Physics 2B Master Class in Electricity & Magnetism

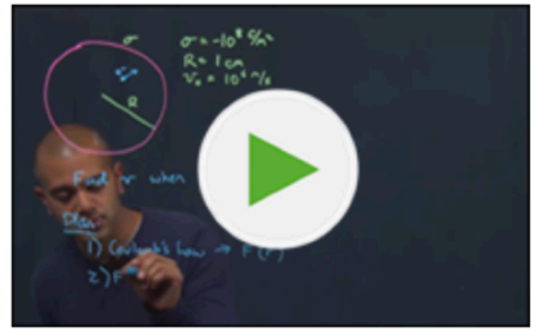
Welcome to the Physics 2B Master Class in Electricity & Magnetism video site.

Before watching each video, please download the supplementary solution so that you can follow along with the algebra which will be skimmed over in the video. You are encouraged to pause the video at the problem statement, read the problem a few times, and attempt it on your own to see where you get stuck. Please be sure to leave feedback about each video so that we can gear the videos towards whatever you find most useful. We hope these will be a great tool for you throughout the course!

Coulomb's Law [13:05]

Solution Feedback

Coulomb's Law and Charged Shells [9:34]

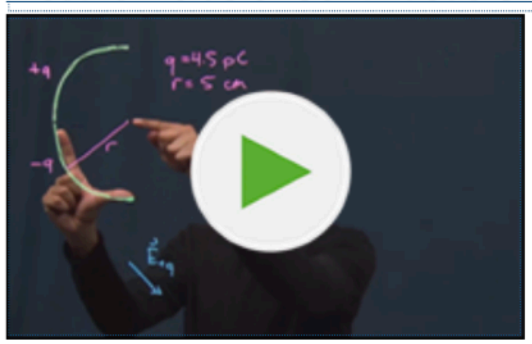


Solution Feedback

Electric Fields of Point Charges [10:45]

Solution Feedback

Electric Fields of Continuous Charge [13:59]



Solution Feedback

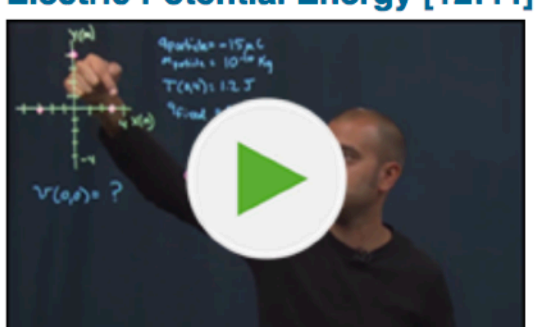
Gauss' Law 1 [11:52]

Solution Feedback

Gauss' Law II [9:27]

Solution Feedback

Electric Potential Energy [12:44]

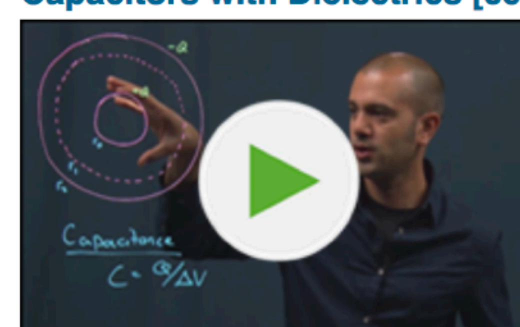


Solution Feedback

Calculating Electric Fields from Electric Potentials [09:09]

Solution Feedback

Capacitors with Dielectrics [09:32]

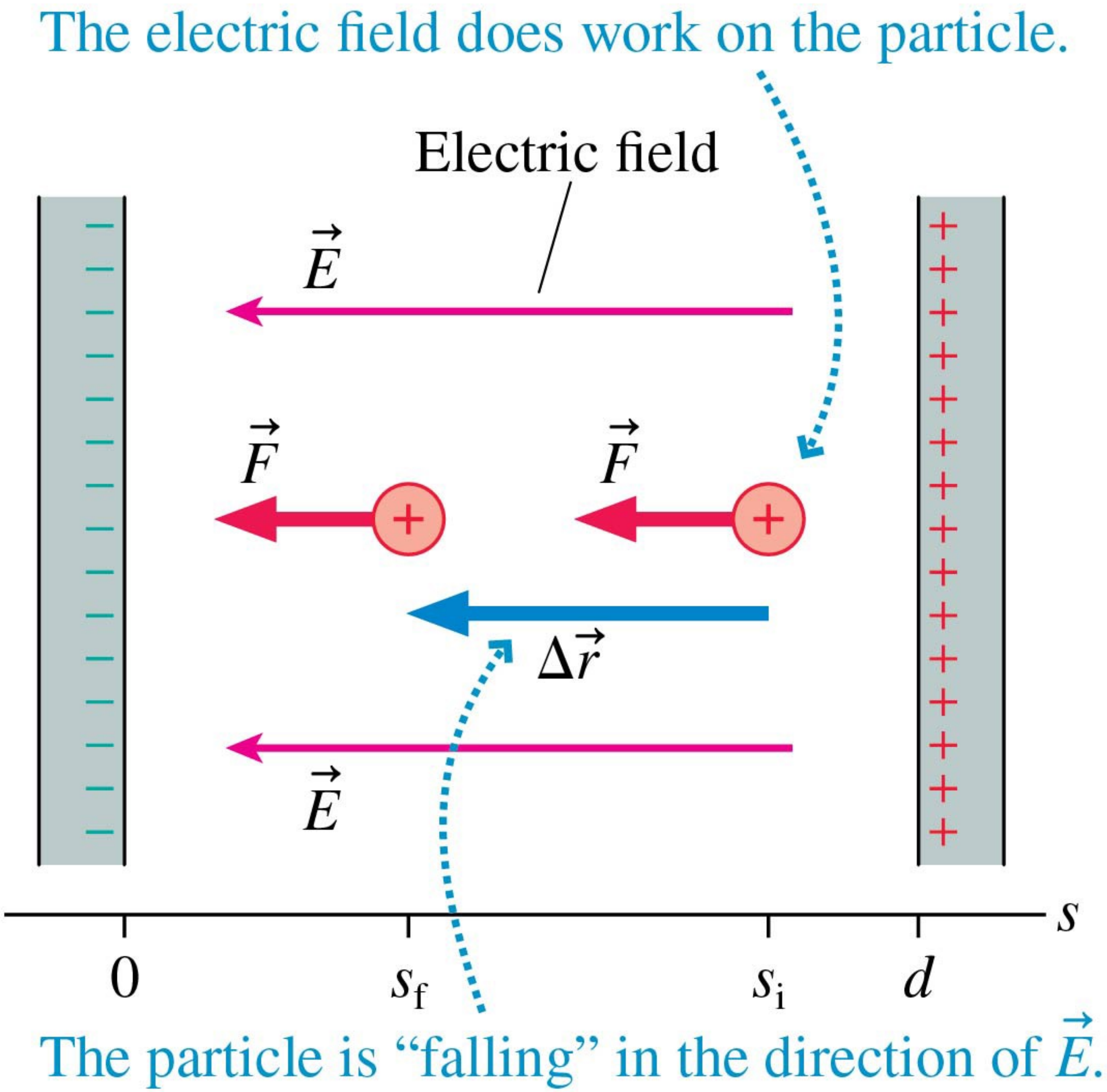


Solution Feedback

Electric Potential **Energy** in a Uniform Field

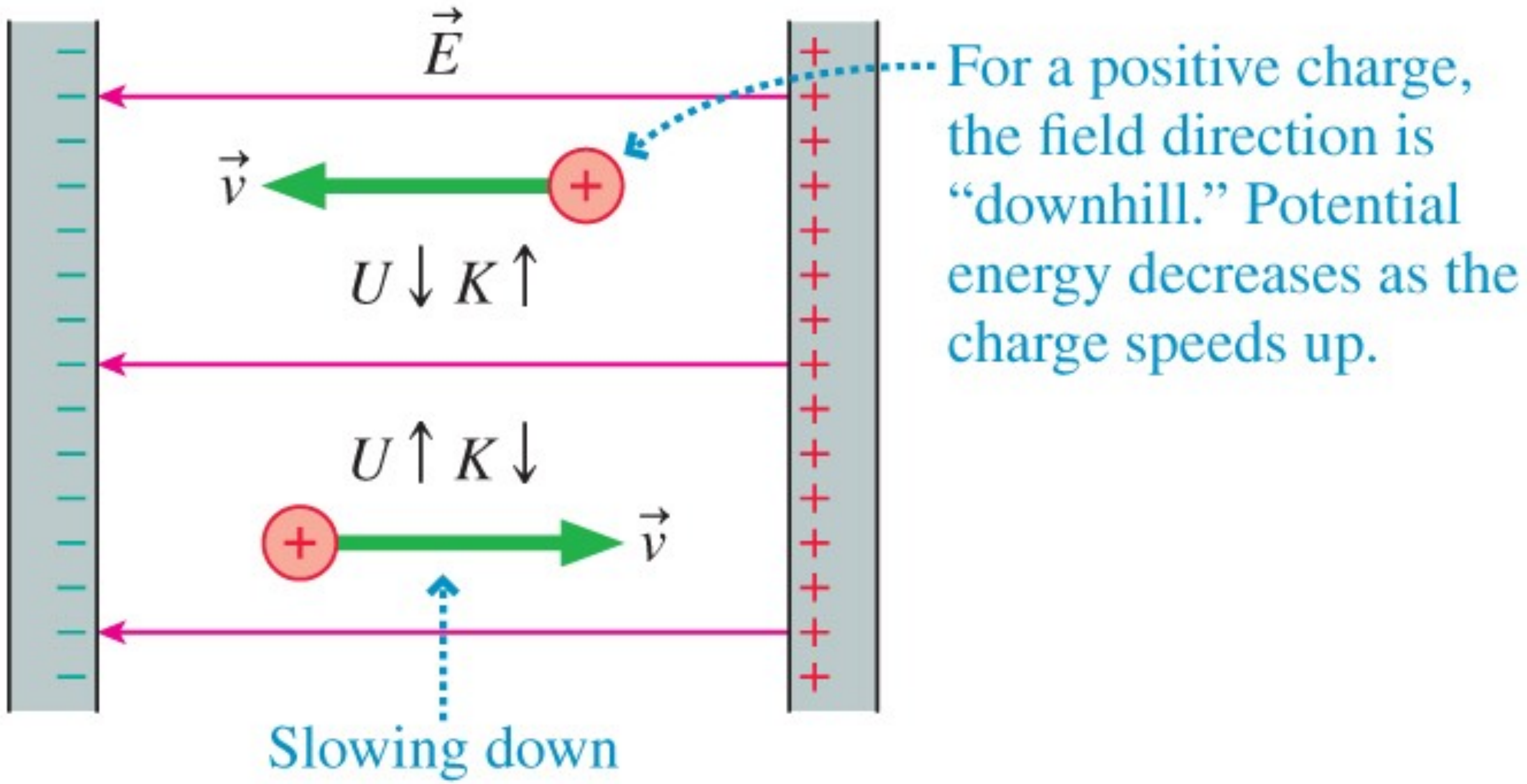
- A positive charge inside a capacitor speeds up and gains kinetic energy as it "falls" toward the negative plate.
- The charge is losing potential energy as it gains kinetic energy.

