11.3. Take $m = 50$, $n = 12$. Using MATLAB's `linspace`, define t to be the m-vector corresponding to linearly spaced grid points from 0 to 1. Using MATLAB's `vander` and `fliplr`, define A to be the $m \times n$ matrix associated with least squares fitting on this grid by a polynomial of degree $n - 1$. Take b to be the function $\cos(4t)$ evaluated on the grid. Now, calculate and print (to sixteen-digit precision) the least squares coefficient vector x by six methods:

(a) Formation and solution of the normal equations, using MATLAB's `\`,

(b) QR factorization computed by `mgs` (modified Gram–Schmidt, Exercise 8.2),

(c) QR factorization computed by `house` (Householder triangularization, Exercise 10.2),

(d) QR factorization computed by MATLAB's `qr` (also Householder triangularization),

(e) `x = A\b` in MATLAB (also based on QR factorization),

(f) SVD, using MATLAB's `svd`.

(g) The calculations above will produce six lists of twelve coefficients. In each list, shade with red pen the digits that appear to be wrong (affected by rounding error). Comment on what differences you observe. Do the normal equations exhibit instability? You do not have to explain your observations.

12.2. In Example 11.1 we remarked that polynomial interpolation in equispaced points is ill-conditioned. To illustrate this phenomenon, let $x_1, \ldots, x_n$ and $y_1, \ldots, y_m$ be n and m equispaced points from -1 to 1, respectively.

(a) Derive a formula for the $m \times n$ matrix A that maps an n-vector of data at $\{x_j\}$ to an m-vector of sampled values $\{p(y_j)\}$, where p is the degree $n-1$ polynomial interpolant of the data (see Example 1.1).

(b) Write a program to calculate A and plot $\|A\|_\infty$ on a semilog scale for $n = 1, 2, \ldots, 30$, $m = 2n-1$. In the continuous limit $m \to \infty$, the numbers $\|A\|_\infty$ are known as the *Lebesgue constants* for equispaced interpolation, which are asymptotic to $2^n/(e\,(n-1)\log n)$ as $n \to \infty$.

(c) For $n = 1, 2, \ldots, 30$ and $m = 2n-1$, what is the ∞-norm condition number κ of the problem of interpolating the constant function 1? Use (12.6).

(d) How close is your result for $n = 11$ to the bound implicit in Figure 11.1?

13.3. Consider the polynomial $p(x) = (x-2)^9 = x^9 - 18x^8 + 144x^7 - 672x^6 + 2016x^5 - 4032x^4 + 5376x^3 - 4608x^2 + 2304x - 512$.

(a) Plot $p(x)$ for $x = 1.920, 1.921, 1.922, \ldots, 2.080$, evaluating p via its coefficients $1, -18, 144, \ldots$.

(b) Produce the same plot again, now evaluating p via the expression $(x-2)^9$.