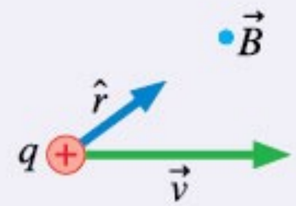
The Magnetic Field

Magnetic Fields

The **Biot-Savart law** for a moving point charge

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$



Magnetic field of a current

MODEL Model wires as simple shapes.

VISUALIZE Divide the wire into short segments.

SOLVE Use superposition:

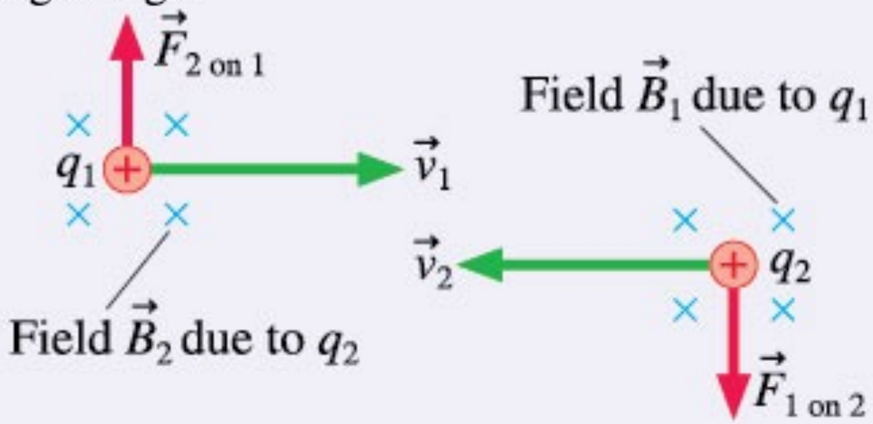
- Find the field of each segment Δs.
- Find $\vec{B}$ by summing the fields of all Δs, usually as an integral

An alternative method for fields with a high degree of symmetr **Ampère's law**:

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{\text{through}}$$

where I_{through} is the current through the area bounded by the integration path.

At its most fundamental level, **magnetism** is an interaction between moving charges. The magnetic field of one moving charge exerts a force on another moving charge.

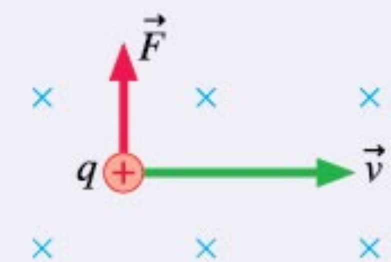


Magnetic Forces

The magnetic force on a moving charge is

$$\vec{F} = q\vec{v}\times\vec{B}$$

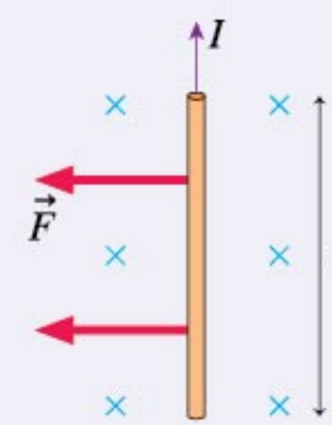
The force is perpendicular to $\vec{v}$ and $\vec{B}$.



The magnetic force on a current-carrying wire is

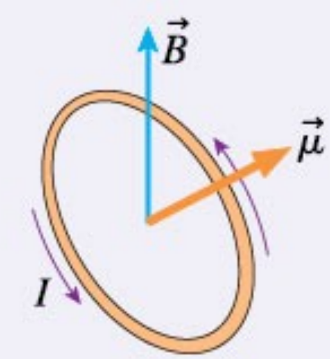
$$\vec{F} = I\vec{l}\times\vec{B}$$

$\vec{F} = \vec{0}$ for a charge or current moving parallel to $\vec{B}$.



The magnetic torque on a magnetic dipole is

$$\vec{\tau} = \vec{\mu}\times\vec{B}$$



Resources to succeed

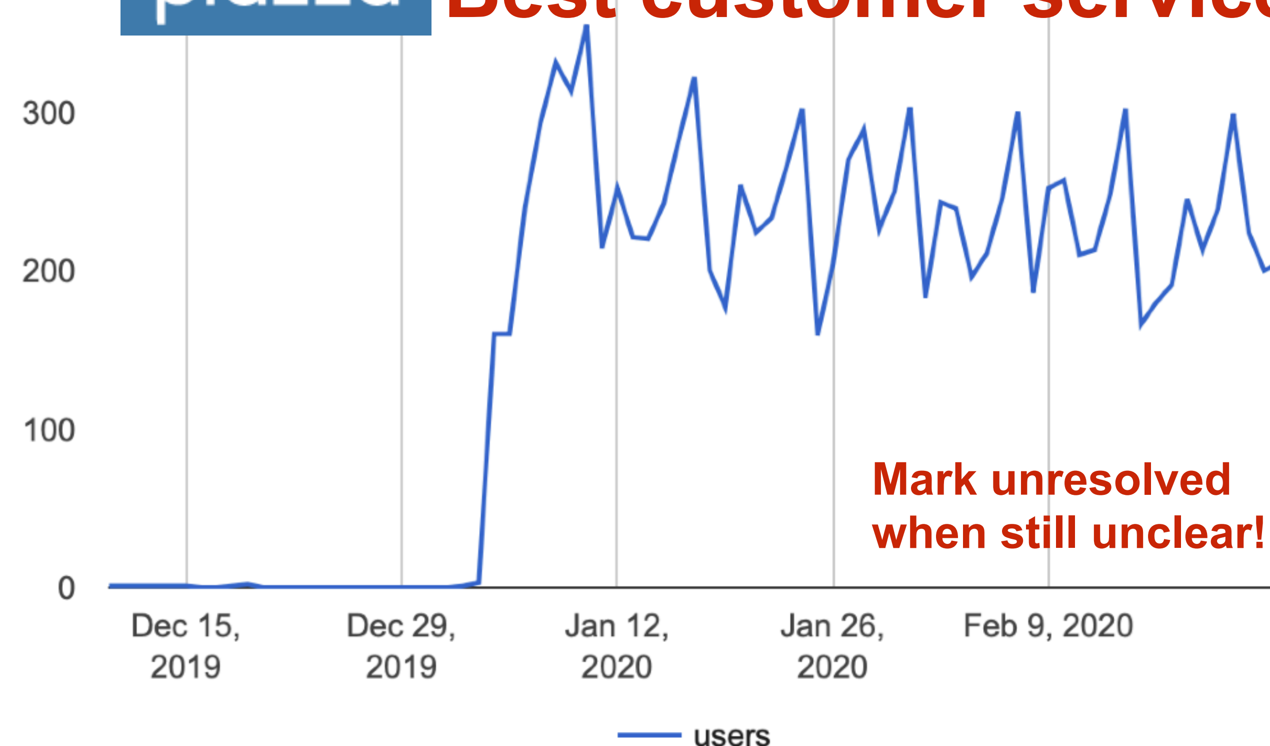
Please take advantage of all the resources we have for you:

- piazza (24/7)
- many office hours (12+ hours, one for each section @271)
- discussion sessions (1 each, @254),
- tutorial center (10 additional hours by your discussion session TAs @271),
- OASIS,
- IDEA, engineering student center, PHYS 2B Wed 10-11:50
- Teaching + Learning Commons, Drop-in Geisel PHYS 2B
- Academic Achievement Hub (see image below)
- Library reserves (yes, now the physical book must be there)

Including class: 12+12+10+2+3 > 39 hours/week of help available

628 total posts
3493 total contributions
426 instructors' responses
403 students' responses
8 min avg. response time

piazza **Best customer service** 😉



Mark unresolved when still unclear!

You have learned A LOT so far!

$$G = 6.67 \times 10^{-11}\ \mathrm{N\ m^2/kg^2} \qquad g = 9.81\ \mathrm{m/s^2} \qquad k_e = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9\ \mathrm{N\ m^2/C^2}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2} \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\text{proton charge} = +1.60 \times 10^{-19}\ \mathrm{C} \qquad m_p = 1.67 \times 10^{-27}\ \mathrm{kg} \qquad m_e = 9.11 \times 10^{-31}$$

$$\text{surface area of a sphere} = 4\pi r^2 \qquad \text{volume of a sphere} = \frac{4}{3}\pi r^3 \qquad x_f = x_i + v_i t + \frac{1}{2}$$

$$\mathbf{F}_{12} = k_e \frac{q_1 q_2}{r^2}\hat{\mathbf{r}}_{12}; \quad \mathbf{E} \equiv \mathbf{F}_e/q_0; \quad \mathbf{E} = k_e \sum_i \frac{q_i}{r_i^2}\hat{\mathbf{r}}_i; \quad \mathbf{E}_{\text{point charge}} = \frac{q}{4\pi\epsilon_0 r^2}\hat{r};$$

$$\lambda = \frac{dQ}{dL}; \sigma = \frac{dQ}{dA}; \rho = \frac{dQ}{dV}; \quad \mathbf{E}_{\infty\ \text{line}} = \frac{\lambda}{2\pi\epsilon_0 r}; \quad \mathbf{E}_{\text{ring charge}} = \frac{1}{4\pi\epsilon_0}\frac{qz}{(z^2 + R^2)^{3/2}}; \quad \mathbf{E}_{\infty}$$

$$\mathbf{p} = q\mathbf{d},\ (\mathbf{d} : - \to +); \quad \mathbf{E}_{\text{dipole, axis}} = \frac{1}{4\pi\epsilon_0}\frac{2\mathbf{p}}{r^3} \quad \vec{\tau} = \mathbf{p} \times \mathbf{E}; \quad U = -\mathbf{p}\cdot\mathbf{E} = -|pE$$

$$\mu_0 = 4\pi \times 10^{-7}\frac{\mathrm{T\cdot m}}{\mathrm{A}} = 1.26 \times 10^{-6}\frac{\mathrm{T\cdot m}}{\mathrm{A}}; \quad \oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 I_{enclosed}$$

$$\mathbf{B}_{point\ charge} = \frac{\mu_0}{4\pi}\frac{q\mathbf{v}\times\hat{\mathbf{r}}}{r^2}; \quad d\mathbf{B}_{Biot-Savart} = \frac{\mu_0}{4\pi}\frac{Id\mathbf{s}\times\hat{\mathbf{r}}}{r^2}; \quad B_{wire} = \frac{\mu_0}{2\pi}\frac{I}{r}; \quad B_{coil\ center} = \frac{\mu_0}{2}\frac{NI}{R}$$

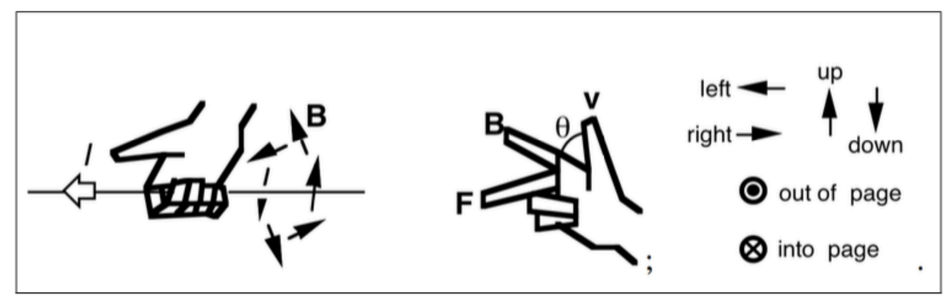
$$B_{solenoid} = \mu_0 I n; \quad \mathbf{B}_{dipole} = \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{z^3}; \quad \mu = NIA; \quad \vec{\tau} = \vec{\mu}\times\mathbf{B}; \quad U = -\vec{\mu}\cdot\mathbf{B}; \quad \mathbf{F} = q\mathbf{v}\times\mathbf{B};$$

$$F_{\text{cent}} = \frac{mv^2}{r}; \quad r = \frac{mv}{qB}; \quad f = \frac{\omega}{2\pi} = \frac{1}{T} = \frac{qB}{2\pi m}; \quad \mathbf{F} = I\mathbf{l}\times\mathbf{B}; \quad F_{\text{wires}} = \frac{\mu_0}{2\pi}\frac{LI_aI_b}{d};$$

$$F_B = ev_dB = e\frac{I}{wtne}B = F_E = eE = e\frac{\Delta V_{\text{Hall}}}{w}$$

$$\Phi_E = \mathbf{E}\cdot\mathbf{A} = EA\cos\phi; \quad \Phi_E = \oint \mathbf{E}\cdot d\mathbf{A} = \frac{q_{\text{encl}}}{\epsilon_0};$$

$$W_{AB} = \int_A^B \mathbf{F}\cdot d\mathbf{s}; \quad V(r) = k_e\frac{q}{r}; \quad \Delta V = \frac{\Delta U}{Q} = -\int_A^B \mathbf{E}(\mathbf{r})\cdot d\mathbf{r};$$



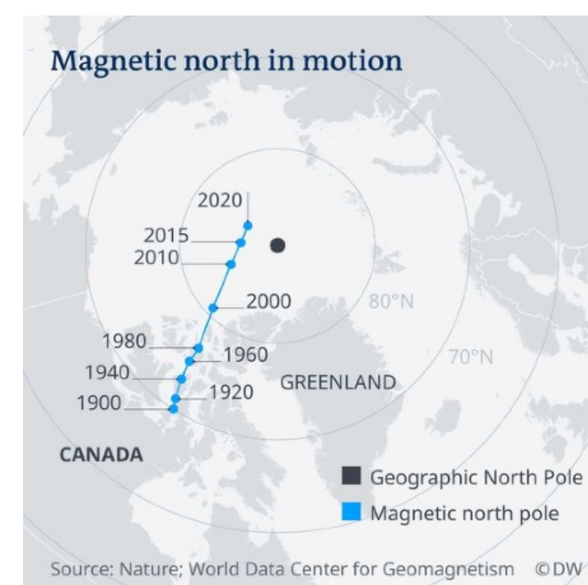
$$C \equiv \frac{Q}{\Delta V} = \frac{\epsilon_0 A}{d}; \quad U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}\frac{Q^2}{C}; \quad C_{\text{total}} = C_1 + C_2 + \cdots; \quad \frac{1}{C_{\text{total}}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$$

$$I = \frac{\Delta V}{R}; \quad E_{wire} = \frac{\Delta V_{wire}}{L}; \quad J = n_e e v_d = \sigma E; \sigma = \frac{n_e e^2 \tau}{m}; R = \frac{\rho L}{A}; \rho = \frac{1}{\sigma}; \sum I_{in} = \sum I_{out}; \sum \Delta V_{loop} = 0$$

$$N_e = i_e\Delta t; \quad Q = I\Delta t; \quad v_d = \frac{e\tau}{m}E; \quad i_e = n_e A v_d; \quad J = I/A; \quad P = (\Delta V)I = I^2R = \frac{(\Delta V)^2}{R};$$

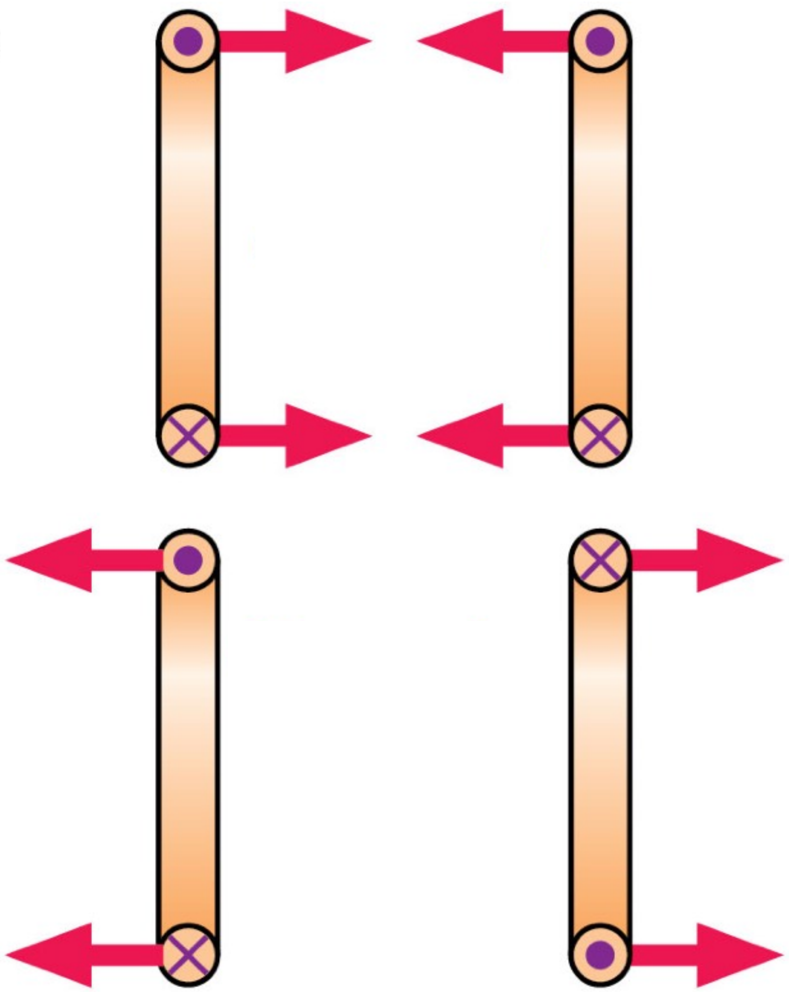
$$R_{\text{total}} = R_1 + R_2 + \cdots; \quad \frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots; \qquad \tau = RC$$

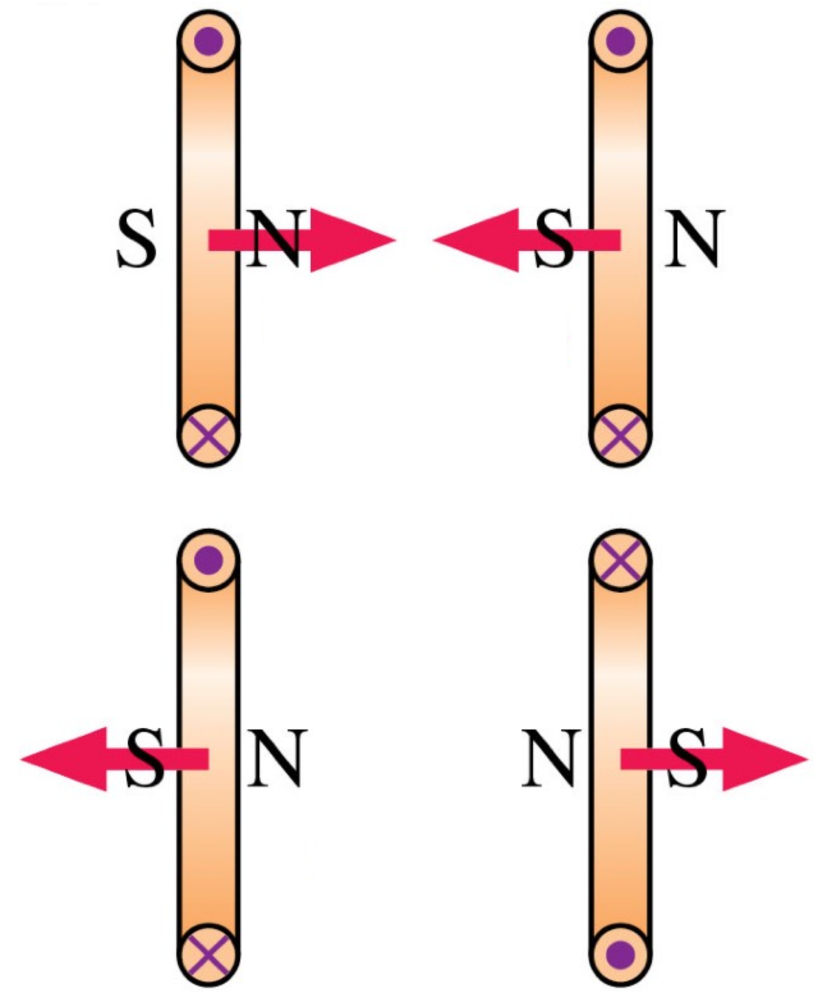
$$q(t) = C\varepsilon(1 - e^{-t/RC}); \quad I(t) = \frac{dq}{dt} = \frac{\varepsilon}{R}e^{-t/RC}; \quad q(t) = q_0 e^{-t/RC}; \quad I(t) = \frac{dq}{dt} = -\left(\frac{q_o}{RC}\right)e^{-t/RC}$$



Forces on Current Loops

- The two figures show alternative but equivalent ways to view magnetic forces between two current loops.