$$U_{\text{elec}} = U_0 + qEs$$



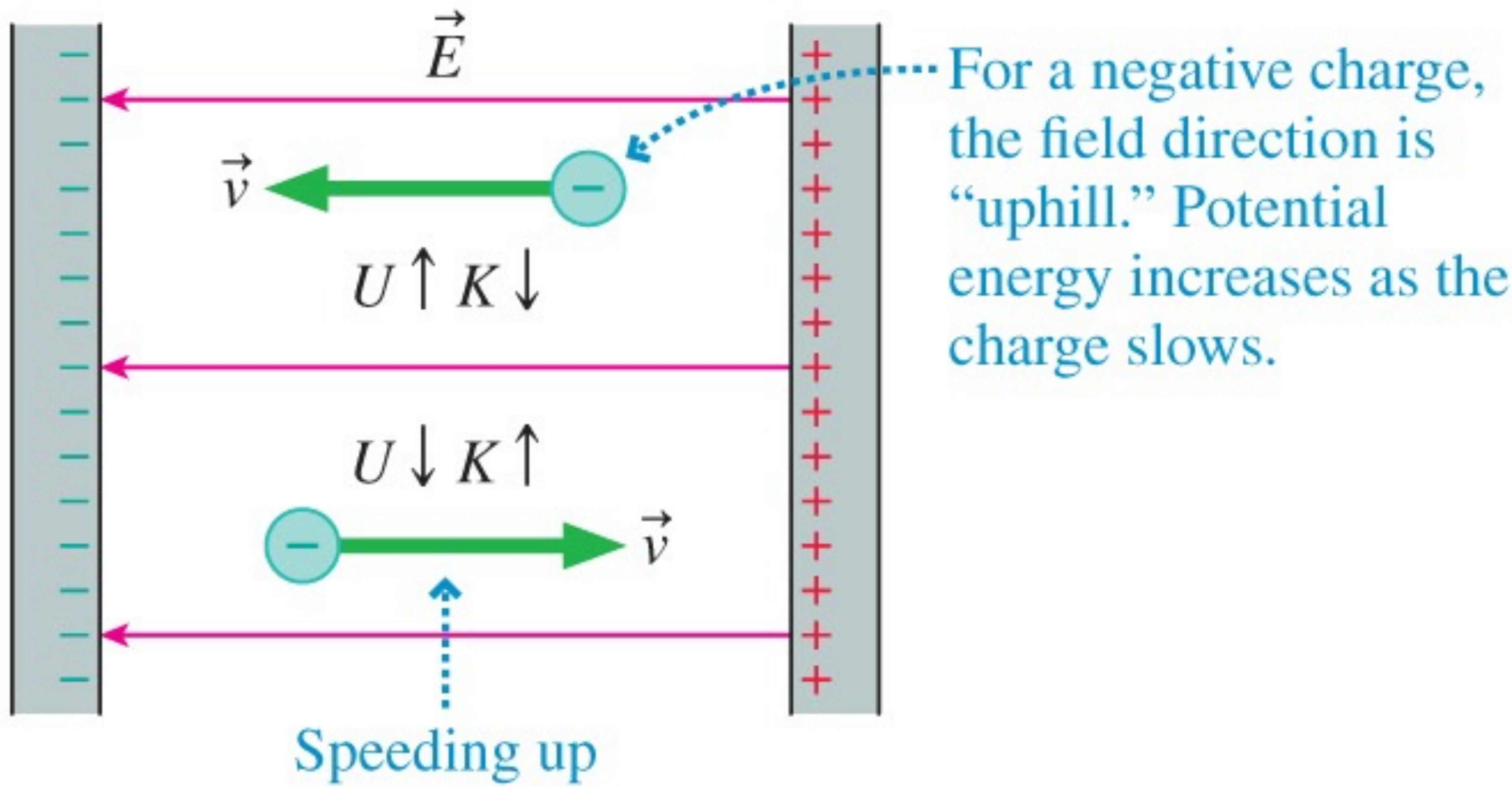
Electric Potential Energy in a Uniform Field

- For a positive (+) charge, U decreases and K increases as the charge moves toward the negative plate.
- A positive charge moving opposite the field direction is going "uphill," slowing as it transforms kinetic energy into electric potential energy.



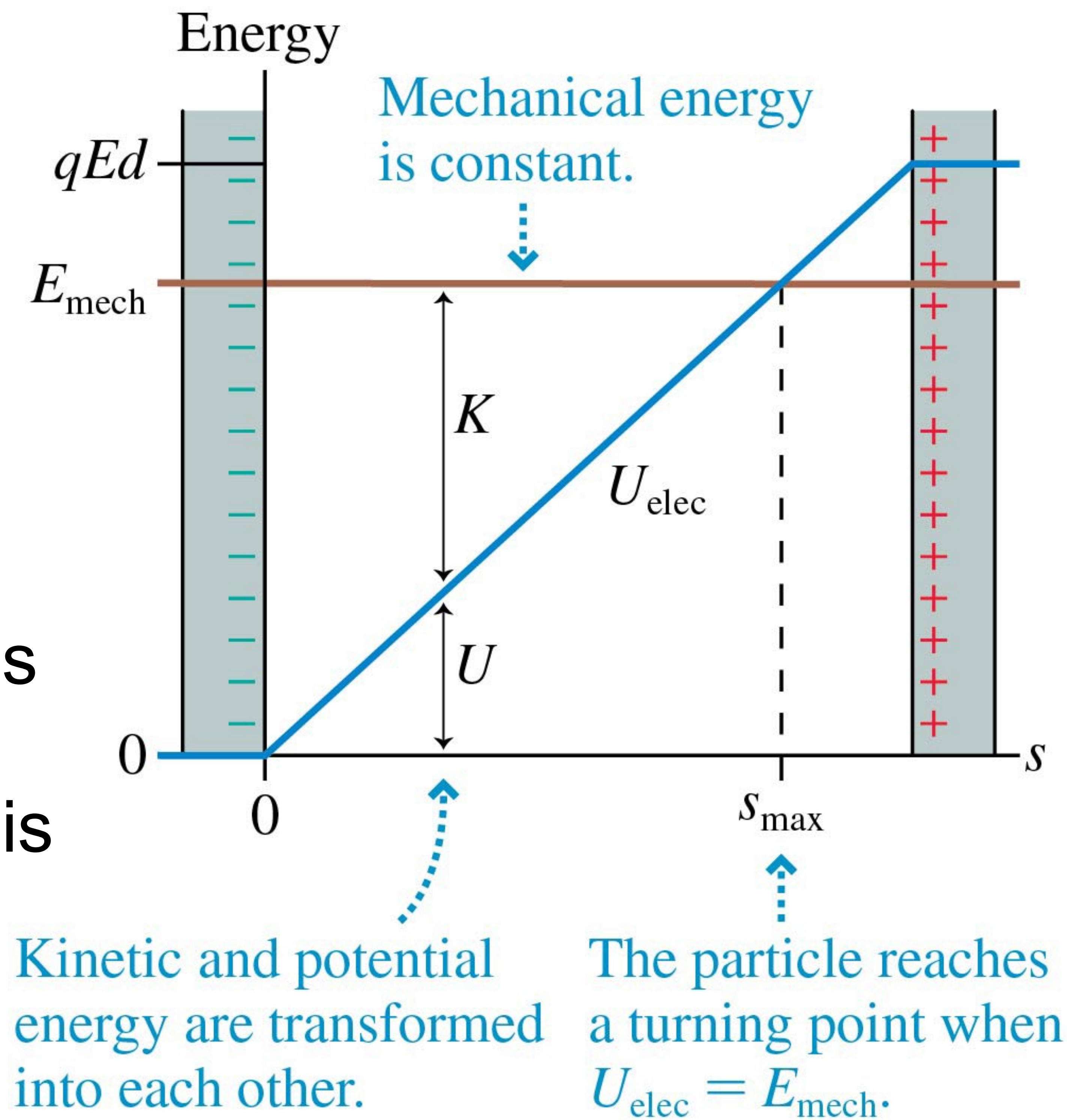
Electric Potential **Energy** in a Uniform Field

- A negative (-) charged particle has *negative* potential energy.
- U increases (becomes less negative) as the negative charge moves toward the negative plate.
- A negative charge moving in the field direction is going "uphill," transforming $K \rightarrow U$ as it slows.



Electric Potential Energy in a Uniform Field

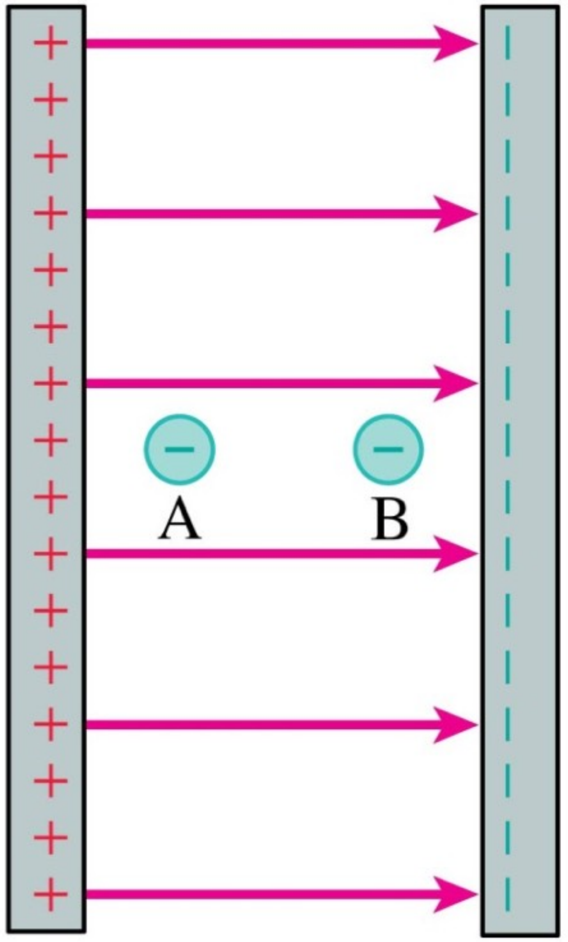
- The figure shows the **energy diagram** for a positively charged particle in a uniform electric field.



The potential energy increases linearly with distance, but the total mechanical energy E_{mech} is fixed.

