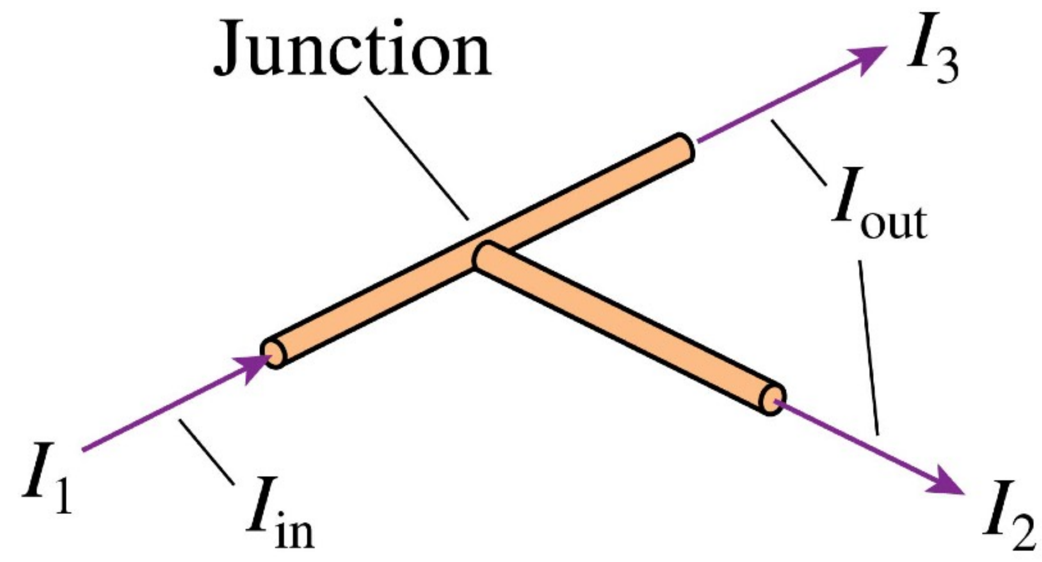
Kirchhoff's laws

- **Kirchhoff's junction law.**

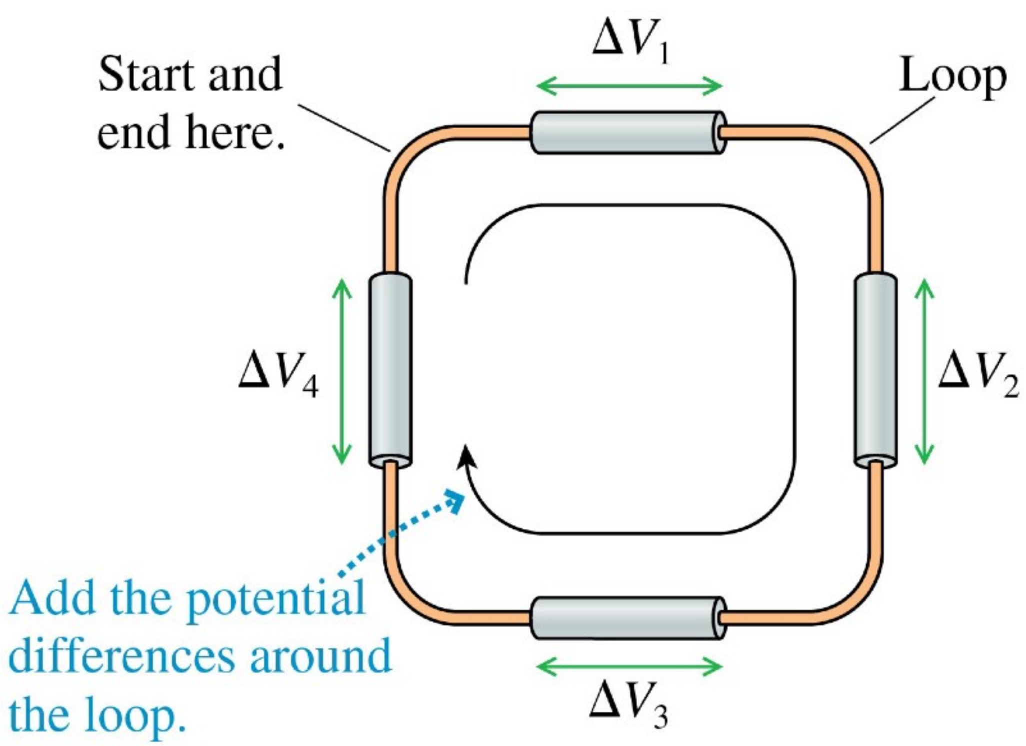
$$\sum I_{\text{in}} = \sum I_{\text{out}}$$



Junction law: $I_1 = I_2 + I_3$

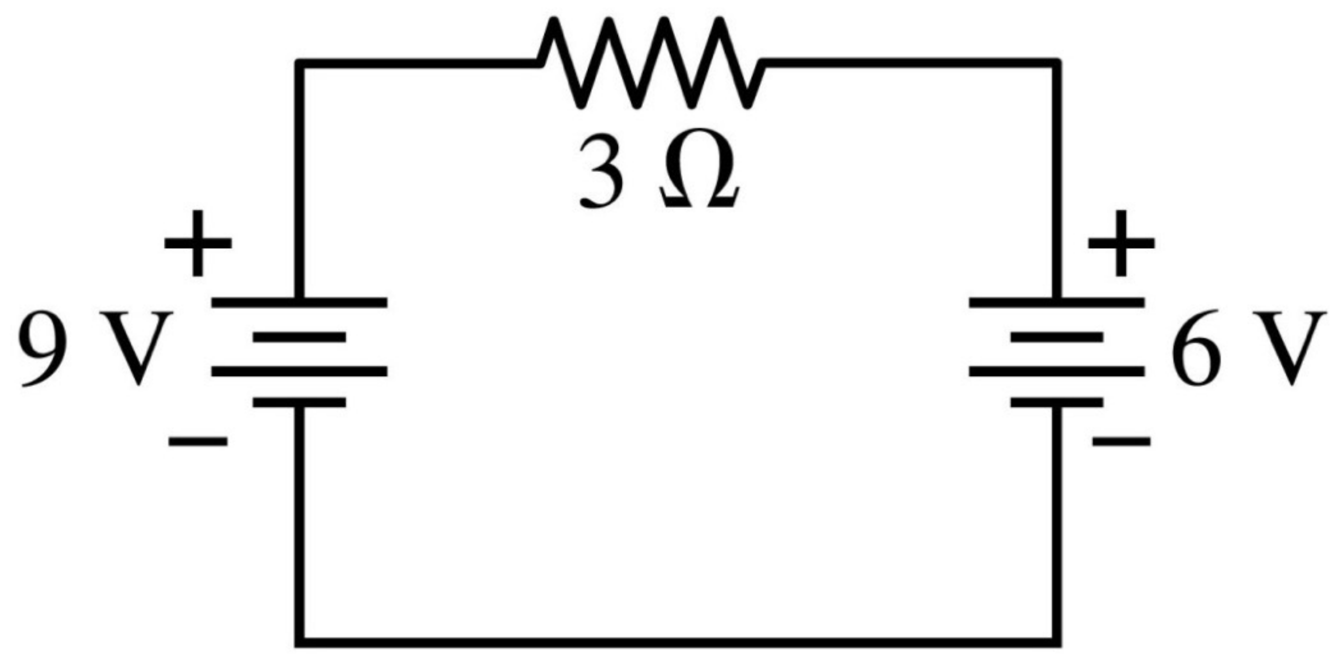
- **Kirchhoff's loop law.**

$$\Delta V_{\text{loop}} = \sum (\Delta V)_i = 0$$



Loop law: $\Delta V_1 + \Delta V_2 + \Delta V_3 + \Delta V_4 = 0$

What is the current through the $3\ \Omega$ resistor?



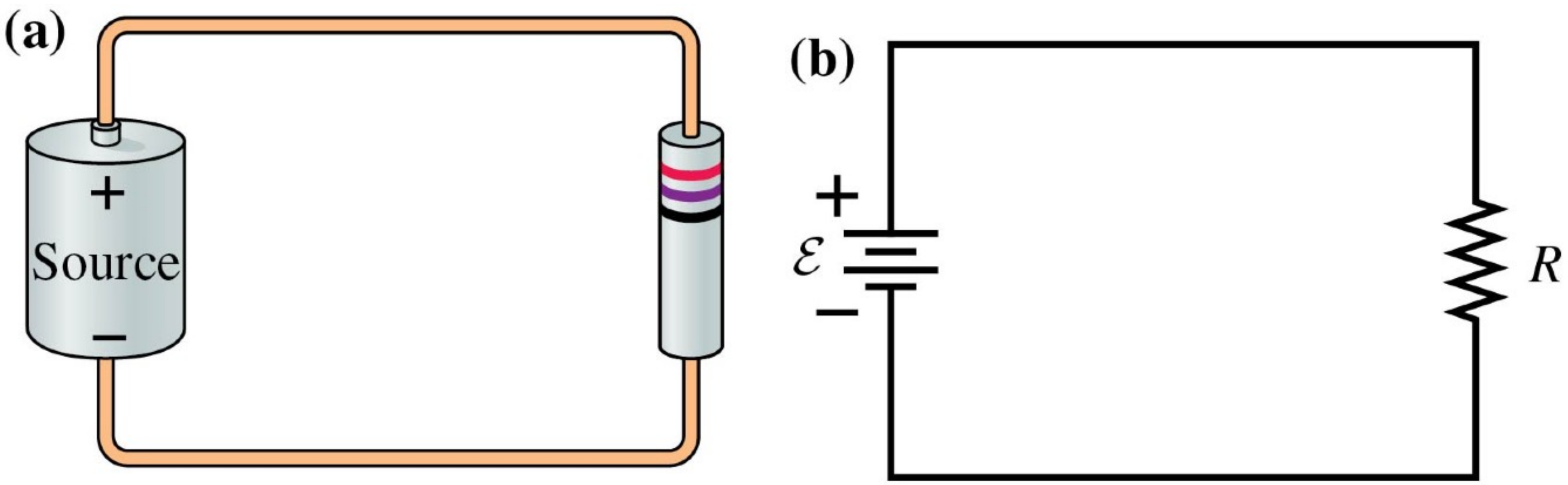
Tactics: Using Kirchhoff's Loop Law, simple circuit

TACTICS BOX 28.1

Using Kirchhoff's loop law

1 Draw a circuit diagram. Label all known and unknown quantities.

- For example:
  The most basic electric circuit is a single resistor connected to the two terminals of a battery.



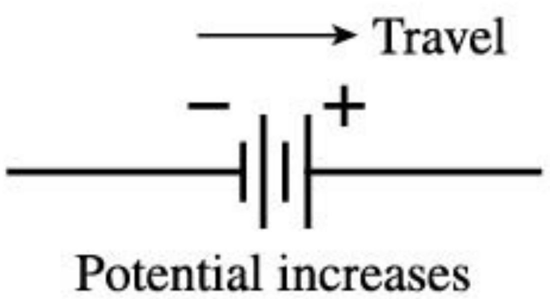
TACTICS BOX 28.1

Using Kirchhoff’s loop law

❸ **“Travel” around the loop.** Start at any point in the circuit, then go all the way around the loop in the direction you assigned to the current in step 2. As you go through each circuit element, ΔV is interpreted to mean $\Delta V = V_{\text{downstream}} - V_{\text{upstream}}$.

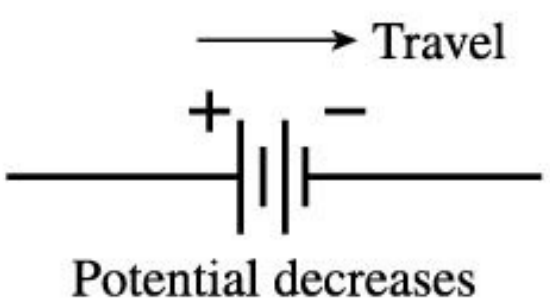
- For an ideal battery in the negative-to-positive direction:

$$\Delta V_{\text{bat}} = +\mathcal{E}$$

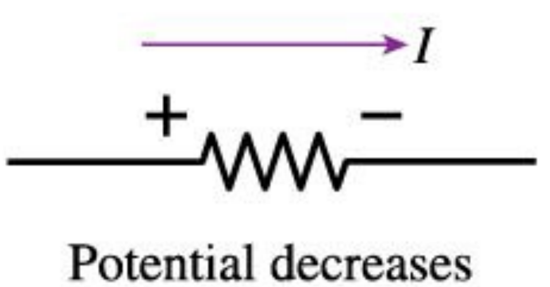


- For an ideal battery in the positive-to-negative direction:

$$\Delta V_{\text{bat}} = -\mathcal{E}$$



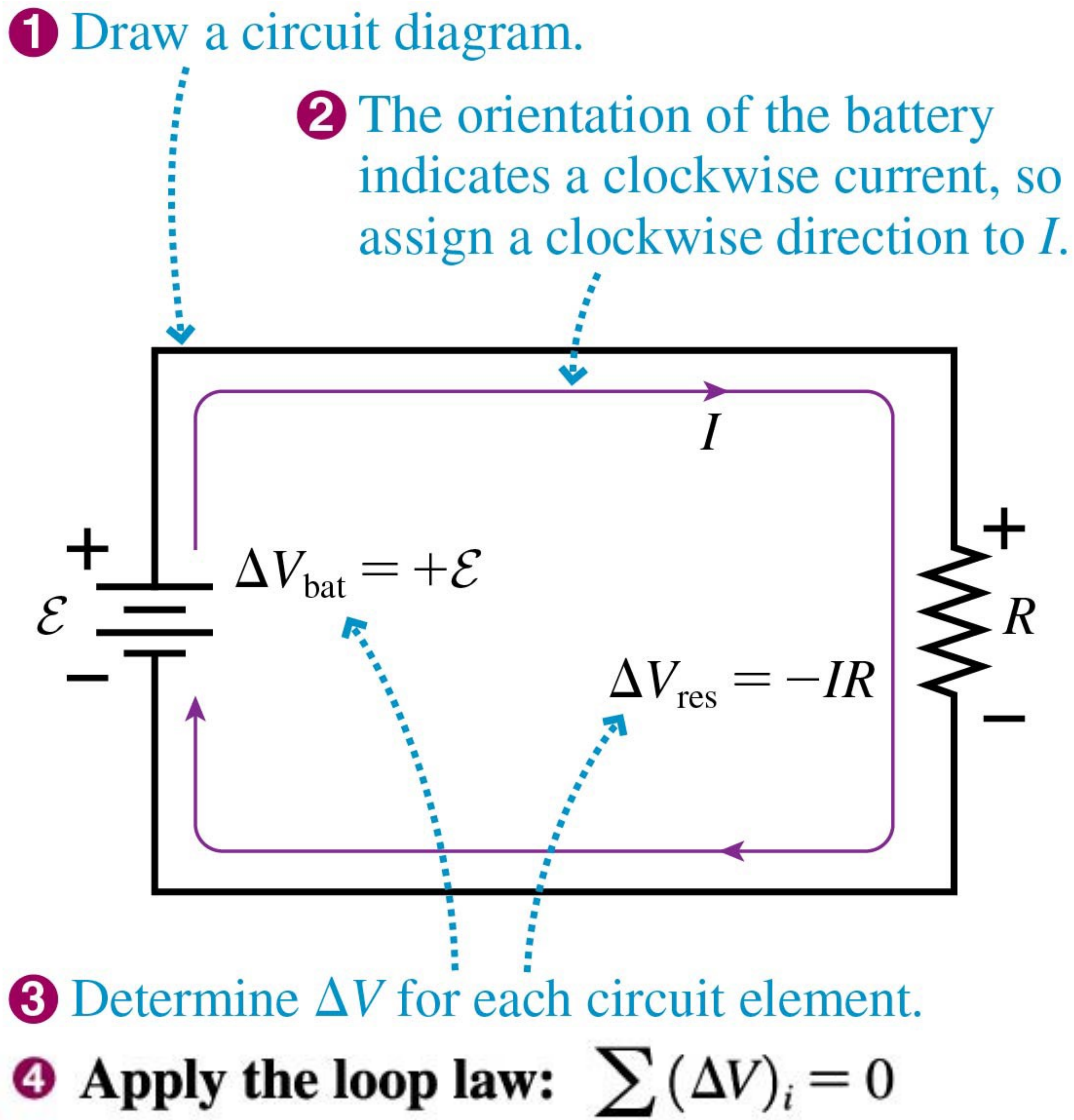
- For a resistor: $\Delta V_{\text{res}} = -\Delta V_{\text{R}} = -IR$