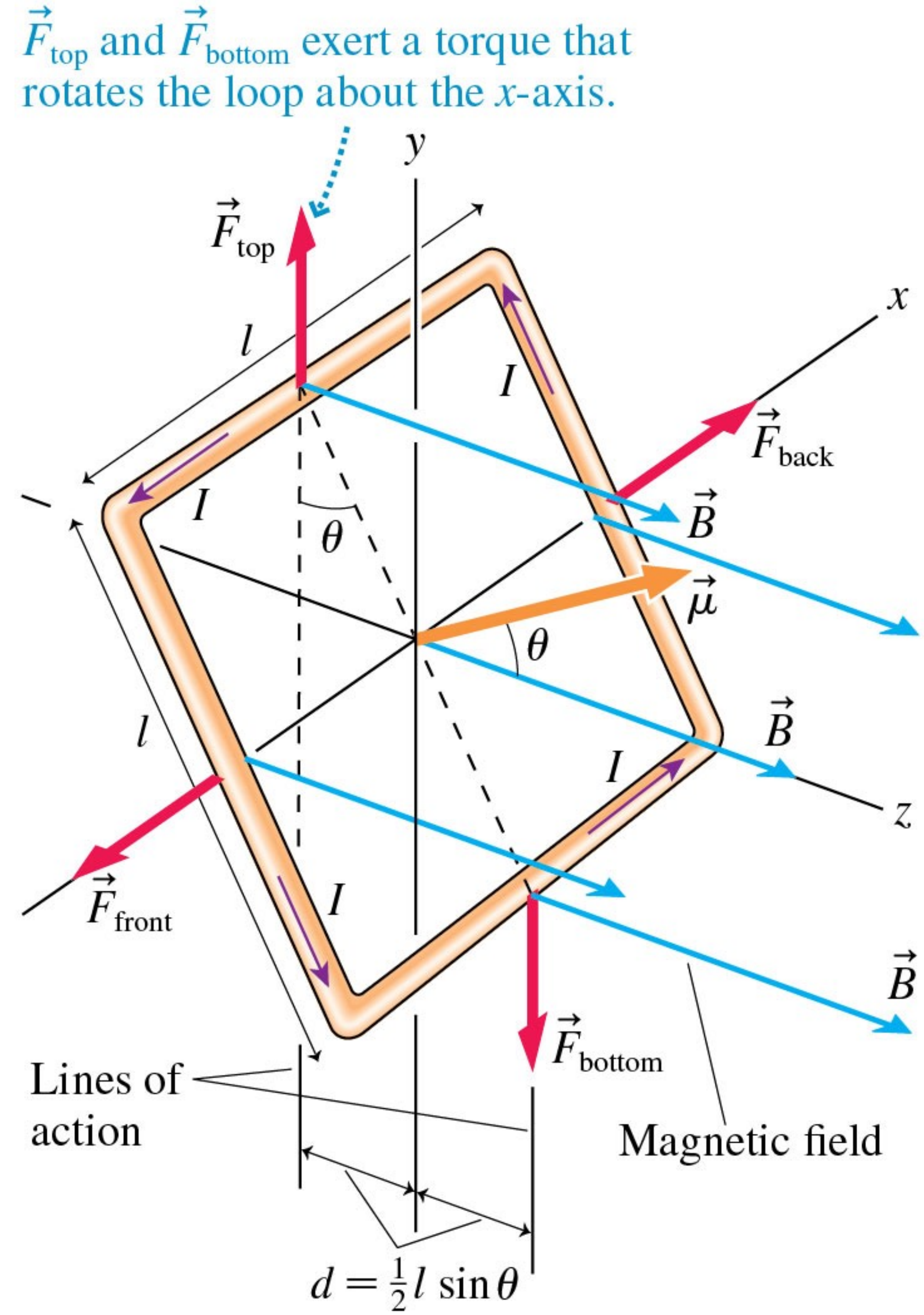




- Parallel currents attract, opposite currents repel.
- Opposite poles attract, like poles repel.

A Uniform Magnetic Field Exerts a Torque on a Square Current Loop

- $\vec{F}_{\text{front}}$ and $\vec{F}_{\text{back}}$ are opposite to each other and cancel.
- Both $\vec{F}_{\text{top}}$ and $\vec{F}_{\text{bottom}}$ exert a force of magnitude $F = I\,l\,B$ around a moment arm $d = ½\,l\,\sin\theta$.



A Uniform Magnetic Field Exerts a Torque on a Square Current Loop

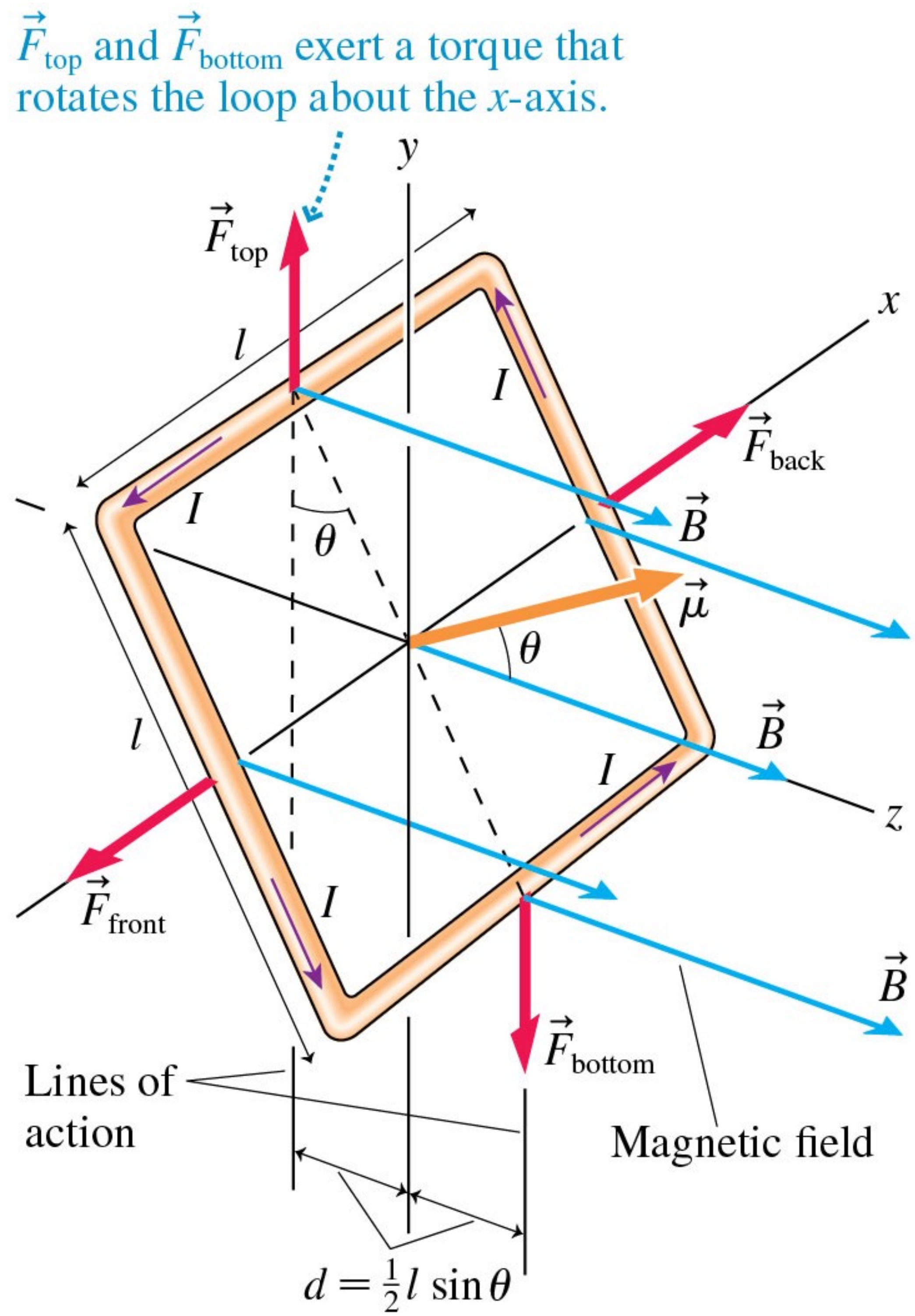
- The total torque is

$$\tau = 2Fd = (Il^2)B\sin\theta = \mu B\sin\theta$$

where $\mu = Il^2 = IA$ is the loop's magnetic dipole moment.

- Although derived for a square loop, the result is valid for a loop of any shape:

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

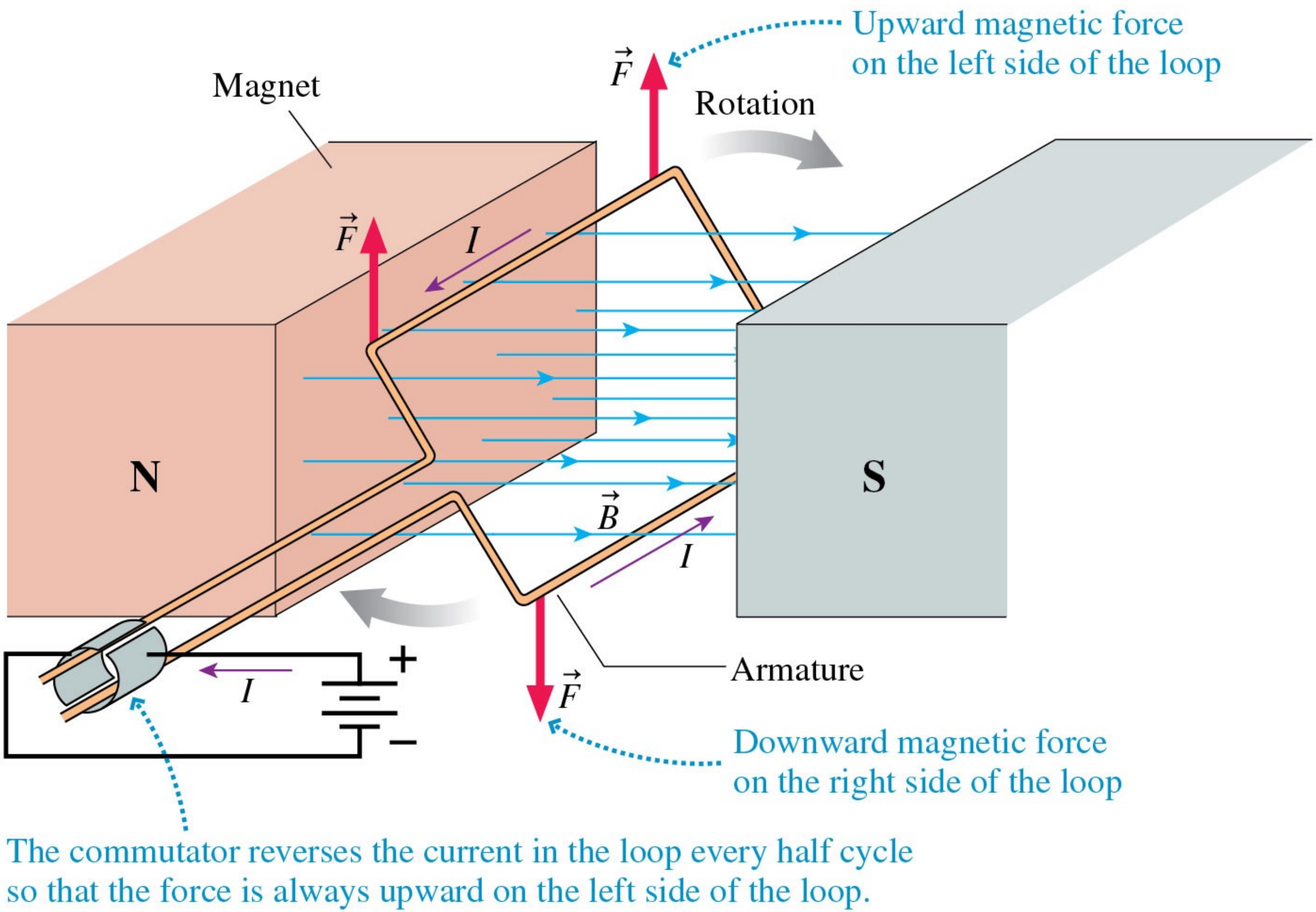


iClicker question #16-1

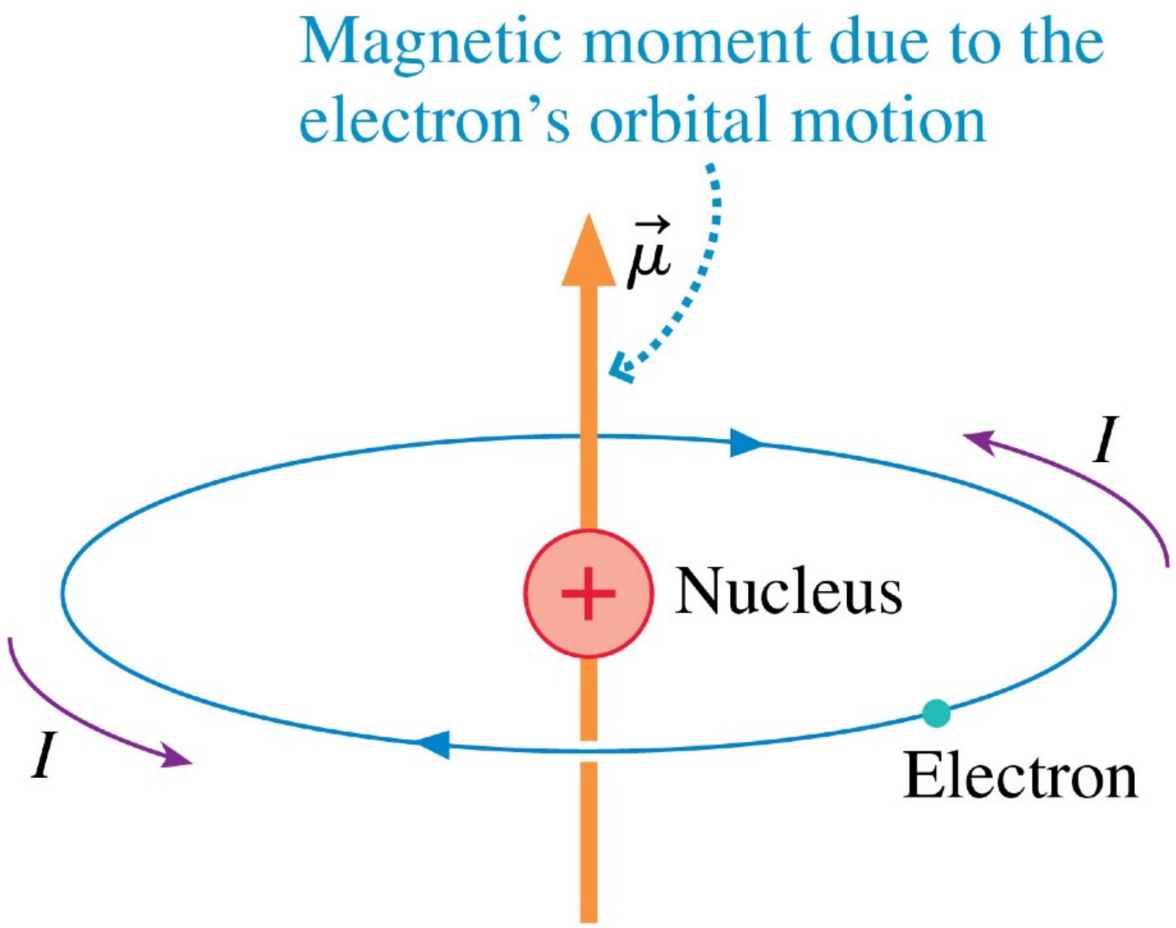
If released from rest, the current loop will

A. Move upward.
B. Move downward.
C. Rotate clockwise.
D. Rotate counterclockwise.
E. Do something not listed here.

A Simple Electric Motor



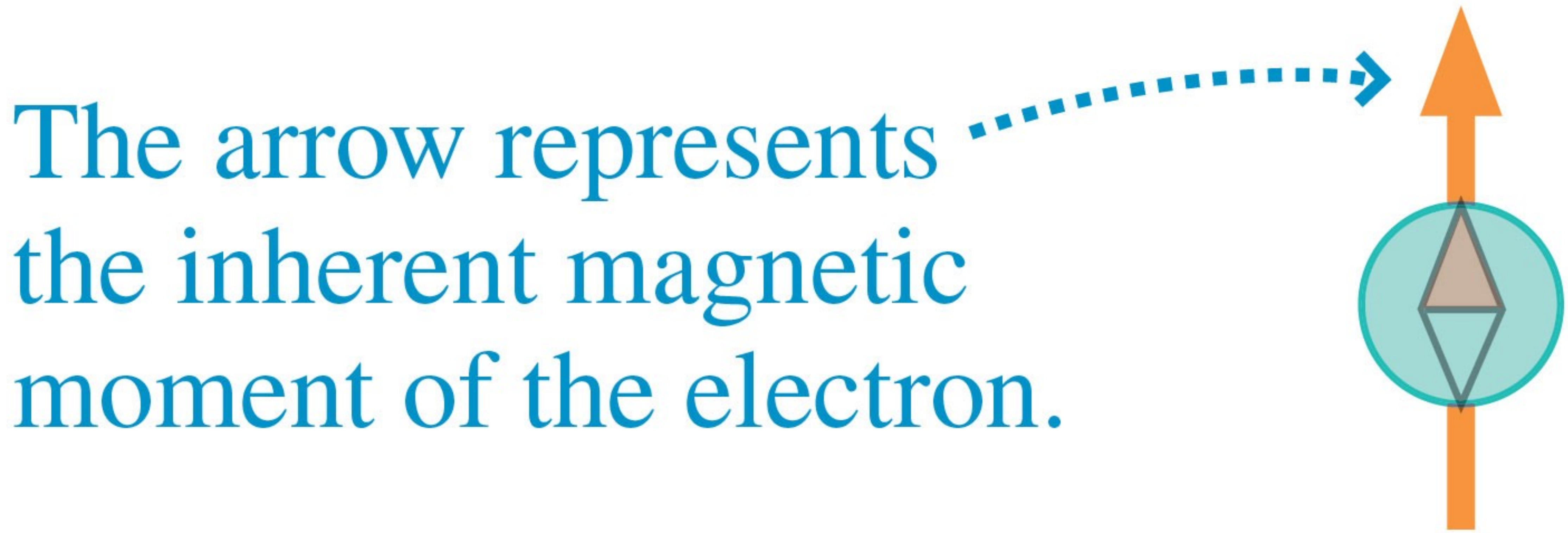
Atomic Magnets



- A plausible explanation for the magnetic properties of materials is the orbital motion of the atomic electrons.
- The figure shows a simple, classical model of an atom in which a negative electron orbits a positive nucleus.

- In this picture of the atom, the electron's motion is that of a current loop!
- An orbiting electron acts as a tiny magnetic dipole, with a north pole and a south pole.

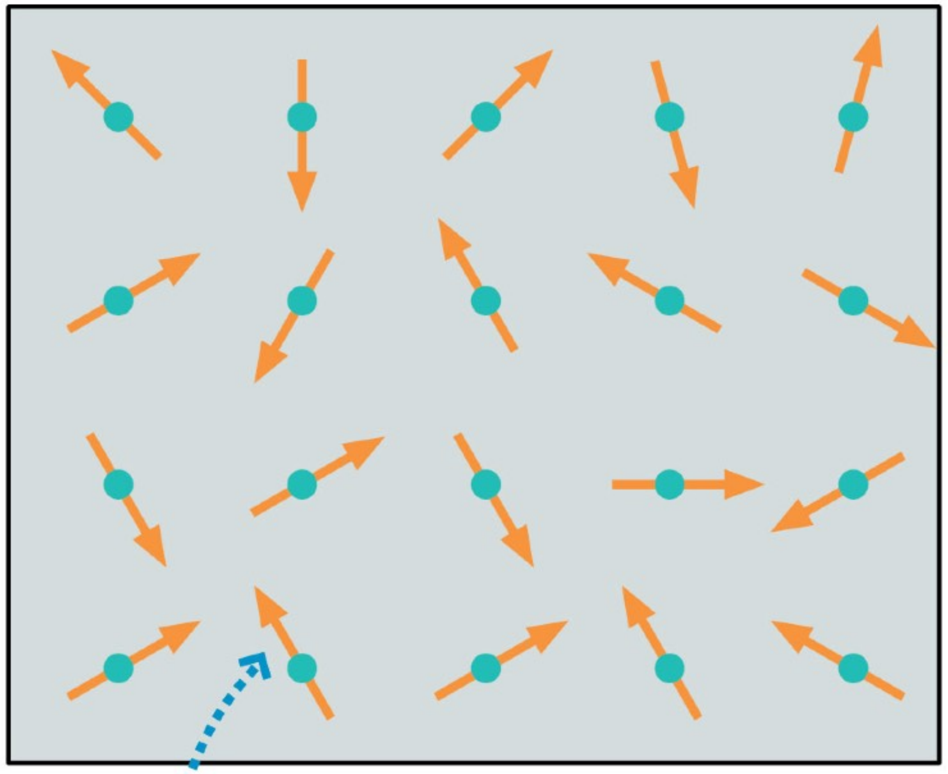
The Electron Spin



- An electron's inherent magnetic moment is often called the electron *spin* because, in a classical picture, a spinning ball of charge would have a magnetic moment.
- While it may not be spinning in a literal sense, an electron really is a microscopic magnet.