iClicker question 7-1

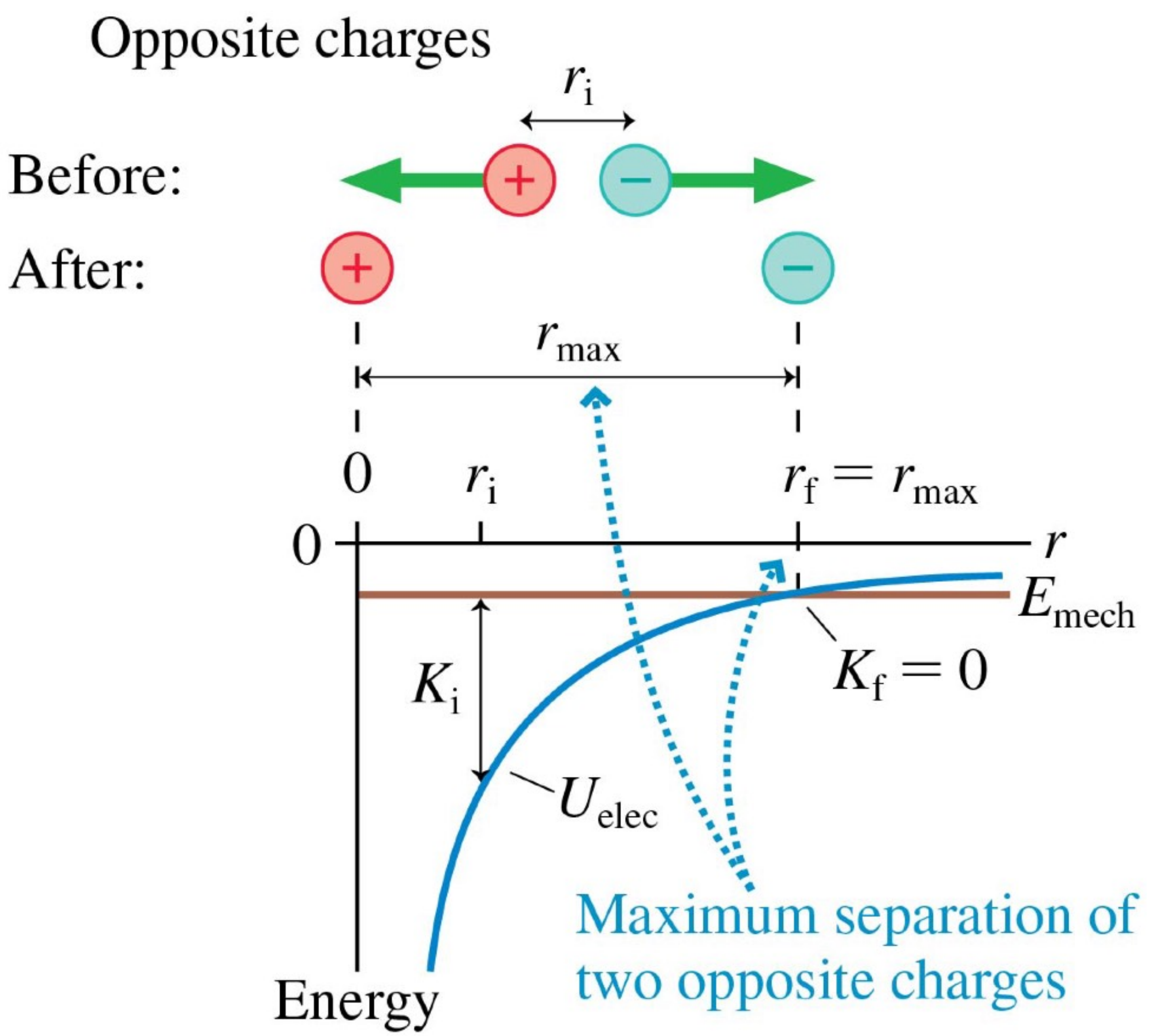
Two negative charges are equal. Which has more electric potential energy?



A. Charge A
B. Charge B
C. They have the same potential energy.
D. Both have zero potential energy.

The Potential **Energy** of Two Point Charges

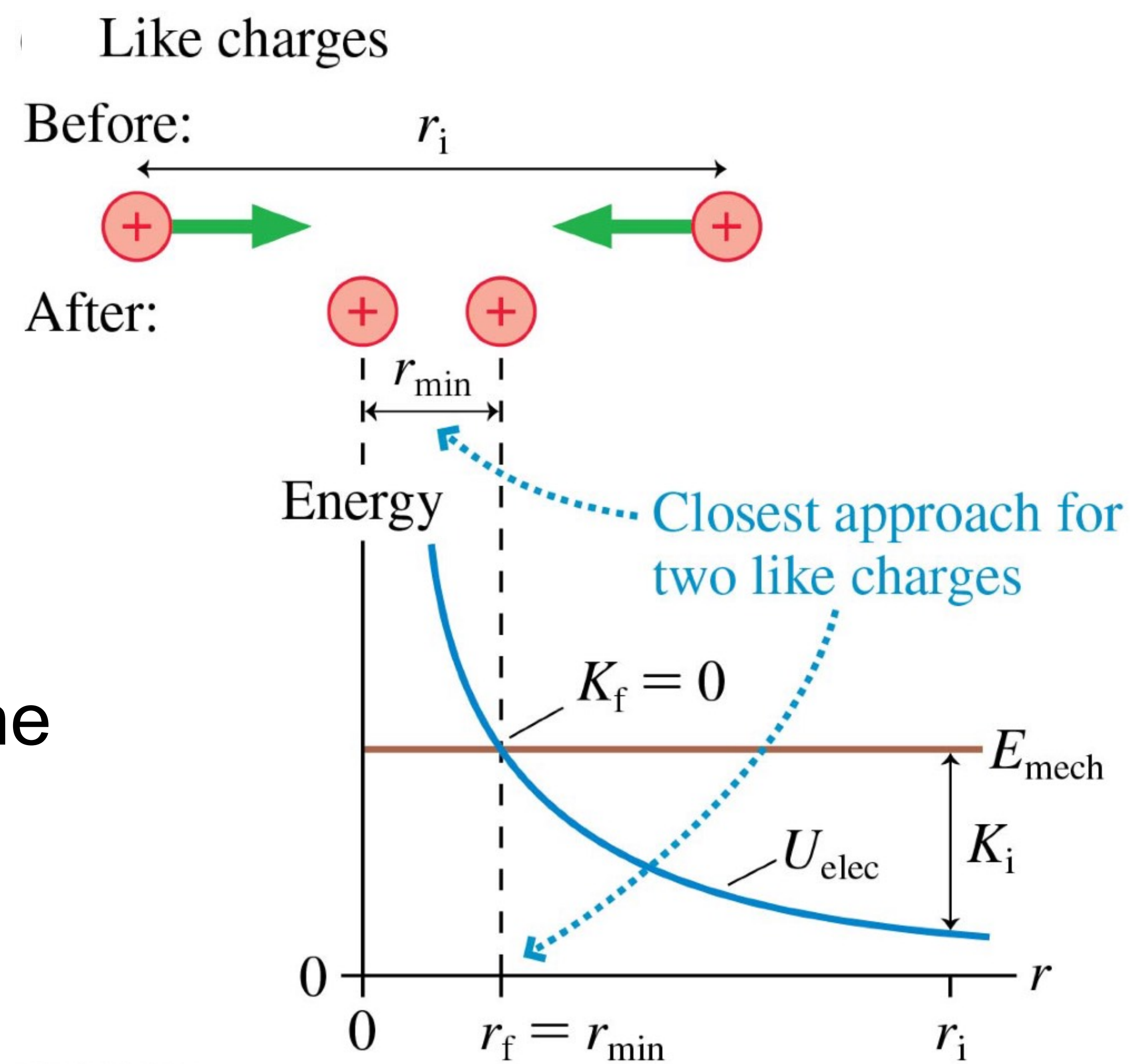
- Two opposite charges are shot apart from one another with equal and opposite momenta.
- Their total energy is $E_{\text{mech}} < 0$.
- They gradually slow down until the distance separating them is r_{max}.
- This is their *maximum separation.*



$$U_{\text{elec}} = \frac{Kq_1q_2}{x}$$

The Potential **Energy** of Two Point Charges

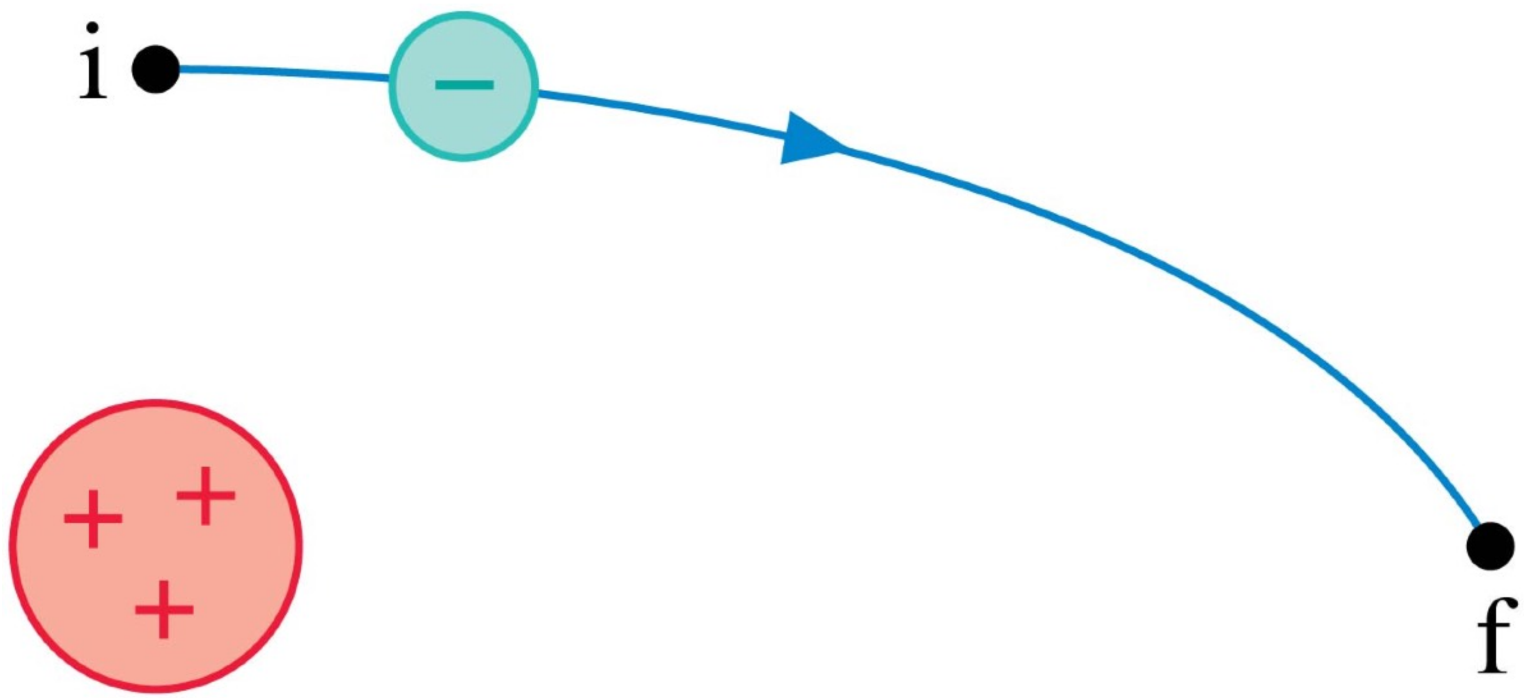
- Two like charges approach each other:



- Their total energy is $E_{mech} > 0$.
- They gradually slow down until the distance separating them is r_{min}.
- This is the *distance of closest approach*.

