

CS6033 – Design and Analysis of Algorithms I

Homework 4

Manuel — NYU (Spring 2021)

Reminders

- Write in a neat and legible handwriting or use \LaTeX
- Clearly explain the reasoning process
- Write in a complete style (subject, verb, and object)
- Be critical on your results

*Questions preceded by a * are optional. Although they can be skipped without any deduction, it is important to know and understand the results they contain.*

Ex. 1 — Time vs. space

The goal of this exercise is to consider the best available hardware and compare the feasibility of heavy computation in terms of (i) time and (ii) memory.

- As of June 2015 the fastest supercomputer publicly known is called NUDT Tianhe-2. Its speed is 33.86 PFLOPS and its storage is 12.4 PB¹.
 - As of August 2015 one of the fastest CPU for desktop computer is the Intel Core i7-5775R Processor which has four core running at maximal frequency of 3.8 GHz².
 - As of August 2015 the largest hard drive is almost 16TB³.
1. How long would it take to perform 2^{64} operations on NUDT Tianhe-2? What about 2^{80} operations?
 2. How many desktop computers would be necessary to perform 2^{64} operations in no more than a day. What about 2^{80} operations in no more than a month?
 3. How many hard drives would be necessary to store 2^{64} bits. What about 2^{80} bits?

Ex. 2 — Critical thinking

Given a set S of n integers, generate a subset S' of S composed of k elements, each selected with probability k/n . Explain how to obtain S' in only one pass.

Ex. 3 — Algorithm and complexity

In the following triangle each entry is the sum of the three entries directly above it.

			1					
		1	1	1				
	1	2	3	2	1			
	1	3	6	7	6	3		
1	4	10	16	19	16	10	4	1

1. Write the pseudo-code of a simple algorithm which returns the sum on all the elements in the i -th line, when given i as input.

¹Source: top500.org.

²Source: intel.com.

³Source: arstechnica.co.uk.

2. Determine the complexity of this algorithm, and prove its correctness.

* **Ex. 4** — *From SAT to 3-SAT*

Rewrite the following SAT formula into a 3-SAT formula.

$$(x_1 \vee x_2 \vee \neg x_3 \vee x_4 \vee x_5 \vee \neg x_6) \wedge (\neg x_1 \vee \neg x_2 \vee x_3 \vee \neg x_4 \vee x_5 \vee x_6) \wedge (x_1 \vee \neg x_2 \vee \neg x_3 \vee x_4 \vee x_5 \vee \neg x_6) \wedge (x_1 \vee \neg x_2).$$

Ex. 5 — *Clique problem*

- * 1. Explain what the Clique problem is.
2. Prove that Clique is in \mathcal{NP} .
3. Given a 3-SAT formula F with k clauses, construct a graph G such that F is satisfiable if and only if G has a k -clique.
4. Conclude on the complexity class of the Clique problem.

Ex. 6 — *IND-SET problem*

- * 1. What is the maximum independent set problem?
2. What is the independent set (IND-SET) decision problem?
3. Prove that IND-SET is in \mathcal{NP} .
4. Construct a graph G' such that " G has a k -clique" is equivalent to " G' has an independent set of size k ".
5. Conclude on the complexity class of the IND-SET problem.

* **Ex. 7** — *Interview problem*

You are given an array A containing positive integers. Find the maximum possible sum of elements of A which is divisible by three.

Example. For $A = [3, 6, 5, 1, 8]$, the maximum sum is 18.