Magnetic Properties of Matter

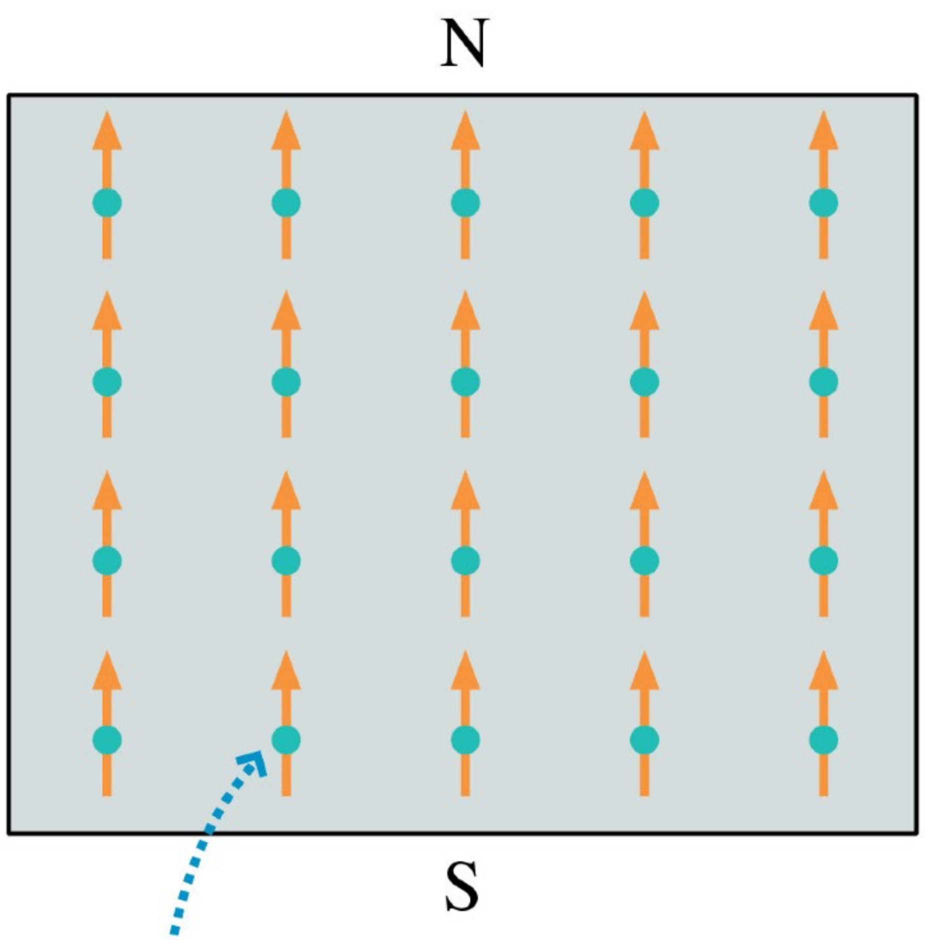
- For most elements, the magnetic moments of the atoms are randomly arranged when the atoms join together to form a solid.
- As the figure shows, this random arrangement produces a solid whose net magnetic moment is very close to zero.



The atomic magnetic moments due to unpaired electrons point in random directions. The sample has no net magnetic moment.

Ferromagnetism

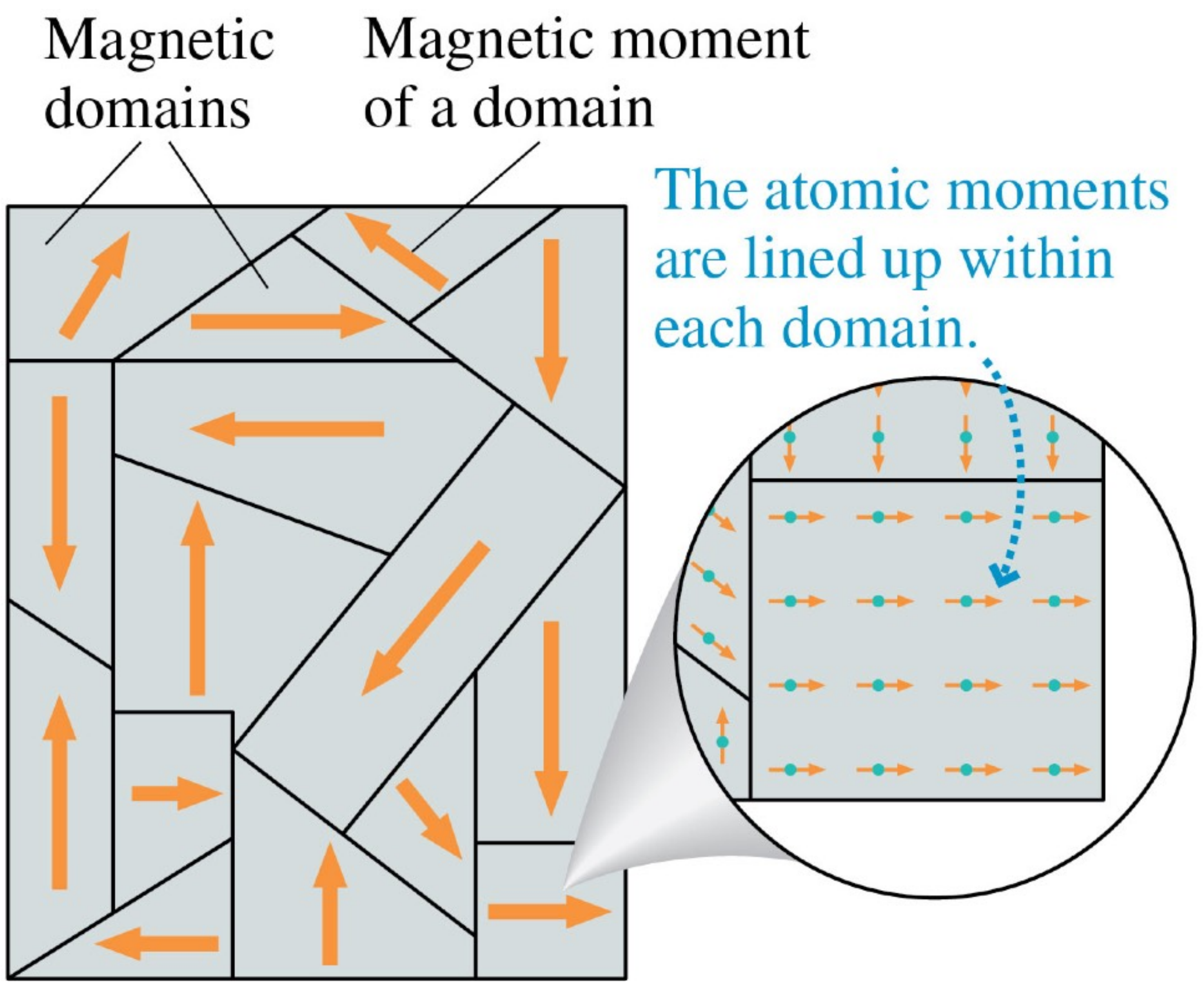
- In iron, and a few other substances, the atomic magnetic moments tend to all line up in the *same* direction, as shown in the figure.
- Materials that behave in this fashion are called **ferromagnetic**, with the prefix *ferro* meaning “iron-like.”



The atomic magnetic moments are aligned.
The sample has north and south magnetic poles.

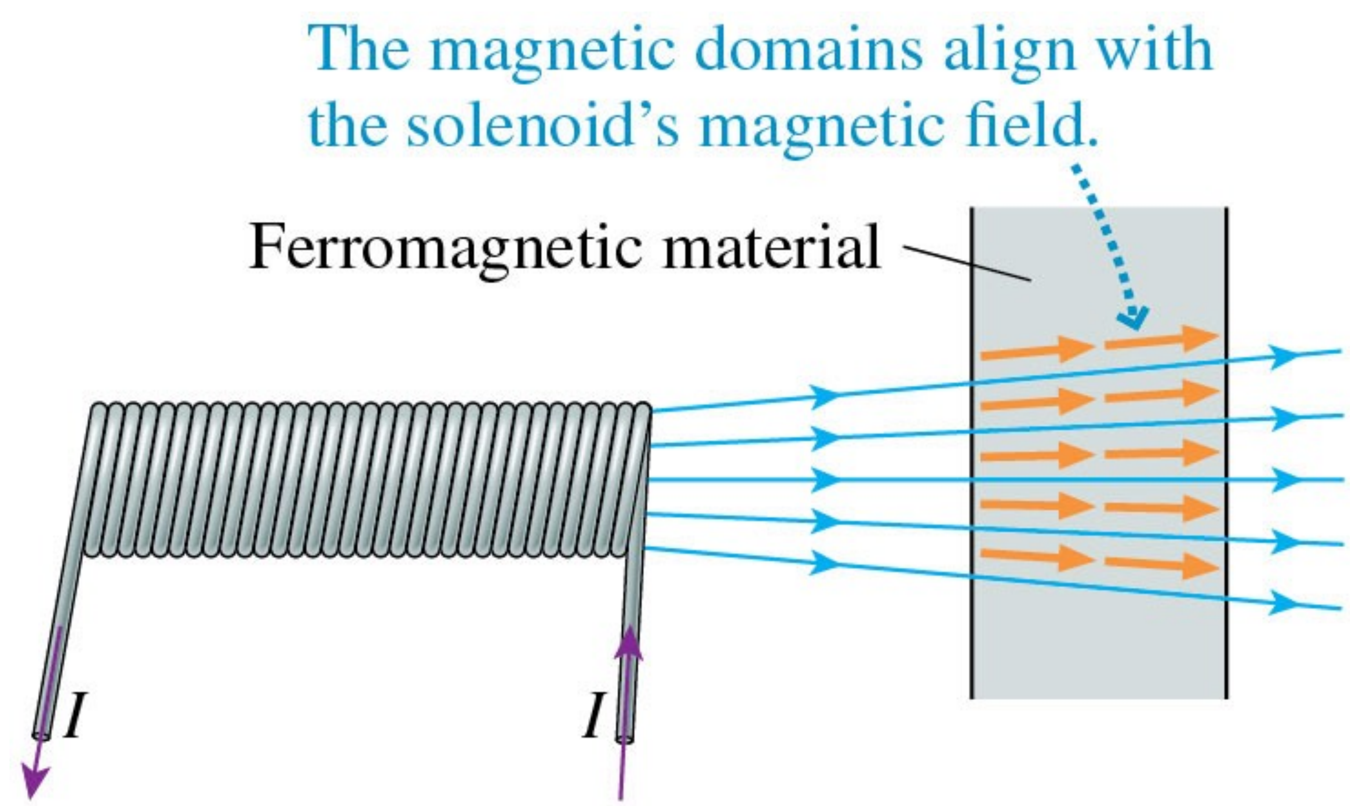
Ferromagnetism

- A typical piece of iron is divided into small regions, typically less than $100\ \mu m$ in size, called **magnetic domains**.
- The magnetic moments of all the iron atoms within each domain are perfectly aligned, so each individual domain is a strong magnet.
- However, the various magnetic domains that form a larger solid are randomly arranged.



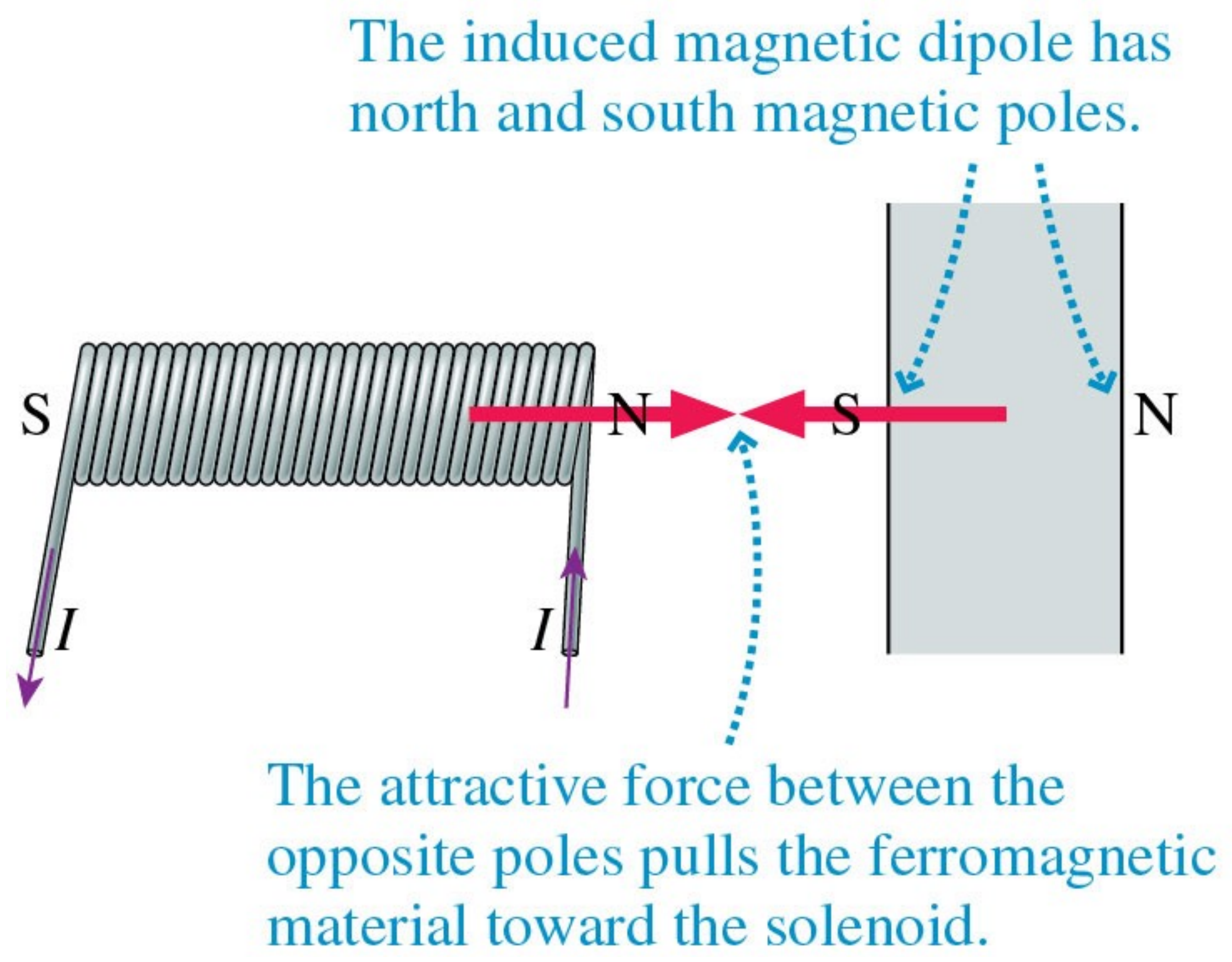
Induced Magnetic Dipoles

- If a ferromagnetic substance is subjected to an *external* magnetic field, the external field exerts a torque on the magnetic dipole of each domain.
- The torque causes many of the domains to rotate and become aligned with the external field.

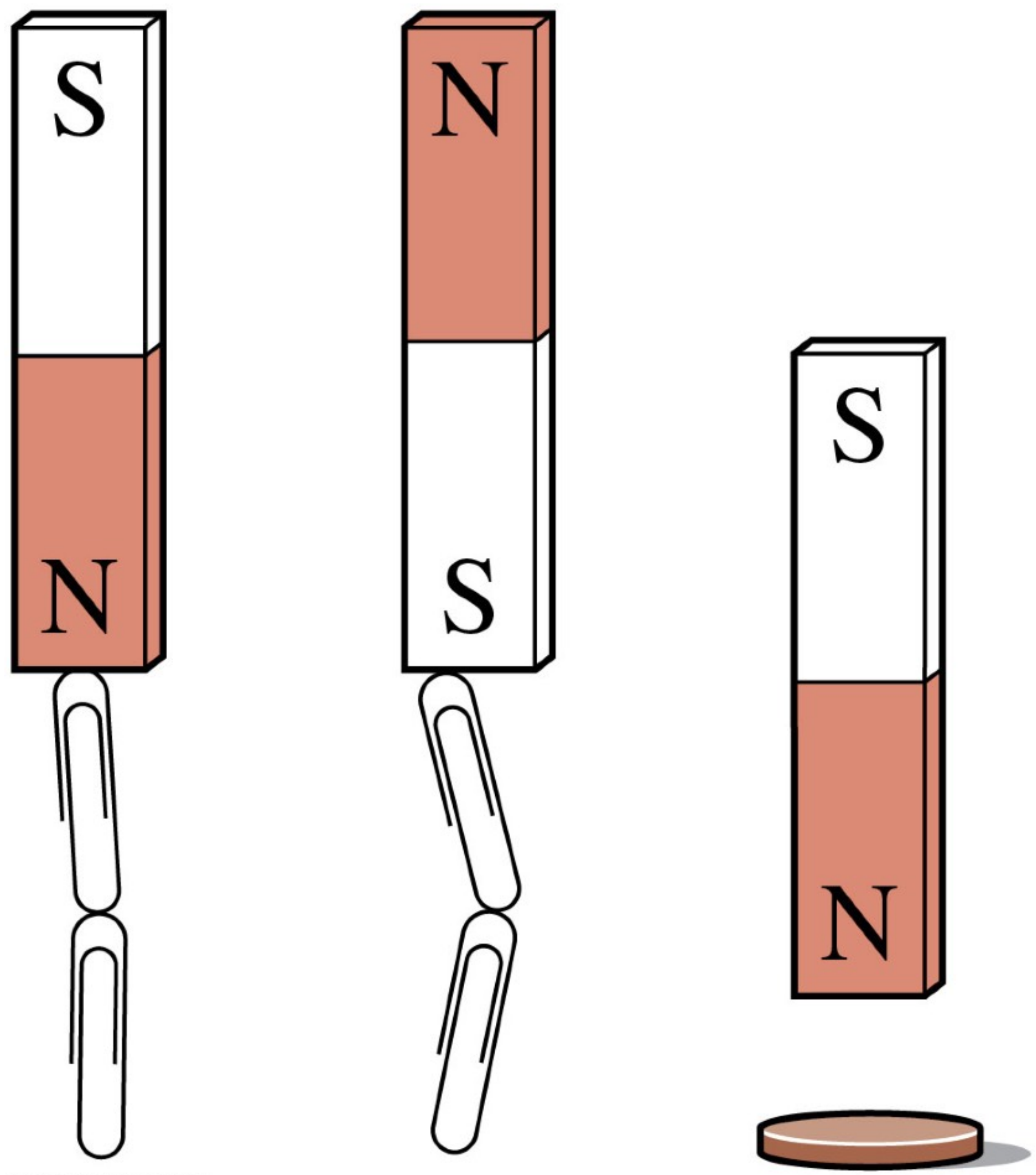


Induced Magnetic Dipoles

- The induced magnetic dipole always has an *opposite* pole facing the solenoid.
- Consequently the magnetic force between the poles *pulls* the ferromagnetic object to the electromagnet.



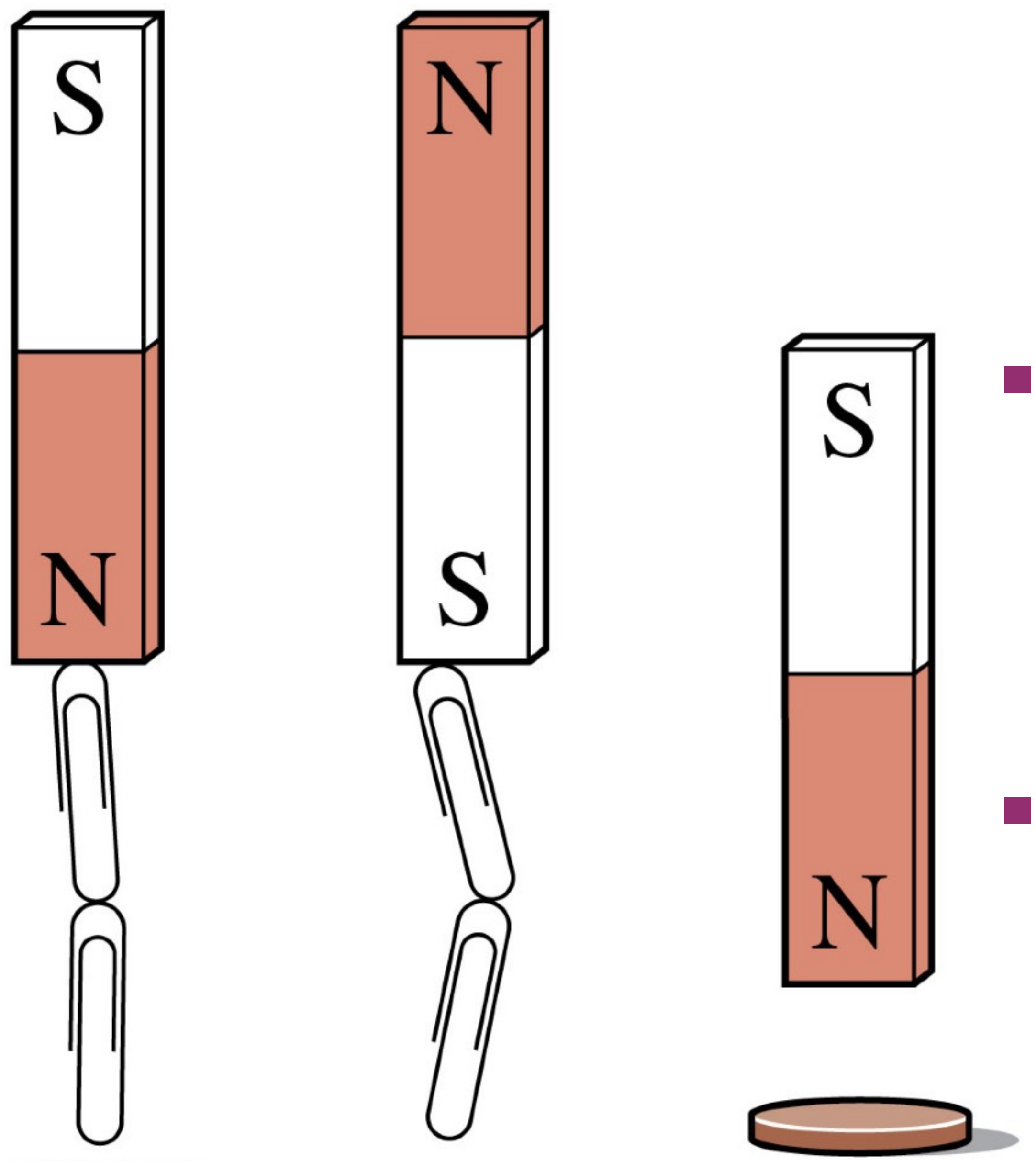
Induced Magnetism



- Now we can explain how a magnet attracts and picks up ferromagnetic objects:

1. Electrons are microscopic magnets due to their spin.
2. A ferromagnetic material in which the spins are aligned is organized into magnetic domains.
3. The individual domains align with an external magnetic field to produce an induced magnetic dipole moment for the entire object.