❹ **Apply the loop law:** $\sum (\Delta V)_i = 0$

Exercises 4–7

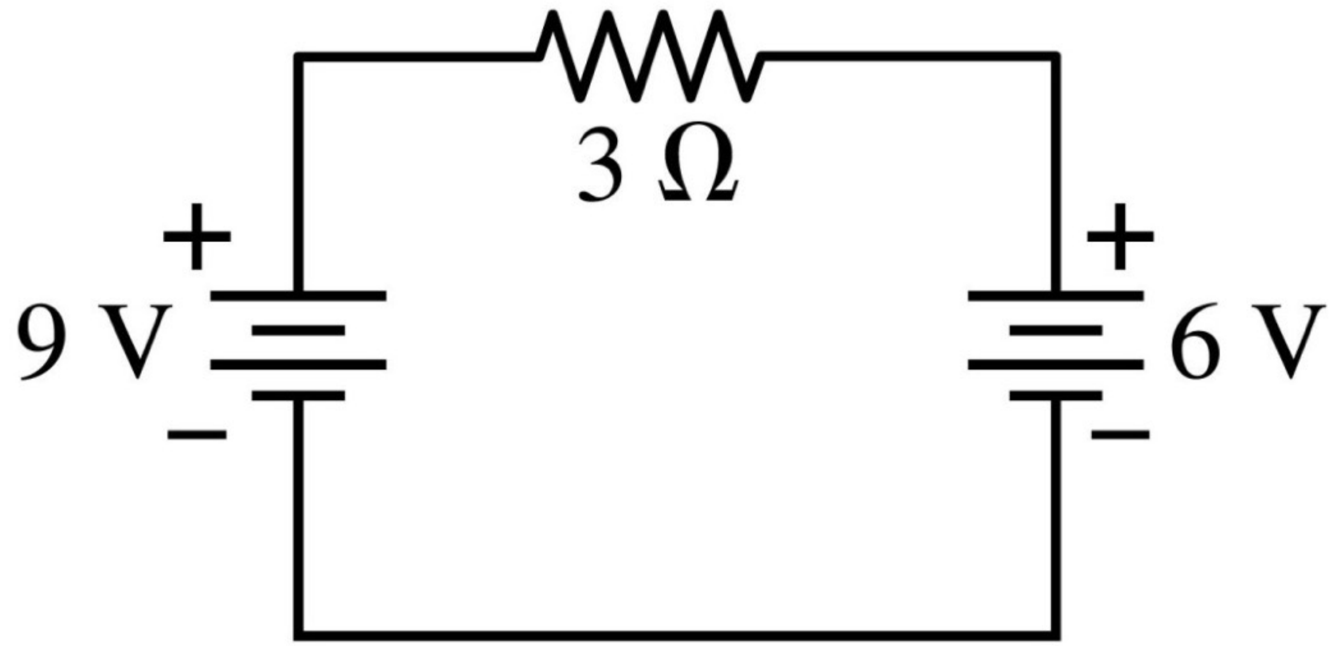
Analyzing the Basic Circuit



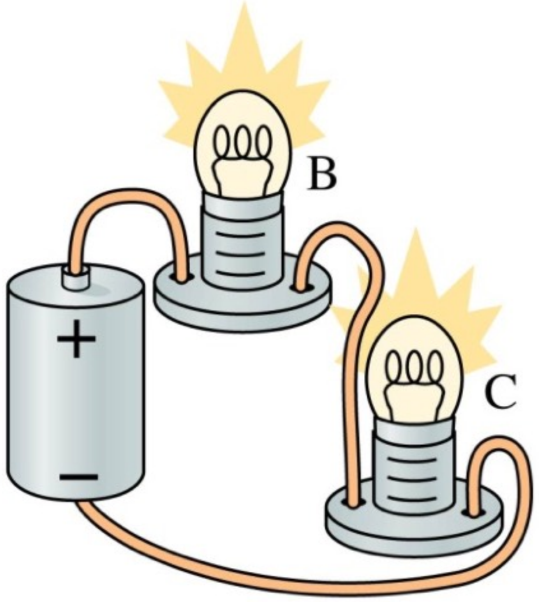
iClicker question 11-4

The current through the $3\ \Omega$ resistor is

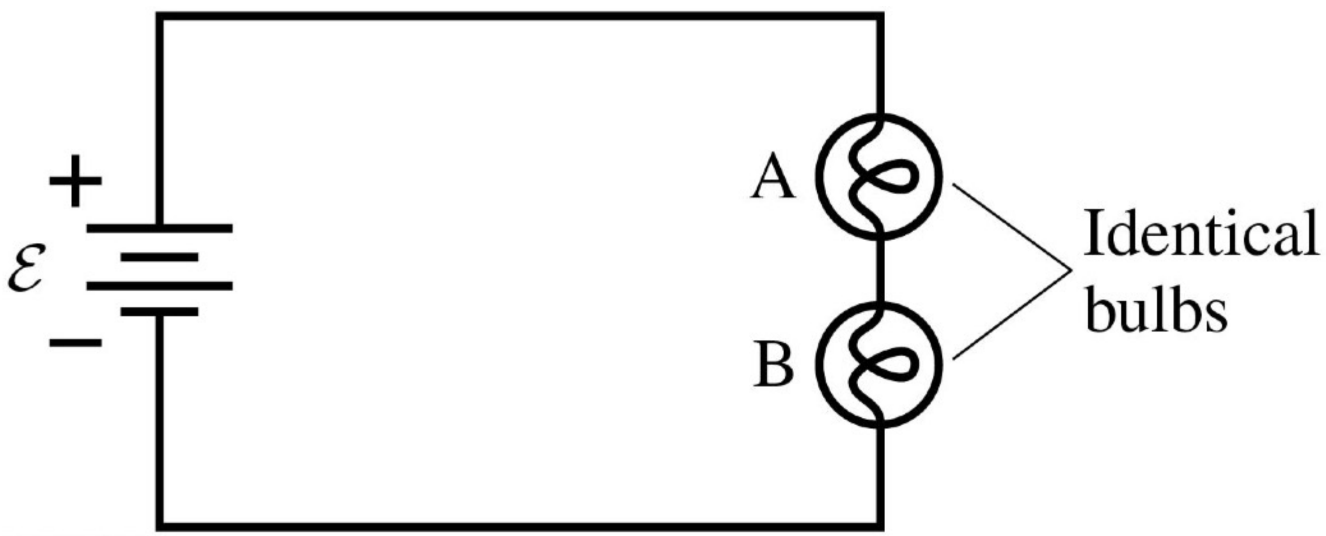
A. 9 A
B. 6 A
C. 5 A
D. 3 A
E. 1 A



Lightbulb Puzzle #1



- The figure shows two identical lightbulbs in a circuit.
- The current through both bulbs is *exactly the same*!
- It's not the current that the bulbs use up, it's *energy*.



- The battery creates a potential difference, which supplies potential energy to the charges.
- As the charges move through the lightbulbs, they lose some of their potential energy, transferring the energy to the bulbs.

Power

- Recall from mechanics that power was a measure of the work done in a given amount of time:

$$\text{Avg. Power} = \mathcal{P}_{avg} = \frac{Work}{time} = \frac{W}{\Delta t}$$

- The SI unit of power is still the Watt (Joule/sec).
- The unit of energy used by the electric company is the kiloWatt-hour.
- Where 1kWh = 3.60×10^6J.
- Since work is also how much your potential energy has changed (assumin[g] KE is constant), then:

$$\mathcal{P} = \frac{\Delta PE}{\Delta t}$$

In summary: Energy and Power

- The power supplied by a battery is

$$P_{\text{bat}} = I\mathcal{E} \qquad \text{(power delivered by an emf)}$$

- The power dissipated by a resistor is

$$P_{\text{R}} = \frac{dE_{\text{th}}}{dt} = \frac{dq}{dt}\Delta V_{\text{R}} = I\,\Delta V_{\text{R}}$$

- Or, in terms of the potential drop across the resistor,

$$P_{\text{R}} = I\,\Delta V_{\text{R}} = I^2 R = \frac{(\Delta V_{\text{R}})^2}{R} \qquad \text{(power dissipated by a resistor)}$$

This is the power that is transferred by electrical energy.

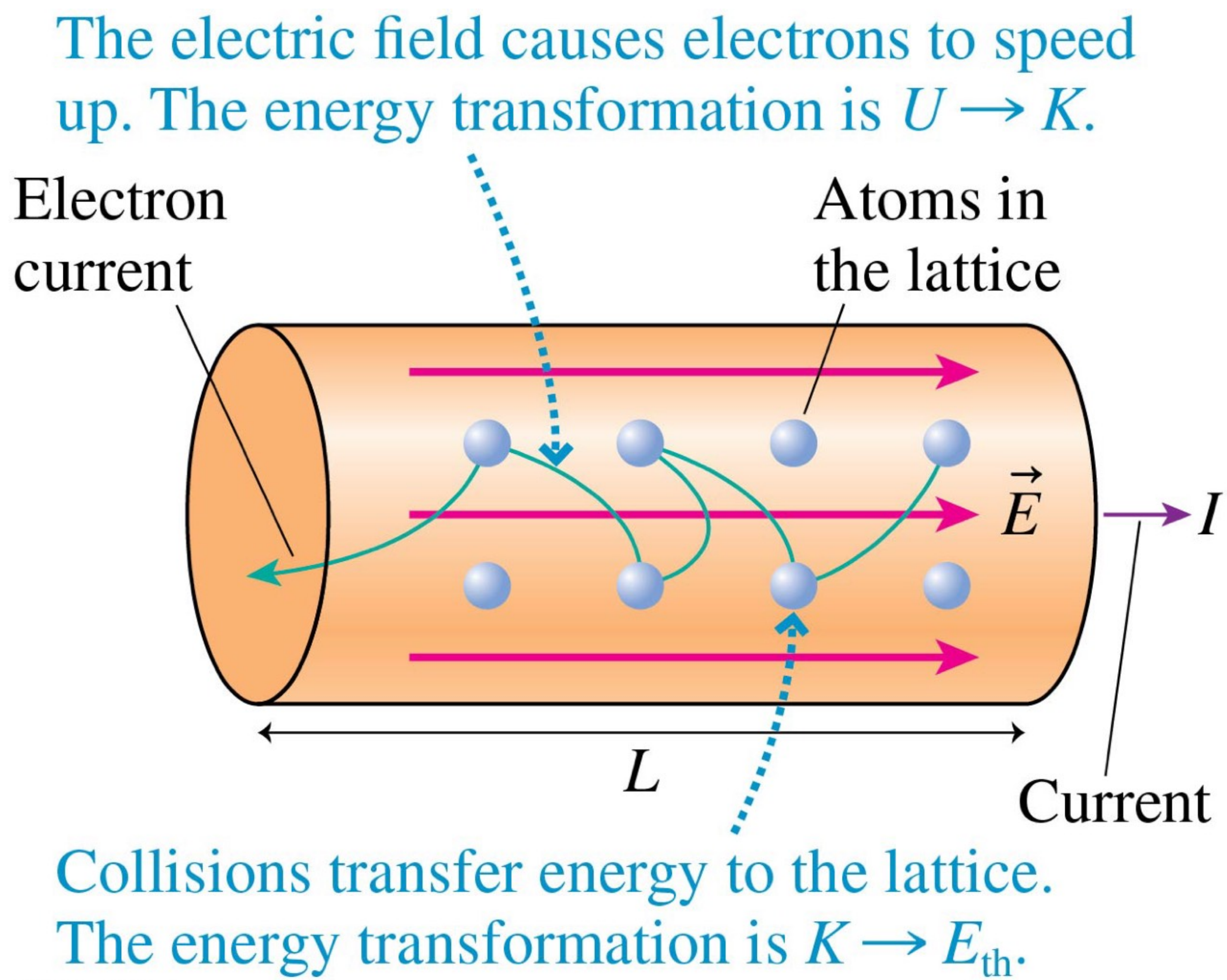
Power dissipation

$$[\mathrm{W}] \quad \boxed{P = I\Delta V} \quad \left[\frac{\cancel{\mathrm{C}}}{\mathrm{s}}\right]\left[\frac{\mathrm{J}}{\cancel{\mathrm{C}}}\right]$$

$$\left(I = \frac{\Delta V}{R}\right) \qquad \left(\Delta V = IR\right)$$

$$\therefore \boxed{P = \frac{(\Delta V)^2}{R}} \qquad \therefore \boxed{P = I^2 R}$$

