

VECTORS IN PHYSICS

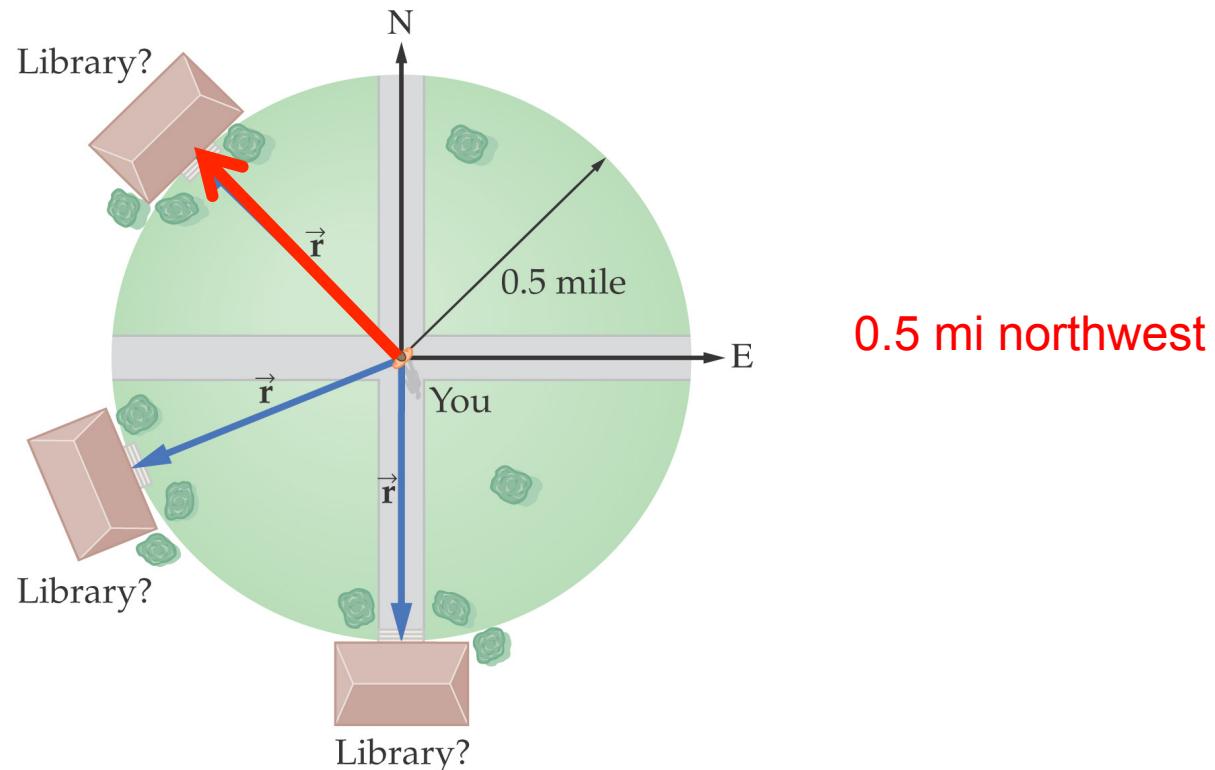
Chapter 3

Units of Chapter 3

- Scalars Versus Vectors
- The components of a vector
- Adding and subtracting vectors
- Unit vectors
- Position, Displacement, Velocity, and Acceleration Vectors
- Relative Motion

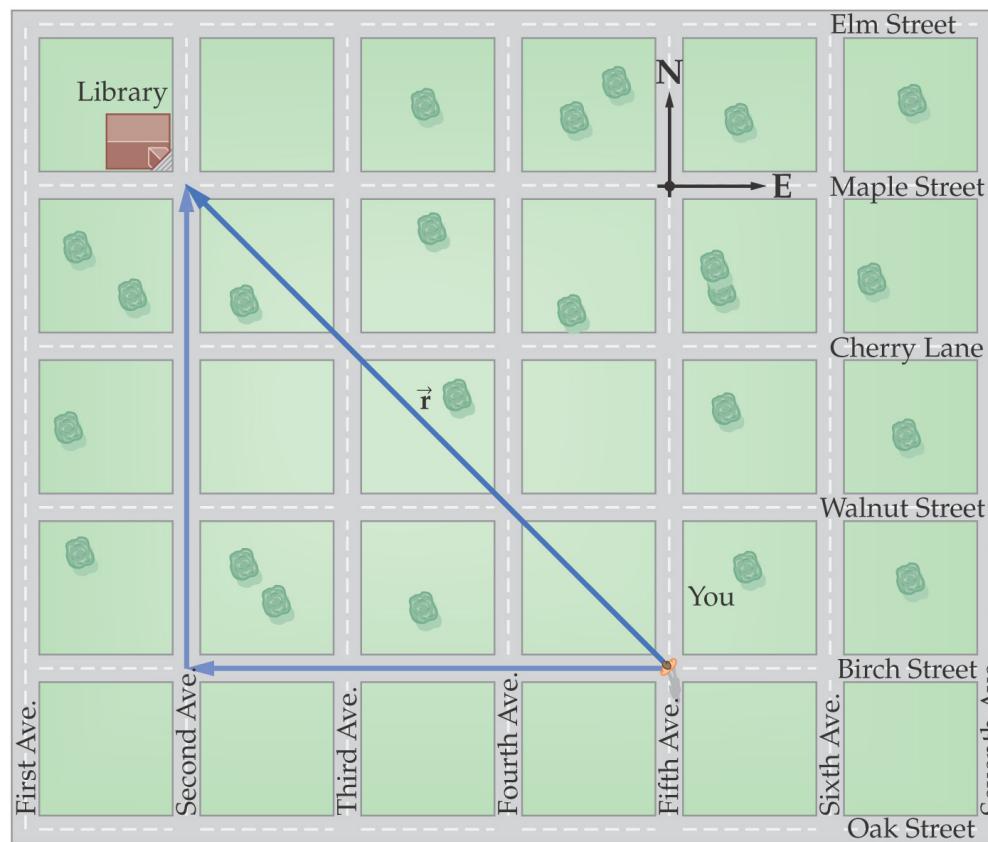
3-1 Scalars versus vectors

- **Scalar**: number with units.
- **Vector**: quantity with magnitude and direction.
- How to get to the library: need to know how far and which way



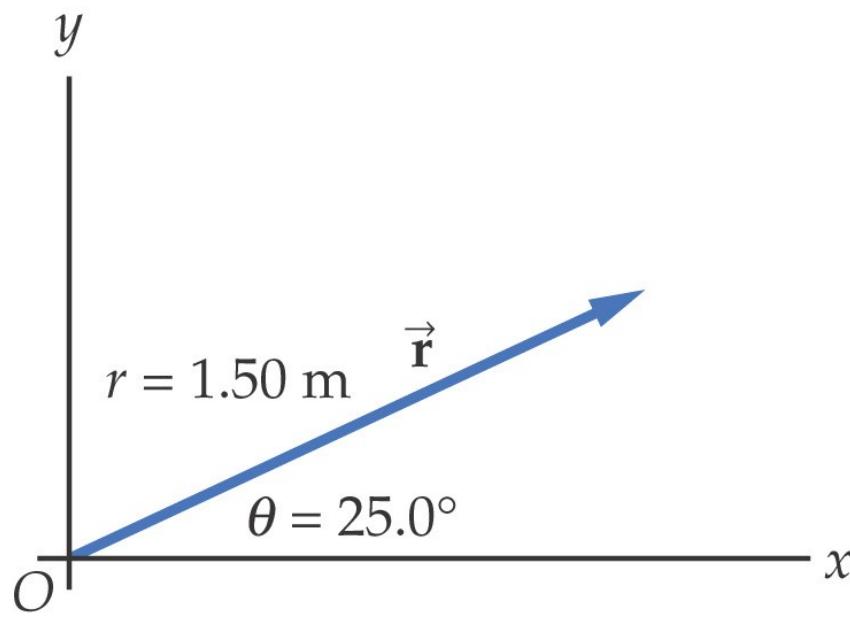
3-2 The Components of a Vector

- Even though you know how far and in which direction the library is, you may not be able to walk there in a straight line:

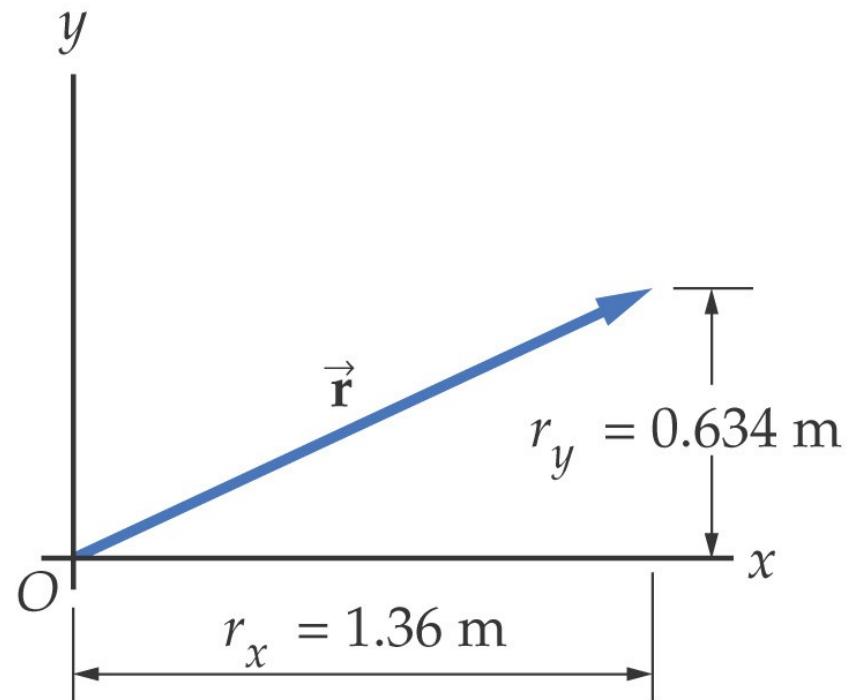


3-2 The Components of a Vector

- Can resolve vector into **perpendicular components** using a two-dimensional coordinate system:



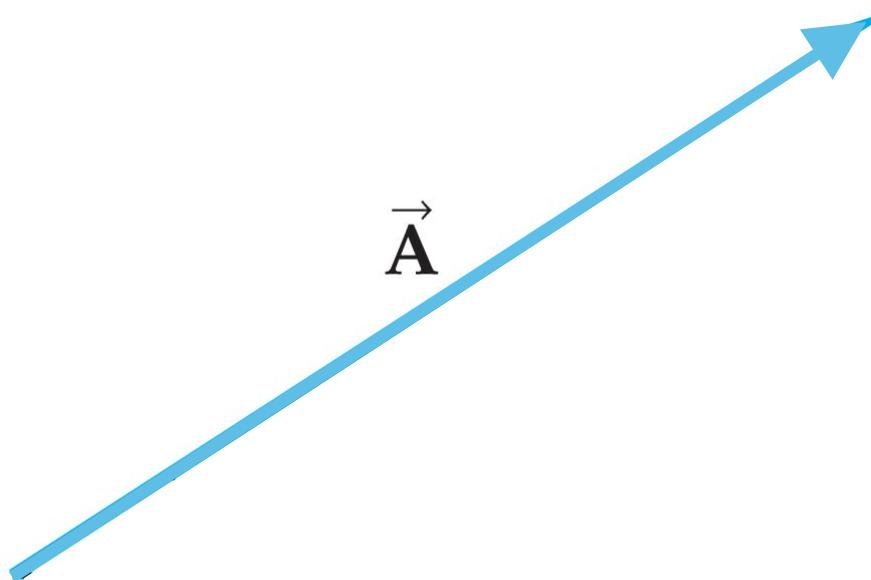
(a)



(b)

3-2 The Components of a Vector

- Length, angle, and components can be calculated from each other using trigonometry:



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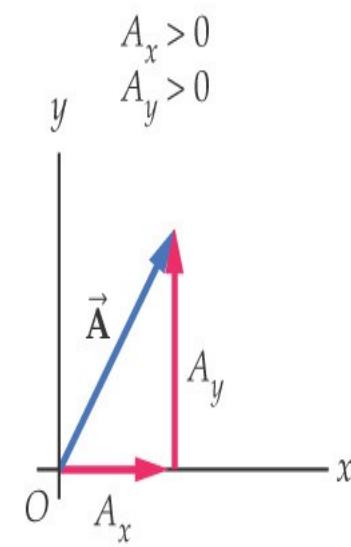
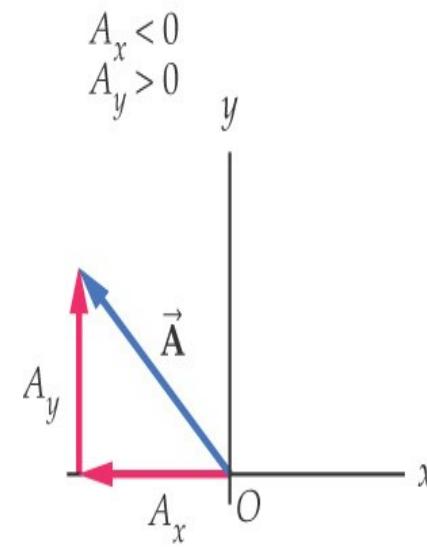
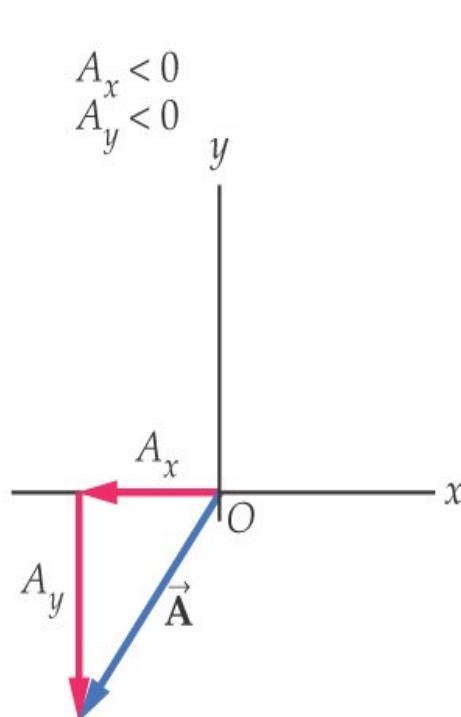
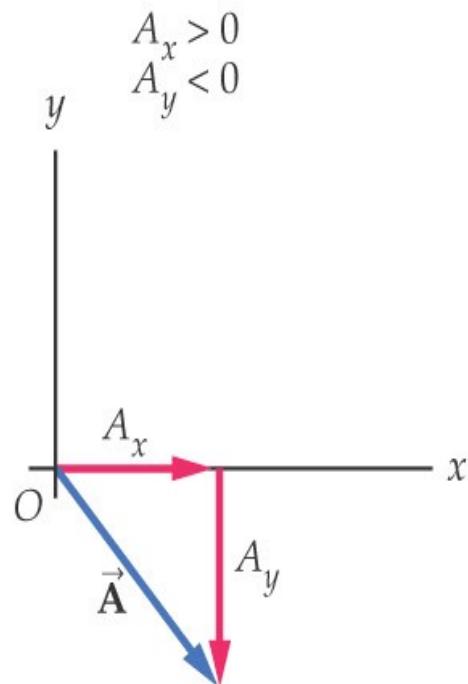
Given the magnitude and direction of a vector,
find its magnitude and direction:

$$A_x = \sqrt{A^2 - A_y^2}$$

$$A_y = A \sin(\theta) / A_x$$

3-2 The Components of a Vector

- Signs of vector components:



(a)

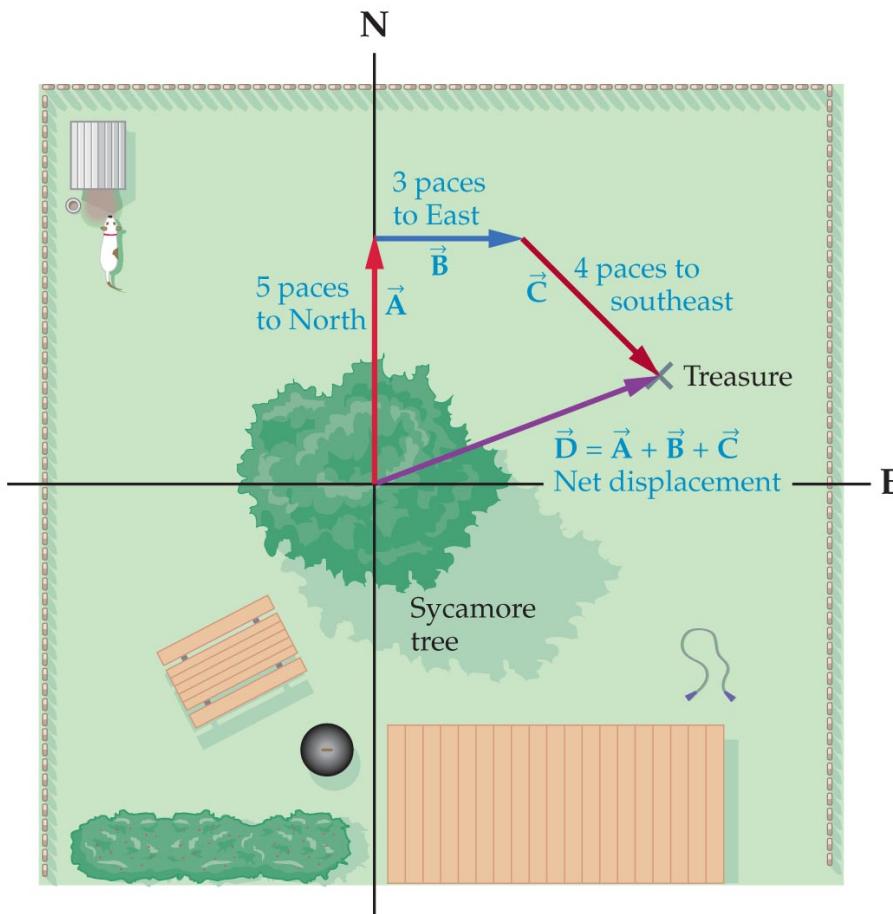
(b)

(c)

(d)

3-3 Adding and Subtracting Vectors

- Adding vectors graphically: Place the tail of the second at the head of the first. The sum points from the tail of the first to the head of the last.