Induced Magnetism

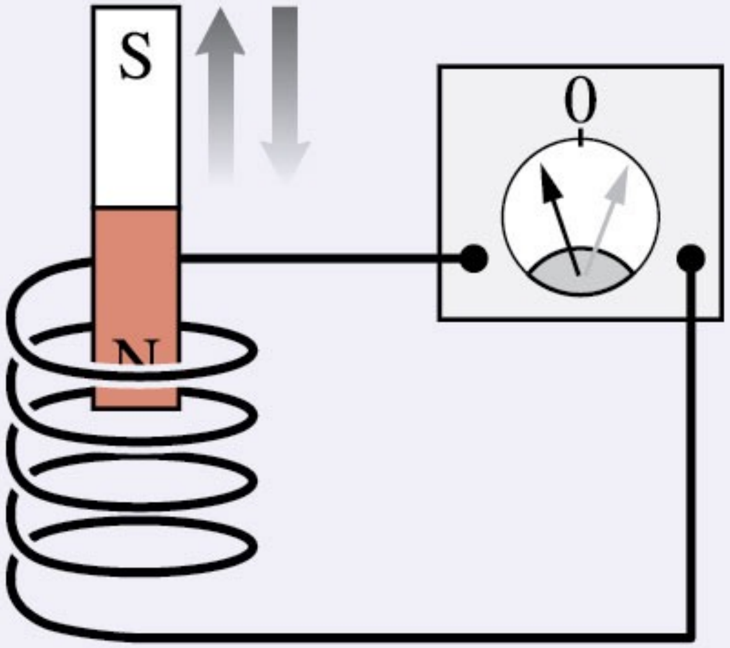


- An object's magnetic dipole may not return to zero when the external field is removed because some domains remain "frozen" in the alignment they had in the external field.
- Thus a ferromagnetic object that has been in an external field may be left with a net magnetic dipole moment after the field is removed.
- In other words, the object has become a **permanent magnet**.

What is an induced current?

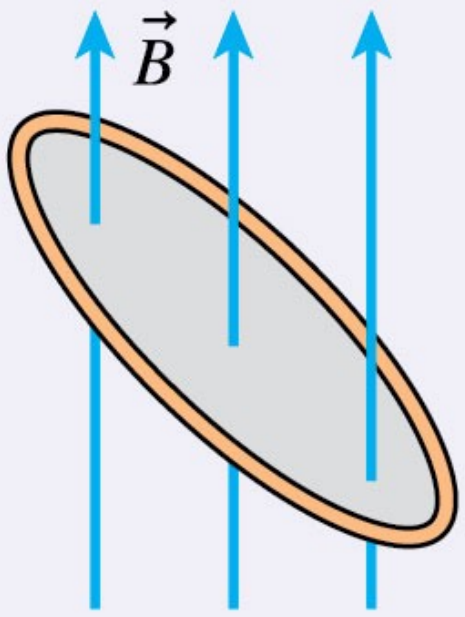
A magnetic field can create a current in a loop of wire, *but only if the amount of field through the loop is changing.*

- This is called an induced current.
- The process is called **electromagnetic induction**.



What is magnetic flux?

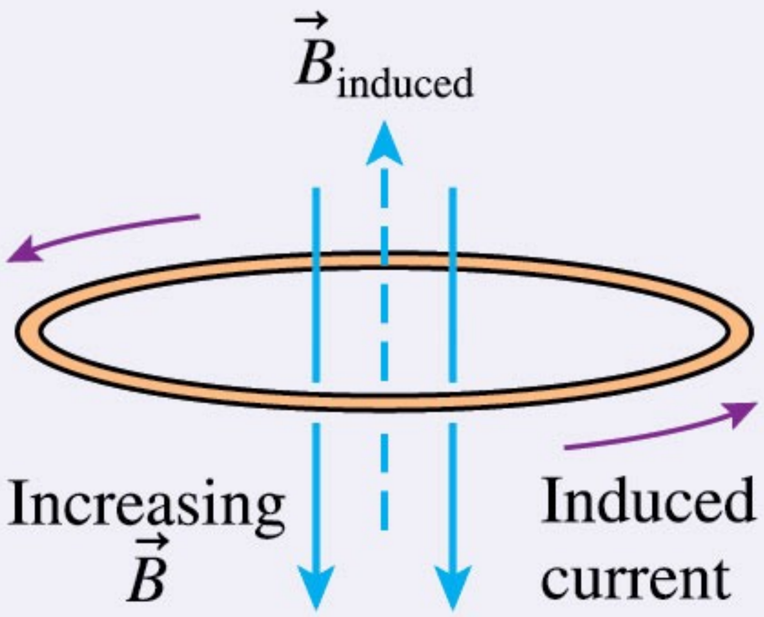
A key idea will be the amount of magnetic field passing through a loop or coil. This is called magnetic flux. Magnetic flux depends on the strength of the magnetic field, the area of the loop, and the angle between them.



« LOOKING BACK Section 24.3 Electric flux

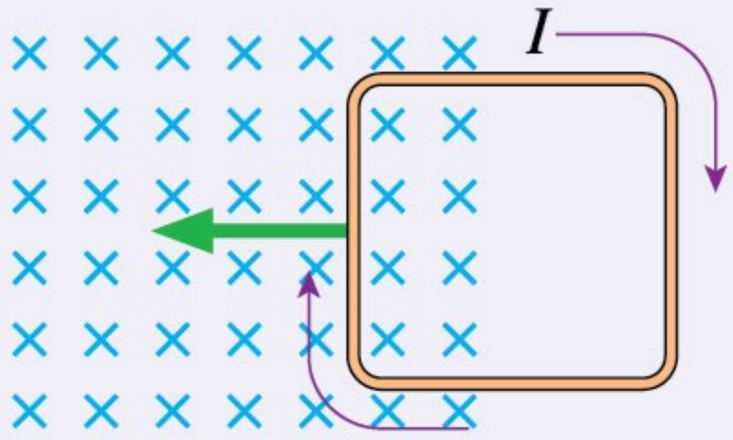
What is Lenz's law?

Lenz's law says that a current is induced in a closed loop if and only if the magnetic flux through the loop is changing. Simply having a flux does nothing; the flux has to change. You'll learn how to use Lenz's law to determine the direction of an induced current around a loop.



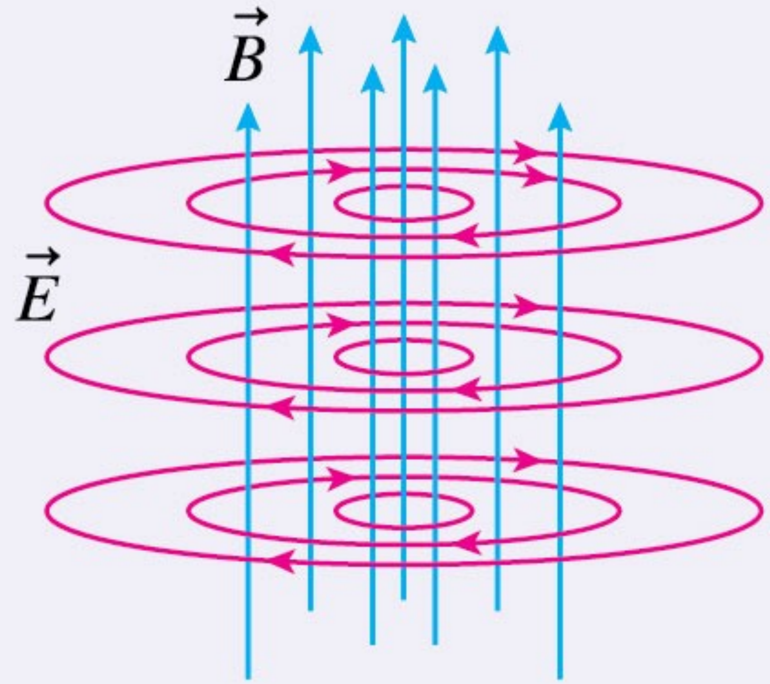
What is Faraday's law?

Faraday's law is the most important law connecting electric and magnetic fields, laying the groundwork for electromagnetic waves. Just as a battery has an emf that drives current, a loop of wire has an induced emf determined by the rate of change of magnetic flux through the loop.



What is an induced field?

At its most fundamental level, Faraday's law tells us that a changing magnetic field creates an induced electric field. This is an entirely new way to create an electric field, independent of charges. It is the induced electric field that drives the induced current around a conducting loop.

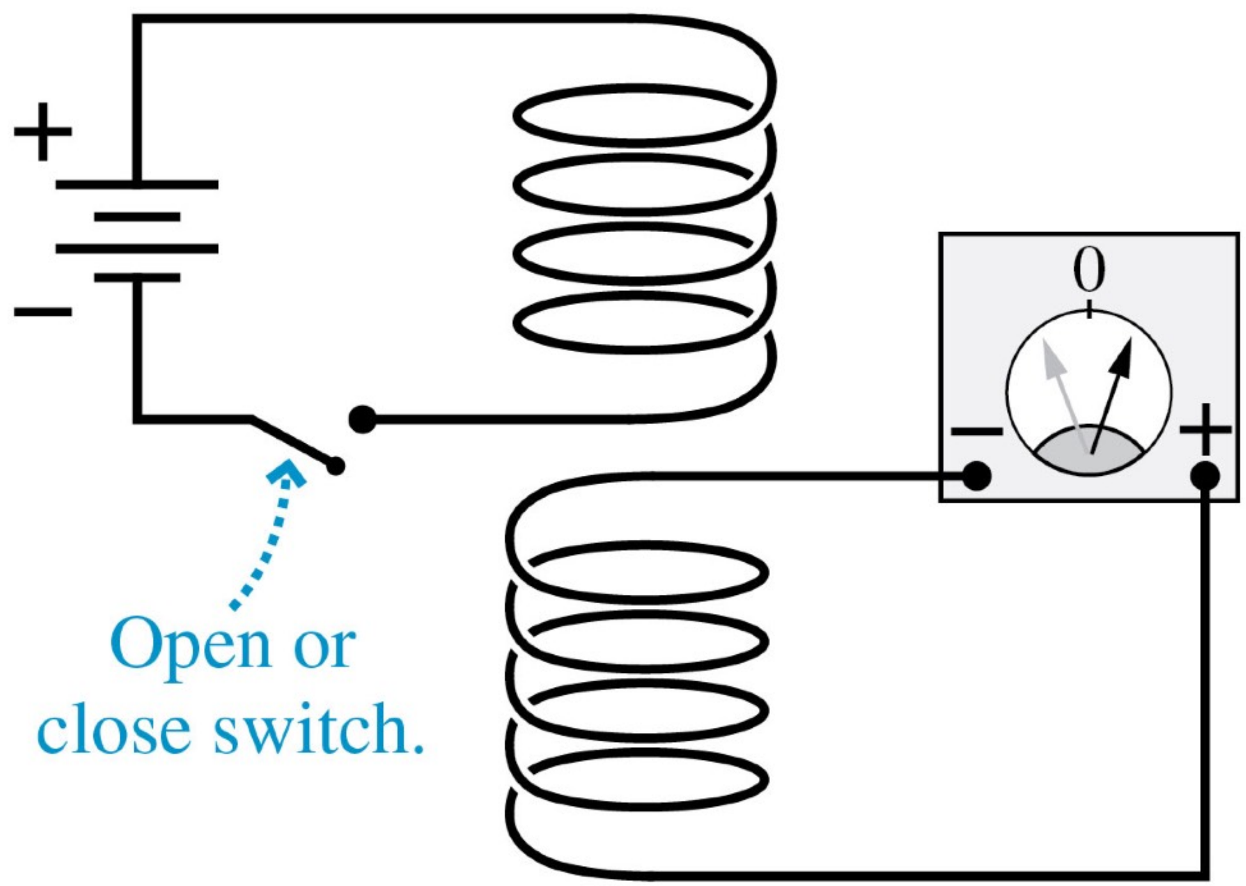


How is electromagnetic induction used?

Electromagnetic induction is one of the most important applications of electricity and magnetism. Generators use electromagnetic induction to turn the mechanical energy of a spinning turbine into electric energy. Inductors are important circuit elements that rely on electromagnetic induction. All forms of telecommunication are based on electromagnetic induction. And, not least, electromagnetic induction is the basis for light and other electromagnetic waves.

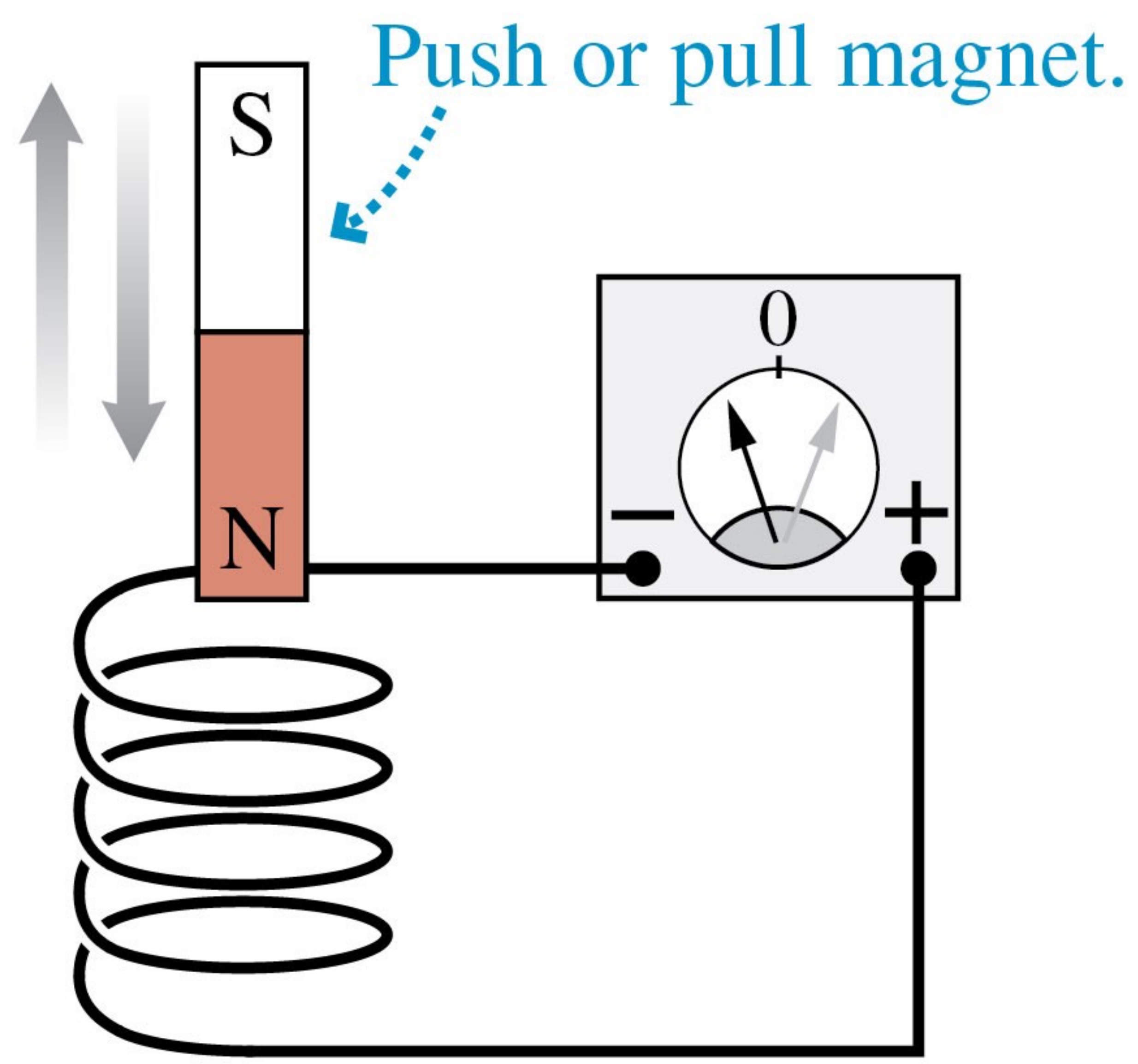
Faraday's Discovery of 1831

- When one coil is placed directly above another, there is no current in the lower circuit while the switch is in the closed position.
- A momentary current appears whenever the switch is opened or closed.



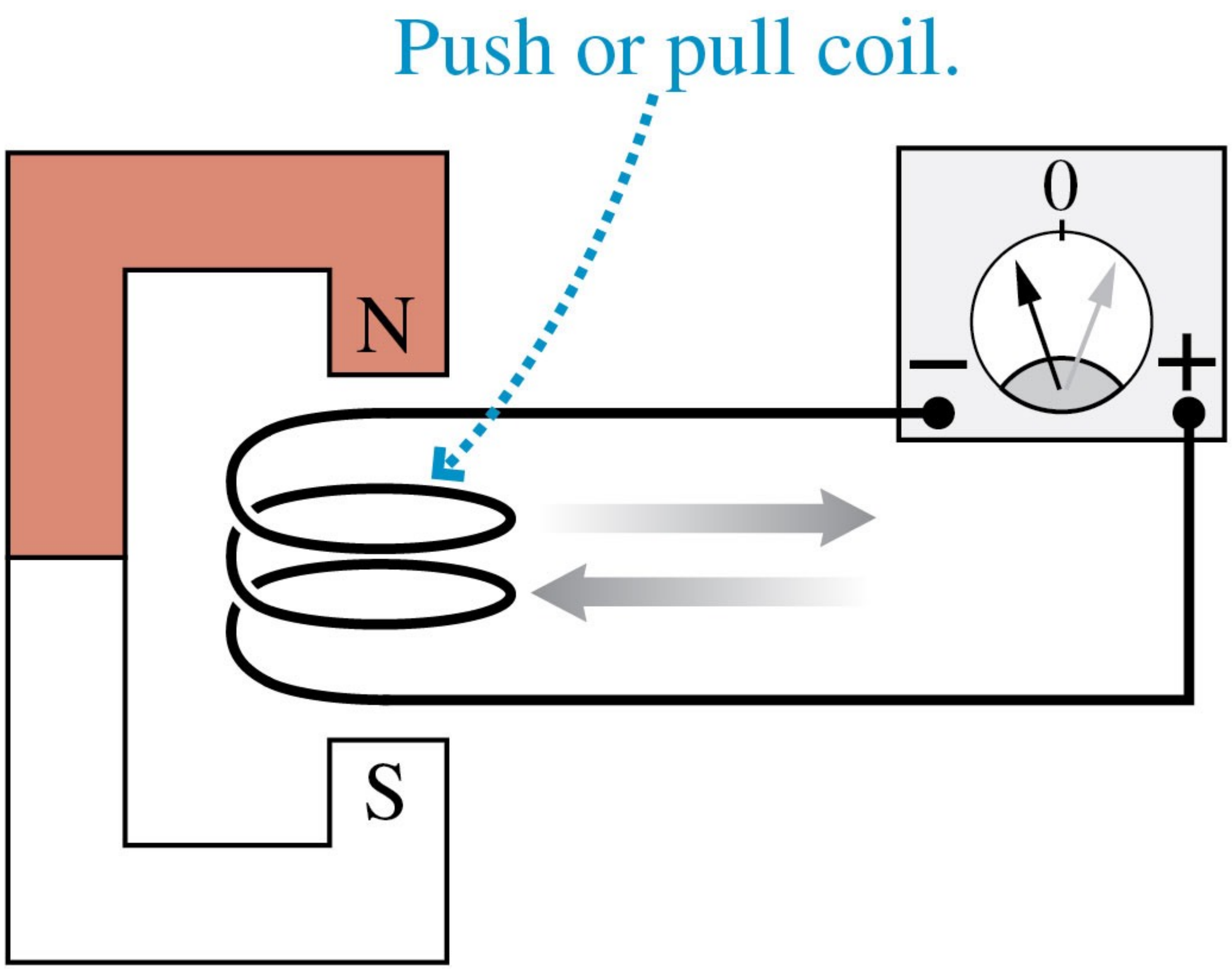
Faraday's Discovery of 1831

- When a bar magnet is pushed into a coil of wire, it causes a momentary deflection of the current-meter needle.
- Holding the magnet inside the coil has no effect.
- A quick withdrawal of the magnet deflects the needle in the other direction.