$$U_{elec} = \frac{Kq_1q_2}{x}$$

An electron follows the trajectory shown from i to f. At point f,



A. $v_f > v_i$

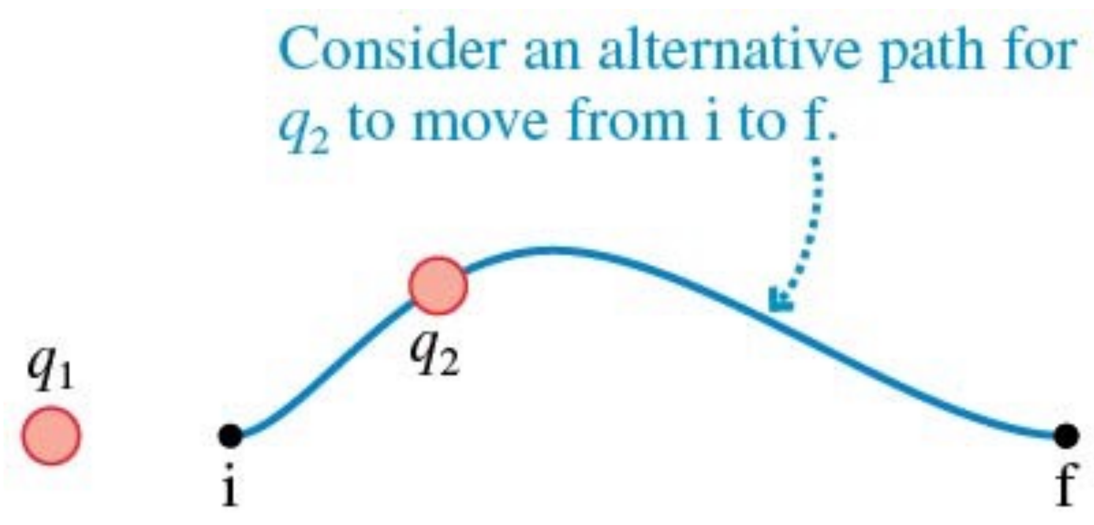
B. $v_f = v_i$

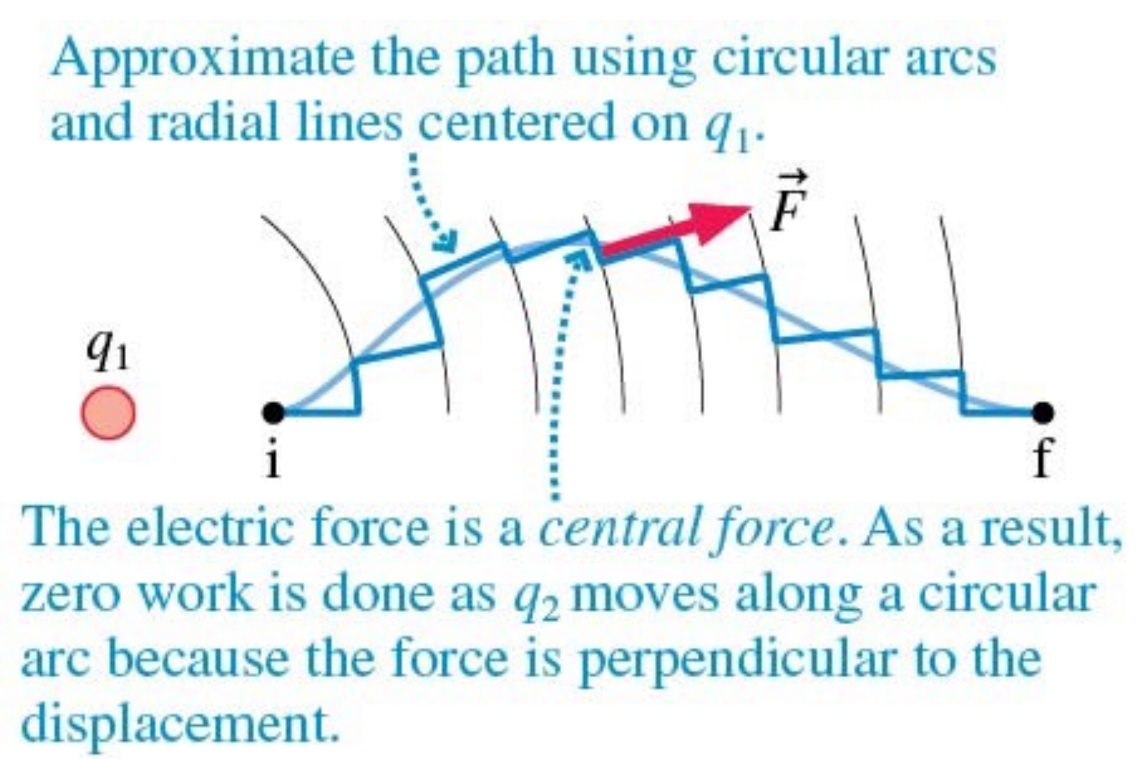
C. $v_f < v_i$

D. Not enough information to compare the speeds at these points.

The Electric Force Is a Conservative Force

- Any path away from q_1 can be approximated using circular arcs and radial lines.
- All the work is done along the radial line segments, which is equivalent to a straight line from i to f.
- Therefore the work done by the electric force depends only on initial and final position, not the path followed.





The Electric Potential

- We define the electric potential V (or, for brevity, just the potential) as

$$V \equiv \frac{U_{q+\text{sources}}}{q}$$

- The unit of electric potential is the joule per coulomb, which is called the **volt** $\mathbf{V}$:

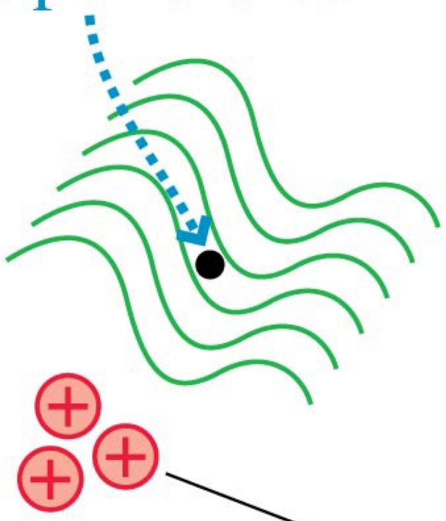
$$1 \text{ volt} = 1 \text{ V} \equiv 1 \text{ J/C}$$

This battery is a source of *electric potential*. The electric potential difference between the + and – sides is 1.5 V.

NEW CONCEPT: The Electric Potential

- Test charge q is used as a probe to determine the electric potential, but the value of V is *independent of* q.

- **The electric potential, like the electric field, is a property of the source charges.**

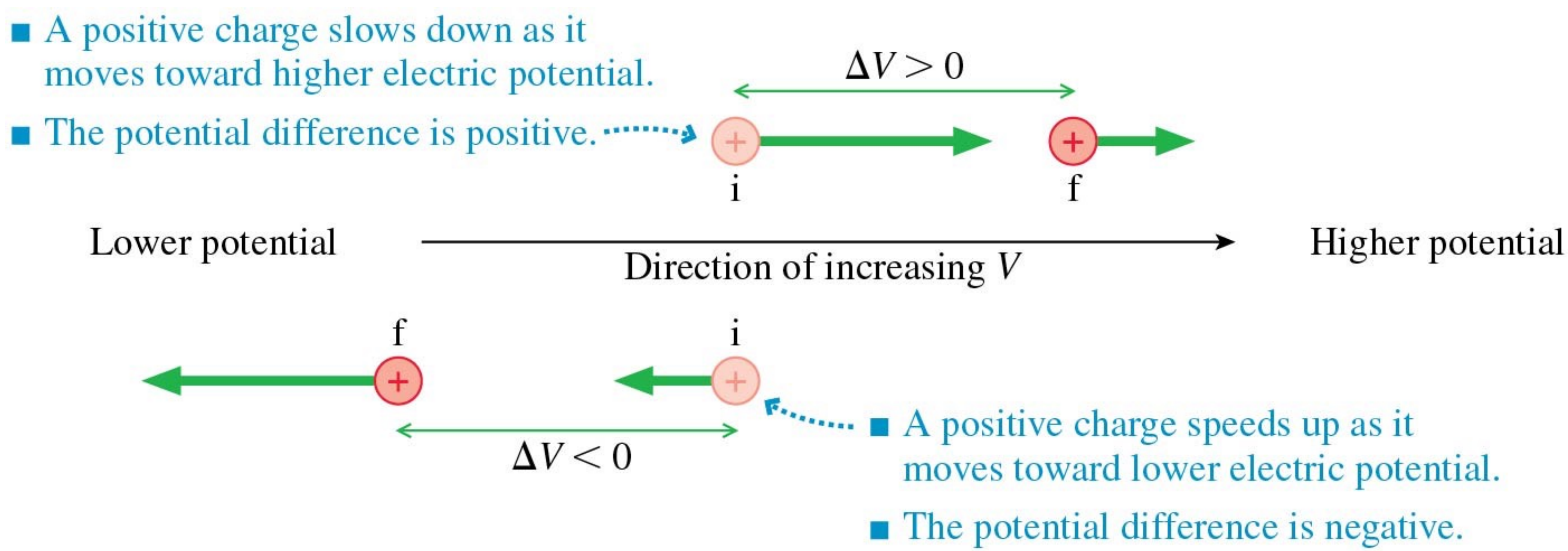


Using the Electric Potential

- As a charged particle moves through a changing electric potential, energy is conserved:

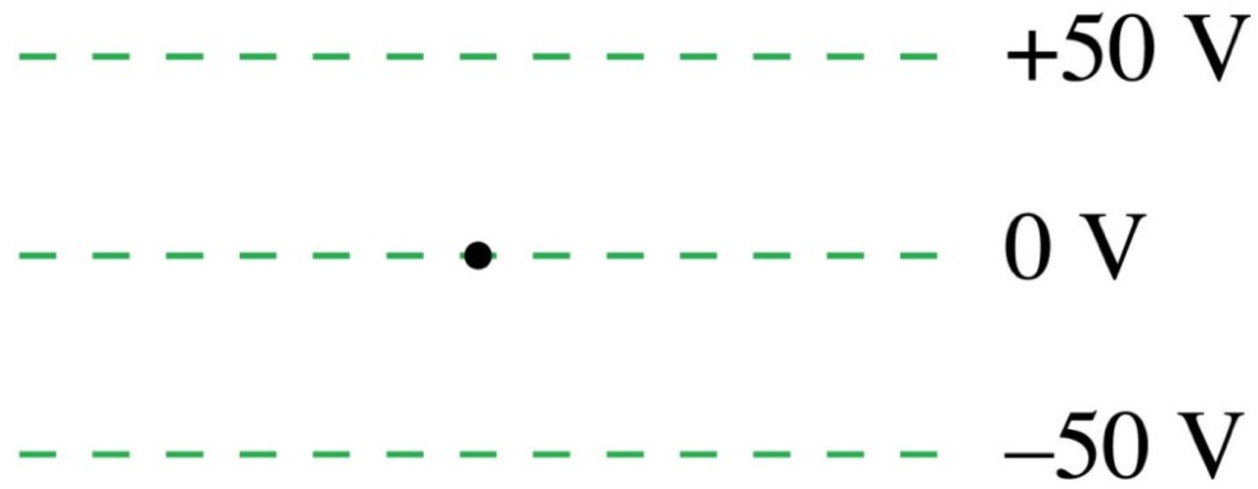
$$K_\mathrm{f} + qV_\mathrm{f} = K_\mathrm{i} + qV_\mathrm{i}$$

	Electric potential	
	Increasing ($\Delta V > 0$)	Decreasing ($\Delta V < 0$)
+ charge	Slows down	Speeds up
− charge	Speeds up	Slows down