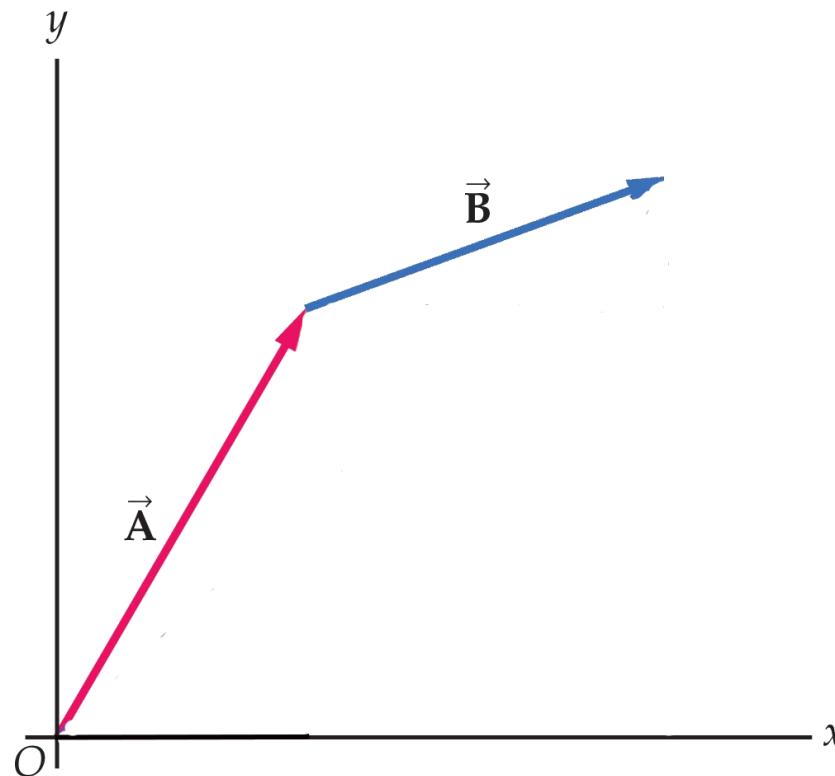


3-3 Adding and Subtracting Vectors

- Adding vectors using components:
 1. Find the components of each vector to be added.
 2. Add the x - and y -components separately.
 3. Find the resultant vector.

3-3 Adding and Subtracting Vectors

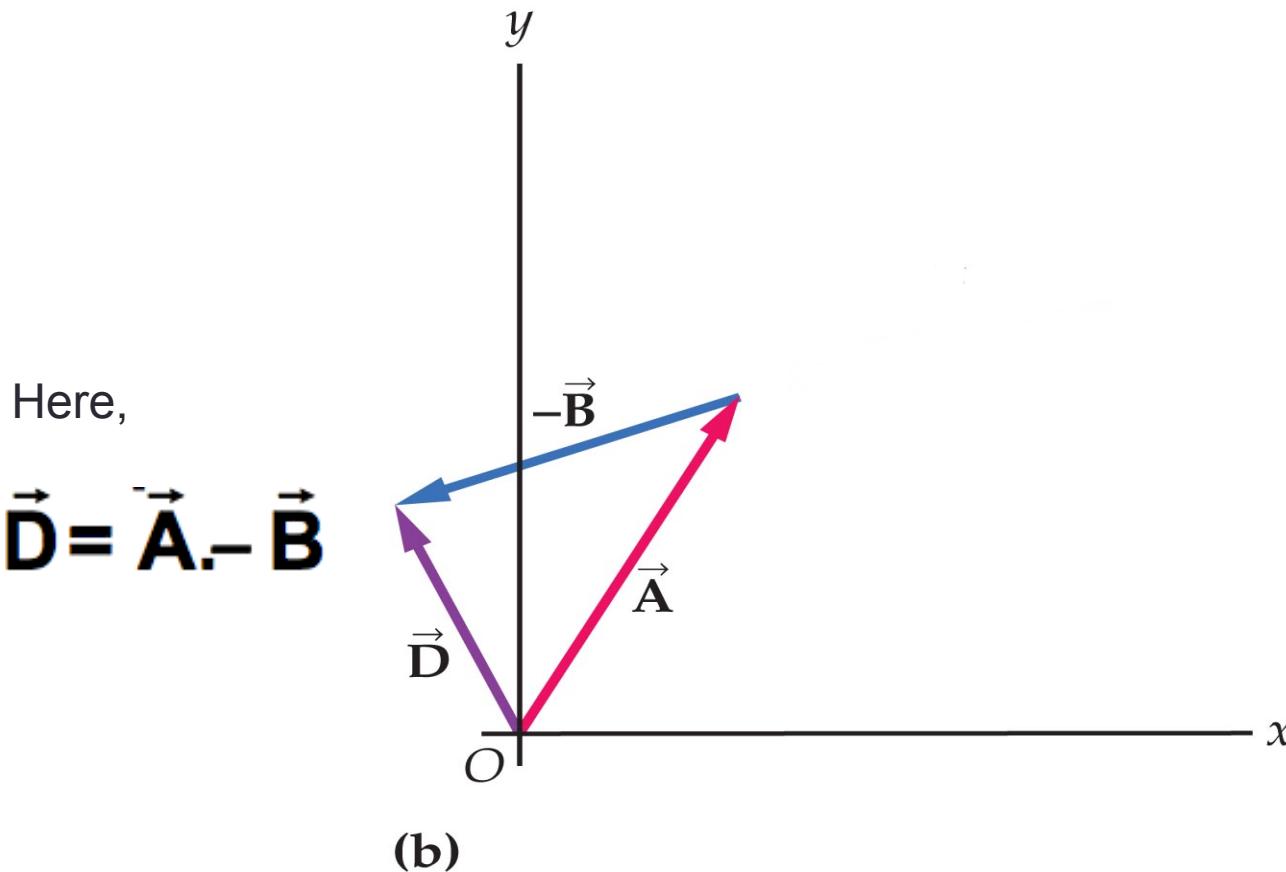
- Adding vectors using components:



(a)

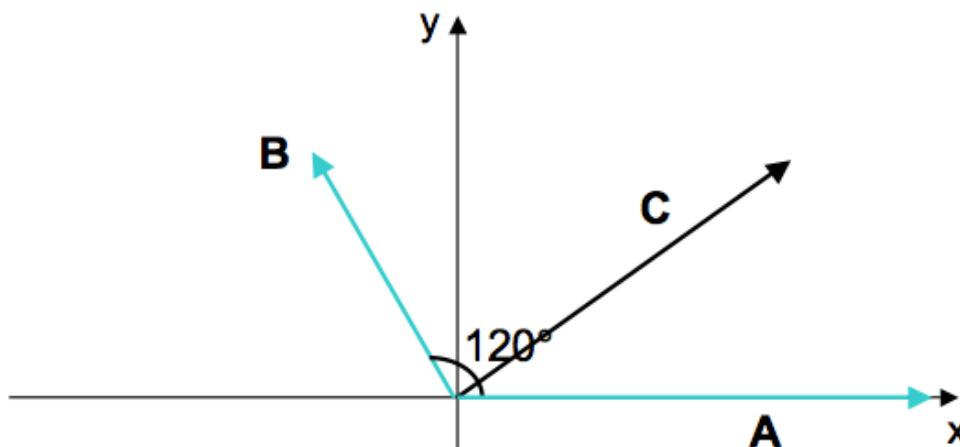
3-3 Adding and Subtracting Vectors

- **Subtracting Vectors:** The negative of a vector is a vector of the same magnitude pointing in the opposite direction.

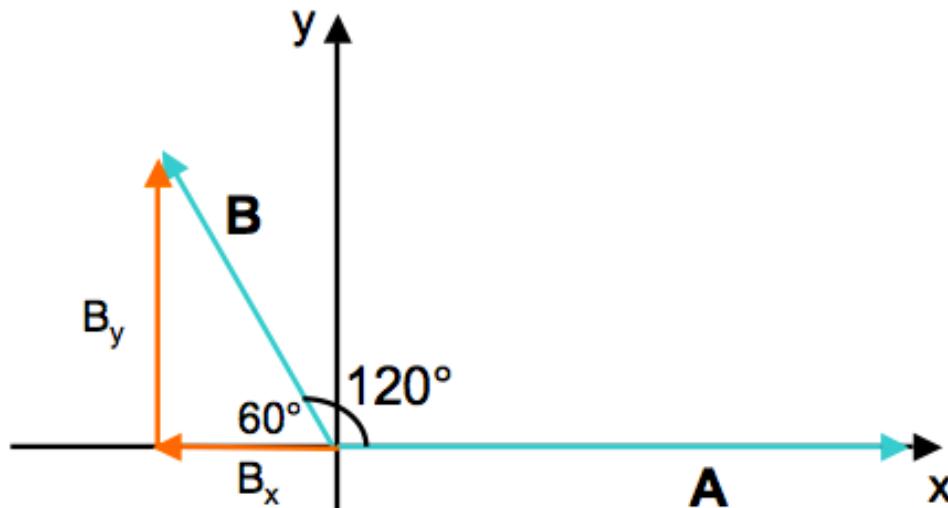


Example

Vector **A** has a length of 5.00 meters and points along the x-axis. Vector **B** has a length of 3.00 meters and points 120° from the +x-axis. Compute **A+B** (=C)



Example continued



$$\sin 60^\circ = \frac{B_y}{B} \Rightarrow B_y = B \sin 60^\circ = (3.00\text{m}) \sin 60^\circ = 2.60 \text{ m}$$

$$\cos 60^\circ = \frac{-B_x}{B} \Rightarrow B_x = -B \cos 60^\circ = -(3.00\text{m}) \cos 60^\circ = -1.50 \text{ m}$$

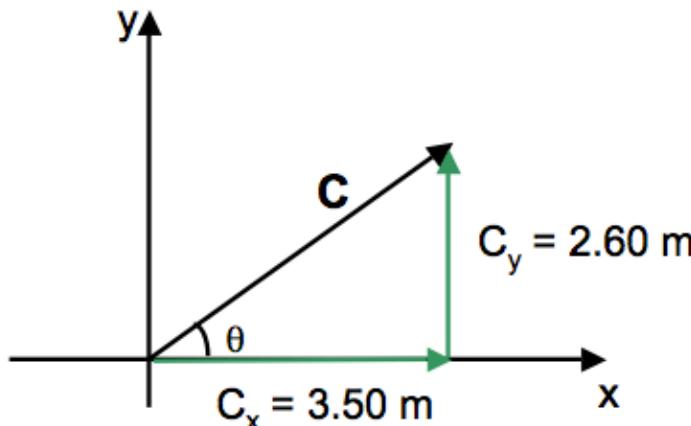
and $A_x = 5.00 \text{ m}$ and $A_y = 0.00 \text{ m}$