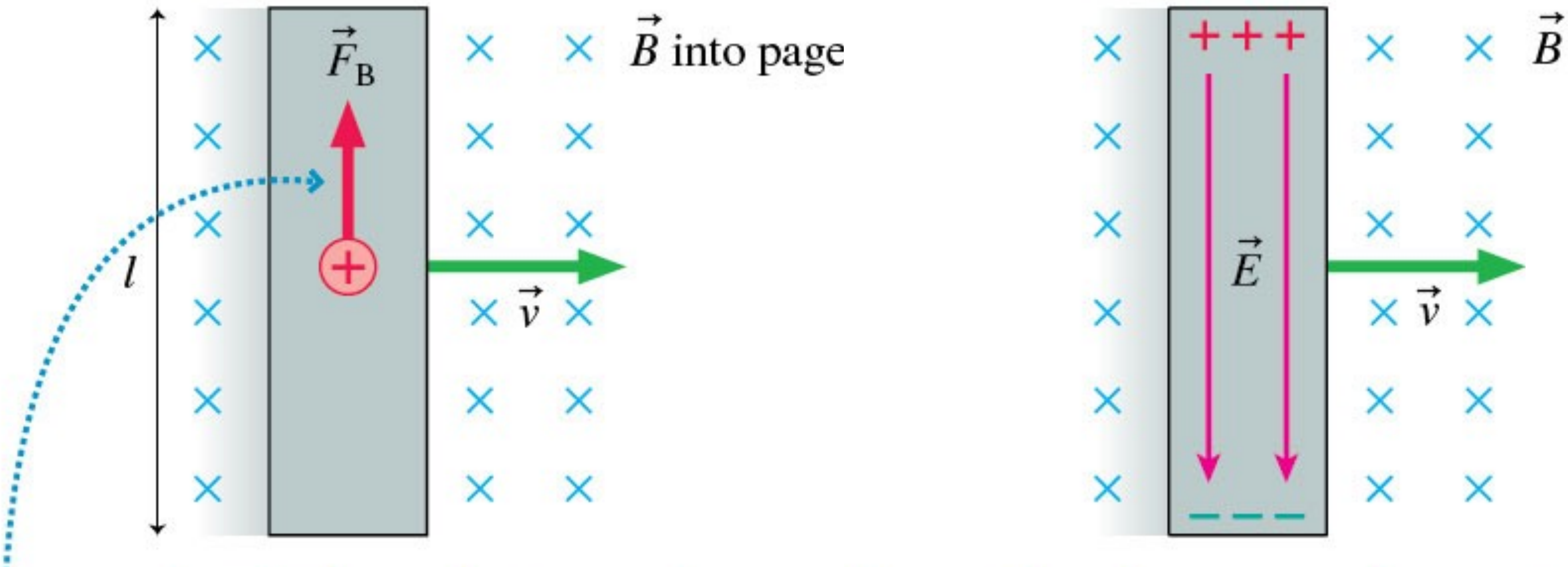


Faraday’s Discovery of 1831

- A momentary current is produced by rapidly pulling a coil of wire out of a magnetic field.
- Pushing the coil into the magnet causes the needle to deflect in the opposite direction.



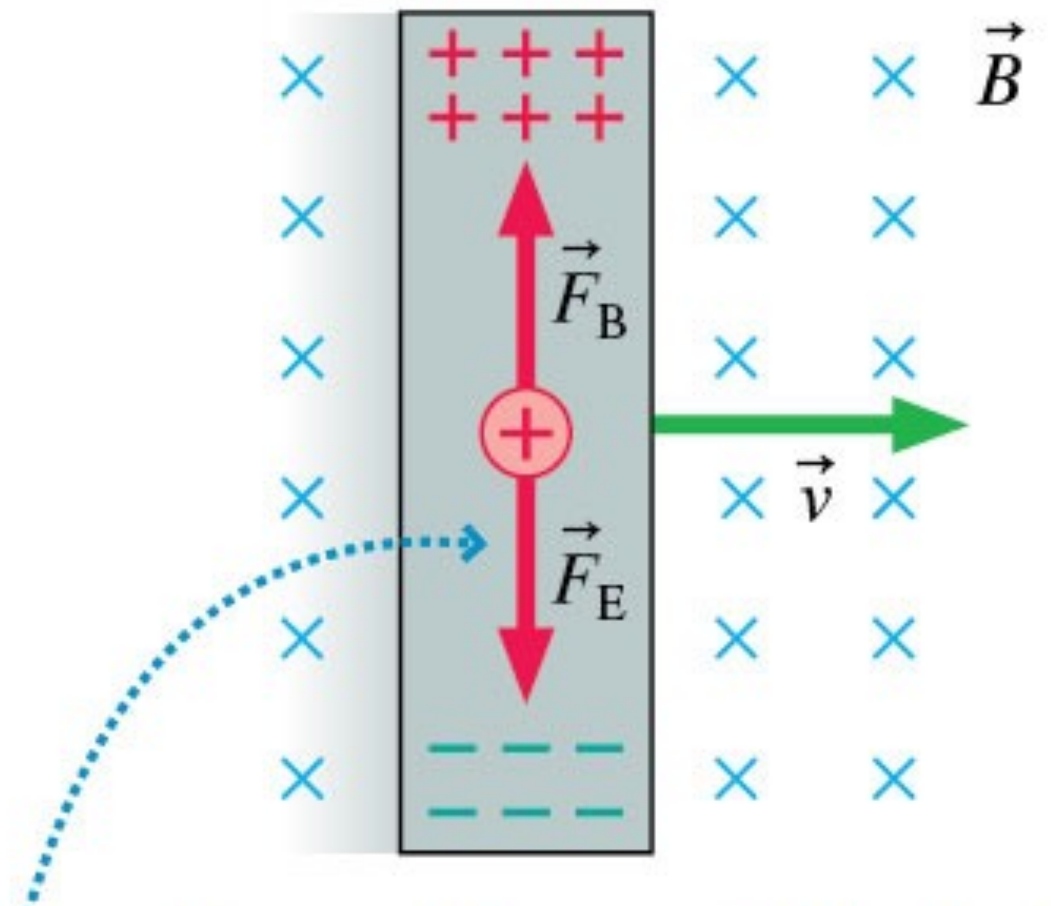
Motional emf



Charge carriers in the conductor experience a force of magnitude $F_B = qvB$. Positive charges are free to move and drift upward.

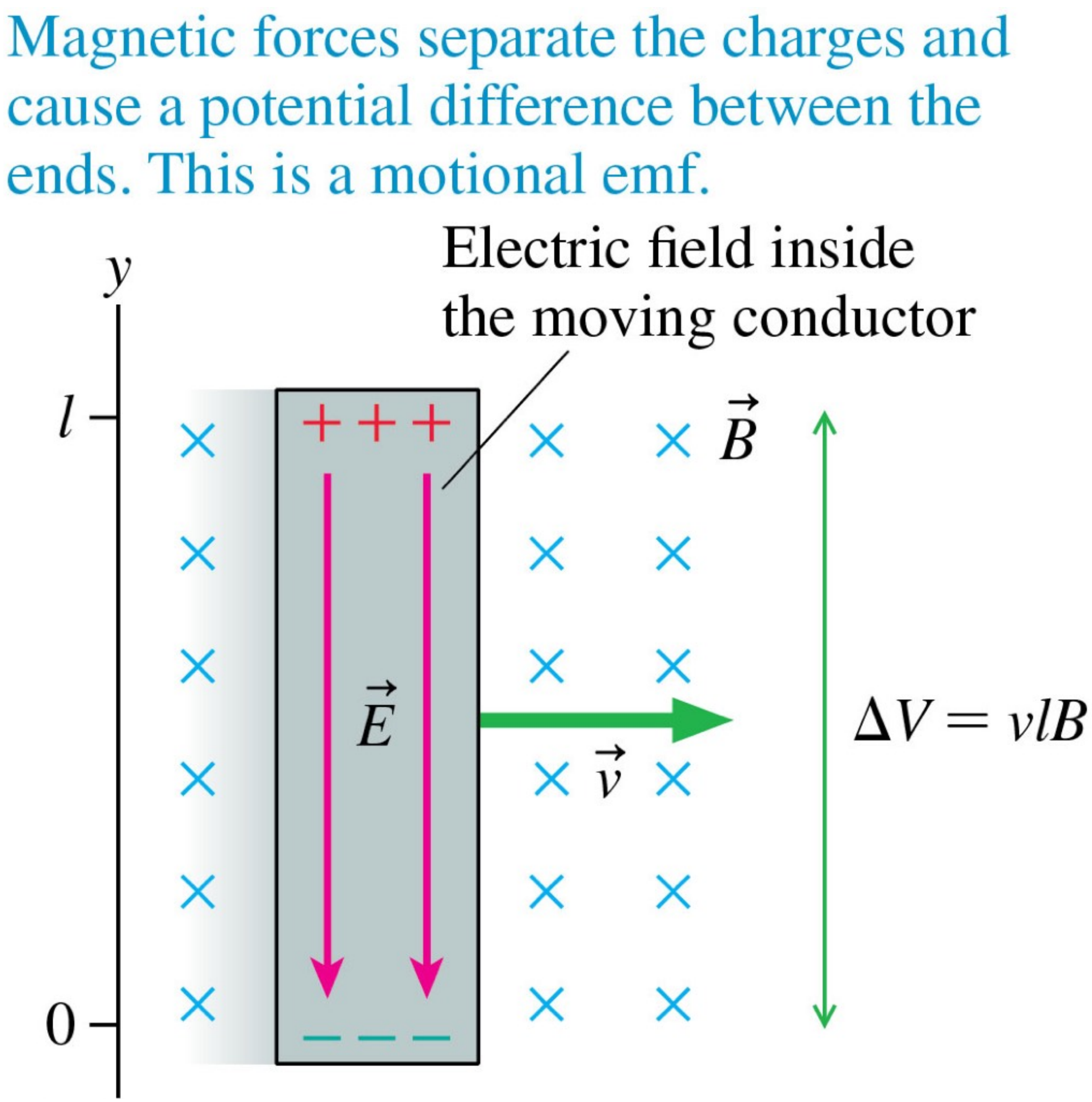
The resulting charge separation creates an electric field in the conductor. $\vec{E}$ increases as more charge flows.

Motional emf



The charge flow continues until the electric and magnetic forces balance. For a positive charge carrier, the upward magnetic force $\vec{F}_B$ is equal to the downward electric force $\vec{F}_E$.

Motional emf

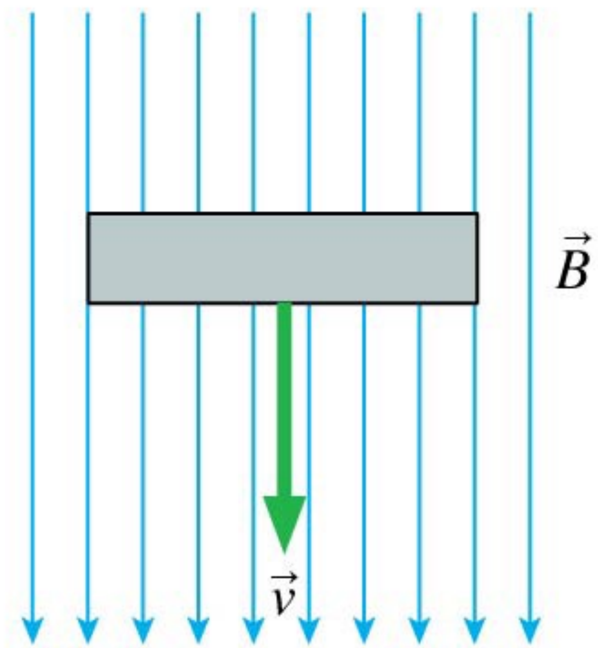


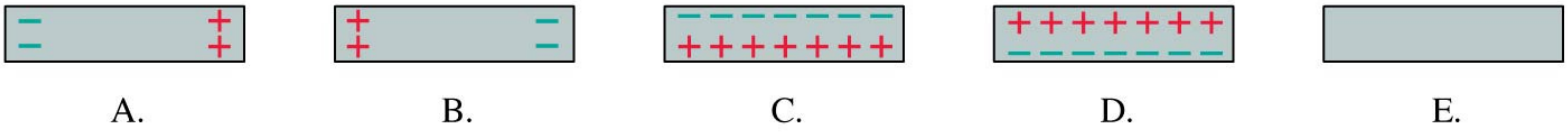
- The magnetic force on the charge carriers in a moving conductor creates an electric field of strength $E = vB$ inside the conductor.
- For a conductor of length l, the motional emf perpendicular to the magnetic field is:

$$\mathcal{E} = vlB$$

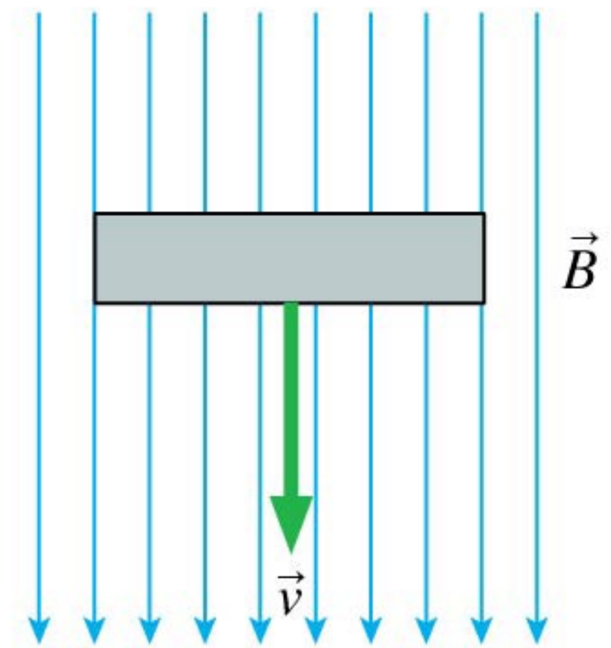
Example

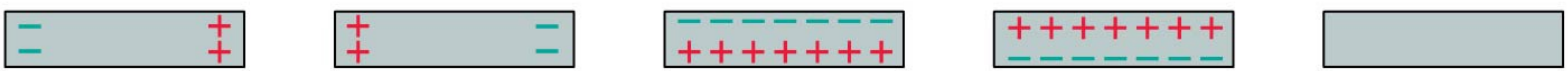
A metal bar moves through a magnetic field. The induced charges on the bar are





A metal bar moves through a magnetic field. The induced charges on the bar are

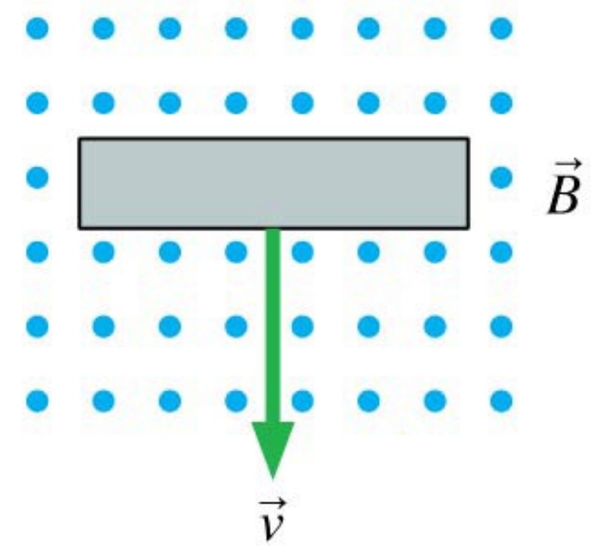


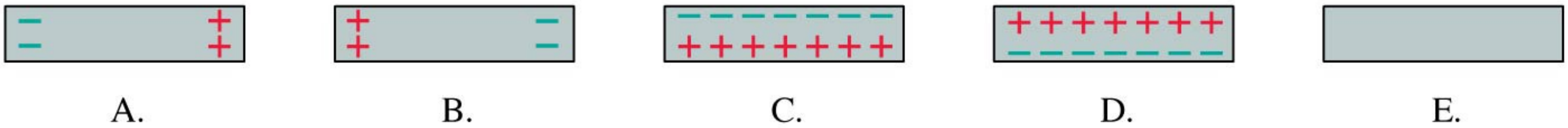


A. B. C. D. **E.**

Example

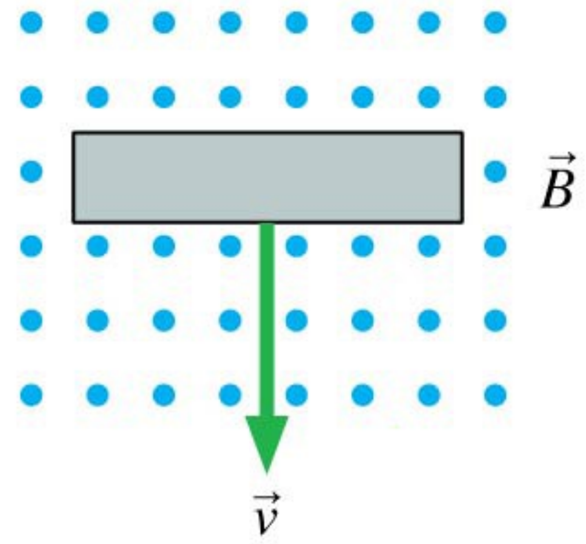
A metal bar moves through a magnetic field. The induced charges on the bar are