Power Dissipation in a Resistor



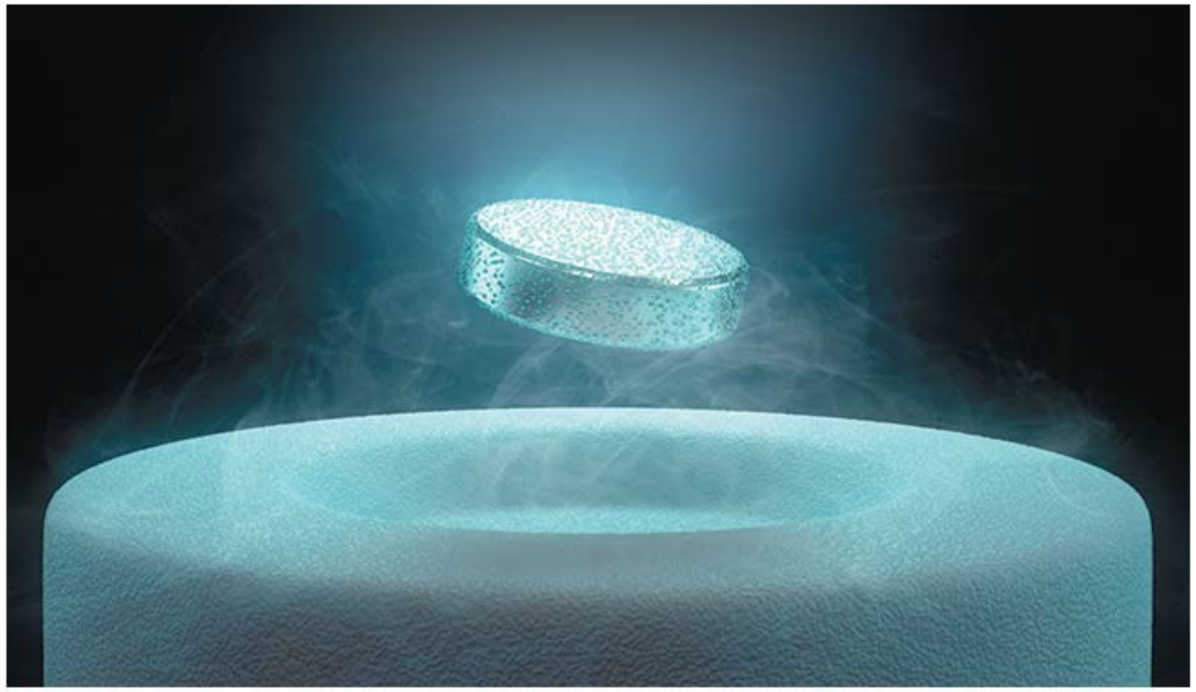
- A current-carrying resistor dissipates power because the electric force does work on the charges.

Kilowatt Hours

- The product of watts and seconds is joules, the SI unit of energy.
- However, most electric companies prefers to use the **kilowatt hour**, to measure the energy you use each month.
- Examples:
  - A $4000\ \mathrm{W}$ electric water heater uses $40\ \mathrm{kWh}$ of energy in 10 hours.
  - A $1500\ \mathrm{W}$ hair dryer uses $0.25\ \mathrm{kWh}$ of energy in 10 minutes.
- The average cost of electricity in the United States is ≈ 10¢ per kWh (\$0.10/kWh).

Superconductivity

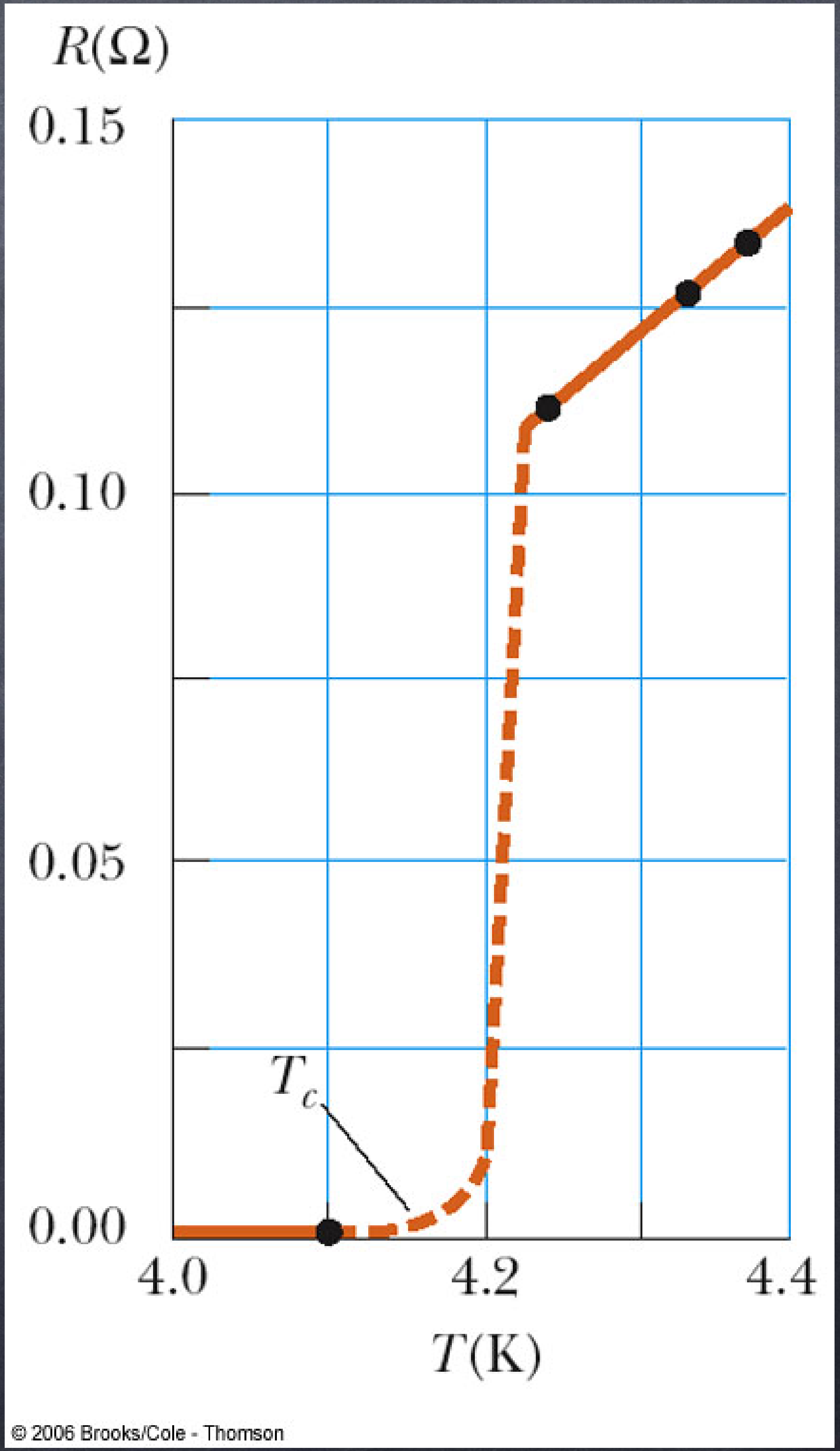
- In 1911, the Dutch physicist Kamerlingh Onnes discovered that certain materials suddenly and dramatically lose *all* resistance to current when cooled below a certain temperature.
- This complete loss of resistance at low temperatures is called **superconductivity**.



Superconductors have unusual magnetic properties. Here a small permanent magnet levitates above a disk of the high temperature superconductor $YBa_2Cu_3O_7$ that has been cooled to liquid-nitrogen temperature.

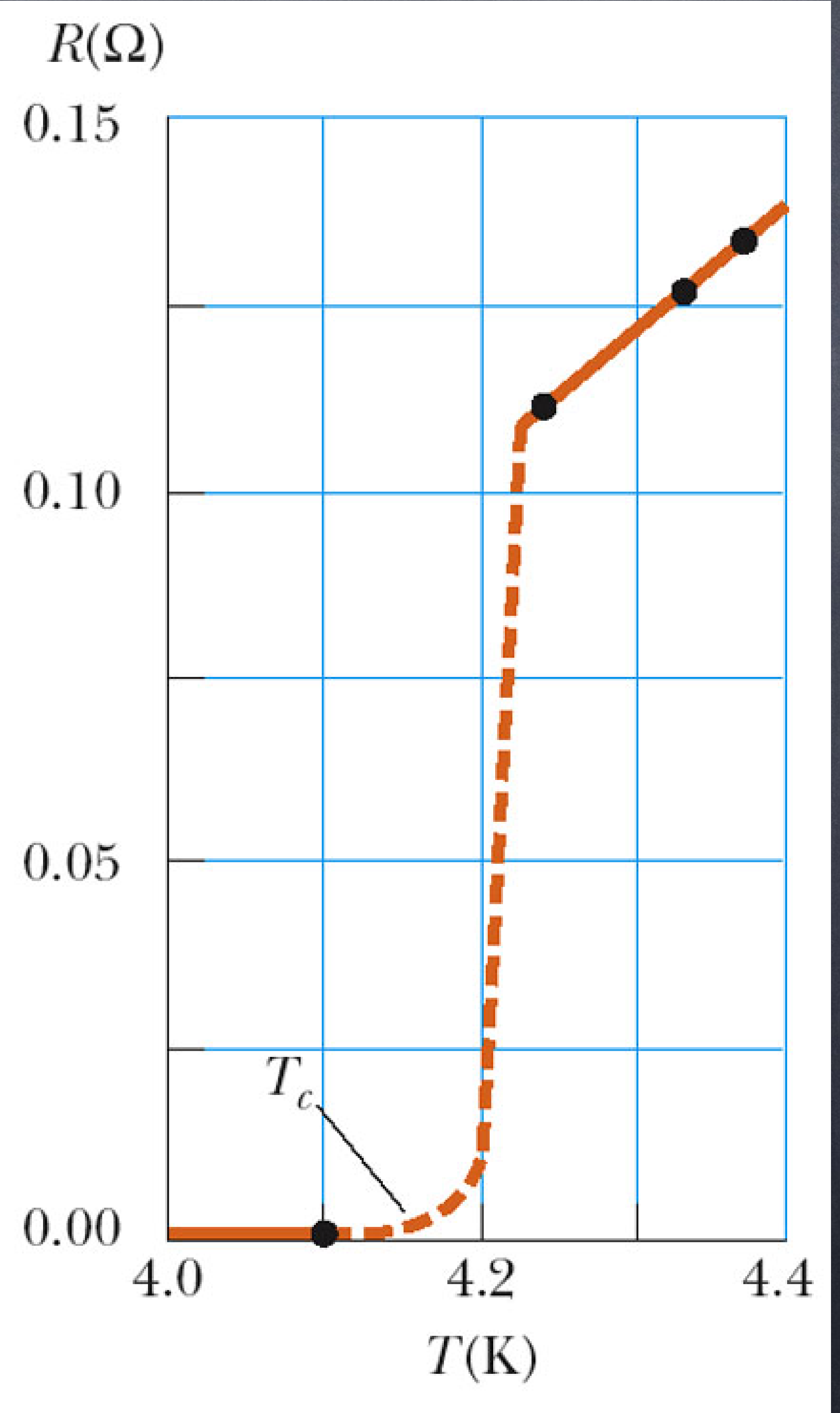
Superconductors

- Supercondutors can help solve power problems.
- Superconductors are materials whose resistance falls to virtually zero below a certain critical temperature, T_C.
  - Once a current is set up in a superconductor, it persists without any applied potential difference.



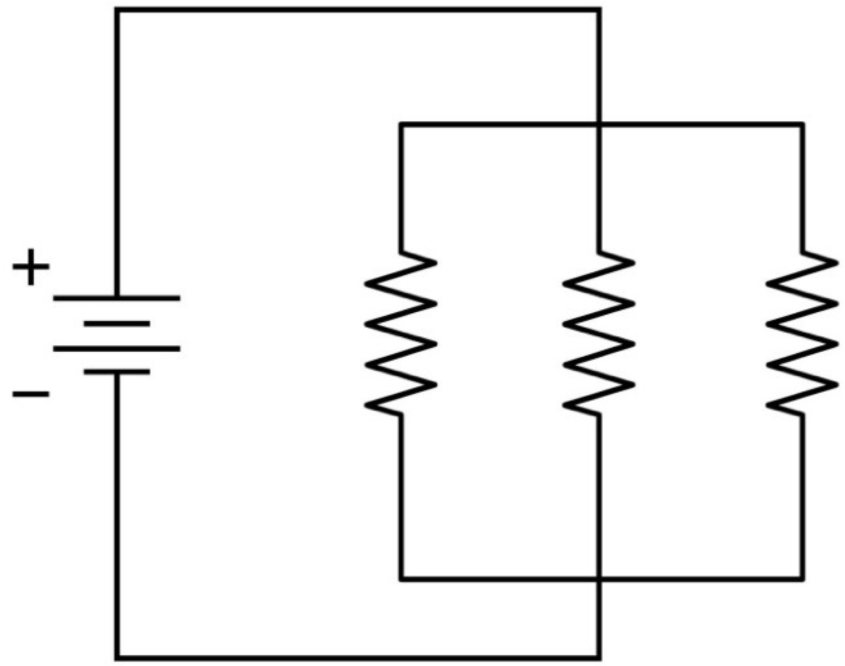
Superconductors

- Above T_c, the superconductor acts as a normal metal.
- Unfortunately superconductors currently only exist at low temperatures (highest ~150K) [−123°C].
- But we are getting closer to room temperature superconductivity. In the mid-80's the highest was near 30K [−243°C].



iClicker question 11-5 (preparation for a power q.)

What things about the resistors in this circuit are the same for all three?

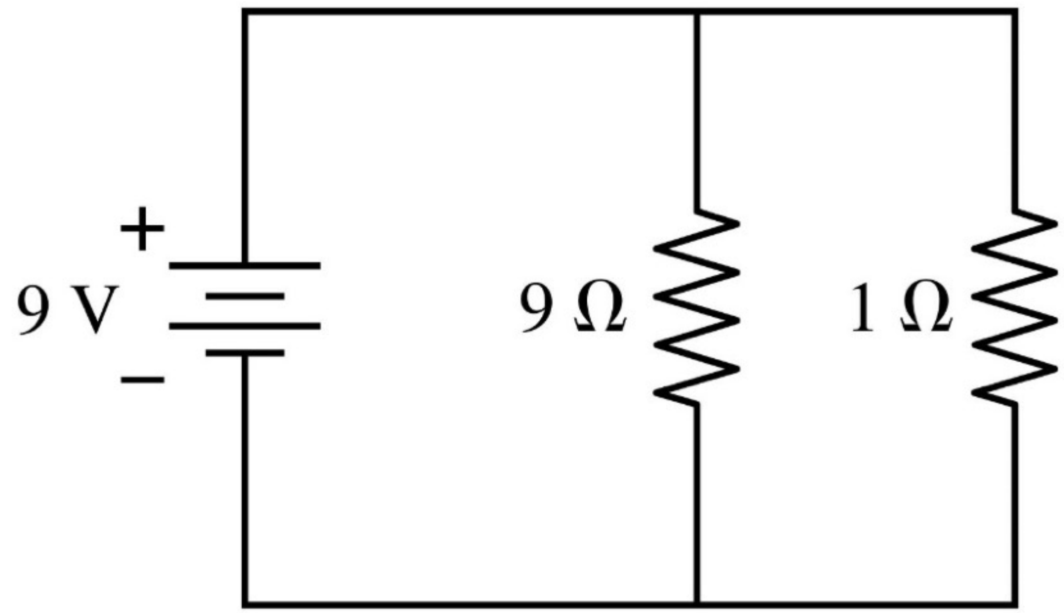


A. Current I
B. Potential difference ΔV
C. Resistance R
D. A and B
E. B and C

iClicker question 11-6

Which resistor dissipates more power?