Example continued

The components of \mathbf{C} :

$$C_x = A_x + B_x = 5.00 \text{ m} + (-1.50 \text{ m}) = 3.50 \text{ m}$$

$$C_y = A_y + B_y = 0.00 \text{ m} + 2.60 \text{ m} = 2.60 \text{ m}$$



The length of \mathbf{C} is:

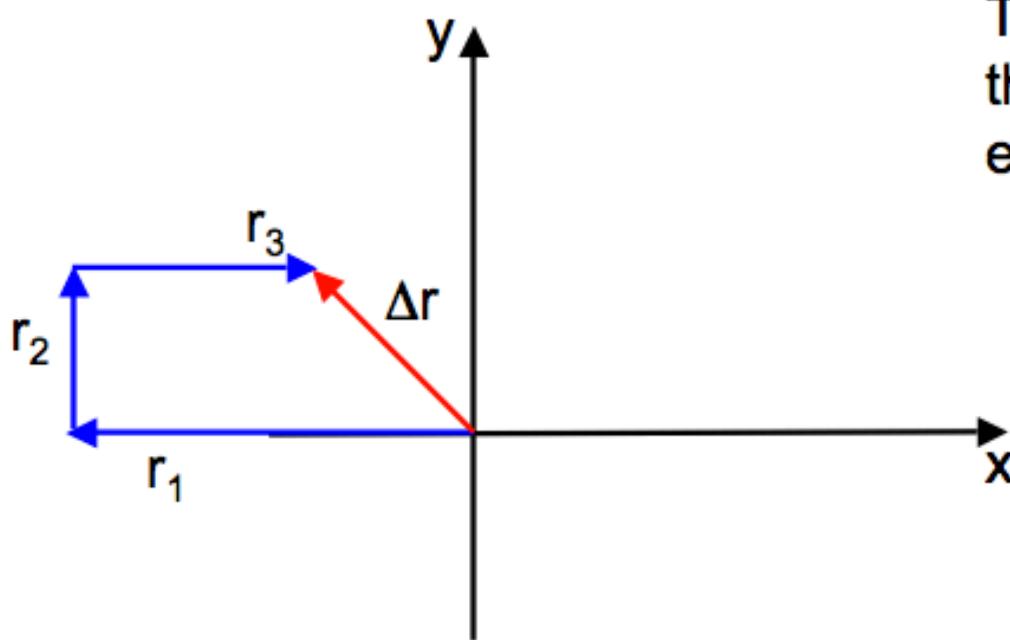
$$\begin{aligned} C &= |\mathbf{C}| = \sqrt{C_x^2 + C_y^2} \\ &= \sqrt{(3.50 \text{ m})^2 + (2.60 \text{ m})^2} \\ &= 4.36 \text{ m} \end{aligned}$$

The direction of \mathbf{C} is: $\tan \theta = \frac{C_y}{C_x} = \frac{2.60 \text{ m}}{3.50 \text{ m}} = 0.7429$

$$\theta = \tan^{-1}(0.7429) = 36.6^\circ \quad \text{From the } +x\text{-axis}$$

Example

- Margaret walks to the store using the following path: 0.500 miles west, 0.200 miles north, 0.300 miles east. What is her total displacement? Give the magnitude and direction.

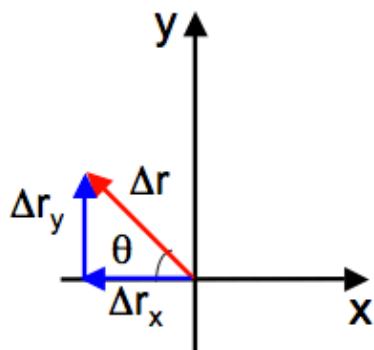


Take north to be in
the $+y$ direction and
east to be along $+x$.

Example continued

The displacement is $\Delta\mathbf{r} = \mathbf{r}_f - \mathbf{r}_i$. The initial position is the origin; what is \mathbf{r}_f ?

The final position will be $\mathbf{r}_f = \mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3$. The components are $r_{fx} = -r_1 + r_3 = -0.2$ miles and $r_{fy} = +r_2 = +0.2$ miles.



Using the figure, the magnitude and direction of the displacement are

$$|\Delta\mathbf{r}| = \sqrt{\Delta r_x^2 + \Delta r_y^2} = 0.283 \text{ miles}$$

$$\tan \theta = \frac{|\Delta r_y|}{|\Delta r_x|} = 1 \text{ and } \theta = 45^\circ \quad \text{N of W.}$$

Example

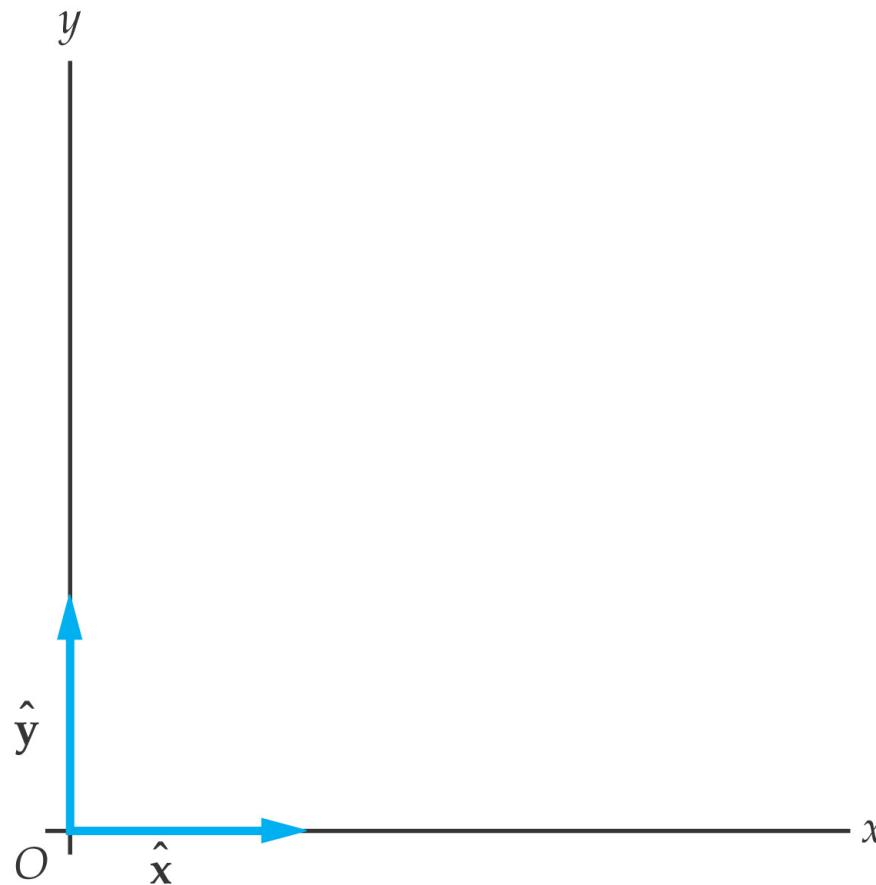
During Sunday's superbowl, the middle linebacker for the Seattle Sea Hawks made the following movements after the ball was snapped on third down. First, he back-pedaled in the southern direction for 2.6 meters. He then shuffled to his left (west) for a distance of 2.2 meters. Finally, he made a half-turn and ran downfield a distance of 4.8 meters in a direction of 240° counter-clockwise from east (30° W of S) before finally knocking the wind out of New England Patriots' wide receiver. Determine the magnitude and direction of his overall displacement.

Example

- Vector **A** is 5.5 cm long and points along the East. Vector **B** is 7.5 cm long and points at $+30^\circ$ North of West. Determine the sum of these two vectors in terms of magnitude and direction.
- A) 2.0 cm at 15° above the x-axis
- B) 3.9 cm at 75° above the x-axis
- C) 7.8 cm at 33° above the x-axis
- D) 13 cm at 17° above the x-axis
- E) 7.5 cm at 30° above the x-axis

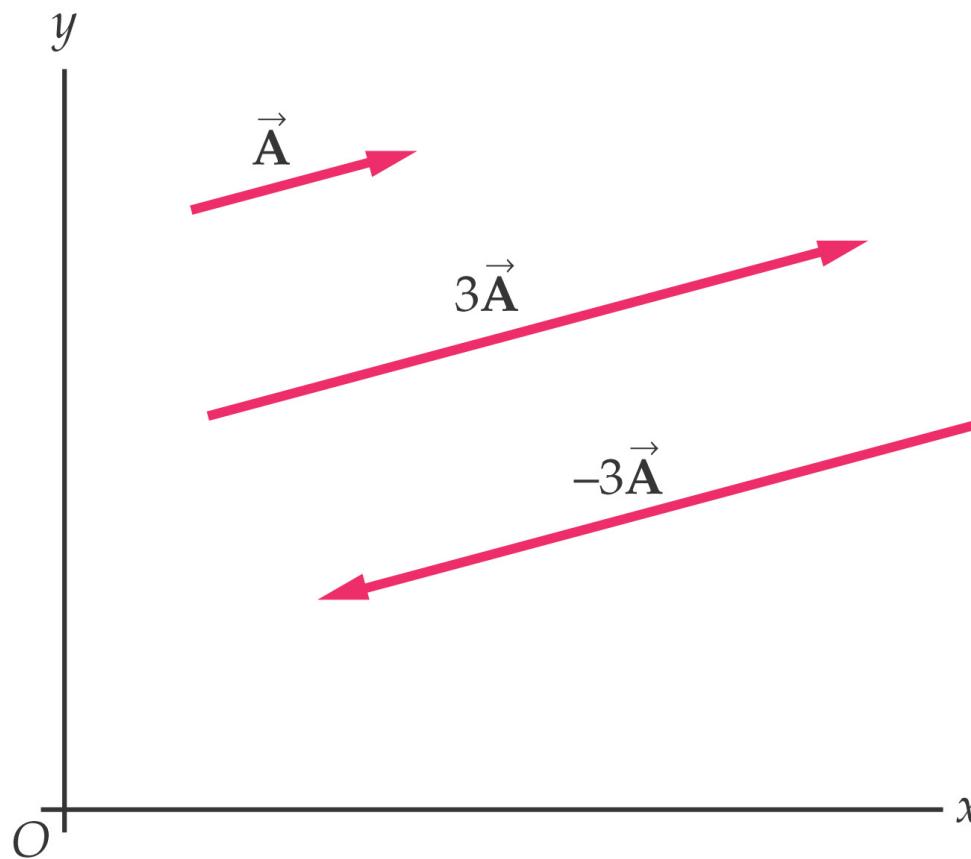
3-4 Unit Vectors

- Unit vectors are dimensionless vectors of unit length.