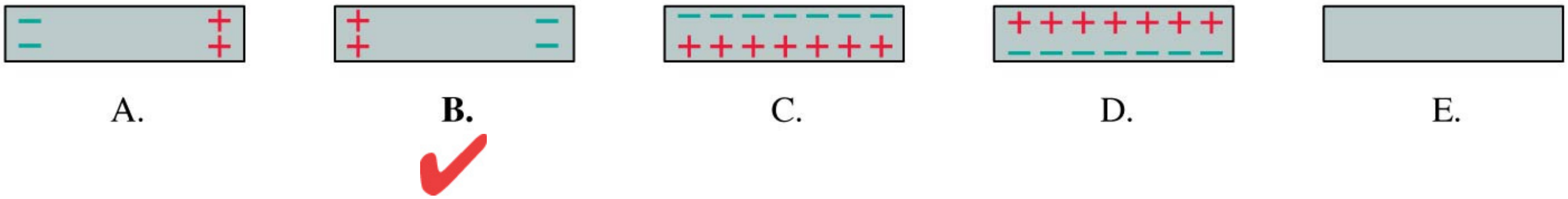




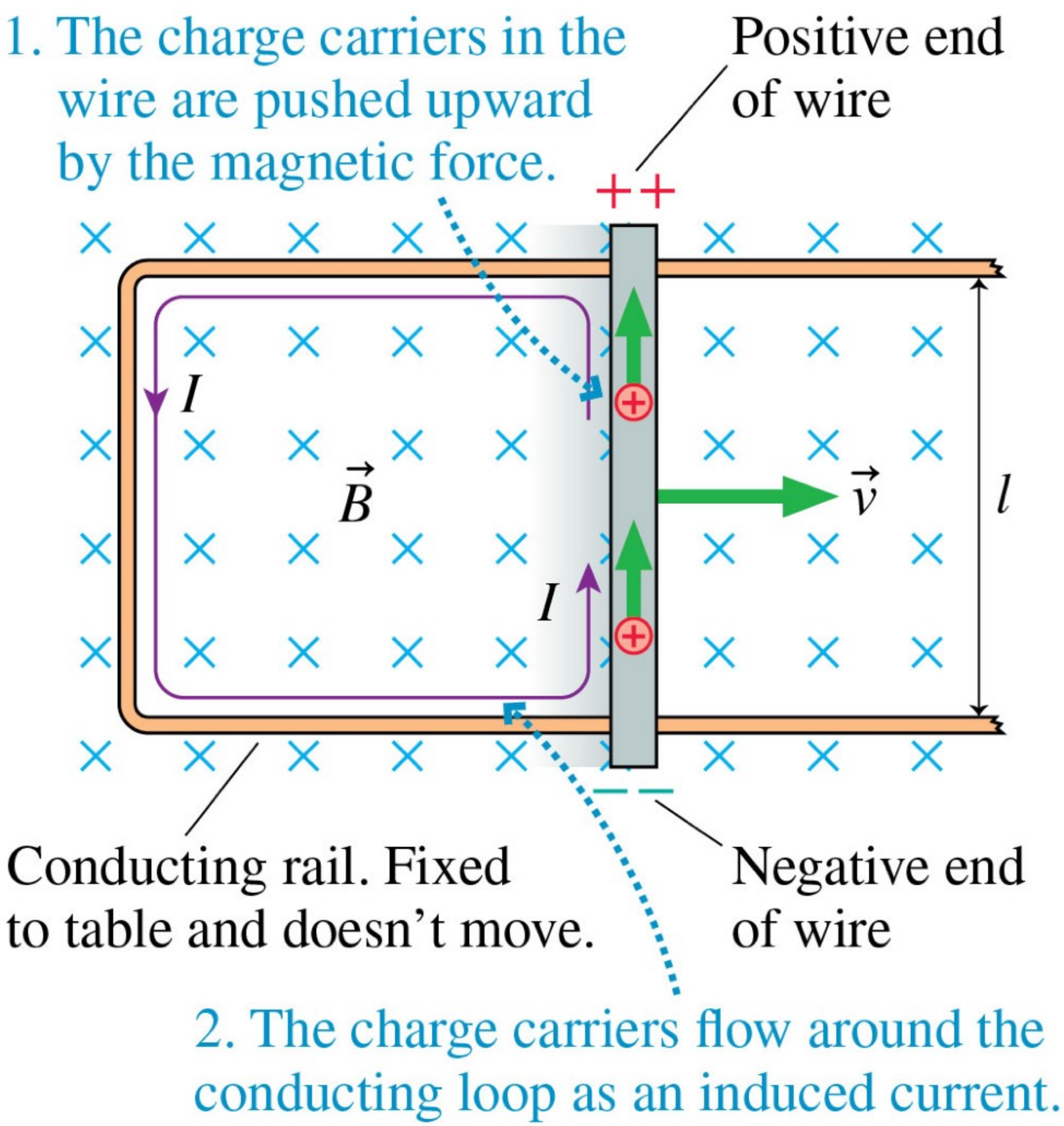
Example

A metal bar moves through a magnetic field. The induced charges on the bar are





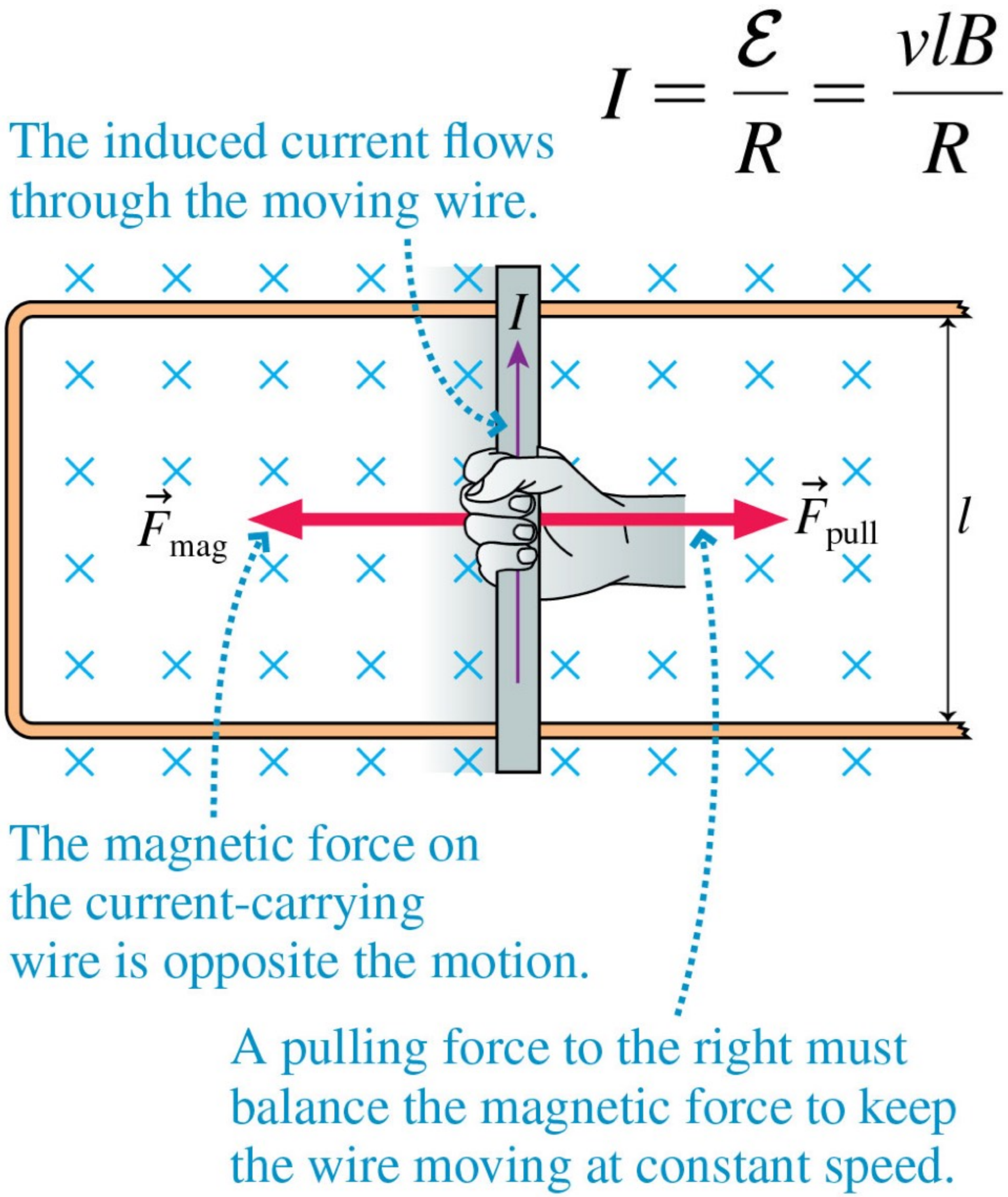
Induced Current



- If we slide a conducting wire along a U-shaped conducting rail, we can complete a circuit and drive an electric current.
- If the total resistance of the circuit is R, the *induced current* is given by Ohm's law as

$$I = \frac{\mathcal{E}}{R} = \frac{vlB}{R}$$

Induced Current

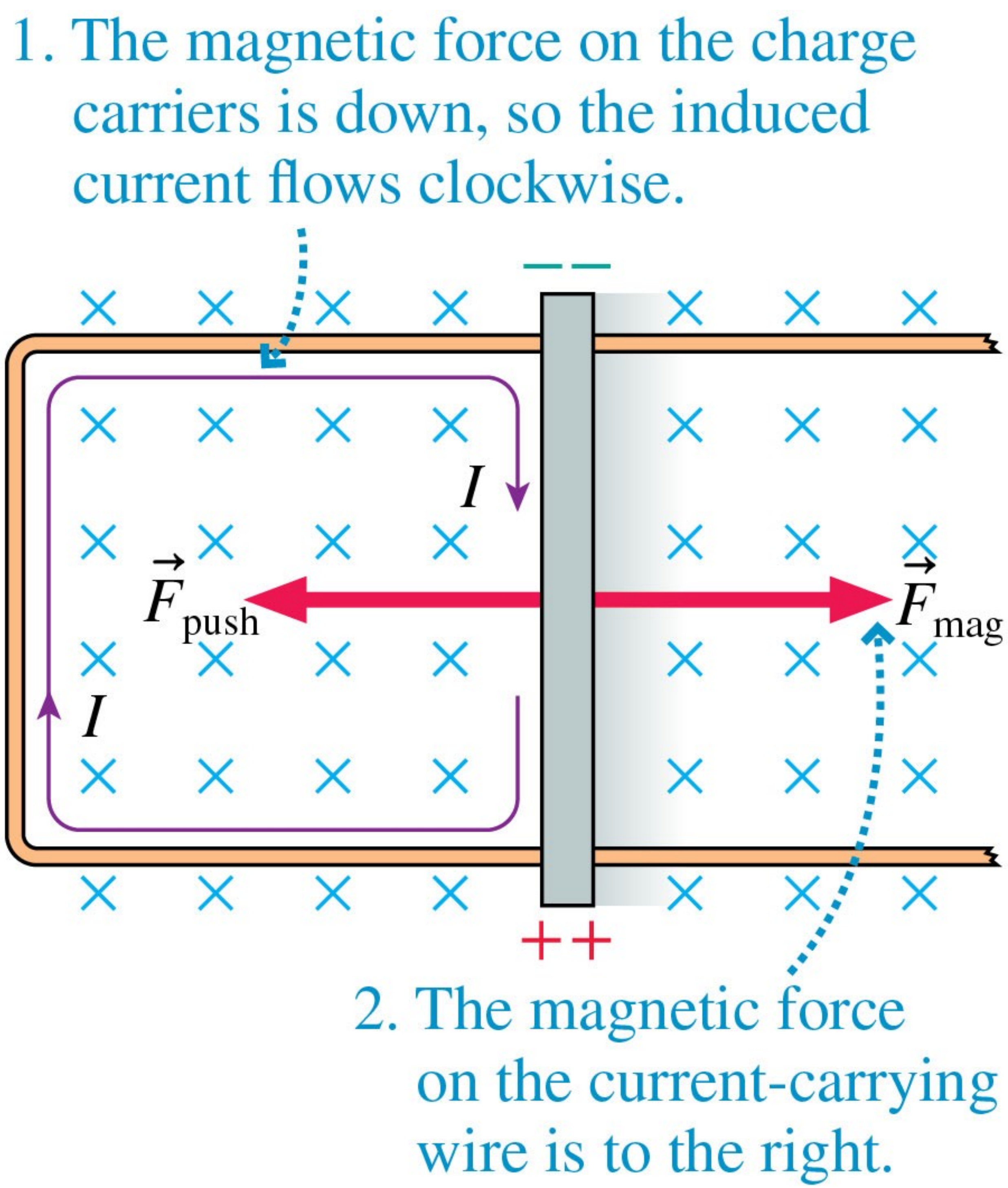


- To keep the wire moving at a constant speed v, we must apply a pulling force $F_{\text{pull}} = I\,lB = vl^2B^2/R$.
- This pulling force does work at a rate

$$P_{\text{input}} = F_{\text{pull}}v = \frac{v^2l^2B^2}{R}$$

- All of this power is dissipated by the resistance of the circuit.

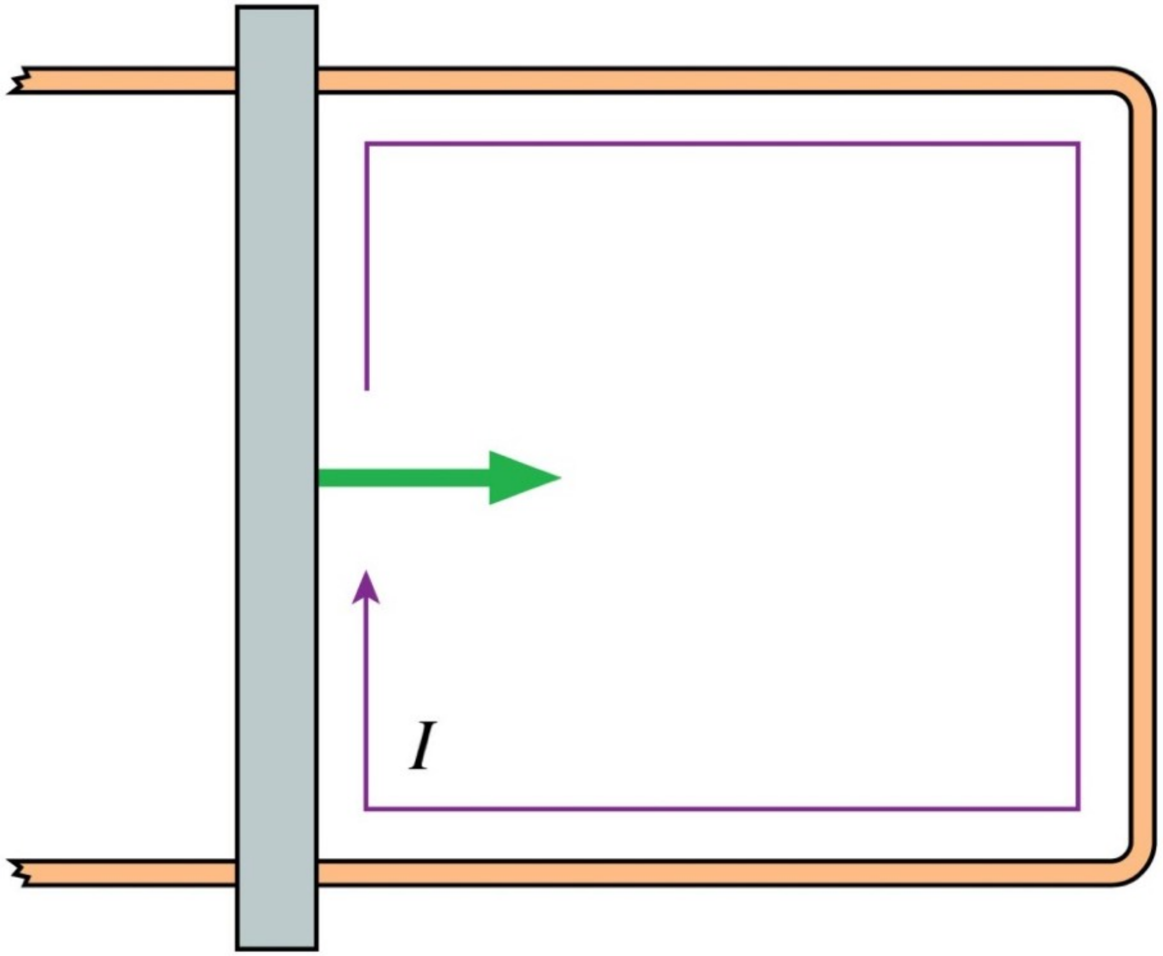
Induced Current



- The figure shows a conducting wire sliding to the *left*.
- In this case, a *pushing* force is needed to keep the wire moving at constant speed.
- Once again, this input power is dissipated in the electric circuit.
- A device that converts mechanical energy to electric energy is called a **generator**.

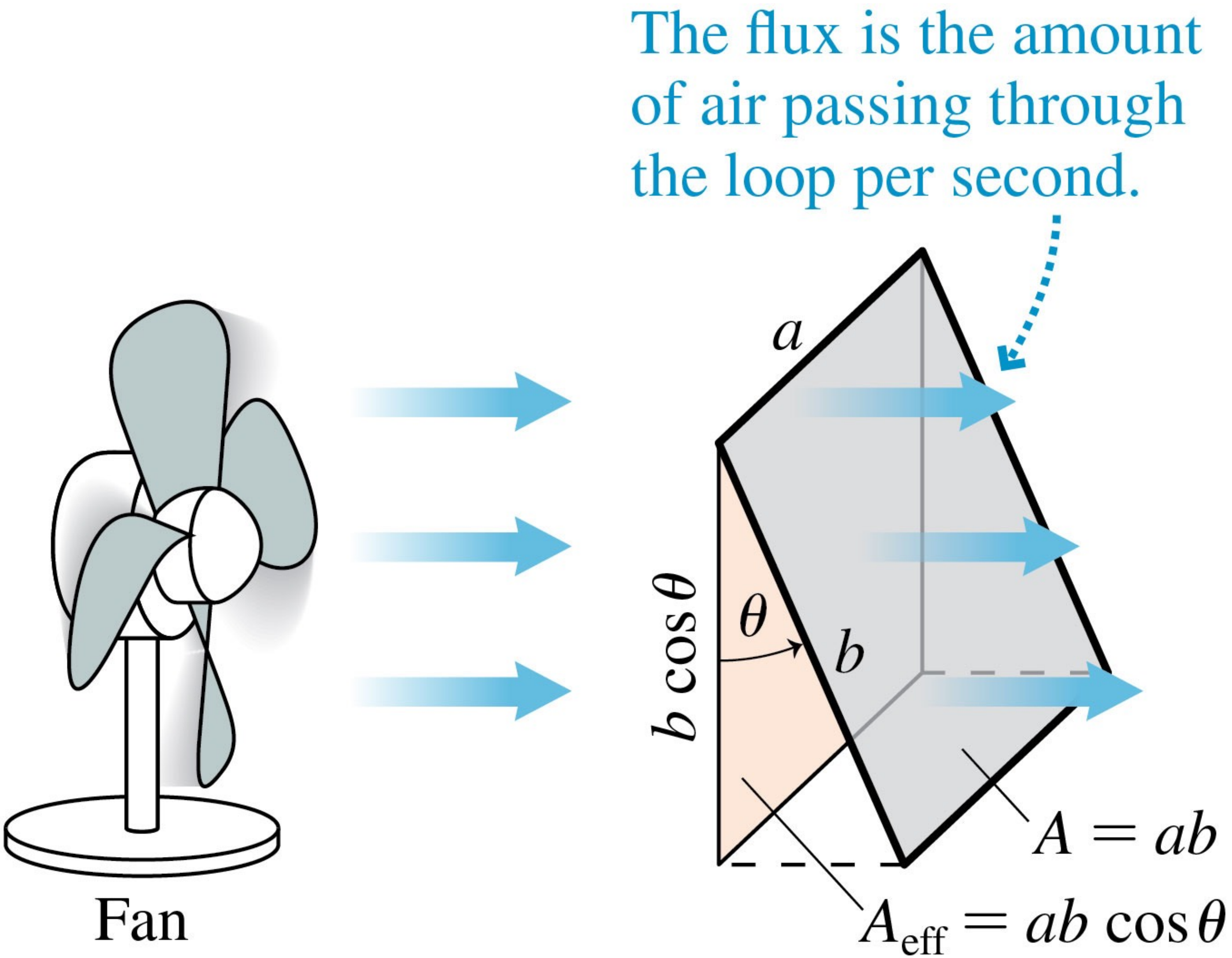
iClicker question #16-2

An induced current flows clockwise as the metal bar is pushed to the right. The magnetic field points

