Math 321 Final Exam

December 15, 1993

Your Name ______________________________
Student Number ________________________
Section Number _________________________

1. Books, notes, and calculators are not allowed.
2. For more space, write on the opposite side. Cross off (instead of erase) the undesired part.
3. Show all your work. Your reason counts most of the points.

Number	Score
1	
2	
3	
4	
5	
6	
Total	

(1) (10 point) Let $c \neq 0$ be a constant.

1. If k and τ are the curvature and the torsion of a curve α, what is the the curvature and the torsion of the similar curve $c\alpha$?
2. If K and H are the Gauss and mean curvatures of a surface ξ, what is the Gauss and mean curvatures of the similar surface $c\xi$?

(2) (15 point) Show there is a parametrized surface with $E = G = \mathrm{e}^u$(not e in the second fundamental form), $F = f = 0$.

(3) (20 point) The curvature of the hyperbola $u^2/a^2 - v^2/b^2 = 1$ is a/b^2 at the tip P. Find the geodesic curvature at the tip P of the hyperbola on the cone of angle θ obtained by intesecting with a plane parallel to and of distance 1 from the axis of the cone.

(4) (20 point) Consider the torus. α and γ are the top and bottom circles, β and δ are the meridians in the opposite position, and A, B, C, D are the four intersection points. If we start with the tangent vector w of δ at A, and then take parallel transportations along α to B, along β to C, along γ to D, and then along δ back to A, what vector do we get? Please explain your answer.

(5) (15 point) Consider parametrized surface

$$x = \frac{a}{2}(u+v), \quad y = \frac{b}{2}(u-v), \quad z = \frac{1}{2}uv.$$

Show that the parametric curves are geodesics.

(6) (20 point) Suppose the sum of interior angles of any geodesic triangle (i.e., the edges are geodesics) on a surface is π. Show that the Gauss curvature $K = 0$.