iClicker question 7-3

A proton is released from rest at the dot. Afterward, the proton

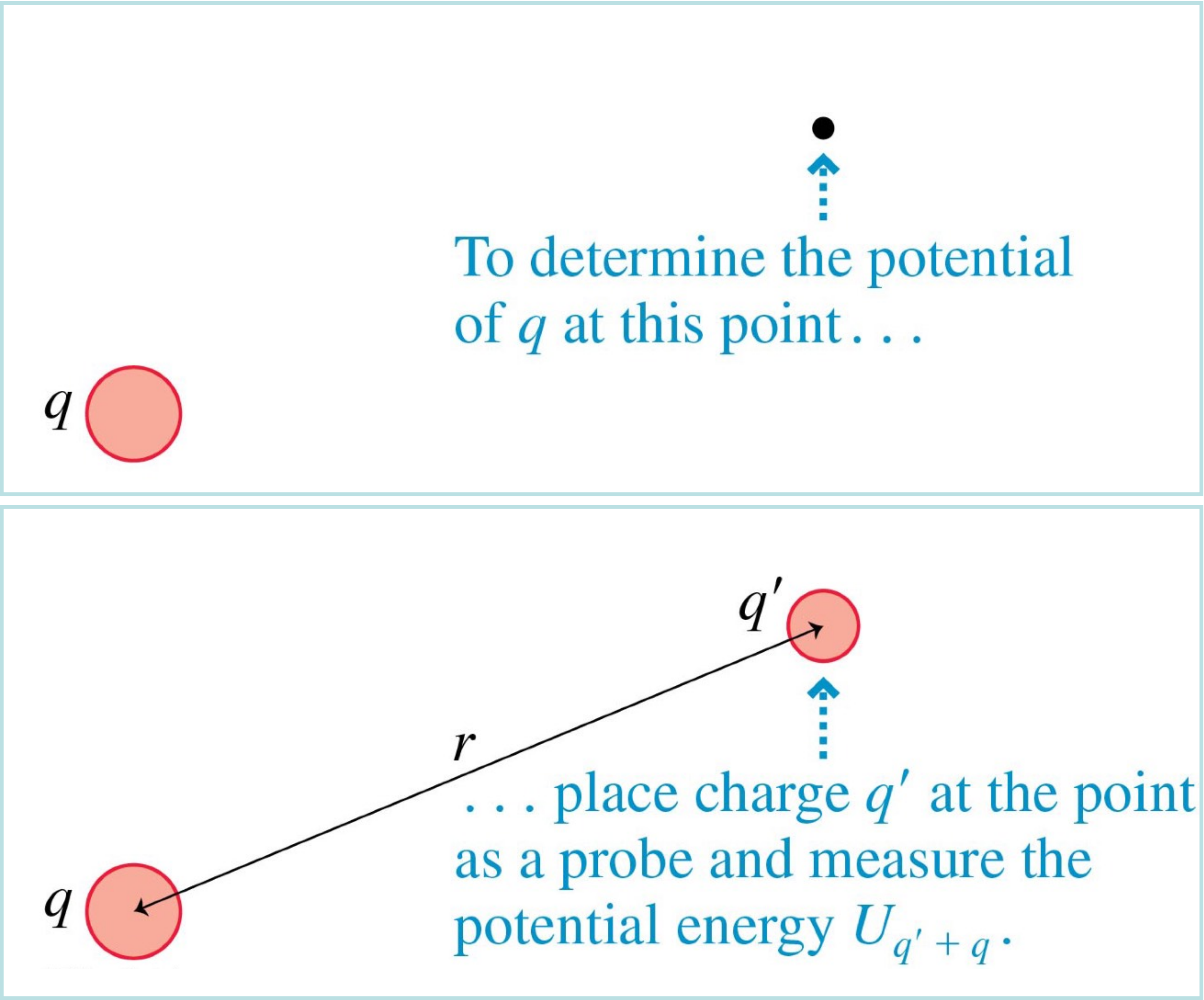


A. Remains at the dot.
B. Moves upward with steady speed.
C. Moves upward with an increasing speed.
D. Moves downward with a steady speed.
E. Moves downward with an increasing speed.

The Electric Potential of a Point Charge

- Let q in the figure be the source charge, and let a second charge q', a distance r away, probe the electric potential of q.
- The potential energy of the two point charges is

$$U_{q'+q} = \frac{1}{4\pi\epsilon_0}\frac{qq'}{r}$$



The Electric Potential of a Point Charge

- The electric potential due to a point charge q is

$$V = \frac{U_{q'+q}}{q'} = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \qquad \text{(electric potential of a point charge)}$$

- The potential extends through all of space, showing the influence of charge q, but it weakens with distance as $1/r$.
- This expression for V assumes that we have chosen $V = 0$ to be at $r = \infty$.

Electric Potential

- The lessons we have learned from examining the electric potential **energy** and the electric potential are that:
- **Electric potential** only depends on your position in the electric field.
- **Electric potential** **energy** will **also** depend on what your test charge is.
- No matter what, if a charge is free to move in an applied electric field, it will move in the direction that lowers its potential energy.

electron Volt

- It becomes very inconvenient to work with Joules when you are dealing with electrons or protons.
- We then introduce a new unit, the electron Volt.
- The electron Volt (eV) is defined as the energy that an electron gains when accelerated through a potential difference of 1Volt.
- $$1\text{eV} = 1.602\times10^{-19}\text{J}$$
- An electron in a normal atom has about 10 eV while gamma rays (light) may have millions of eV.

Voltage

- Voltage is 1.5V
  electron gains 1.5 eV

- High Voltage TV: 15,000V
  electron gains 15 keV

Voltage

- Voltage is 1,000,000 V a million volts! (or 1MV)

  1 electron gains potential energy 1 MeV

Large Electron Accelerators

- Advanced Photon Source, Argonne

(near Chicago)

- Electrons are accelerated to 7 Billion Volts

7,000,000,000 V

or 7GV

- Electron Energy is 7 GeV

Large Electron Accelerators

- Electrons are accelerated to 14 Billion Volts
- Electron Energy is 14 GeV

- Stanford's Linear Coherent Light Source

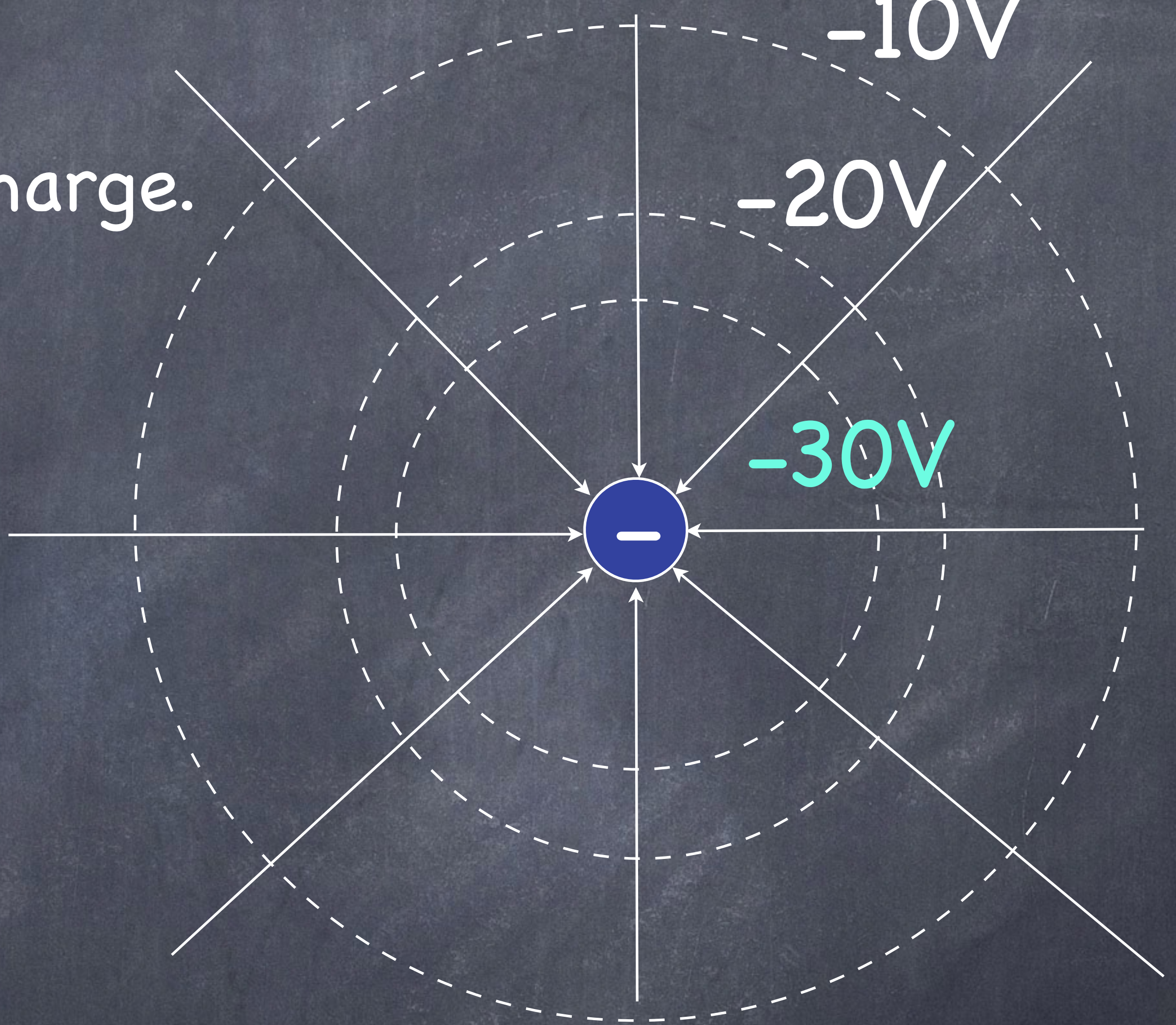
(world's first X-ray Free Electron Laser)

Equipotentials

- An equipotential surface is a surface on which all points are the same potential.
- It takes no work to move a particle along an equipotential surface or line (assume speed is constant).
- The electric field at every point on an equipotential surface is perpendicular to the surface.
- Equipotential surfaces are normally thought of as being imaginary; but they may correspond to real surfaces (like the surface of a conductor).

Equipotentials: lone charge

- Let's construct an equipotential surface for a lone negative charge.



- First, draw the field lines for the charge.

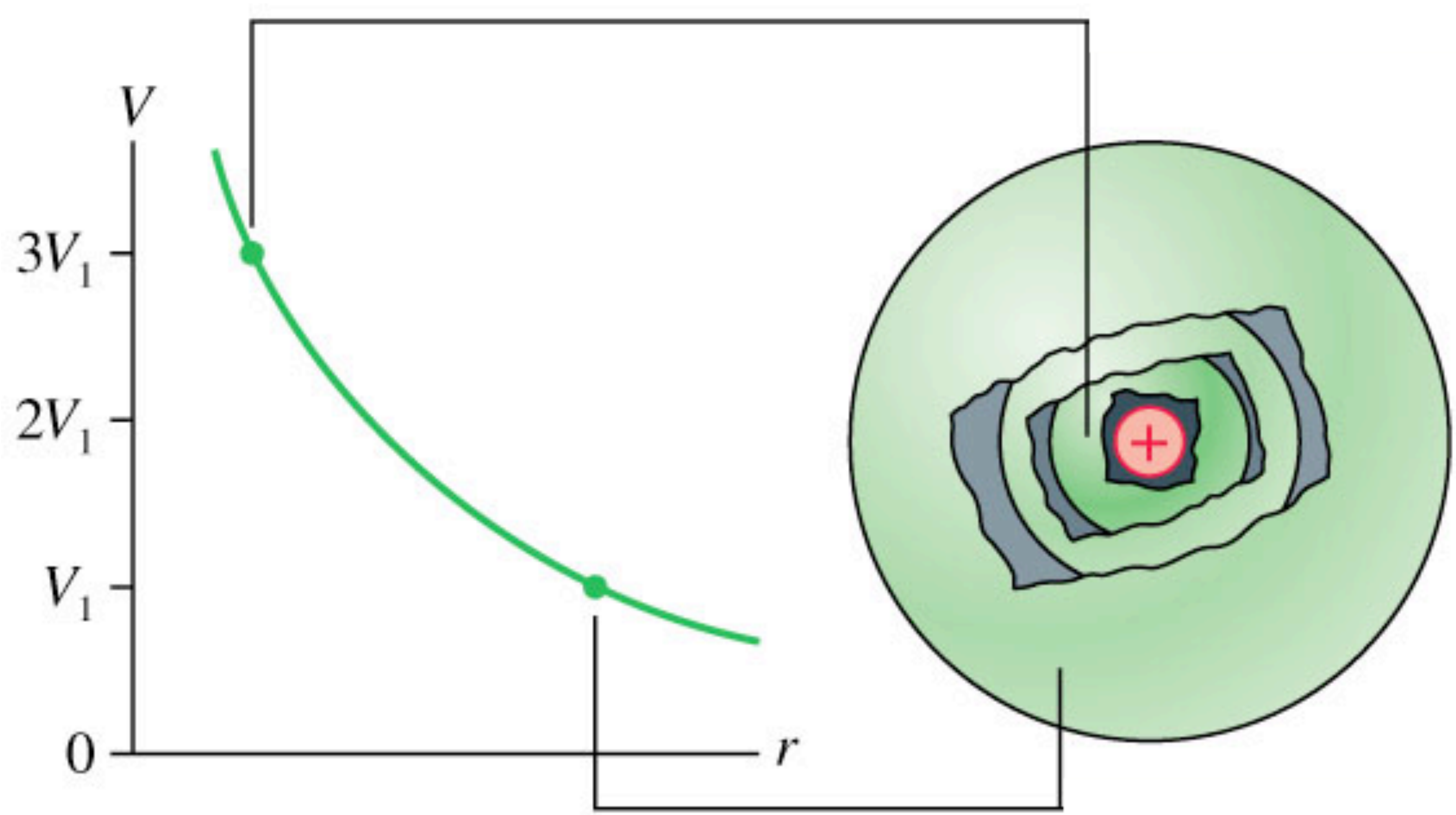
- If I move 1m away would it matter if it was up or down or left or right if I were to calculate potential?

$$V = \frac{U_{q'+q}}{q'} = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

- No, so our equipotential surface would be a sphere.

