CATEGORY THEORY 2025

IVAN DI LIBERTI

EXERCISES

savoir faire: Yoneda, adjunctions and limits

Leinster 1 (▭). 6.2.20

Leinster 2 (▭). 6.2.21

Leinster 3 (▭). 6.3.21(a)

Leinster 4 (▭). 6.3.22

Leinster 5 (▭). 6.3.26

Leinster 6 (▭). 6.3.27

monads

Exercise 7 (▭). Describe the monads (unit and counit) on Set whose algebras are: monoids, groups, semigroups.

Exercise 8 (▭). Consider the free-forgetful adjunction D : **Set** $\leftrightarrows$ Top : U, where D equips a set with the discrete topology over it. Compute the algebras for the induced monad over **Set**.

Exercise 9 (▮). Show that the category Suplat whose objects are suplattices and morphisms are suplattices morphisms is monadic over **Set** via forgetful functor $\mathbb{U}$: Suplat → Set. *Hint*: Guess the monad and prove that an algebra is precisely a suplattice.

Exercise 10 (▭). A monad T on a category C is idempotent if its multiplication if its multiplication is an isomorphism. Show that the fortgetful functor U_T : Alg(T) → C of an idempotent monad is fully faithful.

Exercise 11 (▮). Let C be a category with coproducts and a terminal object. Can you always put a monad structure on the *maybe endofunctor* $c \mapsto c \coprod 1$?

Exercise 12 (▮). Show that the category of fields is not monadic over Set.

Exercise 13 (▮). Show that there is a monad on directed graphs whose algebras are small categories.

Exercise 14 (▮). Show that there is a monad on the category of small categories (and functors) whose algebras are posets.

Jiří's *treat* (▲, ☕). Let Suplat^∇ be the category whose objects are suplattices with a unary operation ∇ satisfying $(\forall x)(x \leq \nabla x)$. Morphisms are suplattices morphisms preserving the unary operation. Show that the forgetful functor

$$\mathbb{U} : \mathsf{Suplat}^\nabla \to \mathsf{Set}$$

preserves limits but does not have a left adjoint.   *Hint*: Show that a free algebra over 1 does not exist.