CS6033 Assign No.1

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1 Hash tables

1.
Picking k from n keys that are in same slot: $\binom{n}{k}$
Probability of k keywords in same slot:$(1/n)^k$
Probability of $(n-k)$ keywords in other slots:$(1-\frac{1}{n})^{n-k}$
So Q_k is equal to:$(\frac{1}{n})^k(1-\frac{1}{n})^{n-k}\binom{n}{k}$

2.
Let X_i to express numbers contained in slot i, A_i to express k keywords in slot i, from question 1 we can know that: $P\{A_i\} = Q_k$.
So $P_k = P_r\{\max_{i \leq i \leq n} X_i = k\} = P_r\{A_1 \cup A_2 \cup \cdots \cup A_n\}$
$\leq P_r\{A_1\} + P_r\{A_2\} + P_r\{A_3\} + \cdots + P_r\{A_n\} = nQ_k$

3.
$Q_k = (\frac{1}{n})^k(1-\frac{1}{n})^{n-k}\binom{n}{k} \leq (\frac{1}{n})^k(\frac{n!}{k!(n-k)!})^k = \frac{n(n-1)(n-2)\ldots(n-k+1)}{n_k k!}$
$\frac{1}{k!} = \frac{1}{\sqrt{2\pi k}(\frac{k}{e})^k(1+\theta(\frac{1}{n}))} \leq \frac{e^k}{k^k}$

5.
$E[M] = \sum_{i=1}^{n} iP_r\{M=i\} = \sum_{i=1}^{\frac{c\lg n}{\lg\lg n}} iP_r\{M=i\} + \sum_{i=1+\frac{c\lg n}{\lg\lg n}}^{n} iP_r\{M=i\}$
$\leq \frac{c\lg n}{\lg\lg n}\sum_{i=1}^{\frac{c\lg n}{\lg\lg n}} P_r\{M=i\} + n * \sum_{i=1+\frac{c\lg n}{\lg\lg n}}^{n} P_r\{M=i\}$
$\leq P_r\{M > \frac{c\lg n}{\lg\lg n}\} * n + P_r\{M < \frac{c\lg n}{\lg\lg n}\} * \frac{c\lg n}{\lg\lg n}$
$E[M] = P_r\{M \leq \frac{c\lg n}{\lg\lg n}\} * \frac{c\lg n}{\lg\lg n} + n * \sum_{i=1+\frac{c\lg n}{\lg\lg n}}^{n} P_r\{M=i\}$,
$= P_r\{M \leq \frac{c\lg n}{\lg\lg n}\} * \frac{c\lg n}{\lg\lg n} + n * \sum_{i=1+\frac{c\lg n}{\lg\lg n}}^{n} P_i < \frac{c\lg n}{\lg\lg n} + n * n * 1/n^2$
$= O(\frac{\lg n}{\lg\lg n})$

2 Minimum Spanning Tree

2.

Algorithm 1 minimum spanning tree solution

Input: Minimum spanning tree T, decreased weight j for nodes (u, v)
Output: new Minimum spanning tree T

Find weight k for nodes (u, v) in T
if $k \geq j$ **then**
    remove k from T
    add j to T
end if
return T

3 Simple Algorithms

2.a

Algorithm 2 mult function

Input: x, y
Output: $mult.(x, y)$

function $mult(x, y)$
    if x=0 or y=0 **then**
        mlt = 0
    else
        return $x * (y \ mod \ 2) + mult.(2x, \lfloor y/2 \rfloor)$
    end if
end function

3.2.b
Solution: First of all, $f(1, 1) = 1{*}1{+}0 = 1$ We can know that $|trunc(y/2)| < |y|$, from the inductive hypothesis, $mult(2x, trunc(y/2)) = 2 * x * trunc(y/2)$.
So the return value is: $2 * x * trunc(y/2) + x * (y \ mod \ 2)$
$= x * (2 * trunc(y/2) + 2 * (y/2 - trunc(y/2)))$
$= x * (2 * y/2)$
$= x * y$

4 Critical Thinking

1. Non of them can solve the Knapsack problem perfectly.
Like a set $S = \{3, 4, 6, 7, 9\}$ and $n = 10$,
if you pick the smallest items first, you'll get $\{3, 4\}$, 7 in total;
if you choose the biggest items first, you'll get $\{9\}$ instead;
While the actual answer could be both $\{3, 7\}$ and $\{4, 6\}$.

3. Imagine you have coins with value$\{30, 10, 7, 8, 1\}$ and you want a total value of 45 with least coins. From greedy algorithm you'll reach local optimum like $\{30, 10, 1, 1, 1, 1, 1\}$ Global optimum should be $\{30, 10, 8, 7\}$.

4. 7 races are necessary.
1st-5th:Divided them into 5*5 groups and race them in groups.
name them $a_1 - a_5, b_1 - b_5, c_1 - c_5, d_1 - d_5, e_1 - e_5$.
6th race for a_1, b_1, c_1, d_1, e_1, name the fastest horse a_1, which is also the fastest among tha pack, e_5 is the slowest in this race.
7th race for a_2, b_1, b_2, c_1, c_2. The fastest two in this race will be 2nd and 3rd horse of the pack.
The first three horses shoud be :a_1, 1st and 2nd horse in the 7th race.