A. The $9\ \Omega$ resistor
B. The $1\ \Omega$ resistor
C. They dissipate the same power.



$$P_R = I\Delta V_R = I^2R = \frac{(\Delta V_R)^2}{R} \qquad \text{(power dissipated by a resistor)}$$

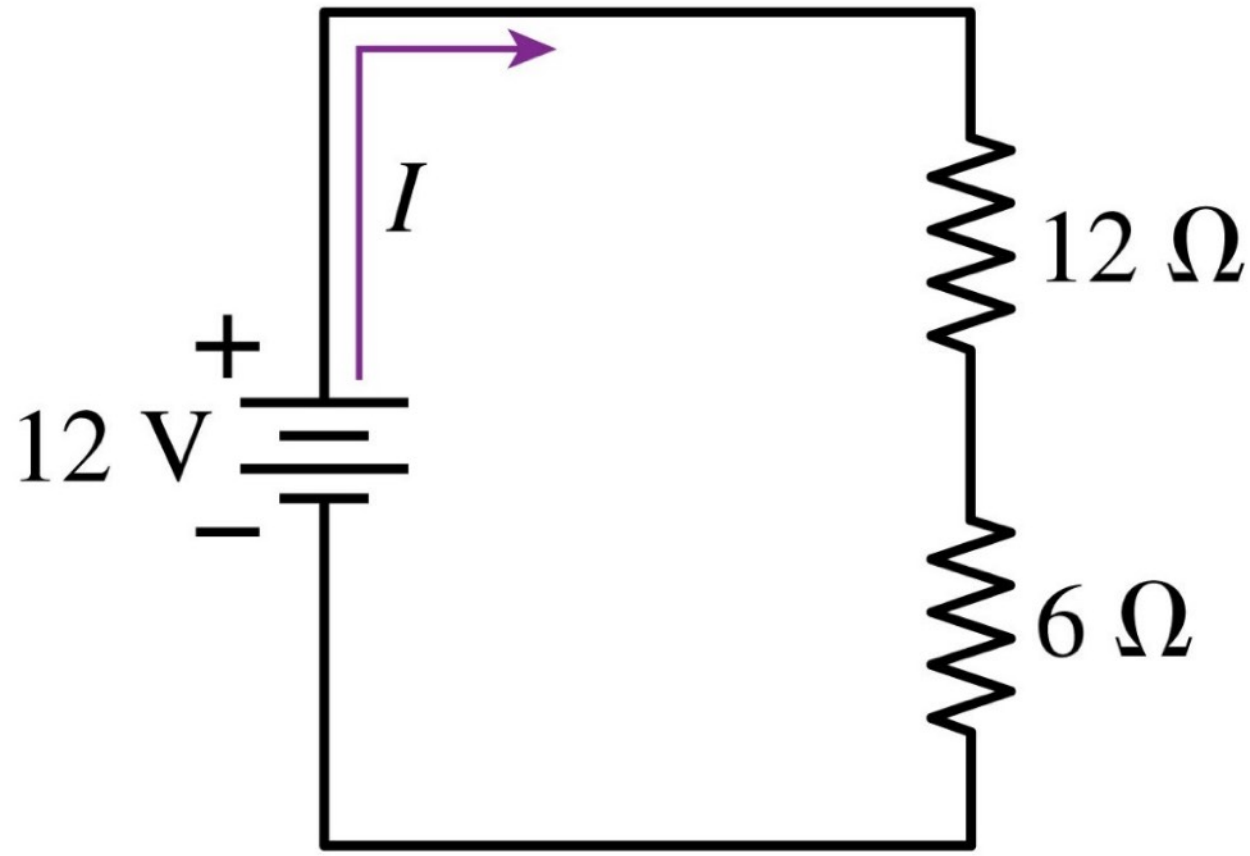
iClicker question 11-7

Which resistor dissipates more power?

A. 12 Ω

B. 6 Ω

C. They dissipate the same power



$$P_R = I\Delta V_R = I^2 R = \frac{(\Delta V_R)^2}{R} \quad \text{(power dissipated by a resistor)}$$

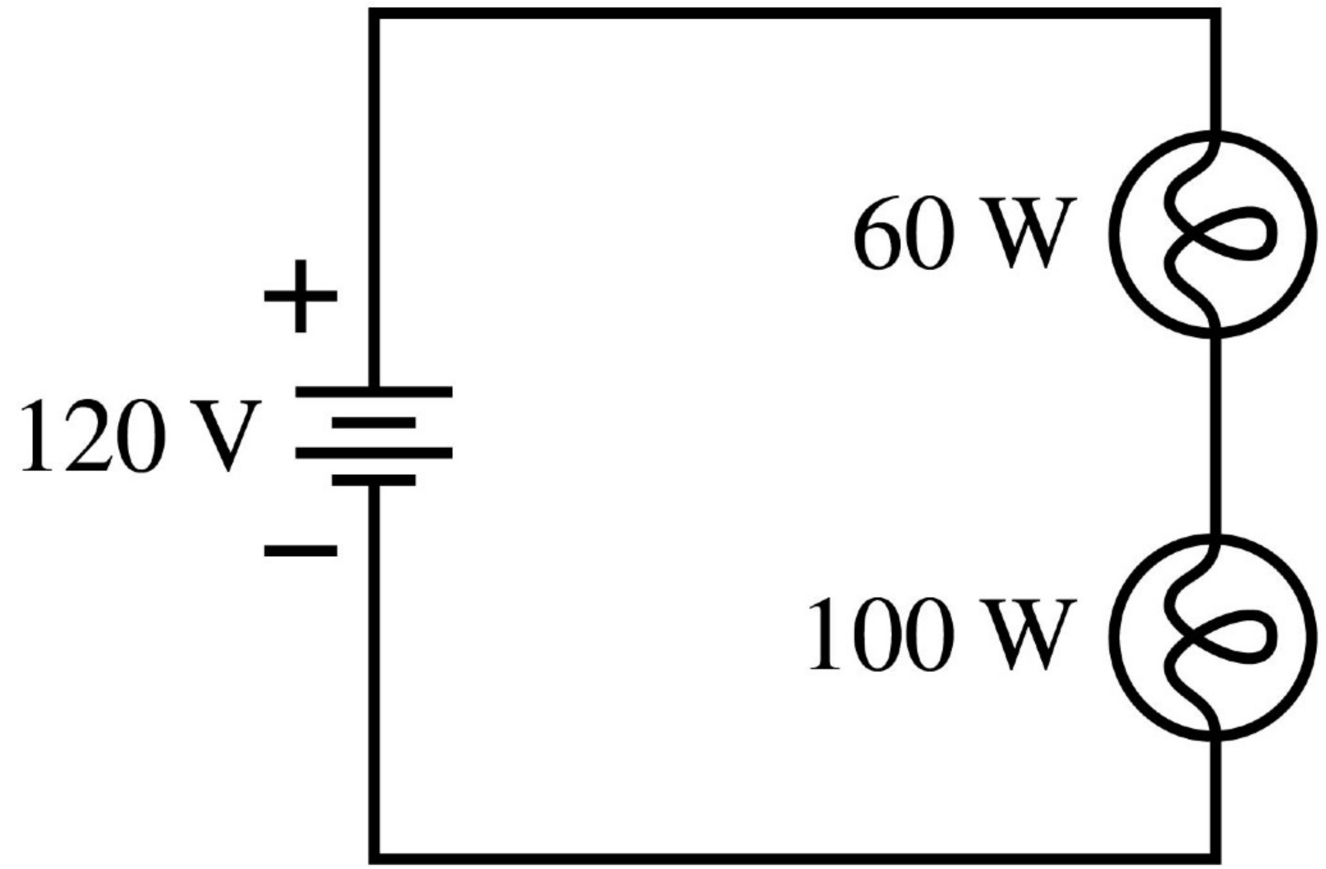
iClicker question 11-7

Which has a larger resistance, a 60 W lightbulb or a 100 W lightbulb?

A. The 60 W bulb
B. The 100 W bulb
C. Their resistances are the same.
D. There’s not enough information to tell.

iClicker question 11-8

Which bulb is brighter?



A. The 60 W bulb
B. The 100 W bulb
C. Their brightnesses are the same.
D. There's not enough information to tell.

previous iClicker questions, unexpected answers!

Which has a larger resistance, a 60 W or a 100 W lightbulb?

✔ **A. The 60 W bulb**

B. The 100 W bulb

C. Their resistances are the same.

D. There's not enough information to tell.

$$P = \frac{(\Delta V)^2}{R} \text{ with both used at } \Delta V = 120 \text{ V}$$

Which bulb is brighter?

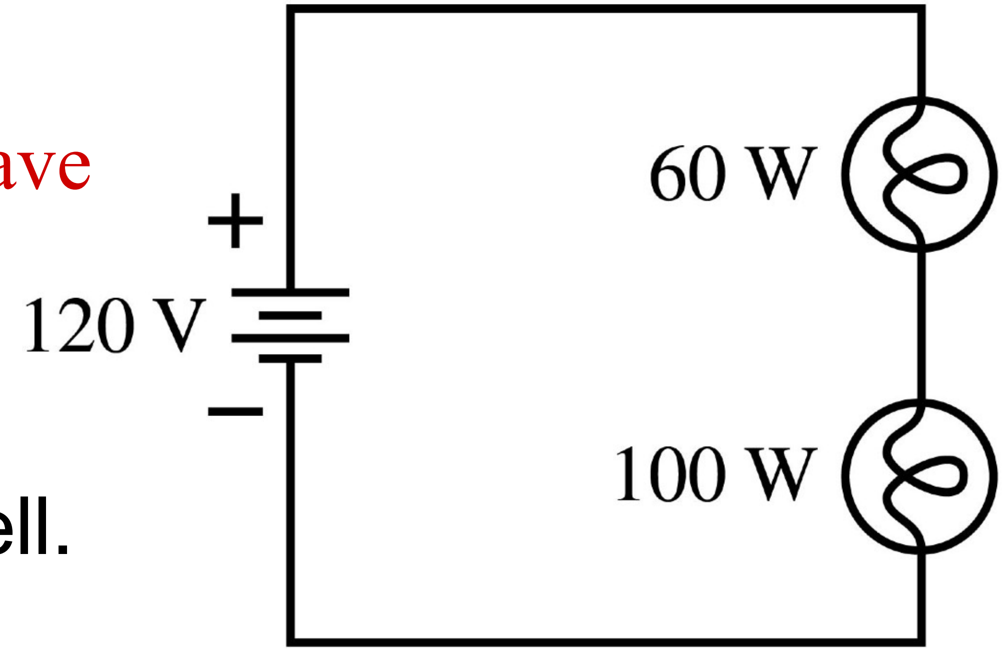
✔ **A. The 60 W bulb**

B. The 100 W bulb

C. Their brightnesses are the same.

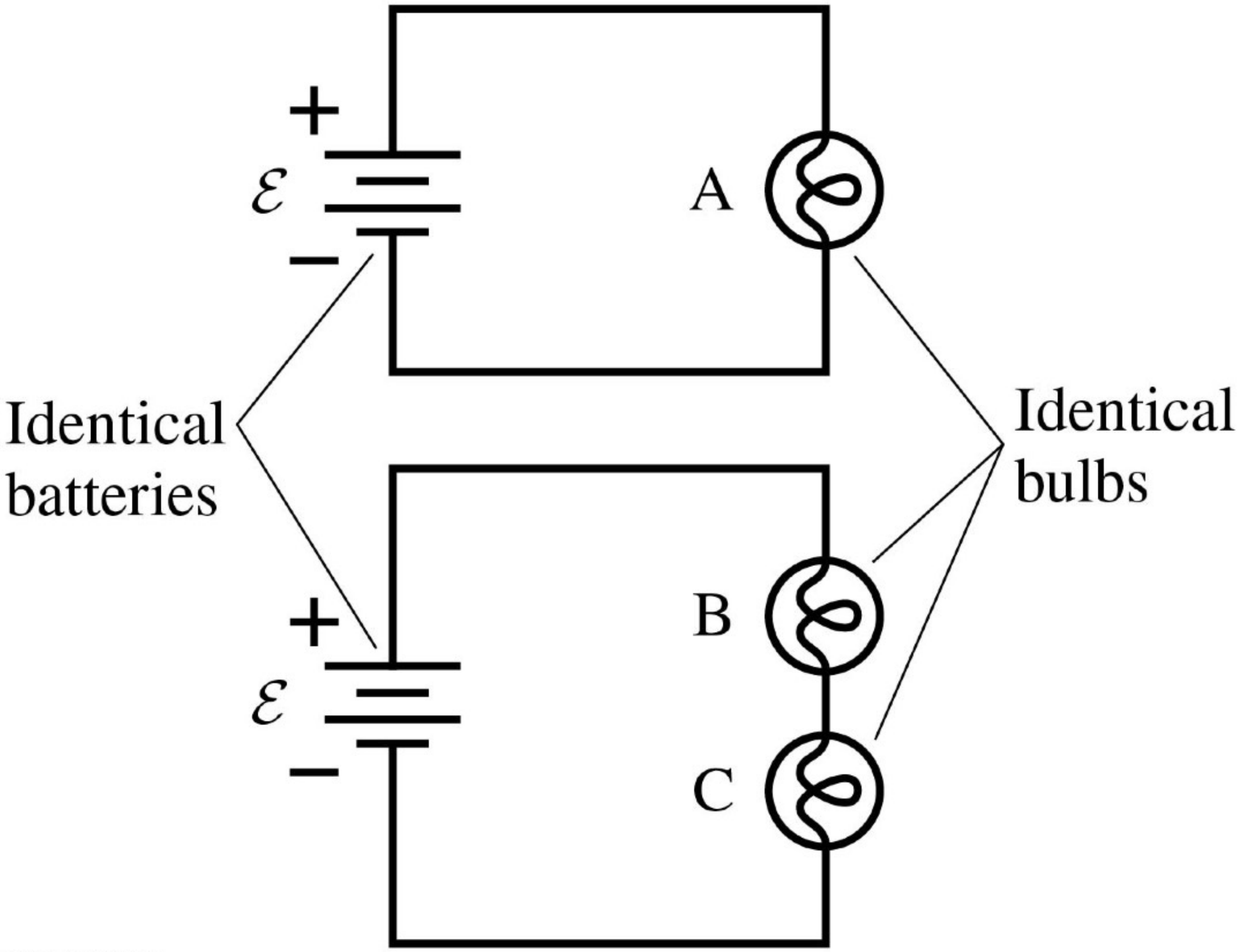
D. There's not enough information to tell.

