Worksheet - 3 Mar 2020 **STAT 50 - Prof. Fitzgerald**

Let X be a a continuous R.V. with Probability Density Function (PDF) $f(x)$.

- The mean of X is:

$$\mu_X = \int_{-\infty}^{\infty} x f(x) dx$$

- The Cumulative Distribution Function (CDF) for X is:

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t) dt$$

1. Answer these in the form $a \leq x \leq b$, for appropriate a, b, possibly ∞.
   (a) What are the possible values for a random variable X?
   (b) What are the possible values for a PDF $f(x)$?
   (c) What are the possible values for a CDF $F(x)$?

2. Let $X \sim \text{Uniform}(0, 1)$.
   Write down and sketch the CDF $F(x)$ for X.

3. Let $X_1 \sim \text{Uniform}(0, 1)$ and $X_2 \sim \text{Uniform}(0, 1)$ be two independent random variables.
   Let $Y = X_1 + X_2$. In class we saw that Y has a PDF shaped like a triangle with vertices at (0,0), (1, 1), (2, 0).
   (a) Write down and sketch the PDF $f(y)$ for Y.
   (b) Write down and sketch the CDF $F(y)$ for Y.
   (c) Show that $\mu_Y = 1$.

4. Is it possible to have a R.V. that is neither continuous or discrete? If so, provide an example of a process that might generate such data. What would the CDF of such a R.V. look like?

5. Let $Z \sim \text{Uniform}(a, b)$, meaning that Z is equally likely to take on any value between a and b. Note $a < b$.
   (a) Write down the PDF $f(z)$ for Z.
   (b) Write Z as a function of X, where $X \sim \text{Uniform}(0, 1)$.