Combinations **STAT 50 - Prof. Fitzgerald**

$$n! = n(n-1)(n-2)...(3)(2)(1) \qquad \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

1. A group of 5 students are flying from Sacramento to New York for a programming competition. Each can choose to take one of 6 different flights.
   (a) How many ways can the students travel on the flights?

   (b) How many ways can the students all be on the same flight?

   (c) If each student randomly chooses a flight, what's the probability they are all on the same flight?

   (d) The students decide to all take the same flight, and they're all waiting in line together for security. How many ways can they line up?

2. You're packing your luggage for the flight.
   (a) You have 12 different shirts, and you want to bring 3. How many different collections of shirts can you bring?

   (b) You have 8 different pants, and you want to again bring 3. How many different collections of pants can you bring?

   (c) How many different collections of shirts and pants can you bring?

3. There are 4 different keys, and 4 matching locks. The problem is, you don't know which key goes into which lock.
   (a) How many ways could the keys match the locks?

   (b) You start trying to unlock the locks, and you keep trying until you find a matching lock. What's the minimum number of times you need to try?

   (c) What's the probability that you try the minimum number of times?

   (d) What's the maximum number of times you need to try?

   (e) What's the probability that you try the maximum number of times?

4. Recall that a standard deck of playing cards has 13 cards with 4 suits for 52 total cards.

   In the popular poker game Texas hold'em, all players share 5 community cards laying face up on the table, and hold 2 cards privately in their hand. From this collection of 7 cards, each player tries to form the best 5 card hand.

   (a) How many ways are there to make a 5 card hand from the 7 cards?

   (b) How many hands are possible in Texas hold'em, if we consider both the community cards and the private cards?

5. In Texas hold'em, bets happen after each of the following steps:

   1. Each player receives their two private cards.
   2. 3 community common cards are laid down (the flop).
   3. Another community common card is laid down (the turn).
   4. The final community common card (the river).

   (a) How many ways are there for the 5 community cards to be dealt?

   (b) The 4th community card is laid down, and all 4 cards are hearts. You hold a 2 of hearts and a jack of diamonds in your hand, so you have the lowest possible flush.

   (c) What's the probability that the river card is a heart?

   (d) The river card is a spade. You're betting against one other player. What's the probability they hold a heart? (Meaning they will win with a higher flush than yours)

   (e) Change the previous question, so that instead of 1 player, you're betting against 4 players. What's the probability another player will beat you with a higher flush?