- Note that the field lines are perpendicular to the equipotential lines at every crossing.

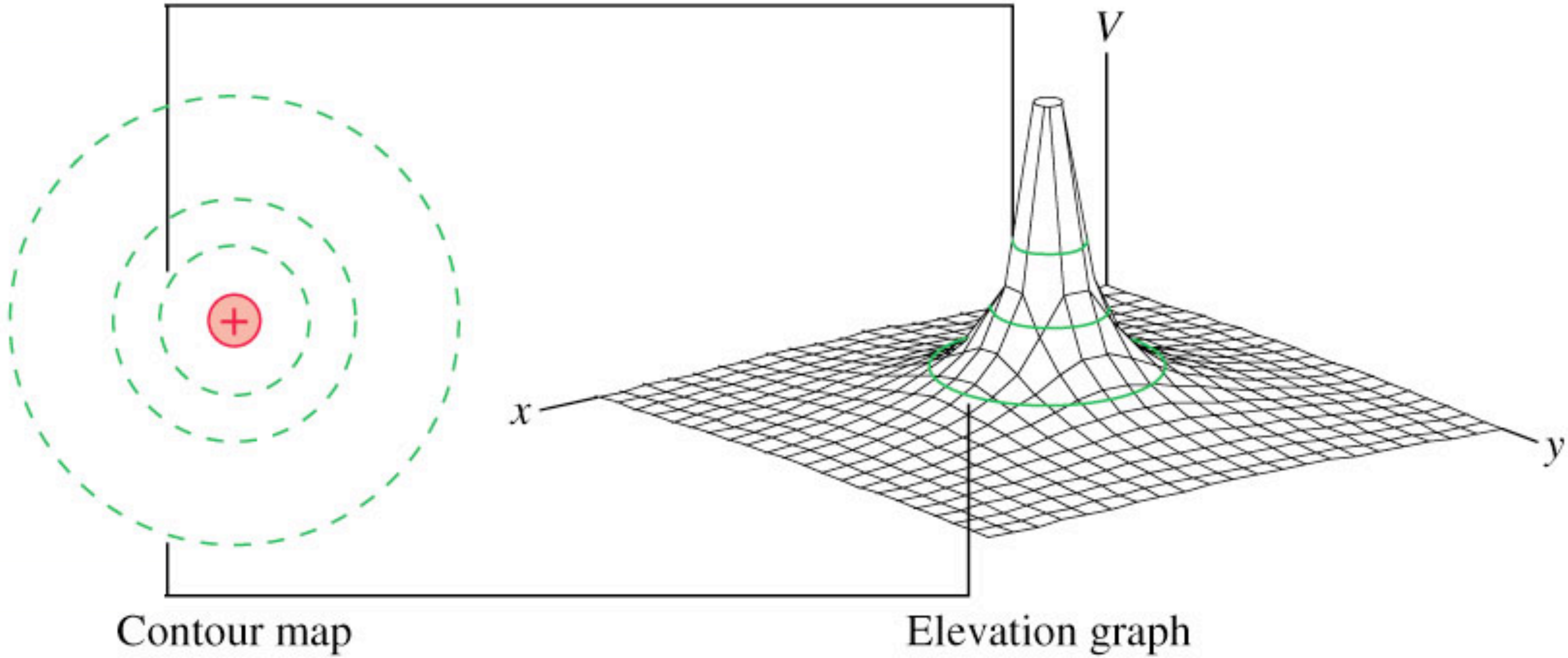
The Electric Potential of a Point Charge



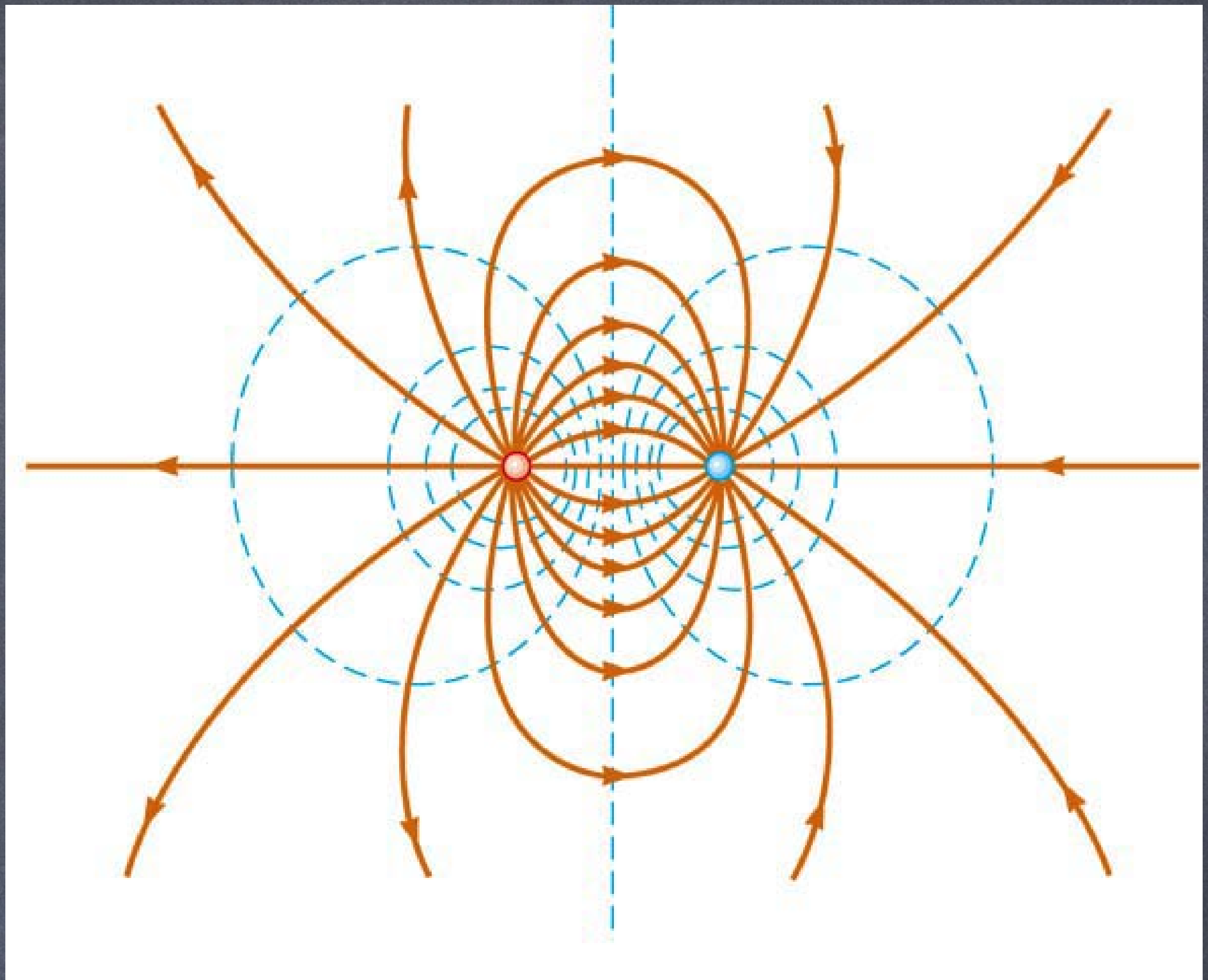
Potential graph

Equipotential surfaces

The Electric Potential of a Point Charge



Equipotentials



- As you increase the number of charges in the distribution the equipotential lines get more complicated.
- Take the electric dipole that we introduced earlier.
- The equipotential lines bunch up between the two charges.
- If you ever have trouble drawing equipotential lines, start by making the electric field lines and make the equipotential lines **perpendicular at each crossing.**

The Electric Potential of ***Many Charges***

- The electric potential V at a point in space is the sum of the potentials due to each charge:

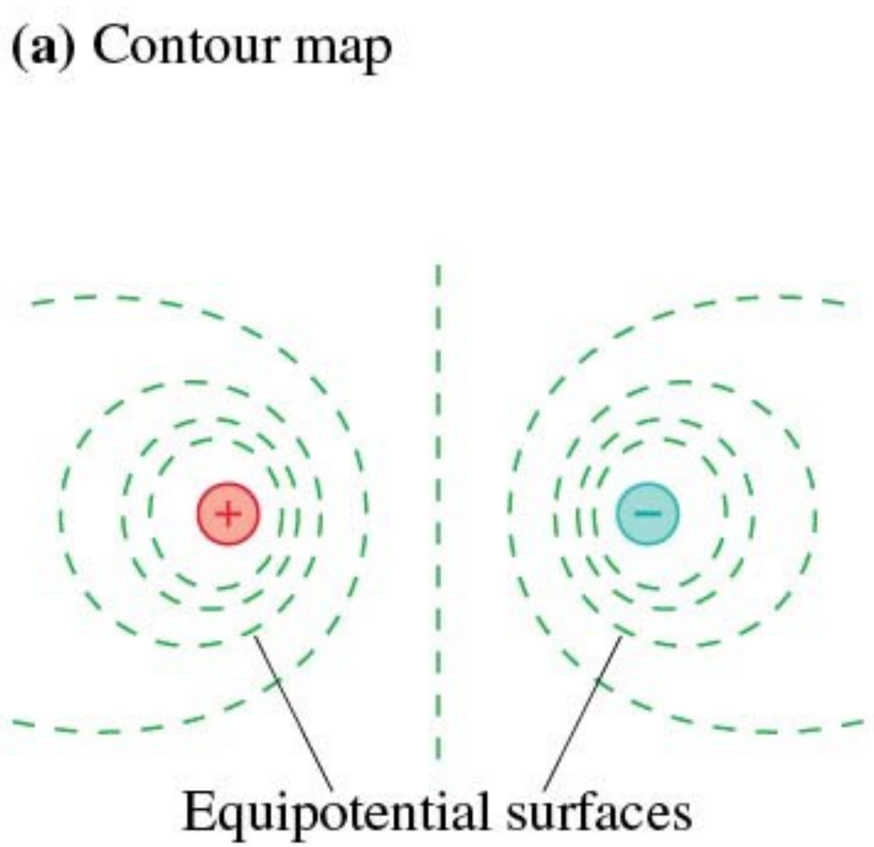
$$V = \sum_i \frac{1}{4\pi\epsilon_0} \frac{q_i}{r_i}$$