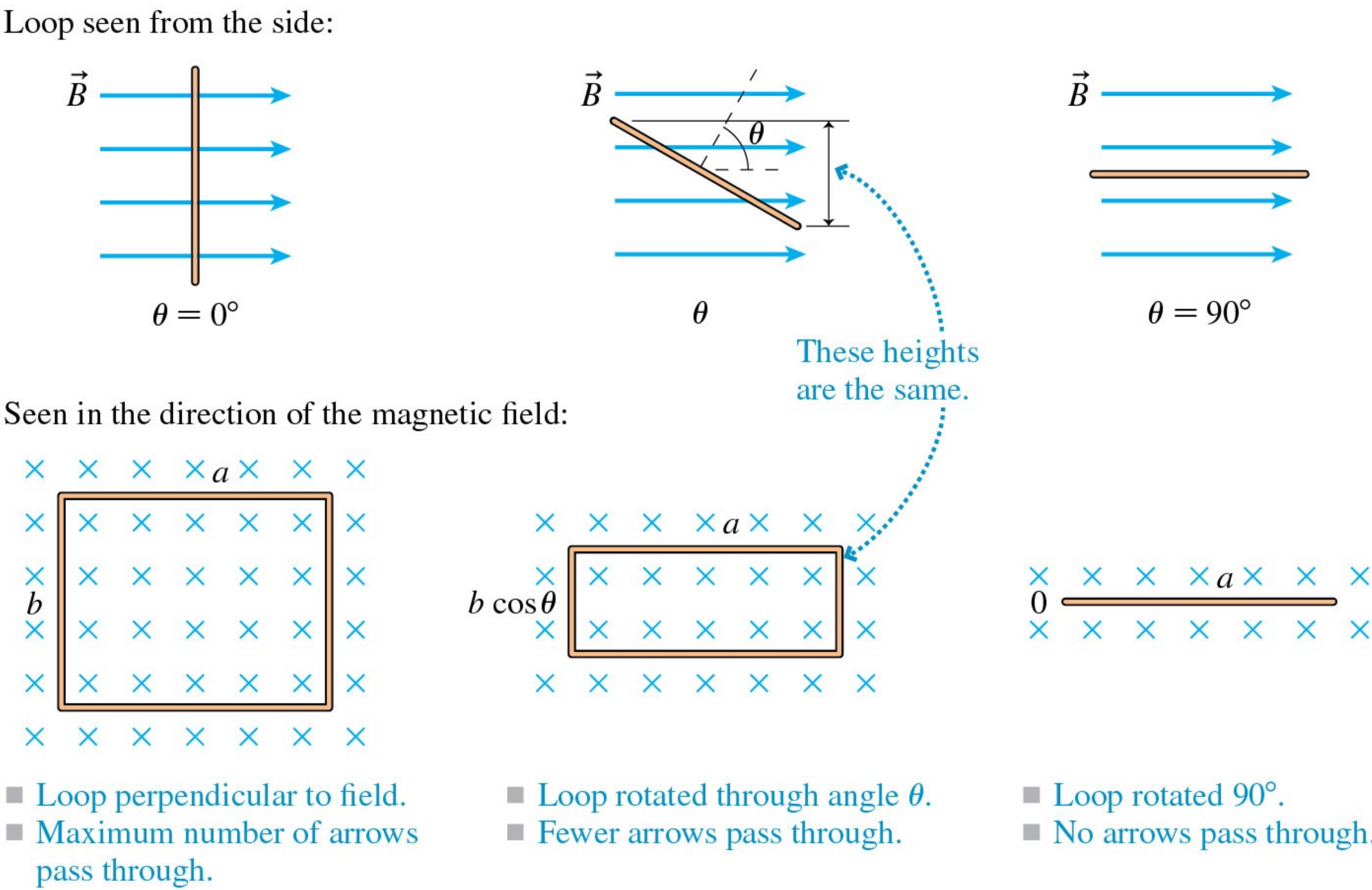


A. Up.
B. Down.
C. Into the screen.
D. Out of the screen.
E. To the right.

Review: The Basic Definition of Flux

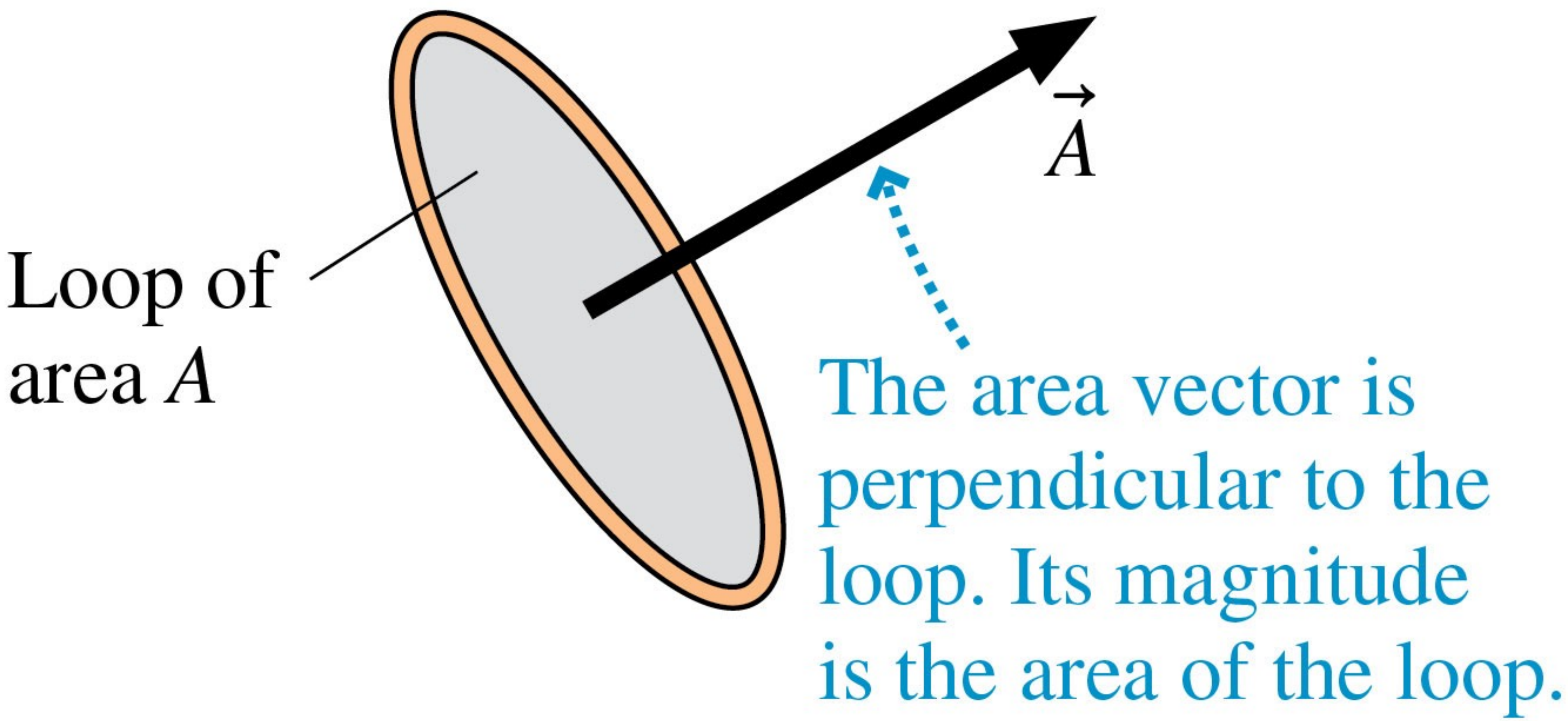


Magnetic Flux Through a Loop

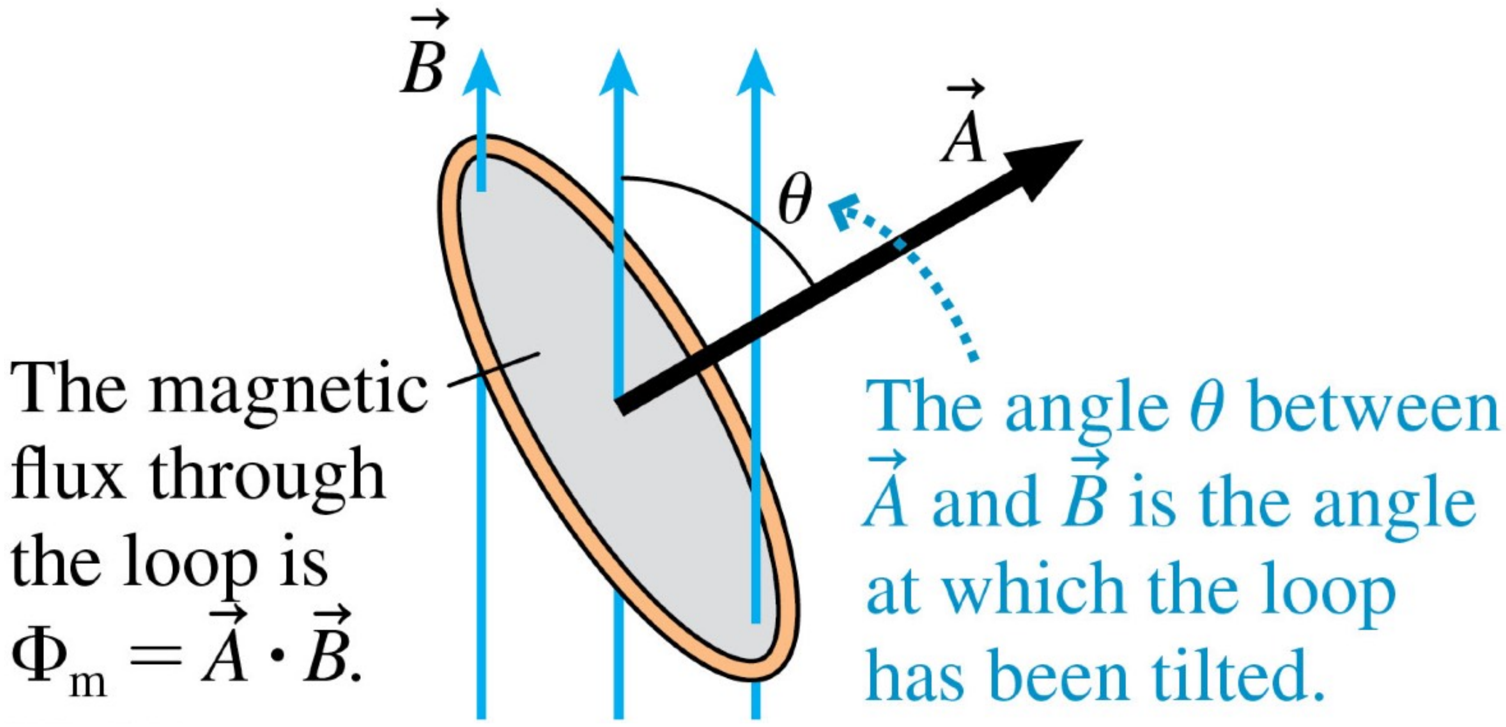


The Area Vector (as used for magn. dipole moment)

- Let's define an area vector $\vec{A} = A\hat{n}$ to be a vector in the direction of, perpendicular to the surface, with a magnitude A equal to the area of the surface.
- Vector $\vec{A}$ has units of m^2.



Magnetic Flux



- The magnetic flux measures the amount of magnetic field passing through a loop of area A if the loop is tilted at an angle θ from the field.

$$\Phi_m = A_{\text{eff}} B = AB\cos\theta$$

- The SI unit of magnetic flux is the **weber**:

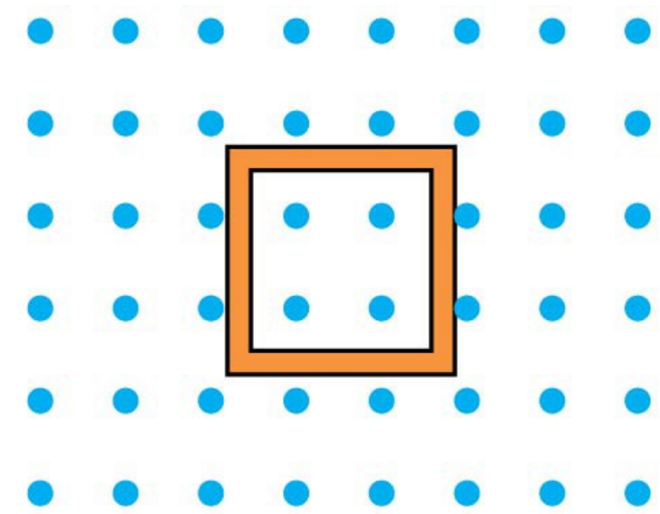
$$1 \text{ weber} = 1 \text{ Wb} = 1 \text{ T m}^2$$

iClicker question #16-3

Which loop has the larger magnetic flux through it?

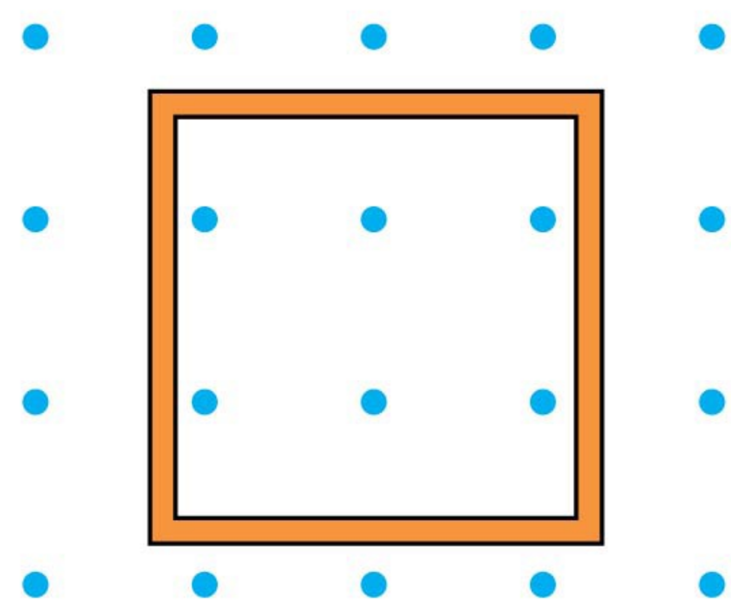
A. Loop A
B. Loop B
C. The fluxes are the same.
D. Not enough information to tell.

A.



This field is twice as strong.

B.



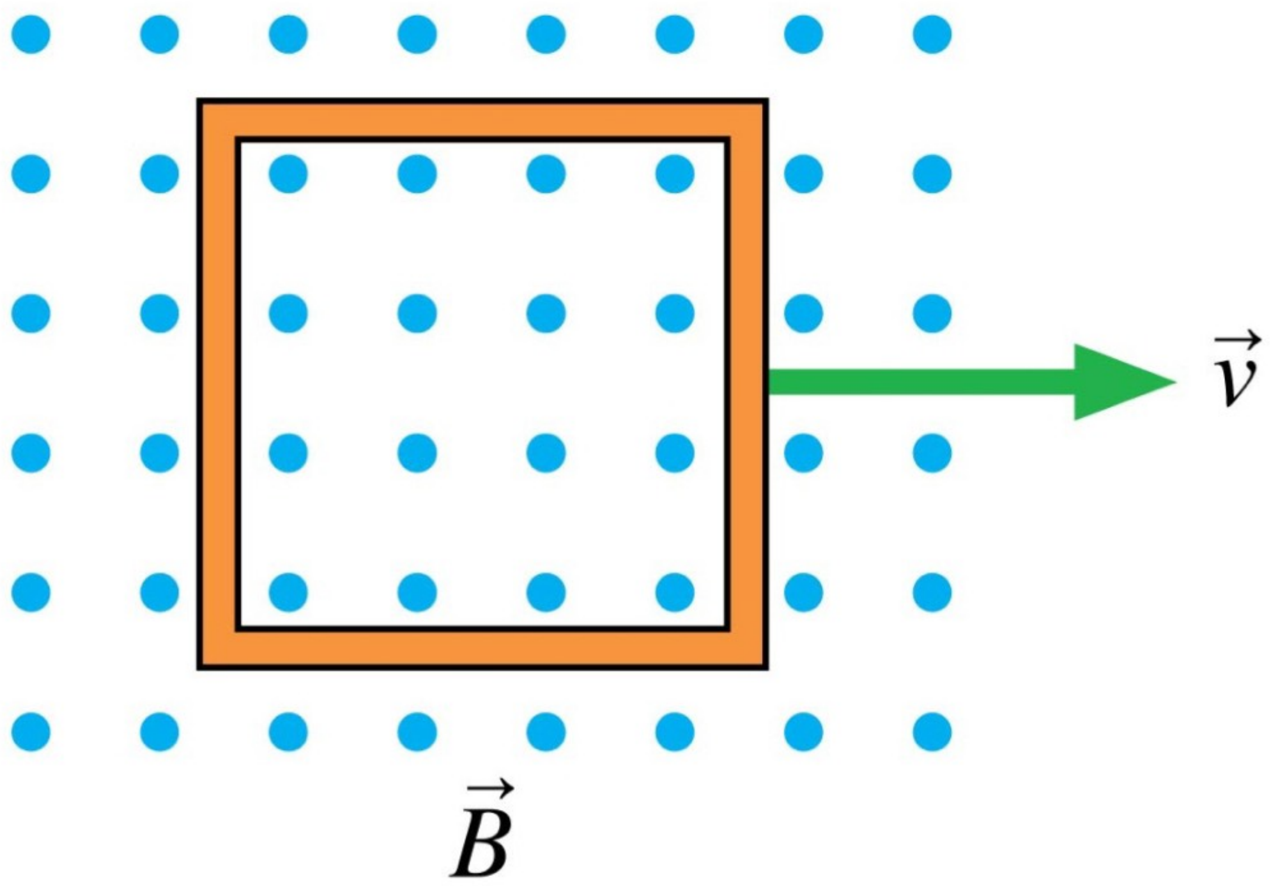
This square is twice as wide.

Example

The metal loop is being pulled through a uniform magnetic field. Is the magnetic flux through the loop changing?

A. Yes

B. No

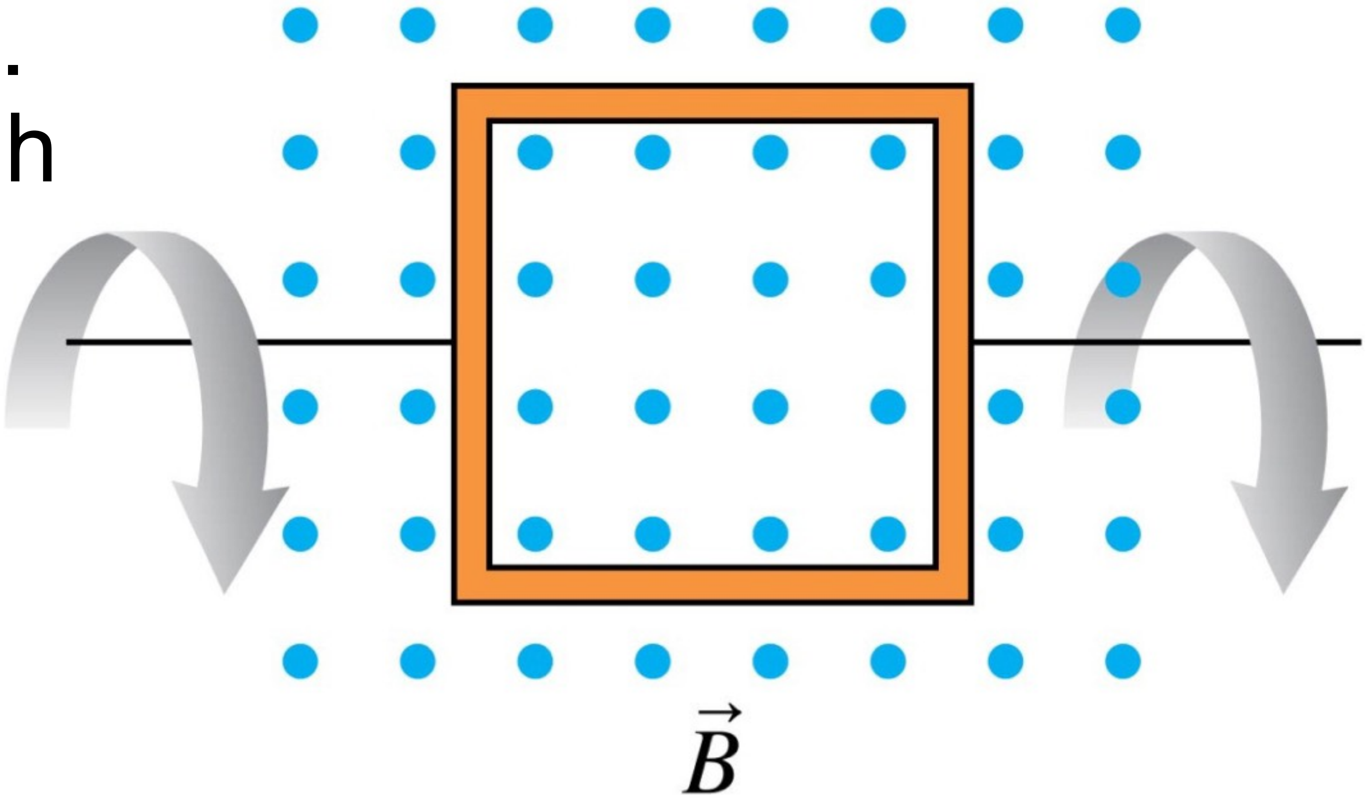


Example

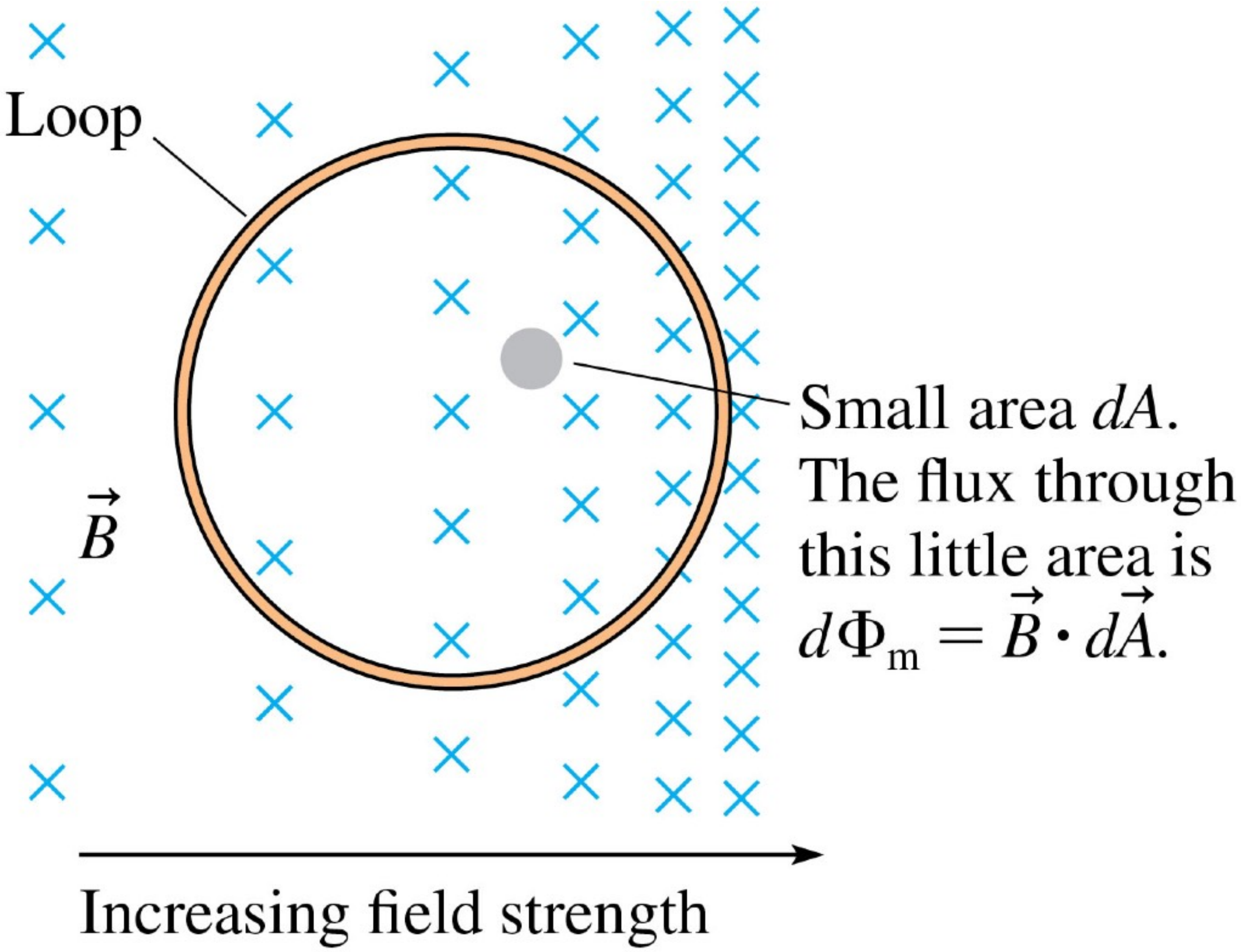
The metal loop is rotating in a uniform magnetic field. Is the magnetic flux through the loop changing?