$P = I^2R$ and both have the same current.



Lightbulb Puzzle #11-1

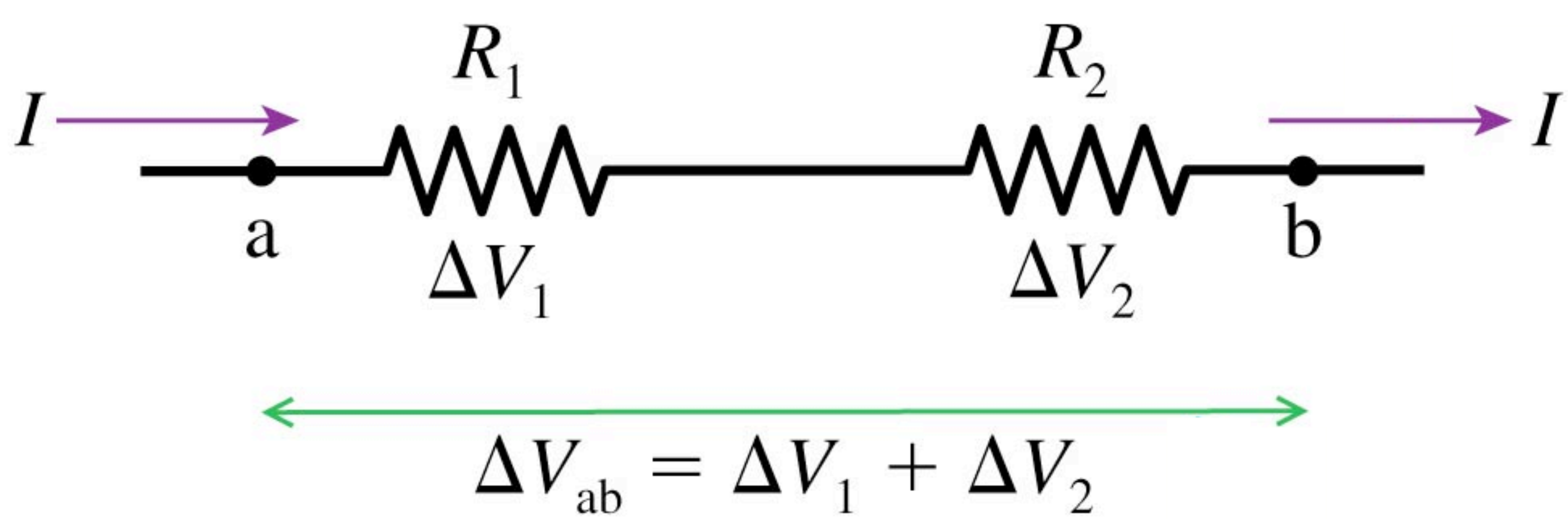
- The figure shows three identical lightbulbs in two different circuits.
- The voltage drop across A is the *same* as the total voltage drop across both B and C.
- More current will pass through Bulb A, and it will be *brighter* than either B or C.



Series Resistors

- The figure below shows two resisters connected *in series* between points a and b.
- The total potential difference between points a and b is the sum of the individual potential differences across R_1 and R_2:

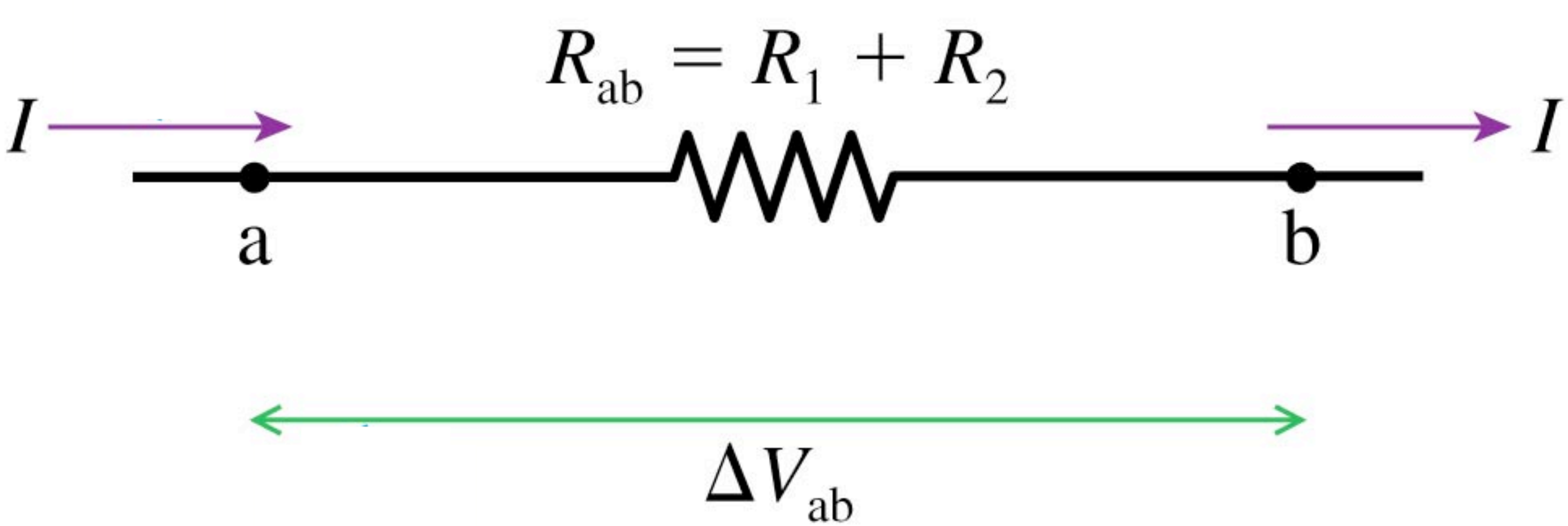
$$\Delta V_{ab} = \Delta V_1 + \Delta V_2 = IR_1 + IR_2 = I(R_1 + R_2)$$



Series Resistors

- Suppose we replace R_1 and R_2 with a single resistor with the same current I and the same potential difference ΔV_{ab}.
- Ohm's law gives resistance between points a and b:

$$R_{\text{ab}} = \frac{\Delta V_{\text{ab}}}{I} = \frac{I(R_1 + R_2)}{I} = R_1 + R_2$$



Series Resistors

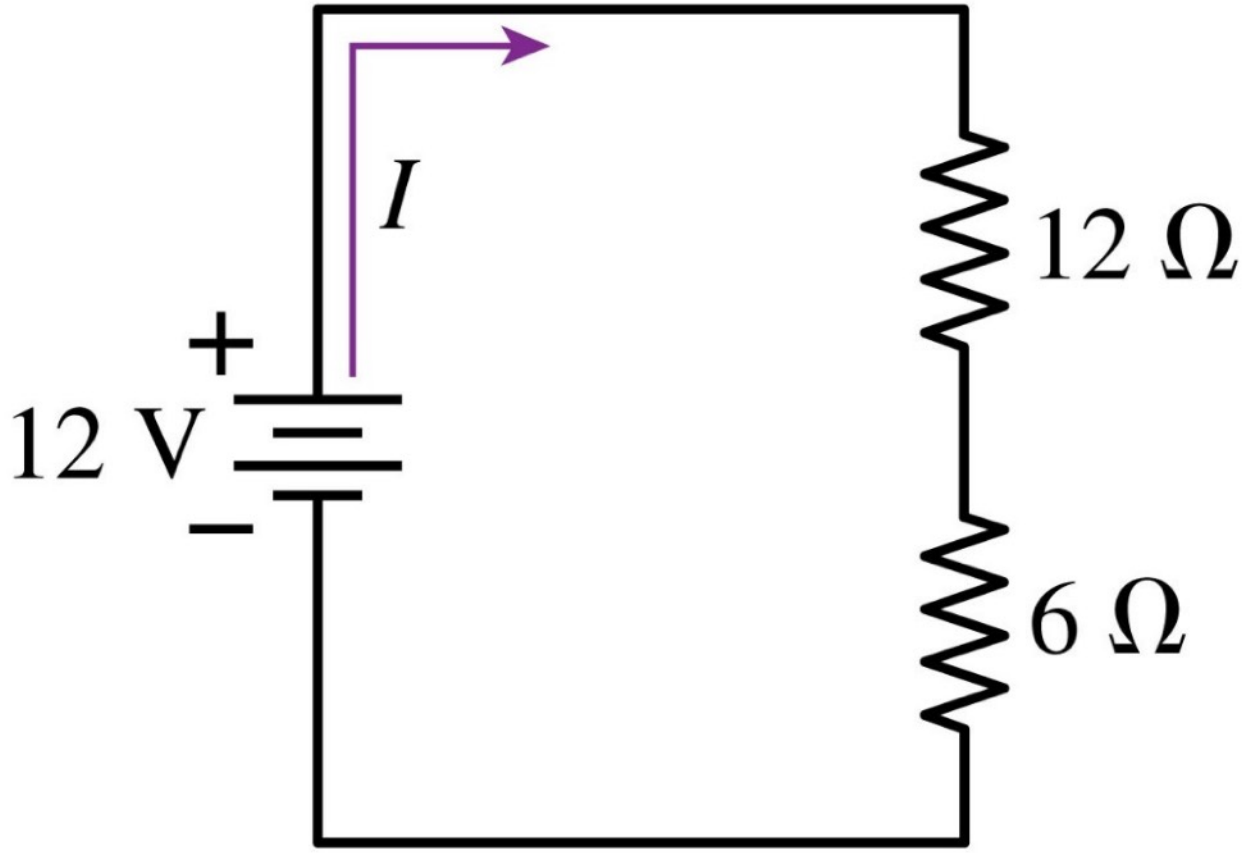
- Resistors that are aligned end to end, *with no junctions between them,* are called **series resistors** or, sometimes, resistors "in series."
- The current I is the same through all resistors placed in series.
- If we have N resistors in series, their **equivalent resistance** is

$$R_{eq} = R_1 + R_2 + \cdots + R_N \qquad \text{(series resistors)}$$

- The behavior of the circuit will be unchanged if the N series resistors are replaced by the single resistor R_{eq}.