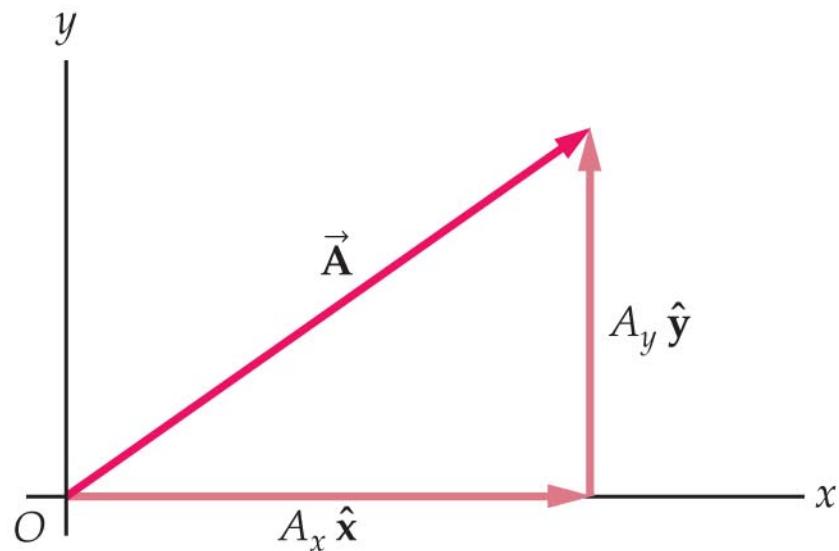
3-4 Unit Vectors

- Multiplying unit vectors by scalars: the multiplier changes the length, and the sign indicates the direction.



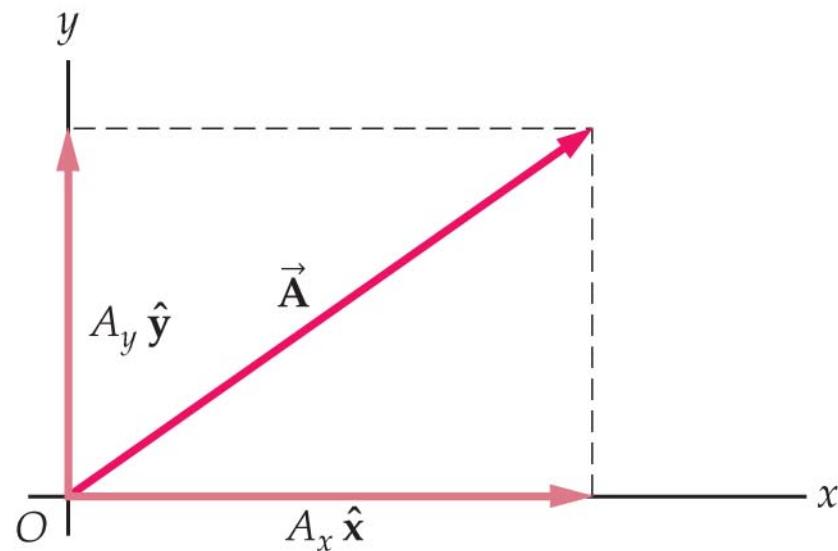
3-4 Unit Vectors

- Unit vectors provide a useful way to keep track of the x and y components of a vector:



(a)

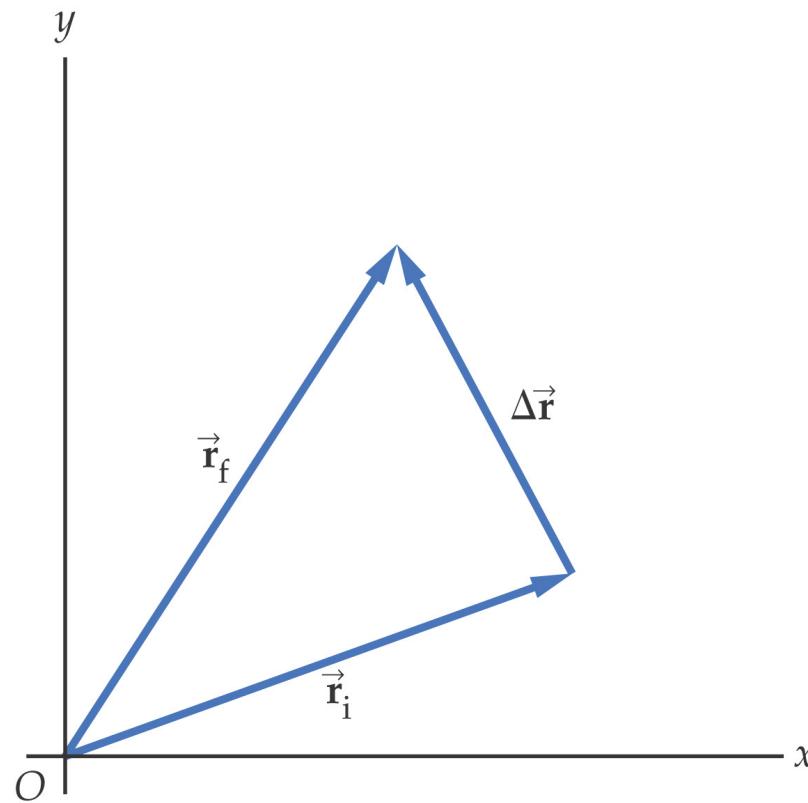
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(b)

3-5 Position, Displacement, Velocity, and Acceleration Vectors

- Position vector \vec{r}_f points from the origin to the location in question.



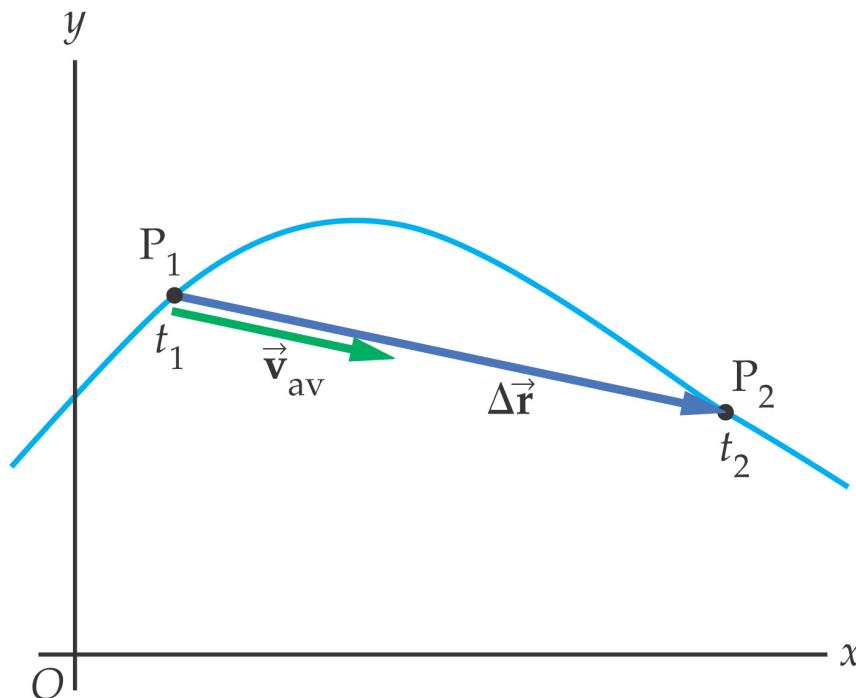
The displacement vector $\Delta\vec{r}$ points from the original position to the final position

3-5 Position, Displacement, Velocity, and Acceleration Vectors

- Average velocity vector:

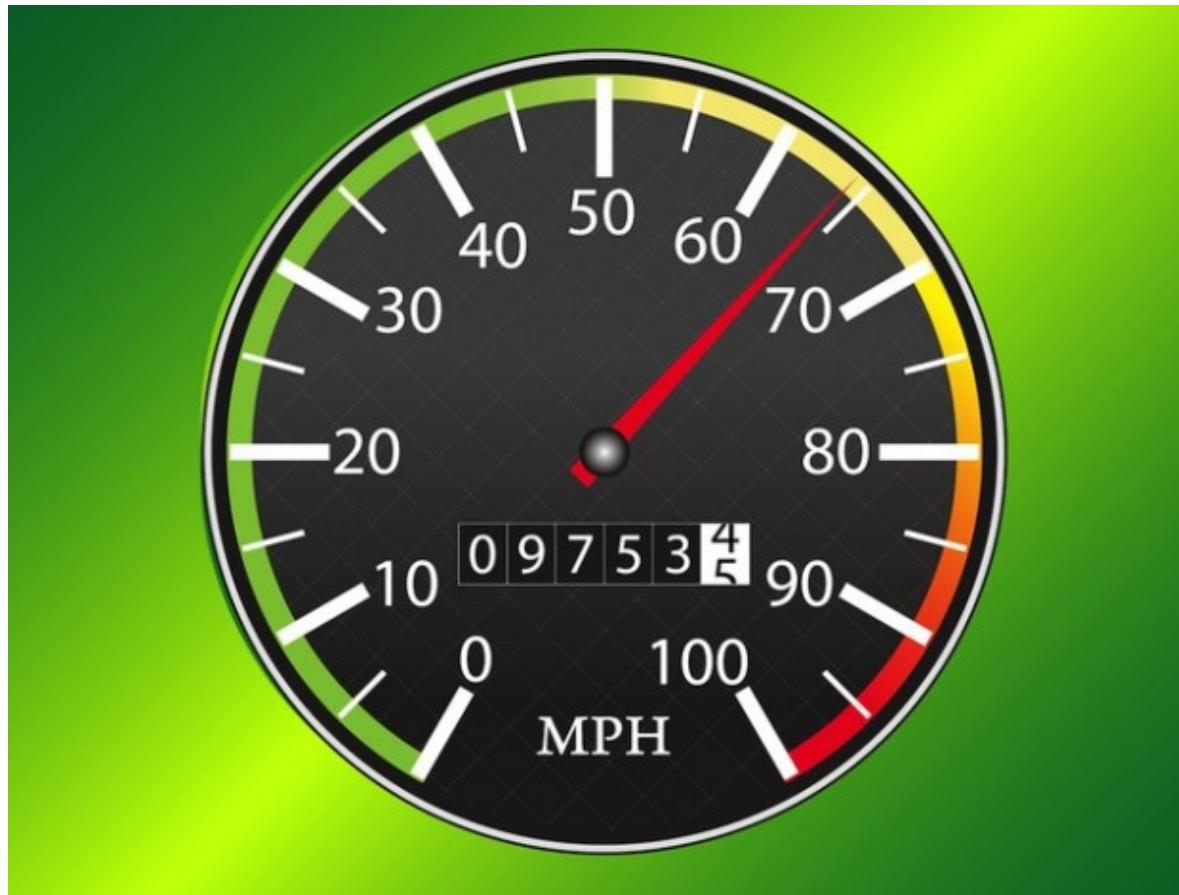
$$\vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t}$$

So \vec{v}_{av} is in the same direction as $\Delta \vec{r}$.



3-5 Position, Displacement, Velocity, and Acceleration Vectors

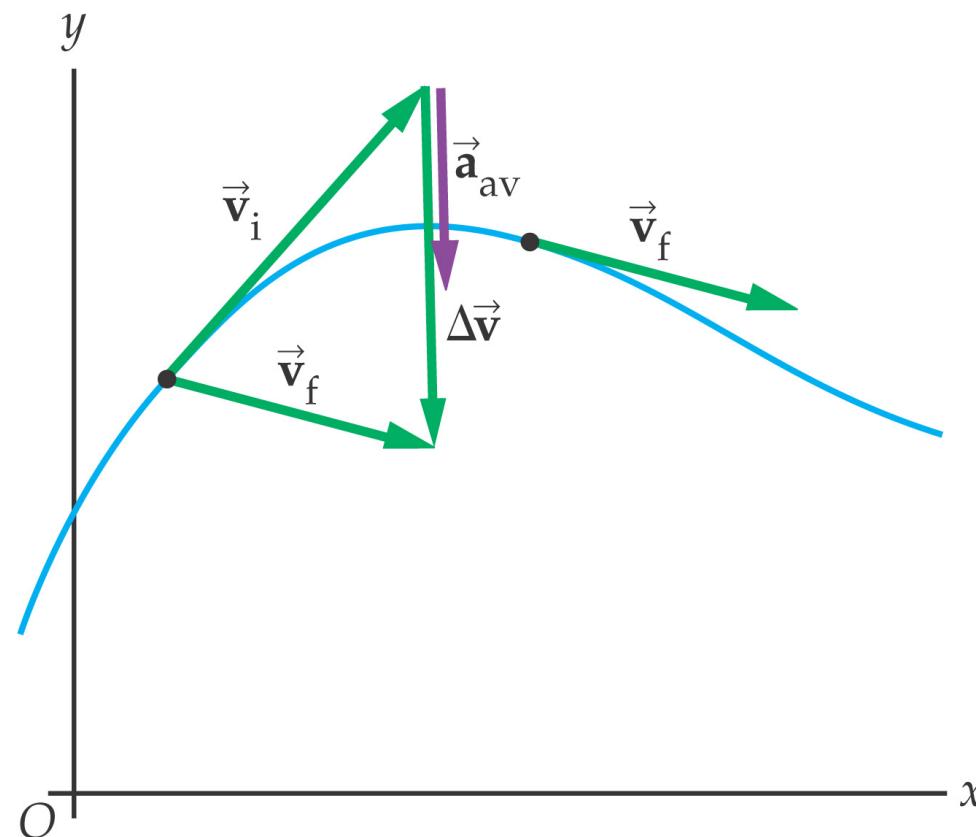
- Instantaneous velocity vector is tangent to the path:



3-5 Position, Displacement, Velocity, and Acceleration Vectors

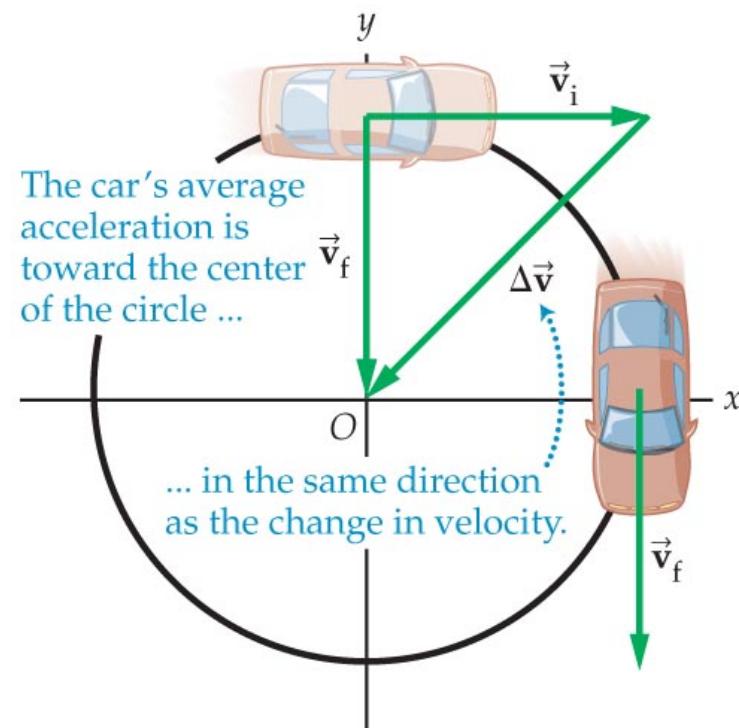
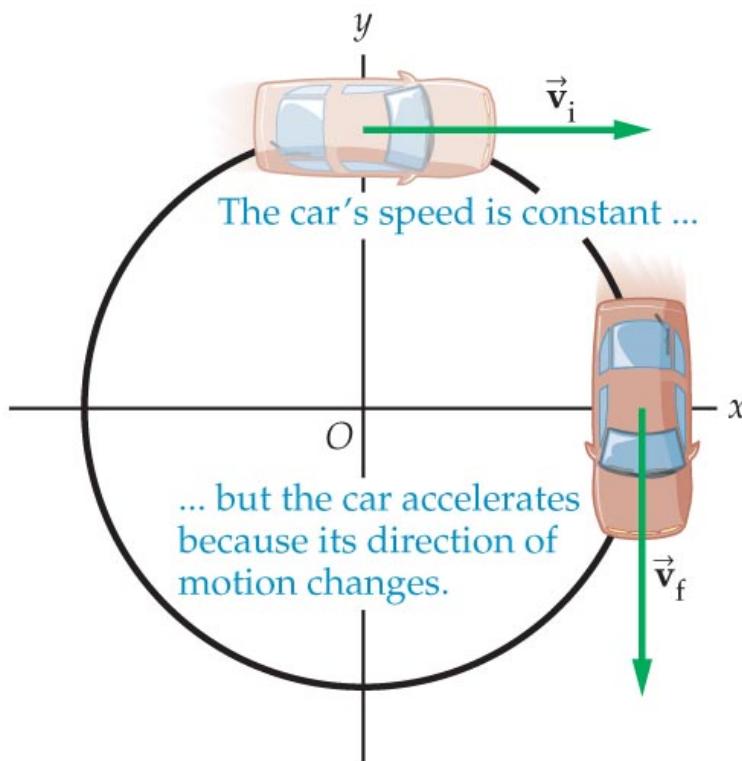
- Average acceleration vector is in the direction of the change in velocity:

$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t}$$



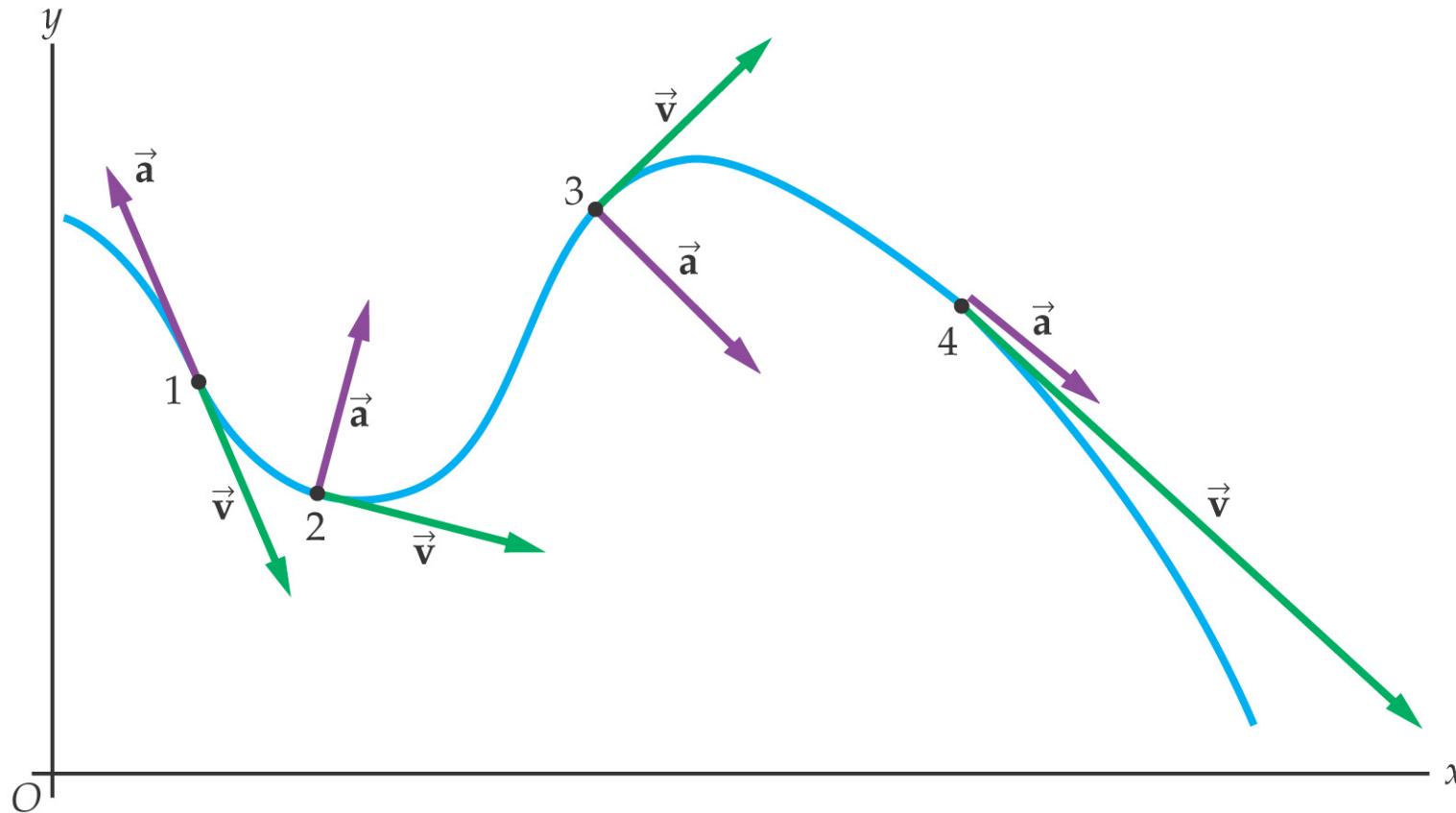
3-5 Position, Displacement, Velocity, and Acceleration Vectors

- Average acceleration is non-zero for constant speed but change of direction:



3-5 Position, Displacement, Velocity, and Acceleration Vectors

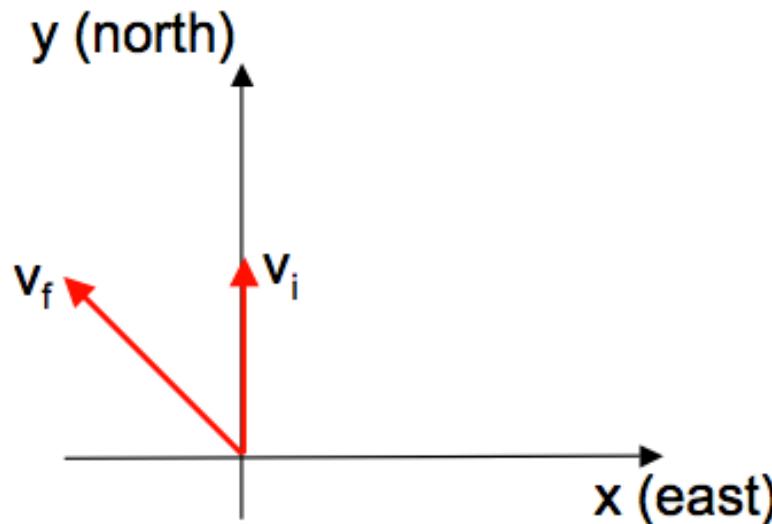
- Velocity vector is always in the direction of motion; acceleration vector can point anywhere:



Example

- At the beginning of a 3 hour plane trip you are traveling due north at 192 km/hour. At the end, you are traveling 240 km/hour at 45° west of north.

(a) Draw the initial and final velocity vectors.



Example continued

(b) Find $\Delta\mathbf{v}$

The components are

$$\Delta v_x = v_{fx} - v_{ix} = -v_f \sin 45^\circ - 0 = -170 \text{ km/hr}$$

$$\Delta v_y = v_{fy} - v_{iy} = +v_f \cos 45^\circ - v_i = -22.3 \text{ km/hr}$$

The magnitude and direction are:

$$|\Delta\mathbf{v}| = \sqrt{\Delta v_x^2 + \Delta v_y^2} = 171 \text{ km/hr}$$

$$\tan \varphi = \frac{|\Delta v_y|}{|\Delta v_x|} = 0.1312 \Rightarrow \phi = \tan^{-1}(0.1312) = 7.5^\circ$$

South of
west

Example continued

(c) What is \mathbf{a}_{av} during the trip?

$$\mathbf{a}_{av} = \frac{\Delta \mathbf{v}}{\Delta t}$$

$$a_{x,av} = \frac{\Delta v_x}{\Delta t} = \frac{-170 \text{ km/hr}}{3 \text{ hr}} = -56.7 \text{ km/hr}^2$$

$$a_{y,av} = \frac{\Delta v_y}{\Delta t} = \frac{-22.3 \text{ km/hr}}{3 \text{ hr}} = -7.43 \text{ km/hr}^2$$

The magnitude and direction are:

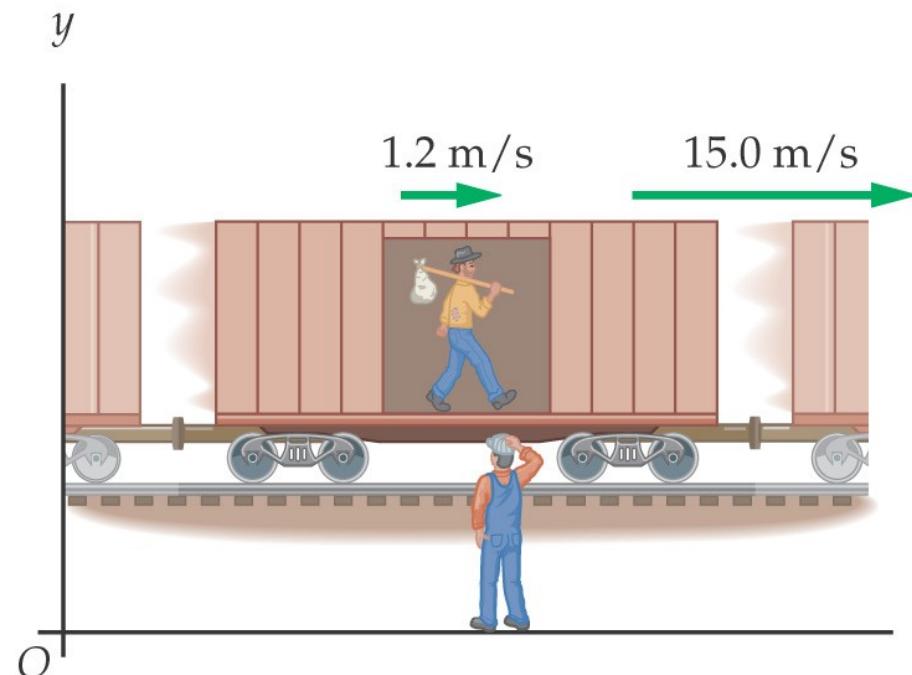
$$|\mathbf{a}_{av}| = \sqrt{a_{x,av}^2 + a_{y,av}^2} = 57.2 \text{ km/hr}^2$$

$$\tan \phi = \frac{|a_{y,av}|}{|a_{x,av}|} = 0.1310 \Rightarrow \phi = \tan^{-1}(0.1310) = 7.5^\circ$$

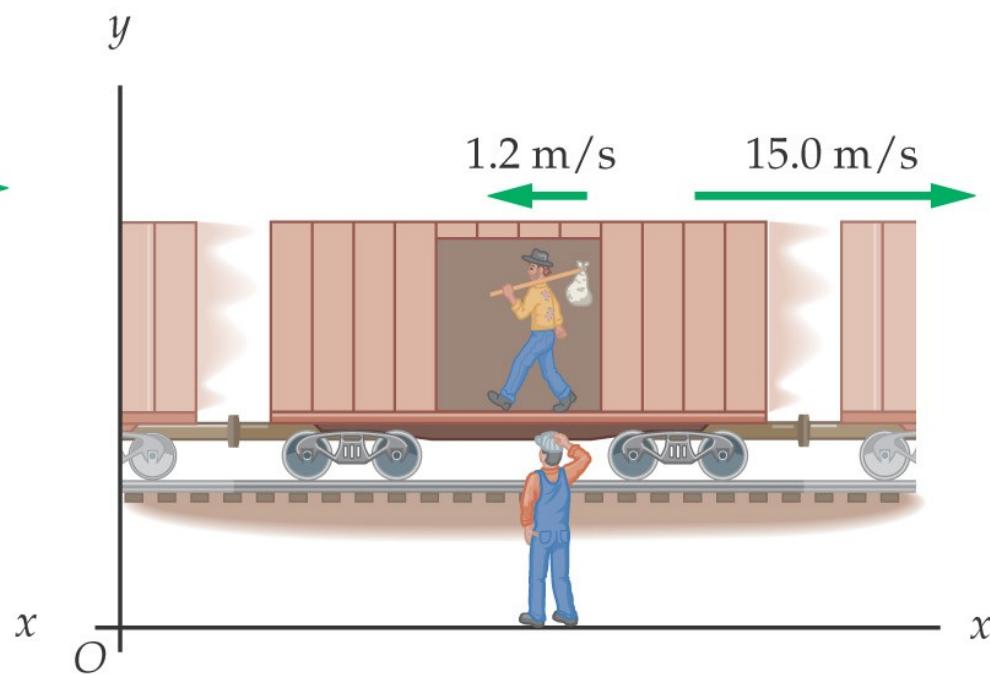
South of west

3-6 Relative motion

- The speed of the passenger with respect to the ground depends on the relative directions of the passenger's and train's speeds:



(a)

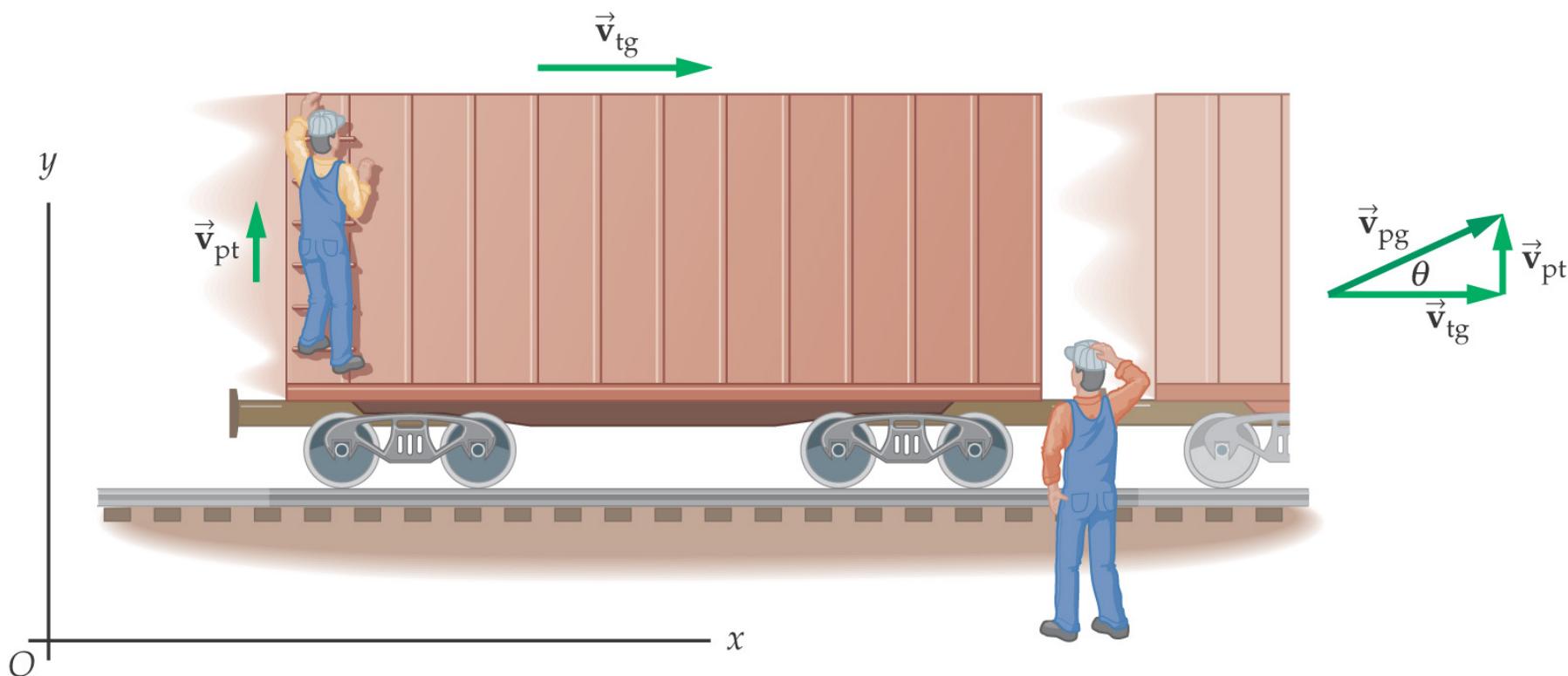


(b)

3-6 Relative motion

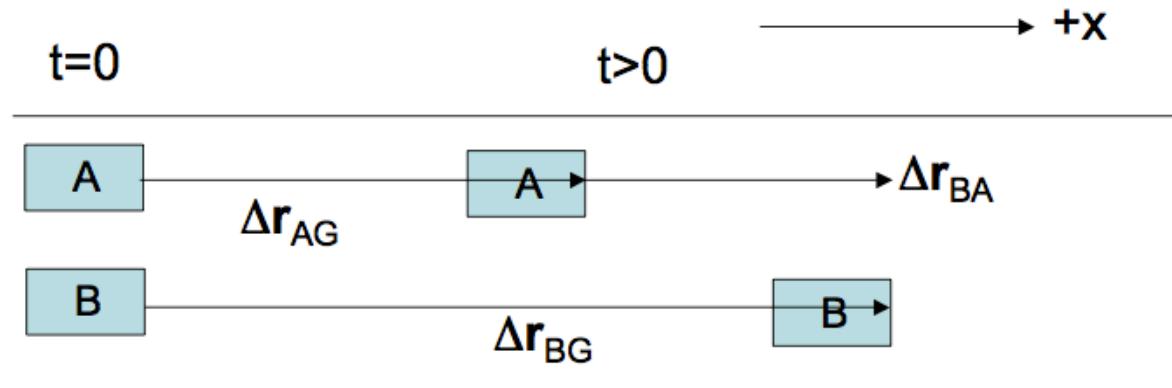
$$\vec{v}_{pg} = \vec{v}_{pt} + \vec{v}_{tg}$$

- This also works in two dimensions:



Example

- You are traveling in a car (A) at 60 miles/hour east on a long straight road. The car (B) next to you is traveling at 65 miles/hour east. What is the speed of car B relative to car A?



$$\text{From the picture: } \Delta r_{BG} = \Delta r_{AG} + \Delta r_{BA}$$

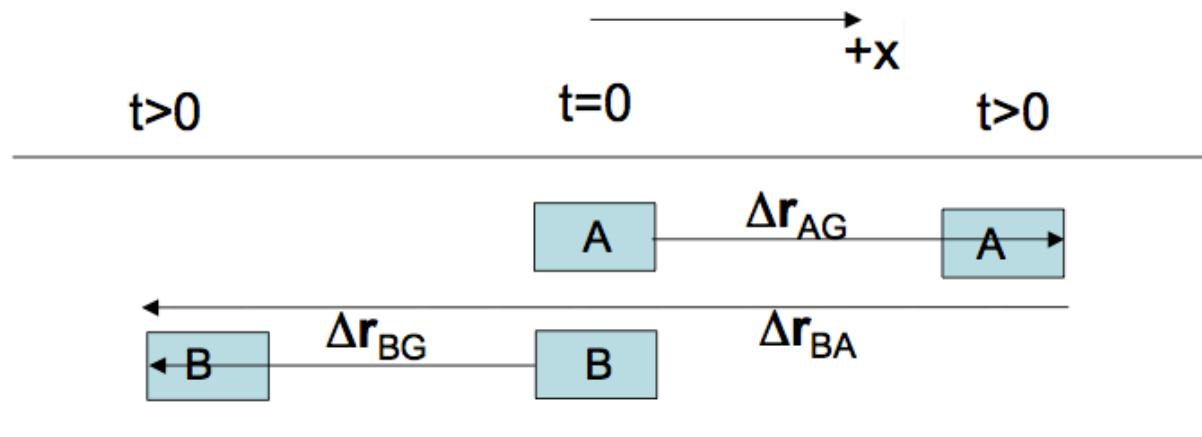
$$\Delta r_{BA} = \Delta r_{BG} - \Delta r_{AG}$$

$$\text{Divide by } \Delta t: \quad \mathbf{v}_{BA} = \mathbf{v}_{BG} - \mathbf{v}_{AG}$$

$$\begin{aligned}\mathbf{v}_{BA} &= 65 \text{ miles/hr east} - 60 \text{ miles/hr east} \\ &= 5 \text{ miles/hour east}\end{aligned}$$

Example

- You are traveling in a car (A) at 60 miles/hour east on a long straight road. The car (B) next to you is traveling at 65 miles/hour west. What is the speed of car B relative to car A?



From the picture: $\Delta r_{BA} = \Delta r_{BG} - \Delta r_{AG}$

$$\begin{aligned}
 \text{Divide by } \Delta t: \quad \mathbf{v}_{BA} &= \mathbf{v}_{BG} - \mathbf{v}_{AG} \\
 &= 65 \text{ miles/hr west} - 60 \text{ miles/hr east} \\
 &= 125 \text{ miles/hr west}
 \end{aligned}$$

Example

- A plane is headed eastward at a speed of 156 m/s relative to the wind. A 20.0 m/s wind is blowing southward at the same time as the plane is flying. The velocity of the plane relative to the ground is:
 - A) 155 m/s at an angle 7.36° south of east
 - B) 155 m/s at an angle 7.36° east of south
 - C) 157 m/s at an angle 7.36° south of east
 - D) 157 m/s at an angle 7.31° south of east
 - E) 157 m/s at an angle 7.31° east of south

Summary of Chapter 3

- Scalar: number, with appropriate units
- Vector: quantity with magnitude and direction
- Vector components: $A_x = A \cos \theta$, $B_y = B \sin \theta$
- Magnitude: $A = (A_x^2 + A_y^2)^{1/2}$
- Direction: $\theta = \tan^{-1} (A_y / A_x)$
- Graphical vector addition: Place tail of second at head of first; the sum points from tail of first to head of last

Summary of Chapter 3

- Component method: add components of individual vectors, then find magnitude and direction
- Unit vectors are dimensionless and of unit length
- Position vector points from origin to location
- Displacement vector points from original position to final position
- Velocity vector points in direction of motion
- Acceleration vector points in direction of change of motion
- Relative motion: $\mathbf{v}_{13} = \mathbf{v}_{12} + \mathbf{v}_{23}$