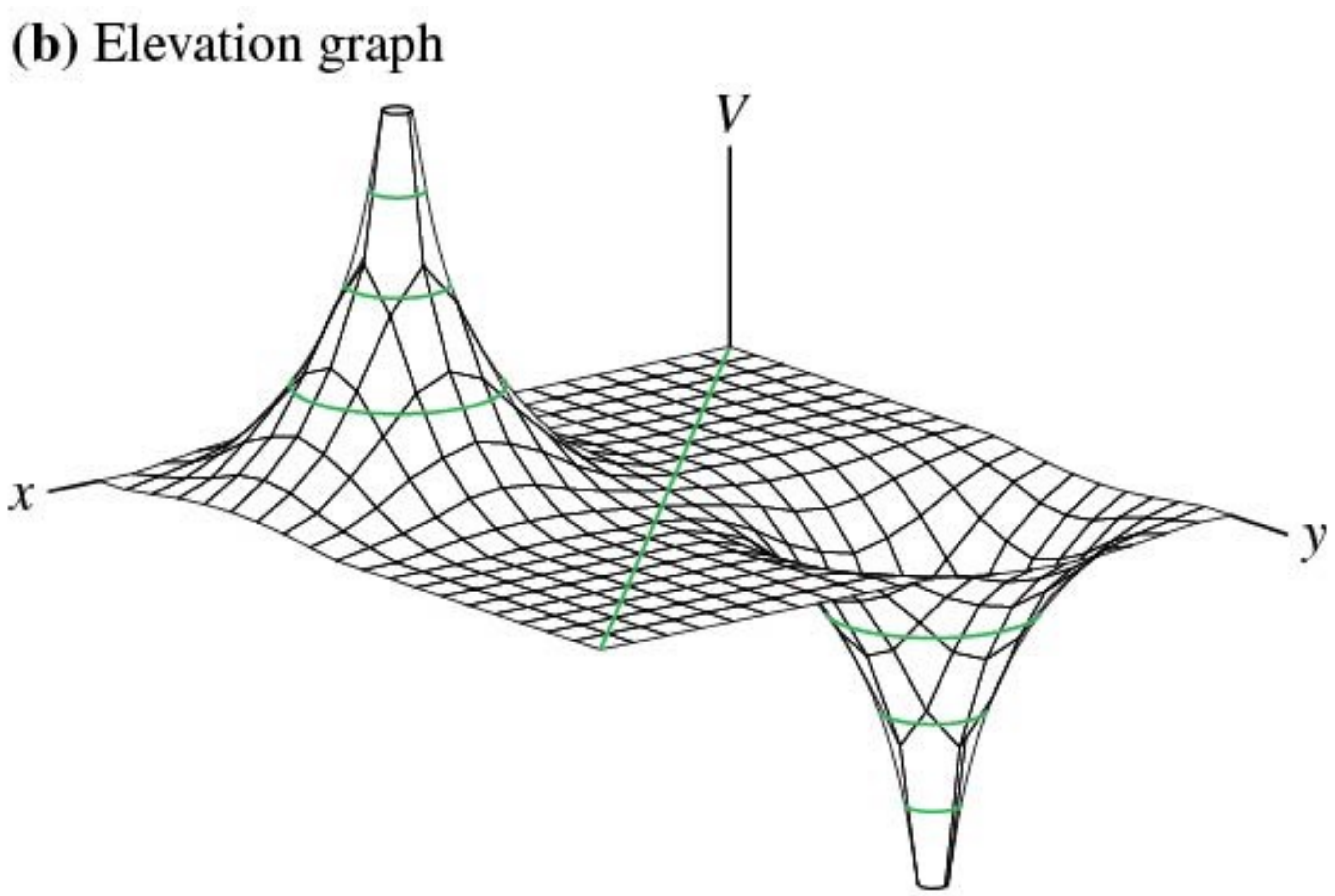
where r_i is the distance from charge q_i to the point in space where the potential is being calculated.

- **The electric potential, like the electric field, obeys the principle of superposition.**

- You do not need to take into account the different components when calculating potential (a scalar!).
- Easier than calculating the electric field due to multiple charges.
- Many times this is the way to eventually calculate the electric field.

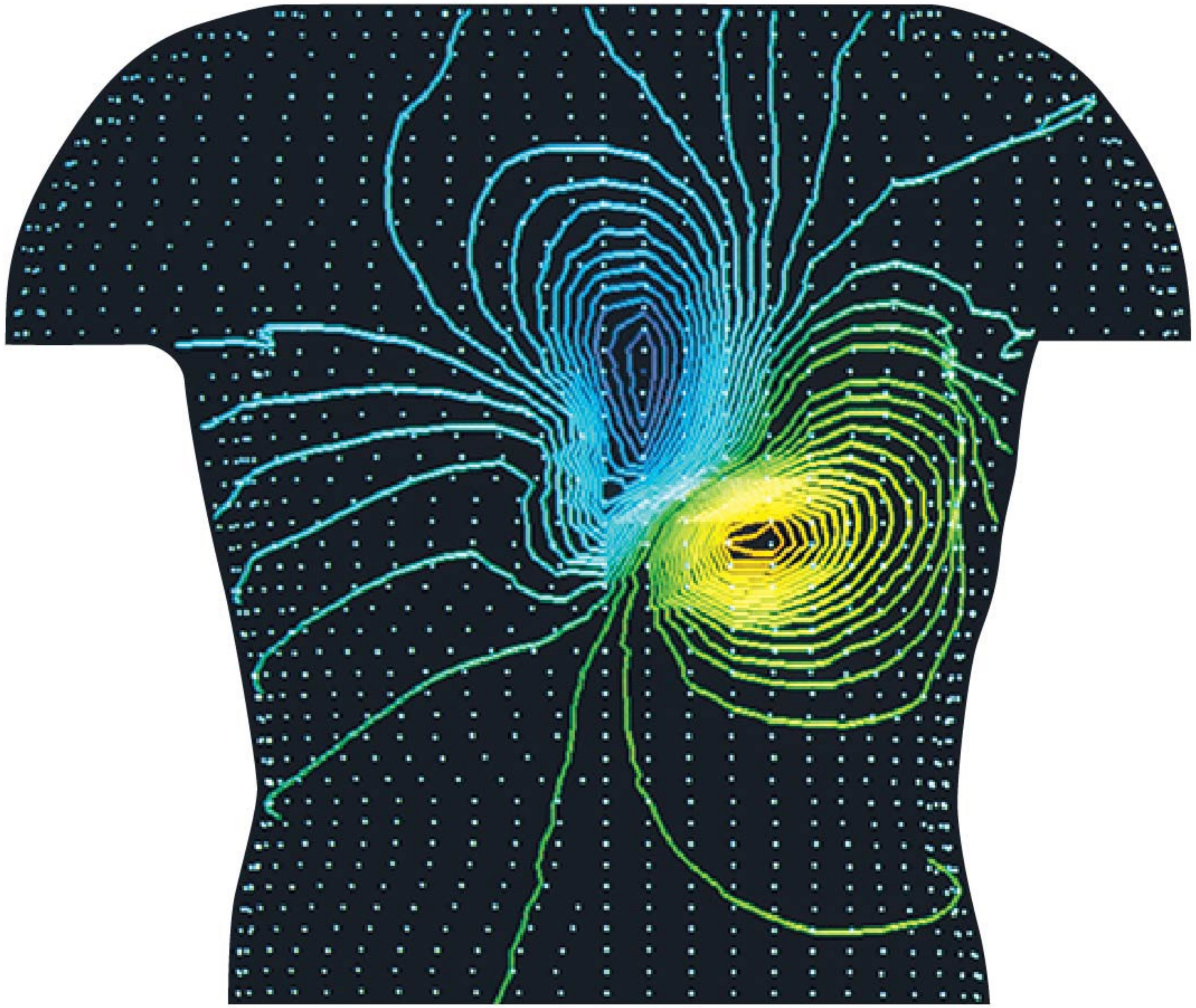
The Electric Potential of an Electric Dipole





The Electric Potential of a Human Heart

- Electrical activity within the body can be monitored by measuring equipotential lines on the skin.
- The equipotentials near the heart are a slightly distorted but recognizable *electric dipole*.



The Potential Energy of Multiple Point Charges

- Imagine bringing charges, q_j, in one at a time
  - The 2nd and subsequent charges (j) interact with all previous charges (i)

$$U_e = \sum_{j=2}^{n} \sum_{i=1}^{j-1} k_e \frac{q_i q_j}{r_{ij}}, \qquad r_{ij} \equiv \left|\mathbf{r}_{ij}\right|$$

all pairs

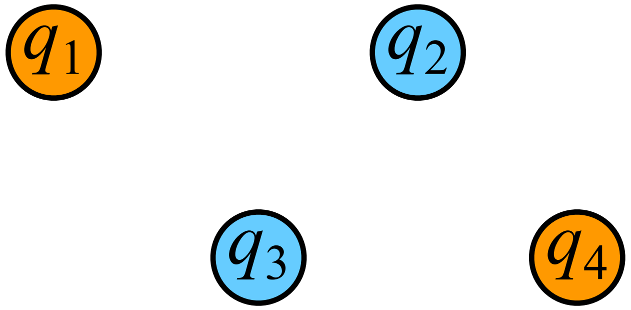
- Also written (as in Knight):

$$U_e = \sum_{i<j} k_e \frac{q_i q_j}{r_{ij}}$$

  - But it's really a *double* sum

- And even:

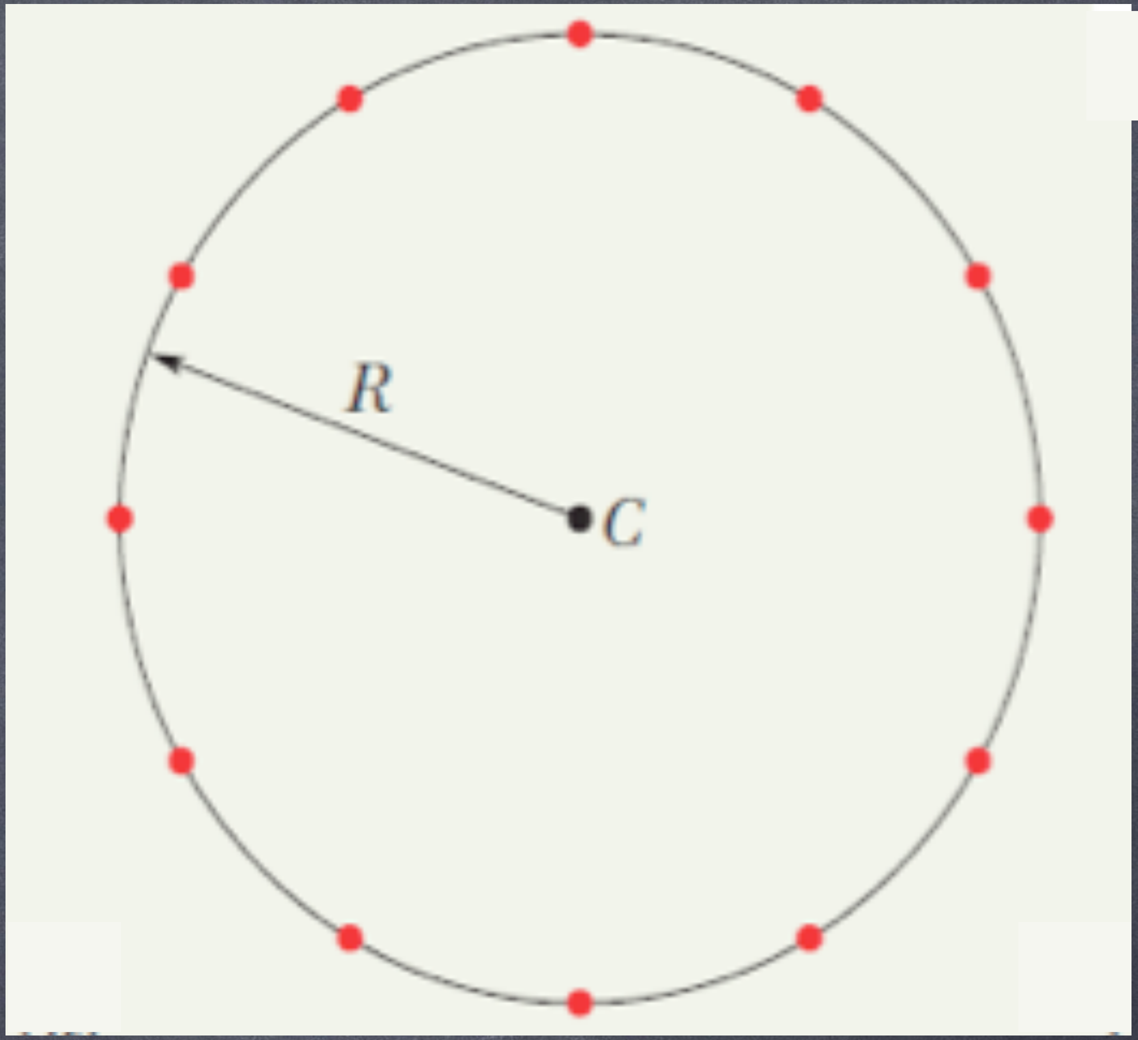
$$U_e = \frac{1}{2} \sum_{i \neq j} k_e \frac{q_i q_j}{r_{ij}}$$