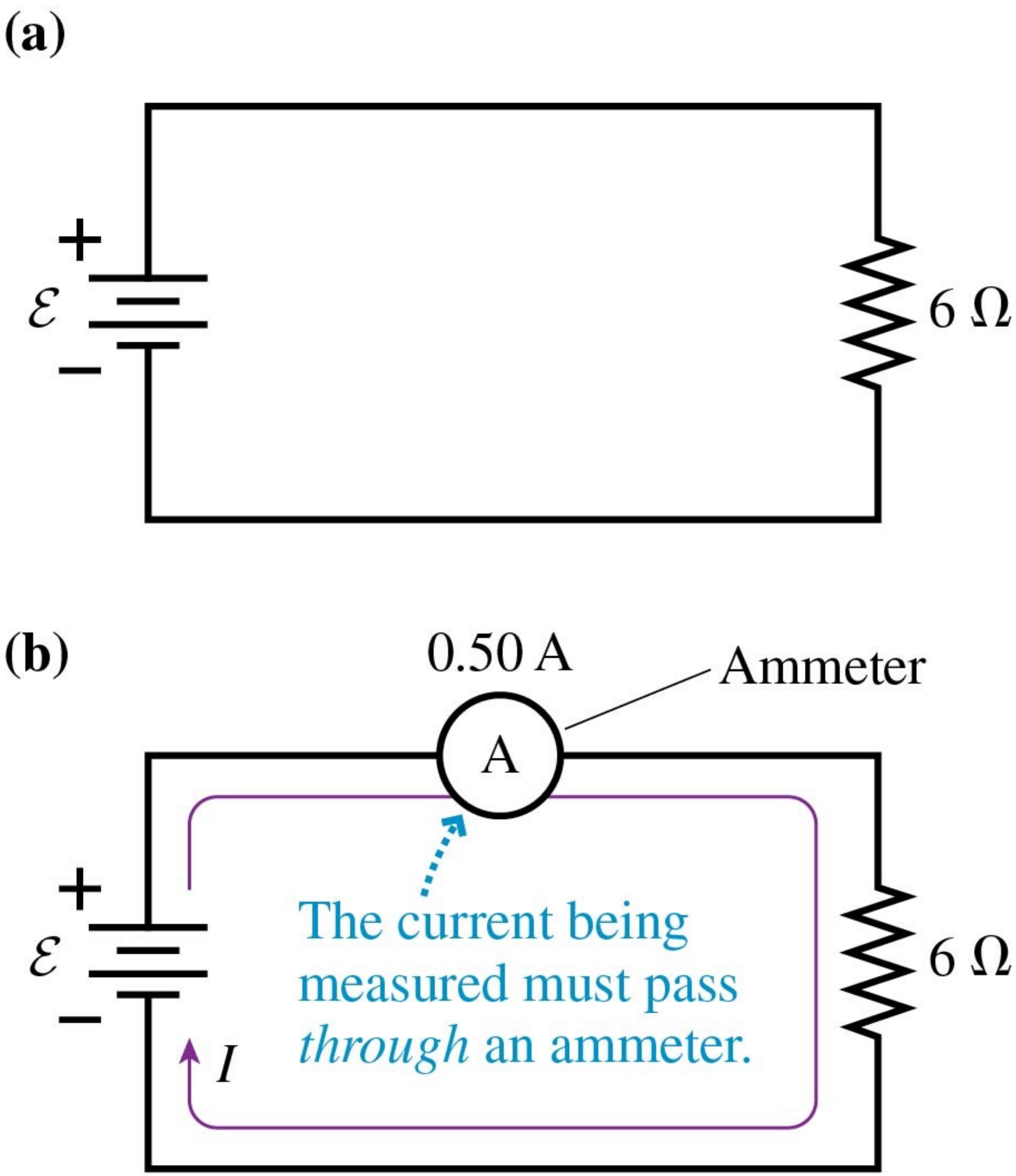
iClicker question 11-9

The battery current I is



A. 3 A
B. 2 A
C. 1 A
D. 2/3 A
E. 1/2 A

Ammeters: current meters

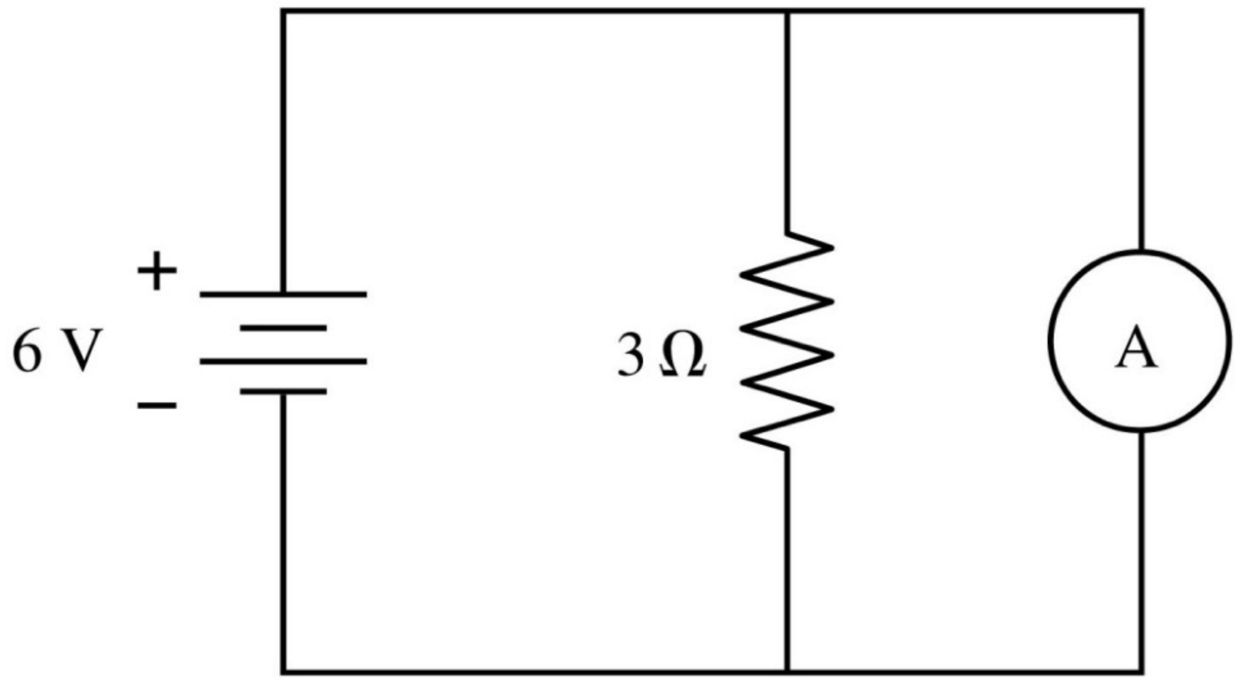


- Figure (a) shows a simple one-resistor circuit.
- We can measure the current by breaking the connection and inserting an ammeter *in series*.
- The resistance of the ammeter is negligible.
- The potential difference across the resistor must be $\Delta V_R = IR = 3.0$ V.
- So the battery's emf must be 3.0 V.

iClicker question 11-10

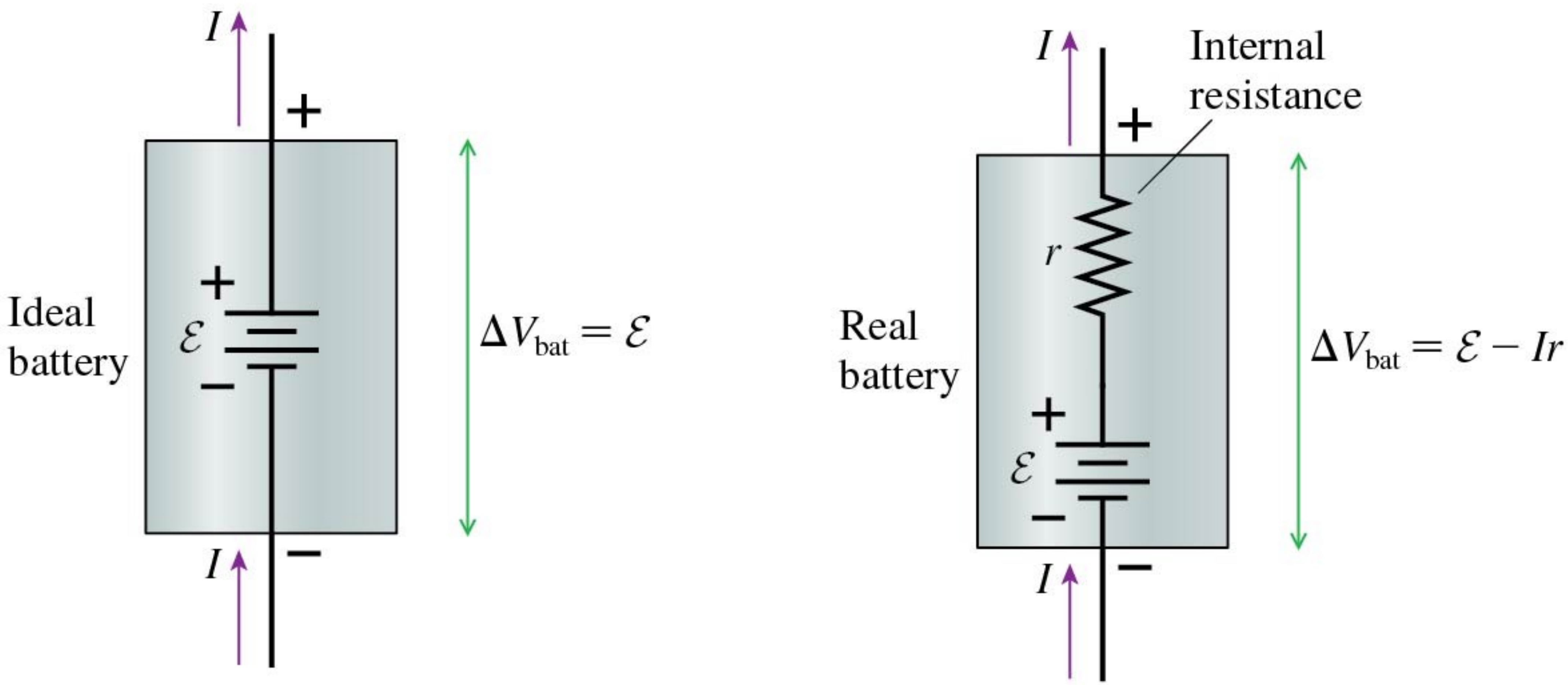
What does the ammeter read?

A. 6 A
B. 3 A
C. 2 A
D. Some other value
E. Nothing because this will fry the meter.



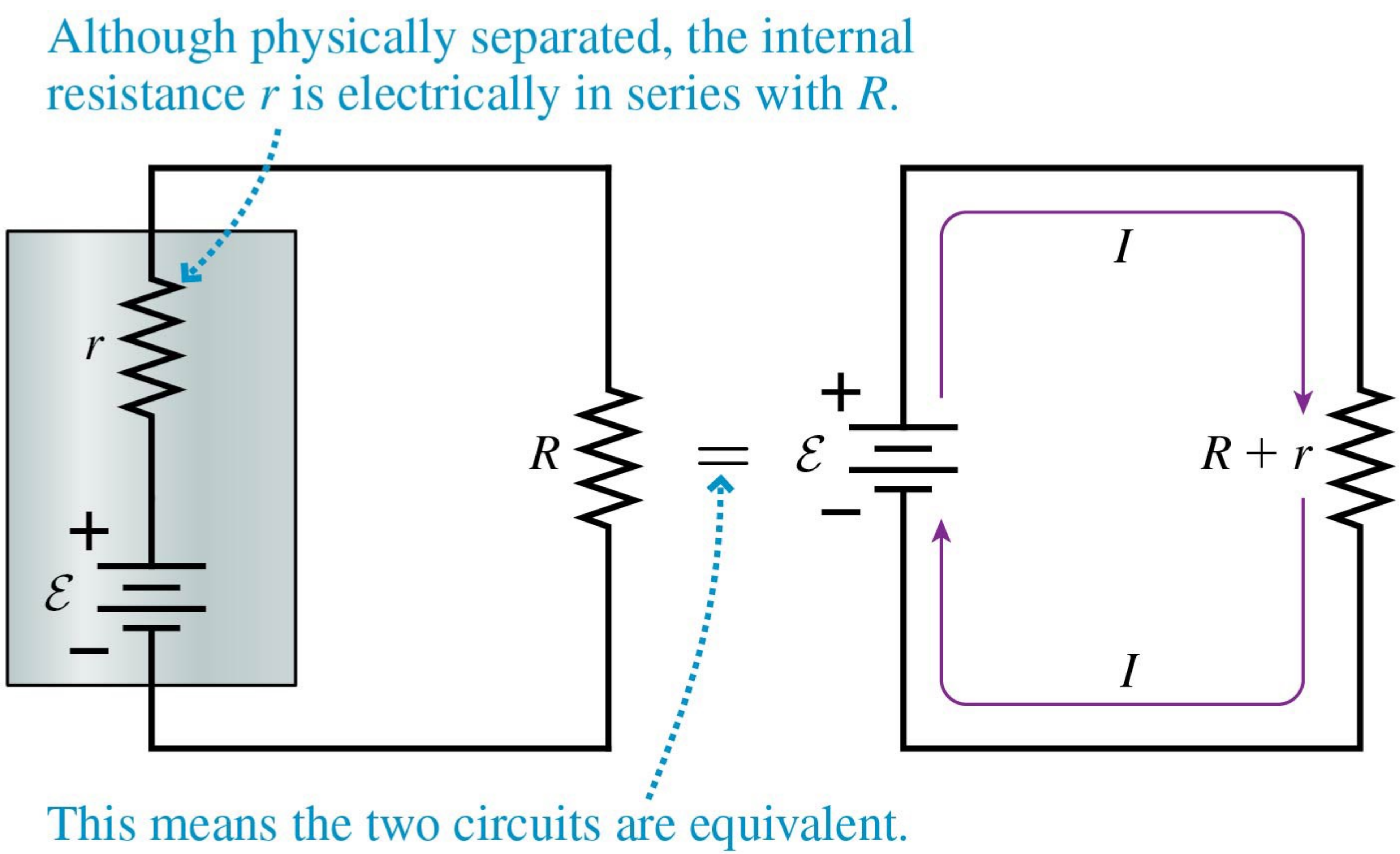
Real Batteries

- Real batteries have what is called an internal resistance, which is symbolized by r.

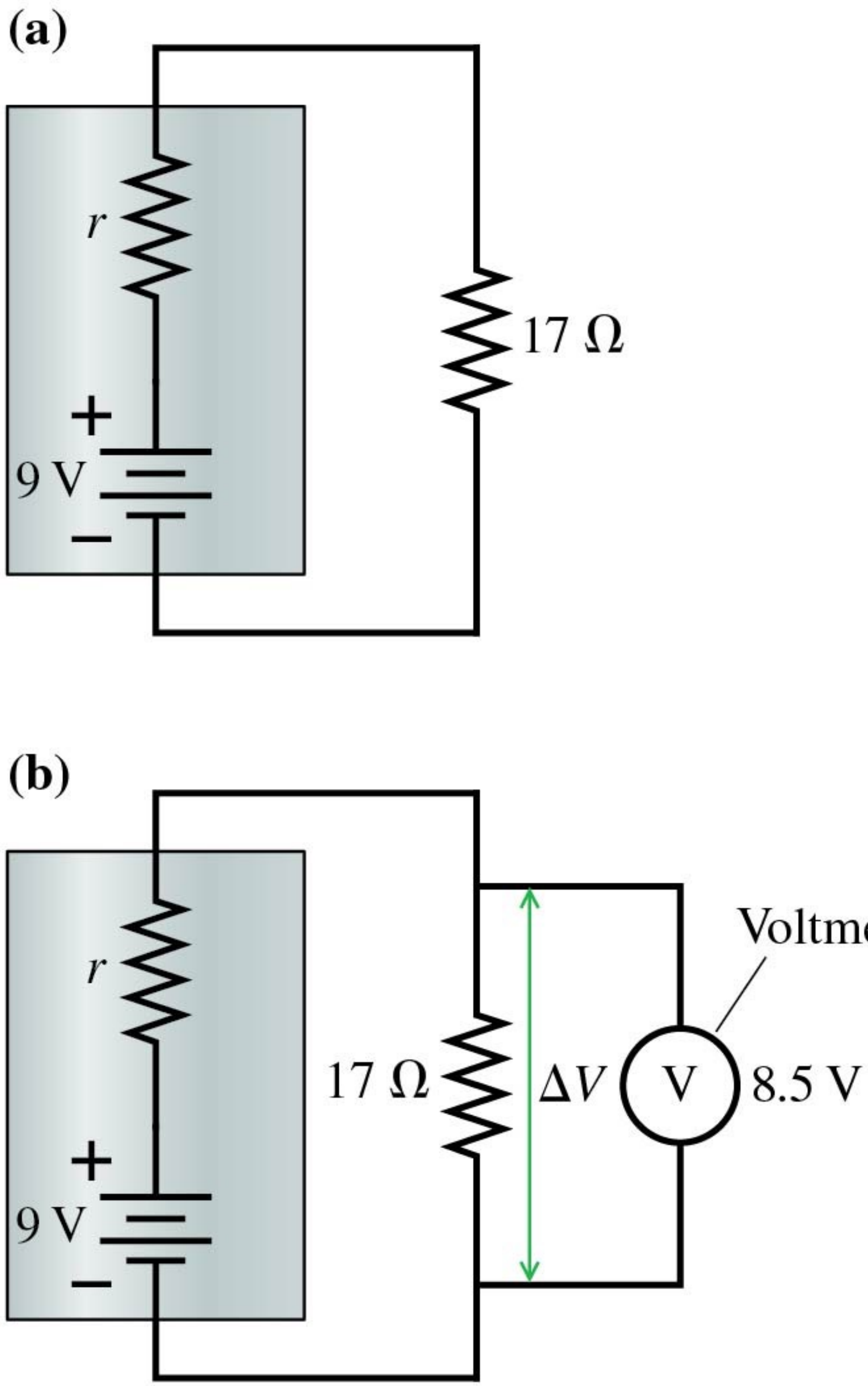


Real Batteries

- A single resistor connected to a real battery is in series with the battery's internal resistance, giving $R_{\text{eq}} = R + r$.