5 Recursive function

1.

$$f(0,0) = 0$$
$$f(1,0) = 0$$
$$f(1,1) = g(f(1,0),1) = g(0,1) = S(g(0,0)) = S(0) = 1$$
$$f(2,1) = g(f(2,0),2) = g(0,2) = S(g(0,1)) = S(1) = 2$$
$$f(2,2) = g(f(2,1),2) = g(g(f(2,0),2),2) = g(2,2) = S(g(2,1))$$
$$= S(S(g(2,0))) = S(S(2)) = 4$$
$$f(2,3) = g(f(2,2),2) = g(g(f(2,1),2),2) = g(g(g(f(2,0),2),2),2)$$
$$= g(g(g(f(2,0),2),2),2) = g(g(g(0,2),2),2) = g(g(S(S(g(0,0))),2),2)$$
$$= g(g(2,2),2) = g(S(S(g(2,0))),2) = g(4,2)$$
$$= S(S(g(4,0))) = S(S(4)) = 6$$

2. $f(x,y) = x * y$,pseudocode should be like:

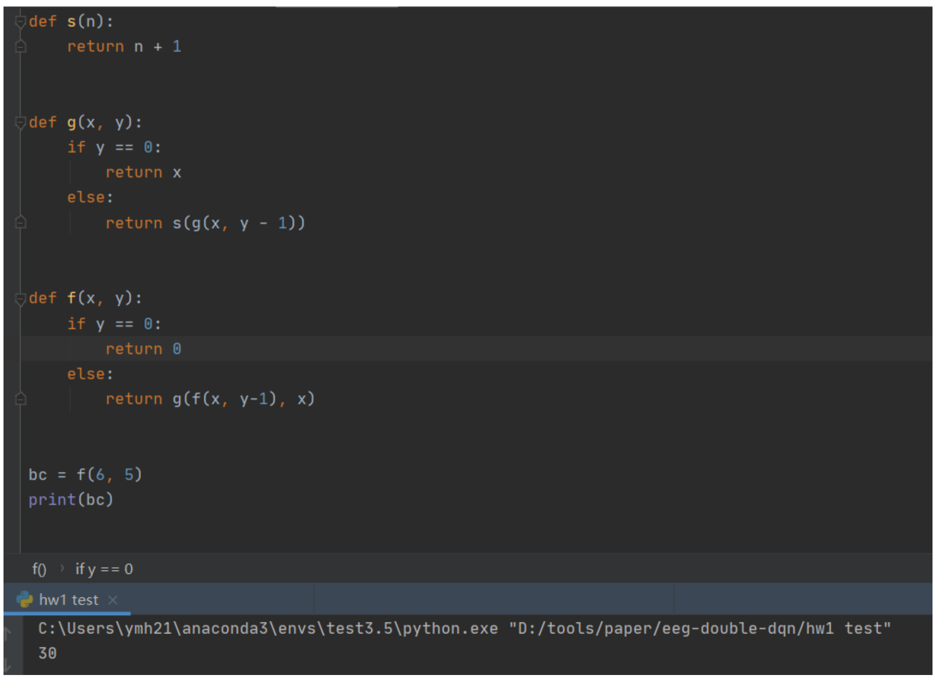
Algorithm 3 function

Input: x, y
Output: $f(x, y)$

```
function f(x, y)
    out = 0
    while y != 0 do
        y = y-1
        out = f(x, y)
        for x ∈ [1, x] do
            out = out + x
        end for
    end while
    return out
end function
```

3.
$f(6,5)$
$= g(f(6,4),6) = g(g(f(6,3),6),6) = g(g(g(f(6,2),6),6),6) = g(g(g(g(f(6,1),6),6),6),6) = g(g(g(g(g(f(6,0),6),6),6),6),6)$

$= g(g(g(g(g(0,6),6),6),6),6) = g(g(g(g(s(g(0,5)),6),6),6),6) = g(g(g(g(s(s(g(0,4))),6),6),6),6) =$
$g(g(g(g(s(s(s(g(0,3)))),6),6),6),6) = g(g(g(g(s(s(s(s(g(0,2))))),6),6),6),6) =$
$g(g(g(g(s(s(s(s(s(g(0,1)))))),6),6),6),6) = g(g(g(g(s(s(s(s(s(s(g(0,0))))))),6),6),6),6) =$
$g(g(g(g(s(s(s(s(s(s(0)))))),6),6),6),6) = g(g(g(g(6,6),6),6),6)$
$= g(g(g(s(s(s(s(s(s(g(6,0))))))),6),6),6) = g(g(g(s(s(s(s(s(s(6)))))),6),6),6) =$
$g(g(g(12,6),6),6) = g(g(18,6),6) = g(24,6)$
$= s(s(s(s(s(s(g(24,0))))))) = 30$



```
def s(n):
    return n + 1


def g(x, y):
    if y == 0:
        return x
    else:
        return s(g(x, y - 1))


def f(x, y):
    if y == 0:
        return 0
    else:
        return g(f(x, y-1), x)


bc = f(6, 5)
print(bc)
```



```
C:\Users\ymh21\anaconda3\envs\test3.5\python.exe "D:/tools/paper/eeg-double-dqn/hw1 test"
30
```

Figure 1: Python Verification