A. Yes

B. No



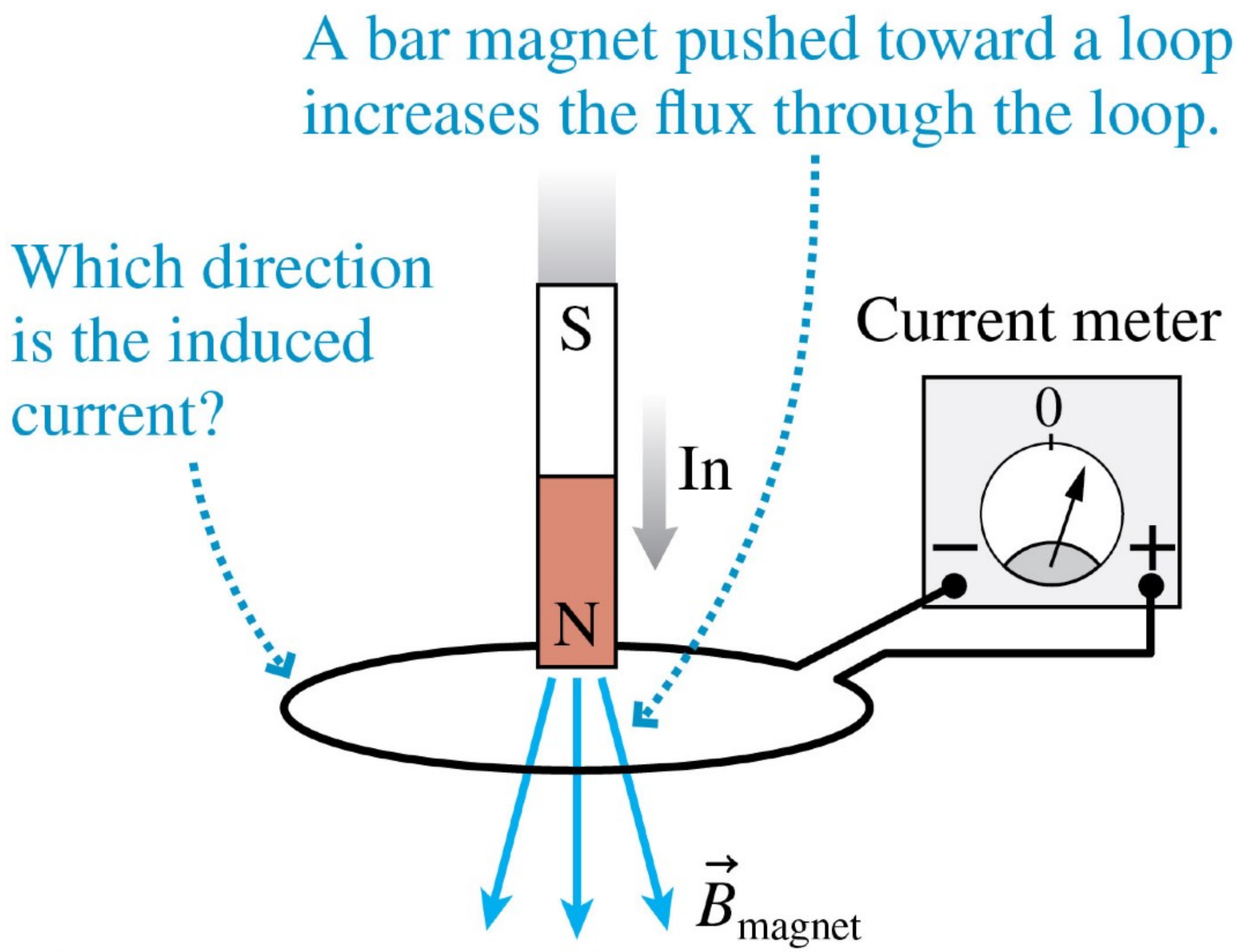
Magnetic Flux in a Nonuniform Field



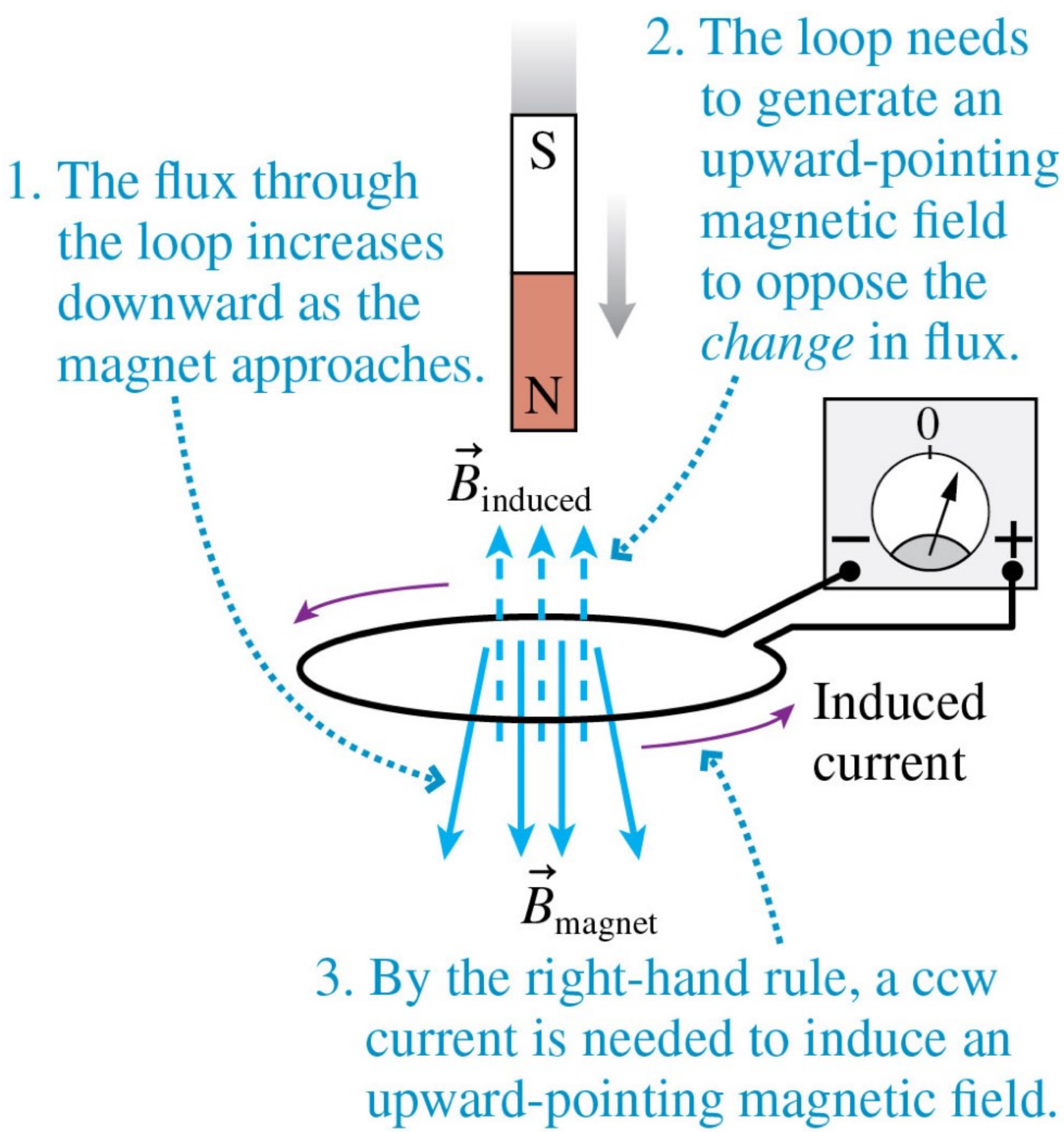
- The figure shows a loop in a nonuniform magnetic field.
- The total magnetic flux through the loop is found with an *area integral*:

$$\Phi_m = \int_{\text{area of loop}} \vec{B} \cdot d\vec{A}$$

Lenz's Law



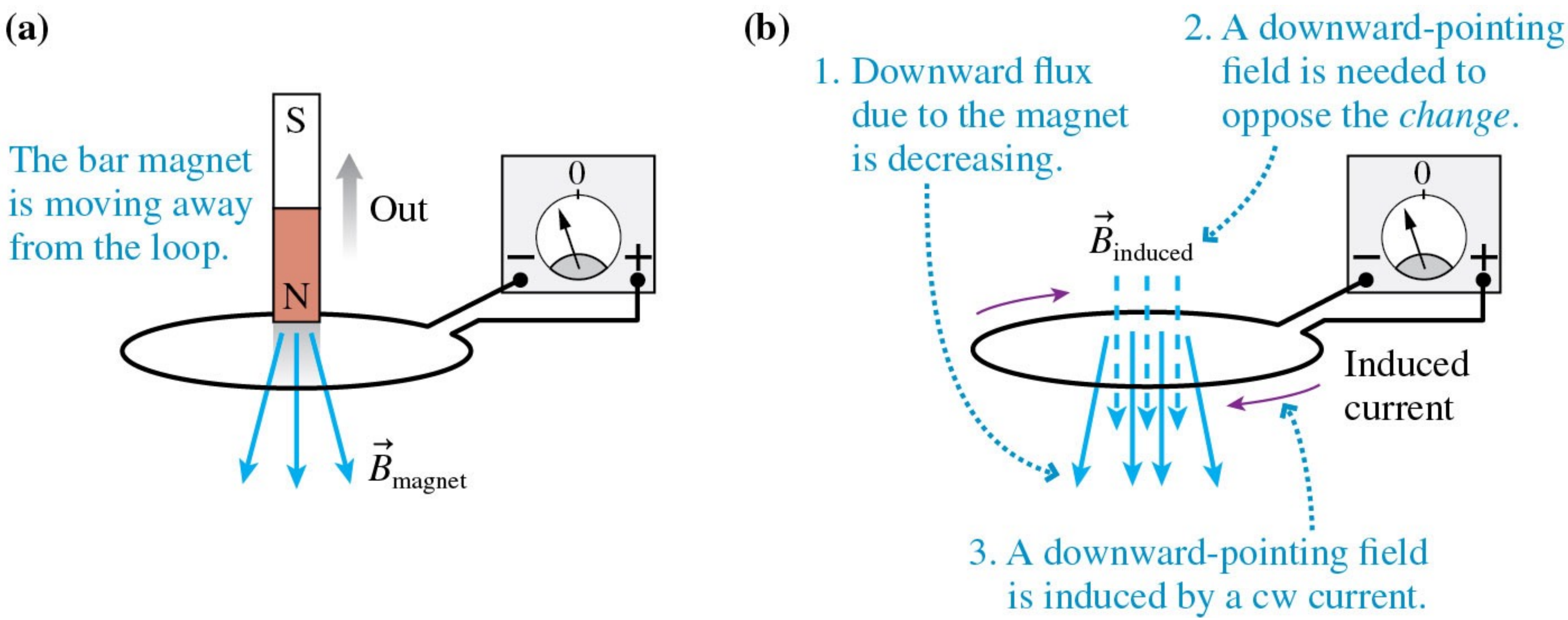
Lenz's law There is an induced current in a closed, conducting loop if and only if the magnetic flux through the loop is changing. The direction of the induced current is such that the induced magnetic field opposes the *change* in the flux.



- Pushing the bar magnet into the loop causes the magnetic flux to *increase* in the downward direction.
- To oppose the *change* in flux, which is what Lenz's law requires, the loop itself needs to generate an *upward*-pointing magnetic field.
- The induced current ceases as soon as the magnet stops moving.

Lenz’s Law

- Pushing the bar magnet away from the loop causes the magnetic flux to *decrease* in the downward direction.
- To *oppose this decrease*, a clockwise current is induced.



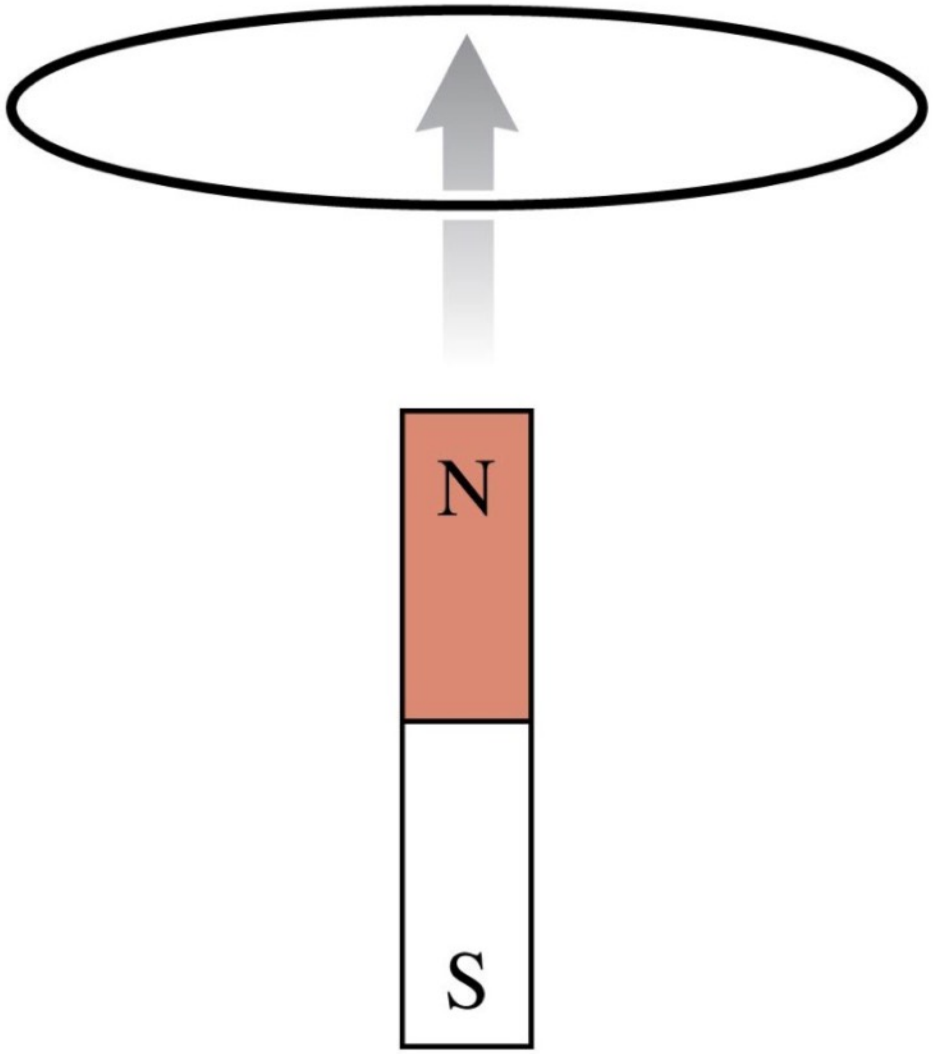
iClicker question #16-4

The bar magnet is pushed toward the center of a wire loop. Which one is true? (Top view)

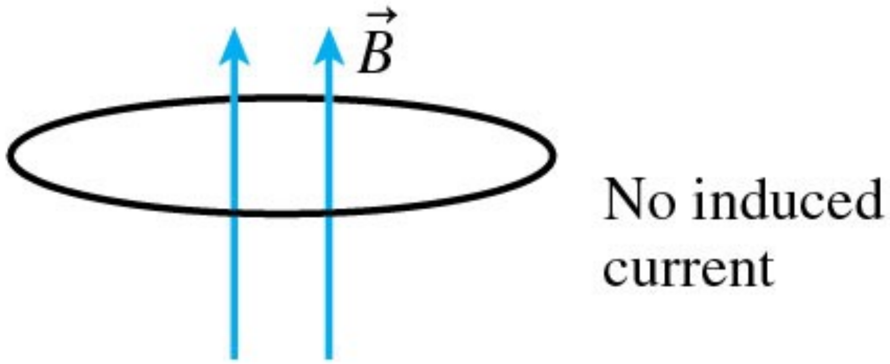
A. There is a clockwise induced current in the loop.

B. There is a counterclockwise induced current in the loop.

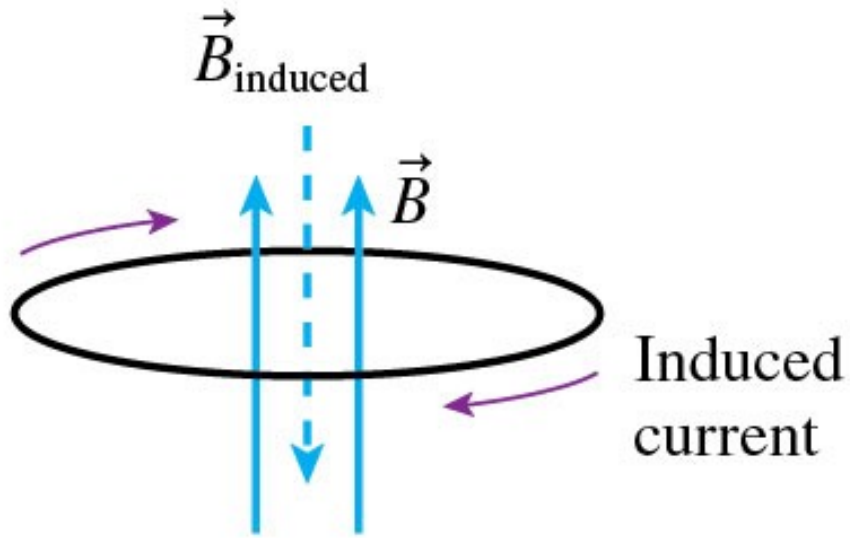
C. There is no induced current in the loop.



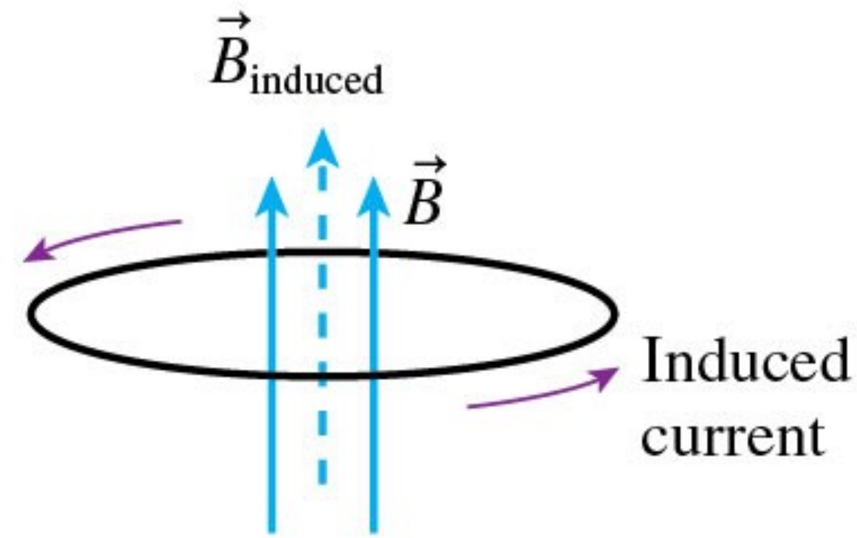
The Induced Current for Six Different Situations:



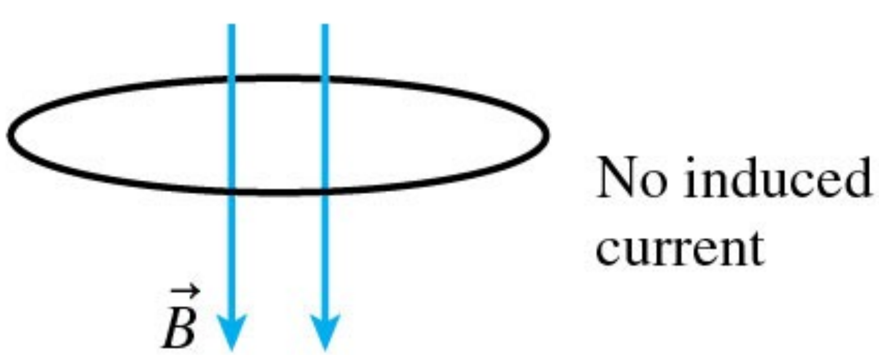
$\vec{B}$ up and steady
- No change in flux
- No induced field
- No induced current



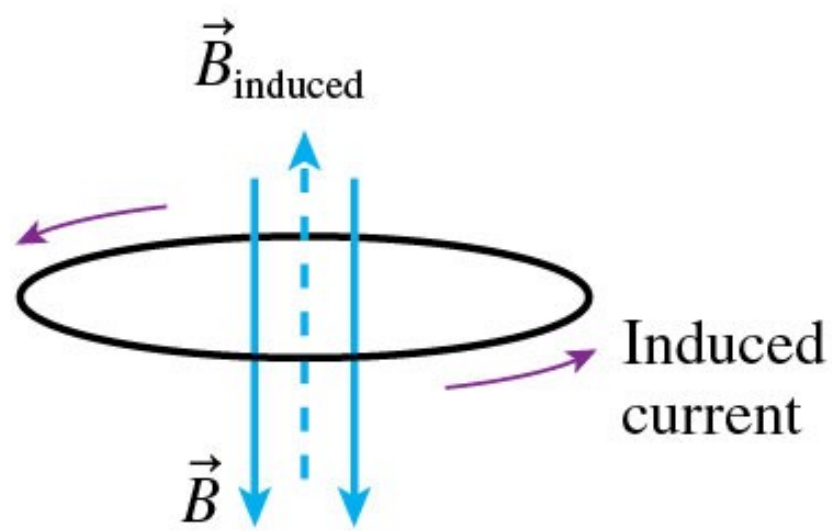
$\vec{B}$ up and increasing
- Change in flux ↑
- Induced field ↓
- Induced current cw



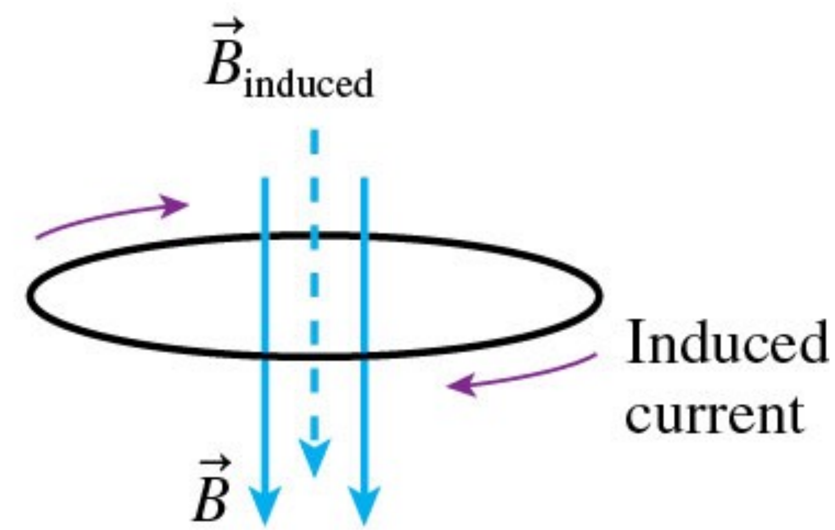
$\vec{B}$ up and decreasing
- Change in flux ↓
- Induced field ↑
- Induced current ccw



$\vec{B}$ down and steady
- No change in flux
- No induced field
- No induced current



$\vec{B}$ down and increasing
- Change in flux ↓
- Induced field ↑
- Induced current ccw



$\vec{B}$ down and decreasing
- Change in flux ↑
- Induced field ↓
- Induced current cw