E&M I Recitation Notes

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You know how in Physics 101, everything was a point particle? Well in E&M we want to build a theory which can talk about point particles (like a single electron), but also a distribution of particles (like a slab of charged metal). You can imagine if we define the point particle in a naive way, the mathematical computation will turn out wrong... This is where we introduce the **delta function**.

> This will be a computational introduction, i.e. how we use and manipulate these functions. If you're more of a proper mathematician, I recommend looking at my friend Panos's notes (https://notes.panos.wiki).

1.2.1 Kronecker Delta (Discrete)

As always, we start discrete and then promote it to be continuous.

Definition 1 (Kronecker Delta)

$$\delta_{ij} = \begin{cases} 1 & \textit{if } i = j \\ 0 & \textit{otherwise} \end{cases} \tag{1.1}$$

where i, j are taken to be integers. It is usually unit-less.

While this notation looks scary, if we interpret the indices as the row/column of a matrix, we realize this is just the identity matrix.

$$\mathbb{I} = \begin{pmatrix} 1 & 0 & 0 & \\ 0 & 1 & 0 & ... \\ 0 & 0 & 1 & \\ & \vdots & & \ddots \end{pmatrix} \implies [\mathbb{I}]_{ij} = \delta_{ij} \tag{1.2}$$

In physics, this scary feeling will come up a lot. You'll be faced with a weird expression with kronecker deltas, etc. but you'll just realize that it ends up being a regular-old matrix operation.

Problem 1 (Properties of Kronecker)
There are a couple fundamental properties of Kronecker deltas. They're pretty easy, so it's your job to show them!

1. Symmetry: $\delta_{ij} = \delta_{ji}$
2. Contraction: $\sum_j a_j \delta_{ij} = a_i$
3. Dimension: $\sum_{i=1}^n \delta_{ii} = n$

Now we can apply our properties to break-down an expression with Kronecker deltas.

Example: Consider the expression $\sum_{i=1}^n \sum_{j=1}^n \delta_{ij} a_i b_j$. If we interpret a_i (b_i) as being the i'th entry from vector $\vec{a}$ ($\vec{b}$). What fundamental vector operation is this expression?

$$\sum_{i=1}^n \sum_{j=1}^n \delta_{ij} a_i b_j = \sum_{i=1}^n a_i b_i \qquad \text{Contraction property} \tag{1.3}$$

$$= \vec{a} \cdot \vec{b} \qquad \text{By definition of dot product} \tag{1.4}$$

Wow it's just a dot product. Another way to see this is to use the identity matrix fact.

$$\sum_{i=1}^n \sum_{j=1}^n \delta_{ij} a_i b_j = \vec{a}^T \, \mathbb{I} \, \vec{b} = \vec{a} \cdot \vec{b} \tag{1.5}$$

Going back to the first proof. We that Kronecker deltas allow us to evaluate sums, this is because it is zero when $i \neq j$, so the only remaining component is $i = j$.

To get you comfortable with this, here's a couple of practice problems.

Problem 2 (Kroneckers are not scary!)
Let A be a matrix, where $A_{ij} \equiv [A]_{ij}$ are the i, j'th entries of the matrix. What are the following expressions in-terms of simple linear algebra operations.

1. $\sum_{ij} \delta_{ij} A_{ij}$
2. $\sum_{k\ell} \delta_{jk} \delta_{i\ell} A_{\ell k}$
3. $\sum_{k\ell} \frac{1}{2} (\delta_{ij} \delta_{j\ell} + \delta_{i\ell} \delta_{jk}) A_{k\ell}$

1.2.2 Dirac Delta (Continuous)

Definition 2 (Dirac Delta / Delta Function)) *The heuristic definition is*

$$\delta(x - x') = \begin{cases} \infty & \textit{if } x = x' \\ 0 & \textit{otherwise} \end{cases} \tag{1.6}$$

$$\textit{s.t.} \quad \int_{\mathbb{R}} \delta(x - x') = 1 \tag{1.7}$$

What is particularly funky, is this is not a function! It is a distribution! For our purposes, this means delta functions are only defined inside an integral. So if you ever see equations with delta functions, these are truly only defined when you integrate both sides of the equation.

1.3 Fourier Analysis

If you're a STEM major, you have probably seen 3Blue1Brown's video on Fourier Analysis (if you haven't, you should watch it now!). Basically he visualizes how complex signals can be decomposed into pure-signals of sines and cosines. I'll try to introduce it again, but lay the notation & computational workflow for accomplishing this.

What we'll learn is that any piecewise-smooth function can be represented as a linear combination of sin and cos

$$f(x) = A_0 + \sum_{n=1}^{\infty} \left[A_n \cos\left(\tfrac{2\pi n}{P} x\right) + B_n \sin\left(\tfrac{2\pi n}{P} x\right)\right] \tag{1.8}$$

$$\text{s.t. } A_0 = \frac{1}{P} \int_P f(a)\ da \tag{1.9}$$

$$A_n = \frac{2}{P} \int_P f(a) \cos\left(\tfrac{2\pi n}{P} a\right)\ da \qquad (n \geq 1) \tag{1.10}$$

$$B_n = \frac{2}{P} \int_P f(x) \sin\left(\tfrac{2\pi n}{P} a\right)\ da \qquad (n \geq 1) \tag{1.11}$$

1.3.1 Fourier Series Expansion

1.3.2 Fourier Transform

1.3.3 Connection to Linear Independence / Basis