

# CATEGORY THEORY 2025

IVAN DI LIBERTI

## EXERCISES

*savoir faire: Yoneda, adjunctions and limits*

**Leinster 1** (■□). 6.2.20

**Leinster 2** (■□). 6.2.21

**Leinster 3** (■□). 6.3.21(a)

**Leinster 4** (■□). 6.3.22

**Leinster 5** (■□). 6.3.26

**Leinster 6** (■□). 6.3.27

*monads*

**Exercise 7** (■□). Describe the monads (unit and counit) on **Set** whose algebras are: monoids, groups, semigroups.

**Exercise 8** (■□). Consider the free-forgetful adjunction  $D : \mathbf{Set} \rightleftarrows \mathbf{Top} : U$ , where  $D$  equips a set with the discrete topology over it. Compute the algebras for the induced monad over **Set**.

**Exercise 9** (■□). Show that the category **Suplat** whose objects are suplattices and morphisms are suplattice morphisms is monadic over **Set** via forgetful functor  $\mathbb{U} : \mathbf{Suplat} \rightarrow \mathbf{Set}$ . *Hint:* Guess the monad and prove that an algebra is precisely a suplattice.

**Exercise 10** (■□). A monad  $T$  on a category  $C$  is idempotent if its multiplication is an isomorphism. Show that the forgetful functor  $U_T : \mathbf{Alg}(T) \rightarrow C$  of an idempotent monad is fully faithful.

**Exercise 11** (■□). Let  $C$  be a category with coproducts and a terminal object. Can you always put a monad structure on the *maybe endofunctor*  $c \mapsto c \amalg 1$ ?

**Exercise 12** (■□). Show that the category of fields is not monadic over **Set**.

**Exercise 13** (■□). Show that there is a monad on directed graphs whose algebras are small categories.

**Exercise 14** (■□). Show that there is a monad on the category of small categories (and functors) whose algebras are posets.

**Jiří's treat** (**A**, **P**). Let  $\text{Suplat}^\nabla$  be the category whose objects are suplattices with a unary operation  $\nabla$  satisfying  $(\forall x)(x \leq \nabla x)$ . Morphisms are suplattices morphisms preserving the unary operation. Show that the forgetful functor

$$\mathbb{U} : \text{Suplat}^\nabla \rightarrow \text{Set}$$

preserves limits but does not have a left adjoint. *Hint:* Show that a free algebra over 1 does not exist.

*rules*

- Hand your exercises before your **oral interview** via email. In order to make my life easier, make sure to include the word **CT25 in the subject**.
  - Pick at least one exercise from each of the yellow groups.
  - You must charge at least **1** batteries and a half!
- Example.* The vector of exercises [2,7,12] would pass this sheet.
- The label **Leinster** refers to the book **Basic Category Theory**, by *Leinster*.
  - The label **Riehl** refers to the book **Category Theory in context**, by *Riehl*.