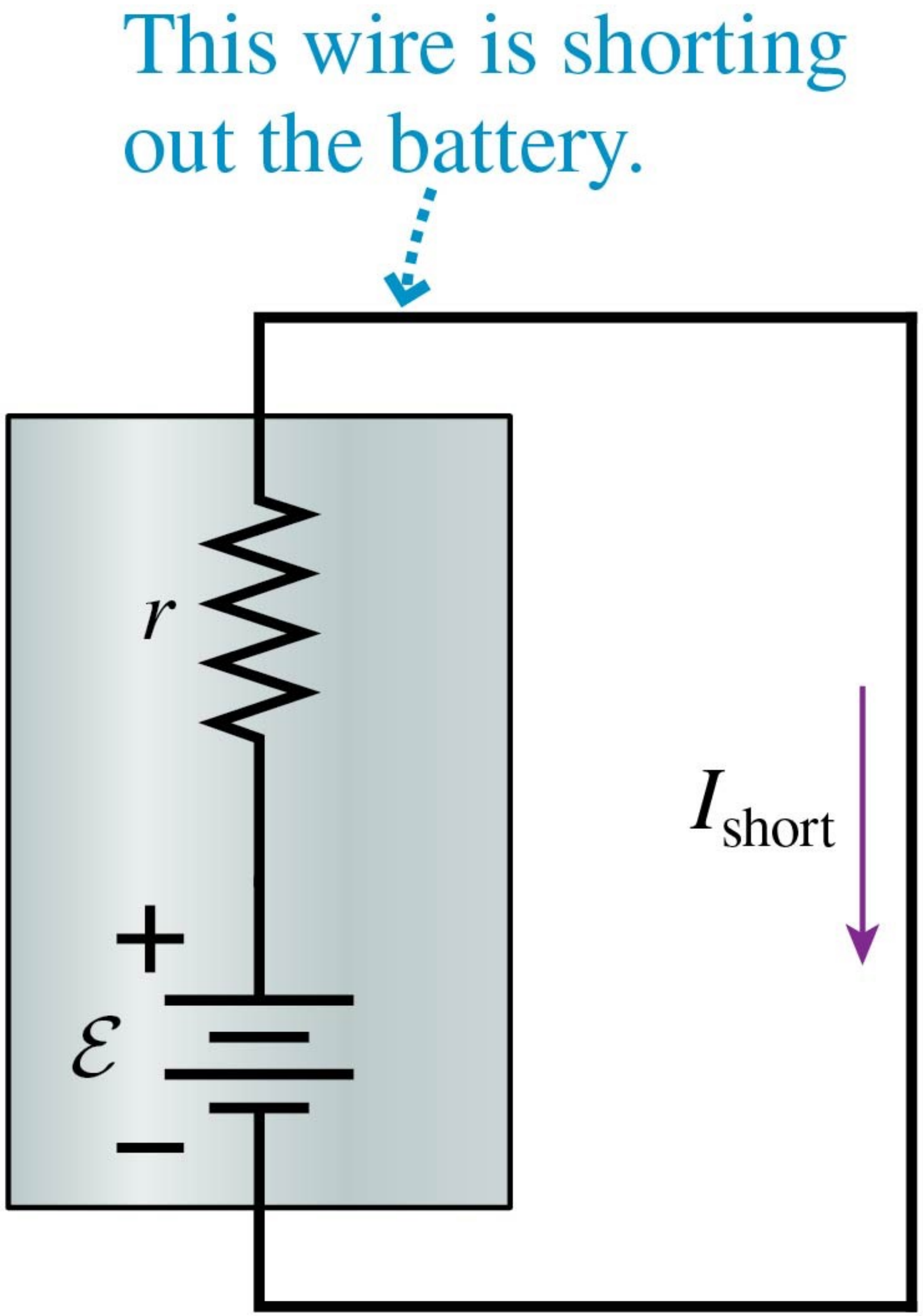
Voltmeters: measure voltage



- Figure (a) shows a simple circuit with a resistor and a real battery.
- We can measure the potential difference across the resistor by connecting a voltmeter *in parallel* across the resistor.
- The resistance of the voltmeter must be very high.
- The internal resistance is

$$r = \frac{\mathcal{E} - \Delta V_R}{\Delta V_R} R = \frac{0.5\ \text{V}}{8.5\ \text{V}} 17\ \Omega = 1.0\ \Omega$$

A Short Circuit

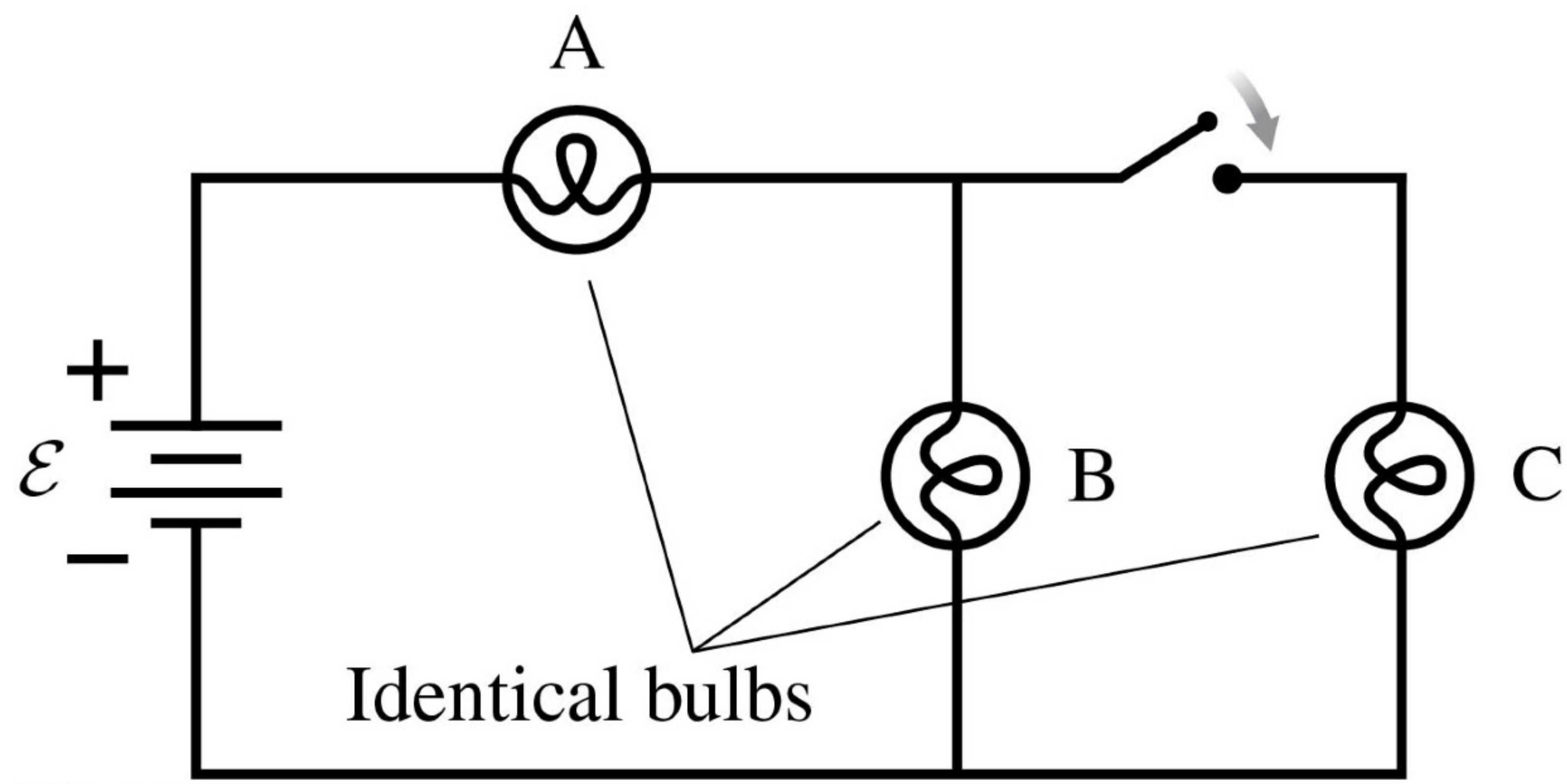


- The figure shows an ideal wire *shorting out* a battery.
- If the battery were ideal, shorting it with an ideal wire ($R = 0\ \Omega$) would cause the current to be infinite!
- In reality, the battery's internal resistance r becomes the only resistance in the circuit.
- The *short-circuit current* is

$$I_{short} = \frac{\mathcal{E}}{r}$$

Lightbulb Puzzle #2

- The figure shows three identical lightbulbs in a circuit.



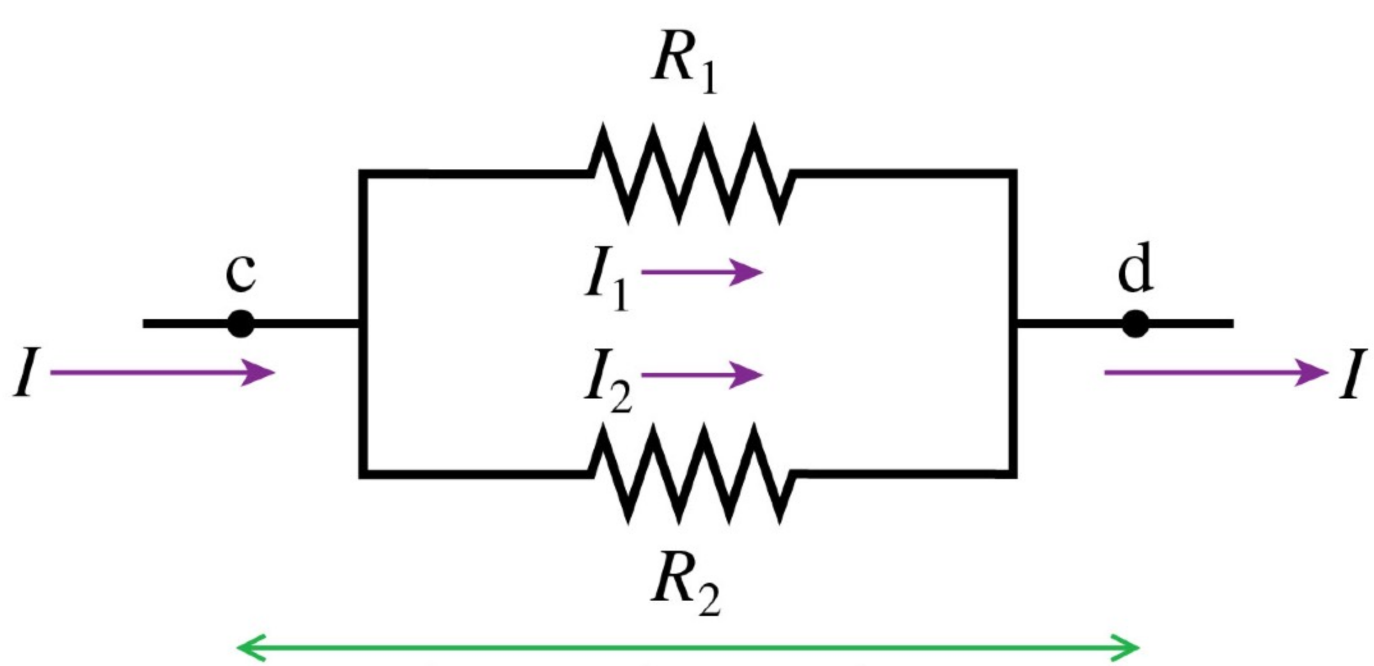
- When the switch is closed,
  an alternate pathway for the current to get from bulb A back to the battery is created.
  - This *decreases* the overall resistance of the circuit, and the <u>brightness of bulb A *increases*</u>.

Parallel Resistors

- The figure below shows two resisters connected *in parallel* between points c and d.
- By Kirchhoff's junction law, the input current is the sum of the current through each resistor: $I = I_1 + I_2$

$$I = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2} = \frac{\Delta V_{\text{cd}}}{R_1} + \frac{\Delta V_{\text{cd}}}{R_2} = \Delta V_{\text{cd}}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Two resistors in parallel

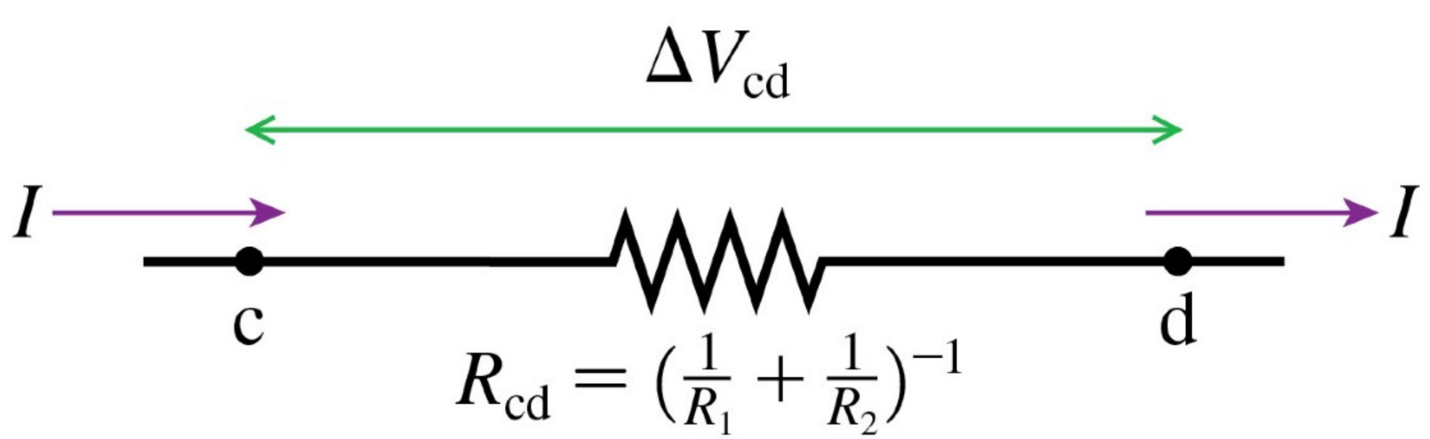


$$\Delta V_1 = \Delta V_2 = \Delta V_{\text{cd}}$$

Parallel Resistors

- Suppose we replace R_1 and R_2 with a single resistor with the same current I and the same potential difference ΔV_{cd}.
- Ohm's law gives resistance between points c and d:

$$R_{\text{cd}} = \frac{\Delta V_{\text{cd}}}{I} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$



An equivalent resistor

Parallel Resistors

- Resistors connected *at both ends* are called **parallel resistors** or, sometimes, resistors "in parallel."
- The left ends of all the resistors connected in parallel are held at the same potential V_1, and the right ends are all held at the same potential V_2.
- The potential differences ΔV are the *same* across all resistors placed in parallel.
- If we have N resistors in parallel, their **equivalent resistance** is

$$R_{\mathrm{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}\right)^{-1} \qquad \text{(parallel resistors)}$$

- The behavior of the circuit will be unchanged if the N parallel resistors are replaced by the single resistor R_{eq}.

Series resistors

$$\boxed{R_{eq} = R_1 + R_2 + \ldots}$$

"because voltage drops add up"

Parallel resistors

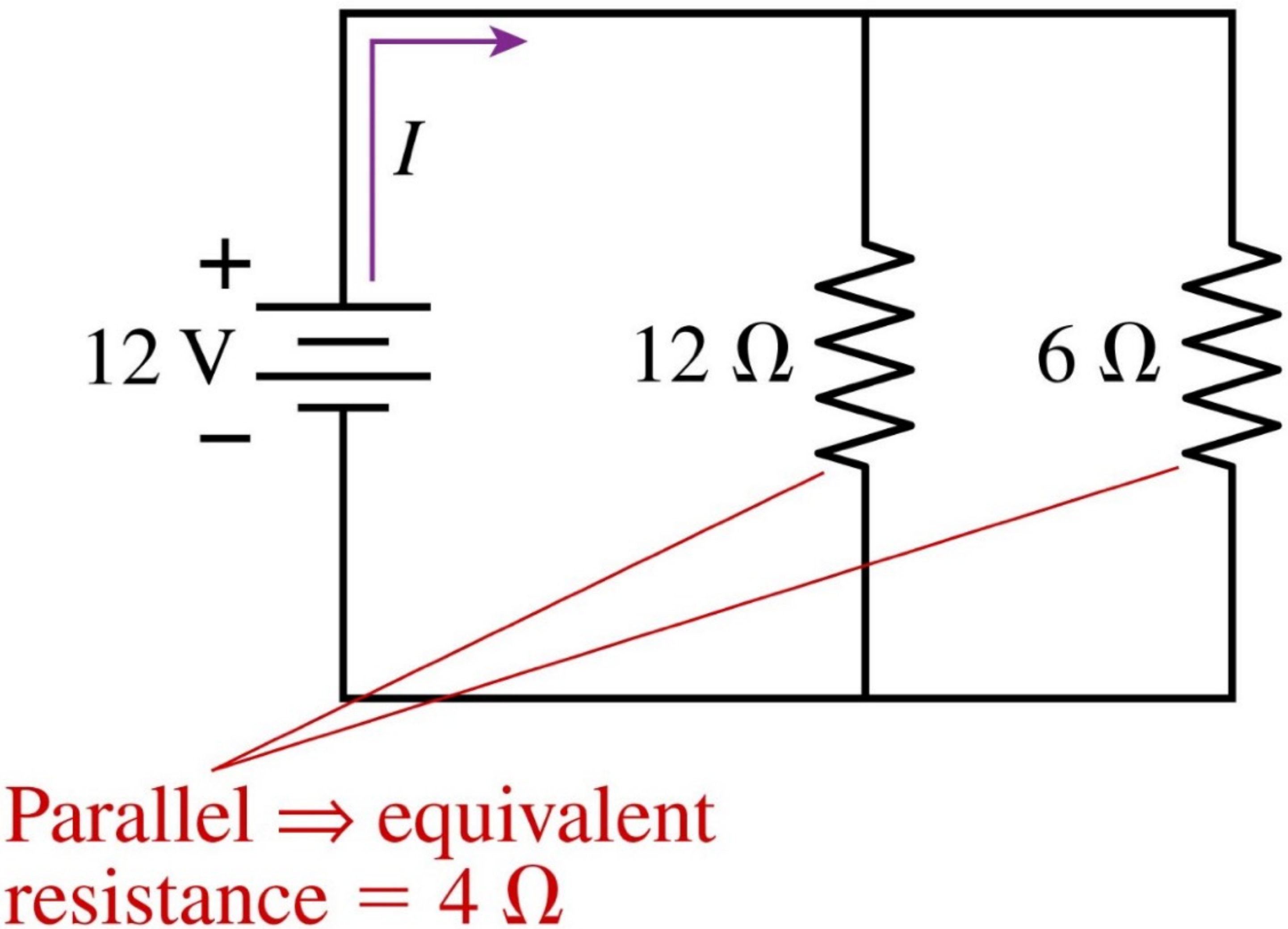
$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots}$$

"because currents at a junction add up to zero"

EXAMPLE:

The battery current I is

- **A. 3 A**
- B. 2 A
- C. 1 A
- D. 2/3 A
- E. 1/2 A



Answer to Lightbulb Puzzle #1

The three bulbs are identical and the two batteries are identical. Compare the brightnesses of the bulbs.

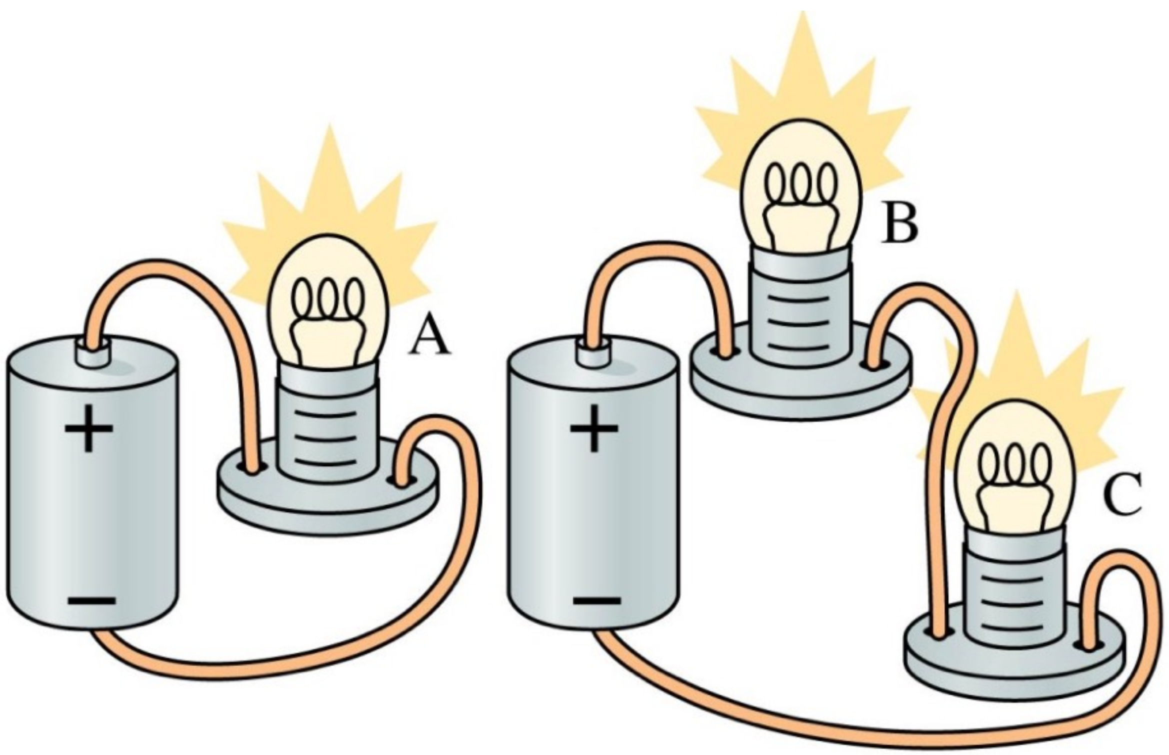
A. $A > B > C$

B. $A > C > B$

C. $\mathbf{A > B = C}$

D. $A < B = C$

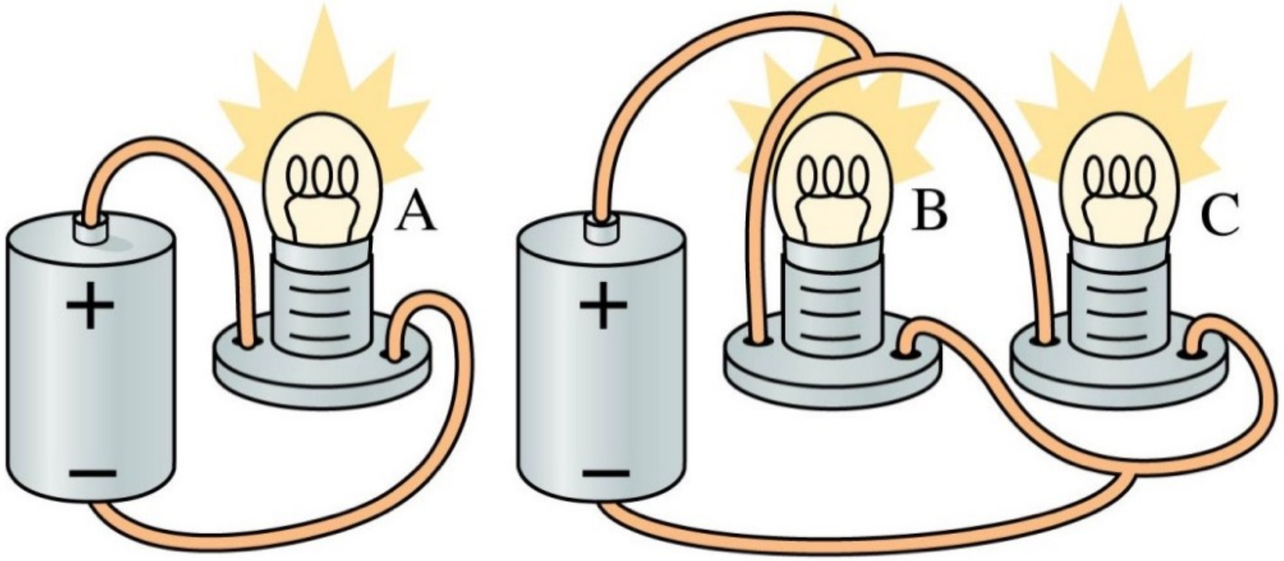
E. $A = B = C$



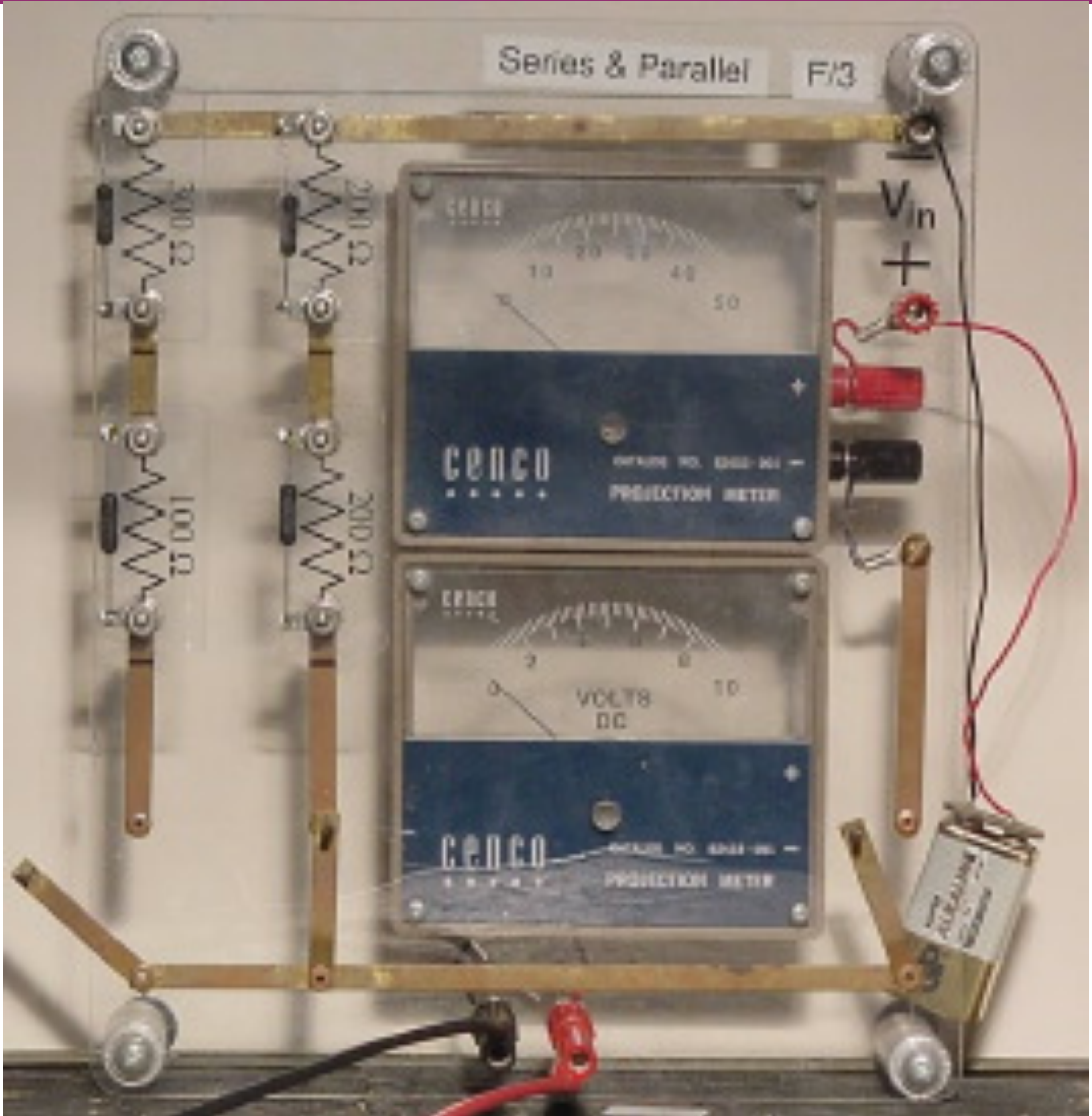
Answer to Lightbulb Puzzle #2

The three bulbs are identical and the two batteries are identical. Compare the brightnesses of the bulbs.

A. A > B > C
B. A > C > B
C. A > B = C
D. A < B = C
✔E. **A = B = C**



Ohm's law demo



iClicker question 11-11

The lightbulbs are identical. Initially both bulbs are glowing. What happens when the switch is closed?

A. Nothing
B. A stays the same;
   B gets dimmer.
C. A gets brighter;
   B stays the same.
D. Both get dimmer.
E. A gets brighter;
   B goes out.