- Don't confuse with the electric potential $V = \sum_i \frac{1}{4\pi\epsilon_0} \frac{q_i}{r_i}$

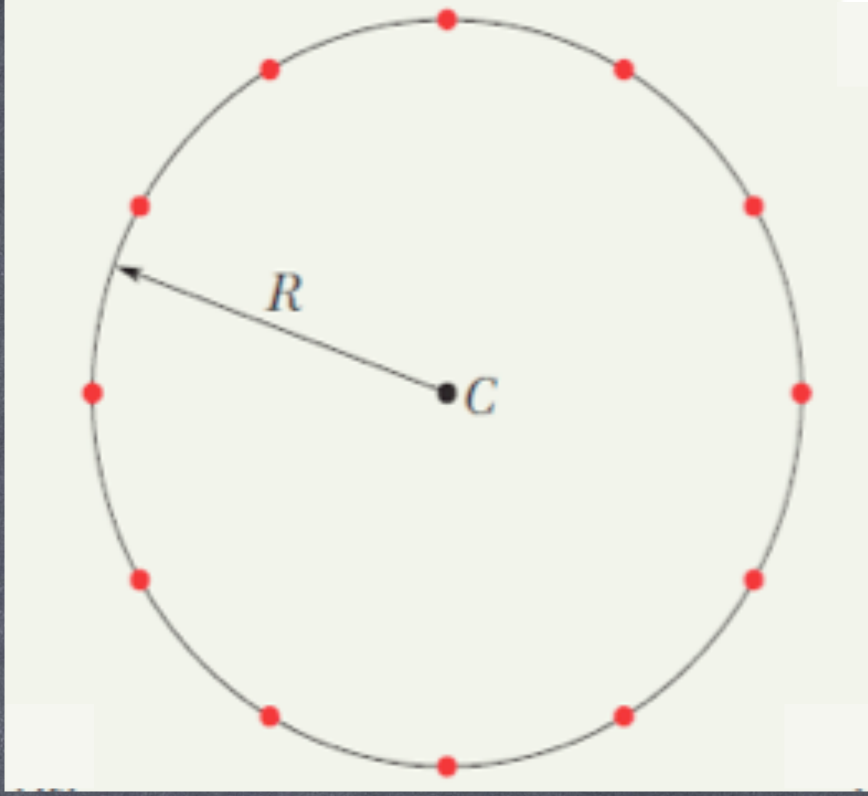
Electric Potential

- Example
- Twelve protons are uniformly placed in a circle of radius R = 1.5cm around point C. What is the electric potential at point C?



- Answer
- Direction doesn't matter when it comes to potential (and all the charges are equidistant from C).

Electric Potential



- Answer
- We can then use:

$$V_{Tot} = \sum_{i=1}^{12} V_i = 12V_1$$

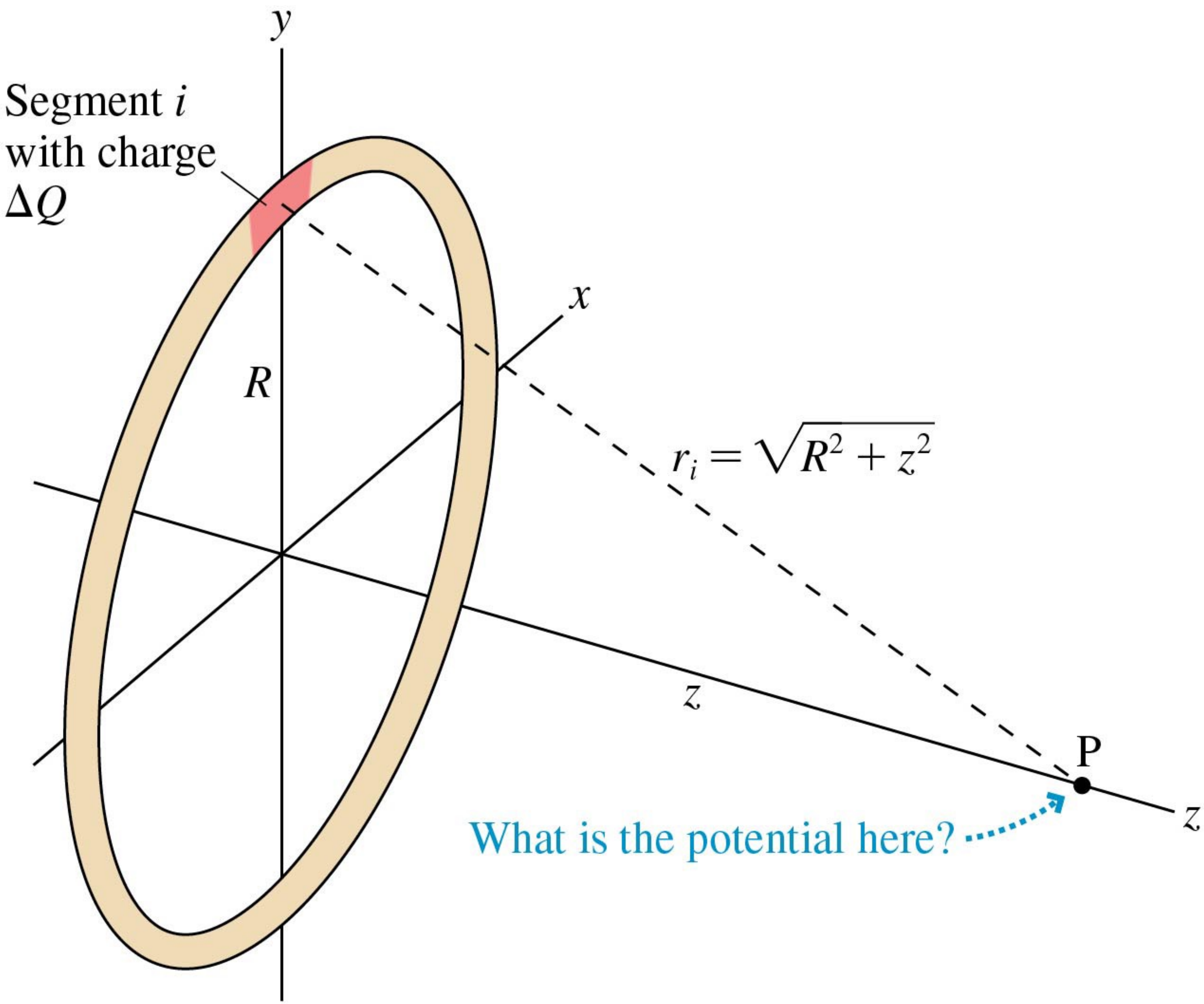
$$V_1 = k\frac{+e}{R}$$

$$V_1 = \left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\frac{\left(1.60\times10^{-19}\ \mathrm{C}\right)}{\left(1.5\times10^{-2}\ \mathrm{m}\right)} = 9.6\times10^{-8}\ \mathrm{V}$$

$$V_{Tot} = 12\left(9.6\times10^{-8}\ \mathrm{V}\right) = 1.2\times10^{-6}\ \mathrm{V}$$

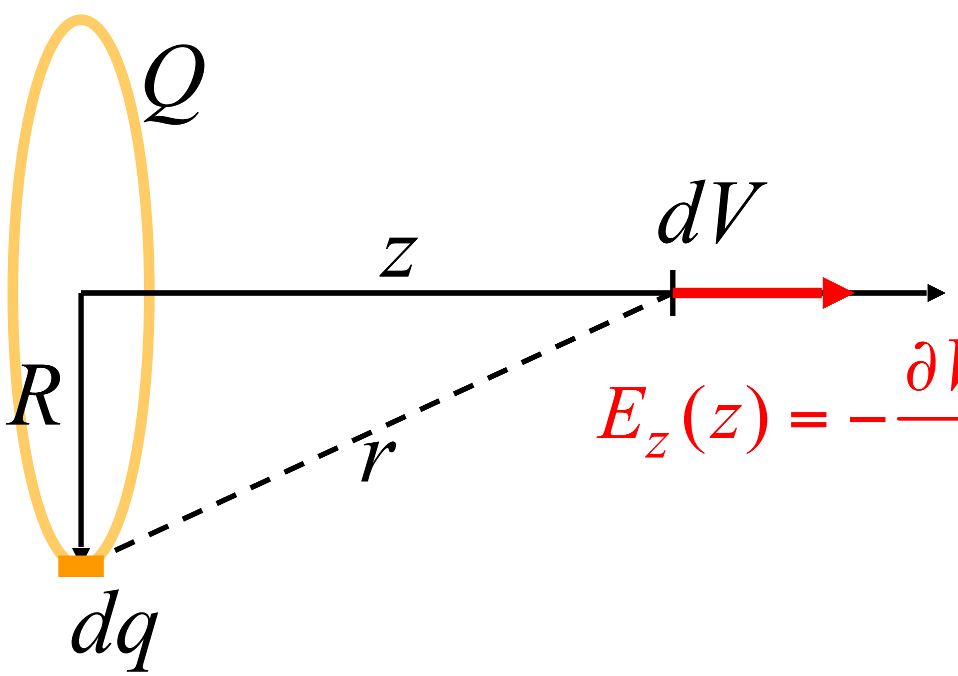
- Much easier to calculate than electric field.
- But this result tells only one piece of information and thus it would be incomplete if we wanted to do something useful with it.

The Potential of a Ring of Charge



E-field of ring of charge: living up to our potential

- What is E-field along the z-axis at distance z from center?
- Start by computing potential (a scalar)
- Axial symmetry implies **E** parallel to axis



$$E_z(z) = -\frac{\partial V(z)}{\partial z}$$

$$dV = k_e \frac{dq}{r} = k_e \frac{dq}{\left(z^2 + R^2\right)^{1/2}} \qquad \text{(no cosines)}$$

$$\int_0^V dV = k_e \int_0^Q \left(z^2 + R^2\right)^{-1/2} dq$$

$$V(z) = k_e \left(z^2 + R^2\right)^{-1/2} Q$$

$$E_z(z) = -k_e Q \left(-\frac{1}{2}\right)\left(R^2 + z^2\right)^{-3/2} 2z$$

$$= k_e \frac{z}{\left(R^2 + z^2\right)^{3/2}} Q, \qquad \mathbf{E}(z) = E_z(z)\hat{\mathbf{z}}$$

What happens when $z = 0$?

When $z = \infty$?

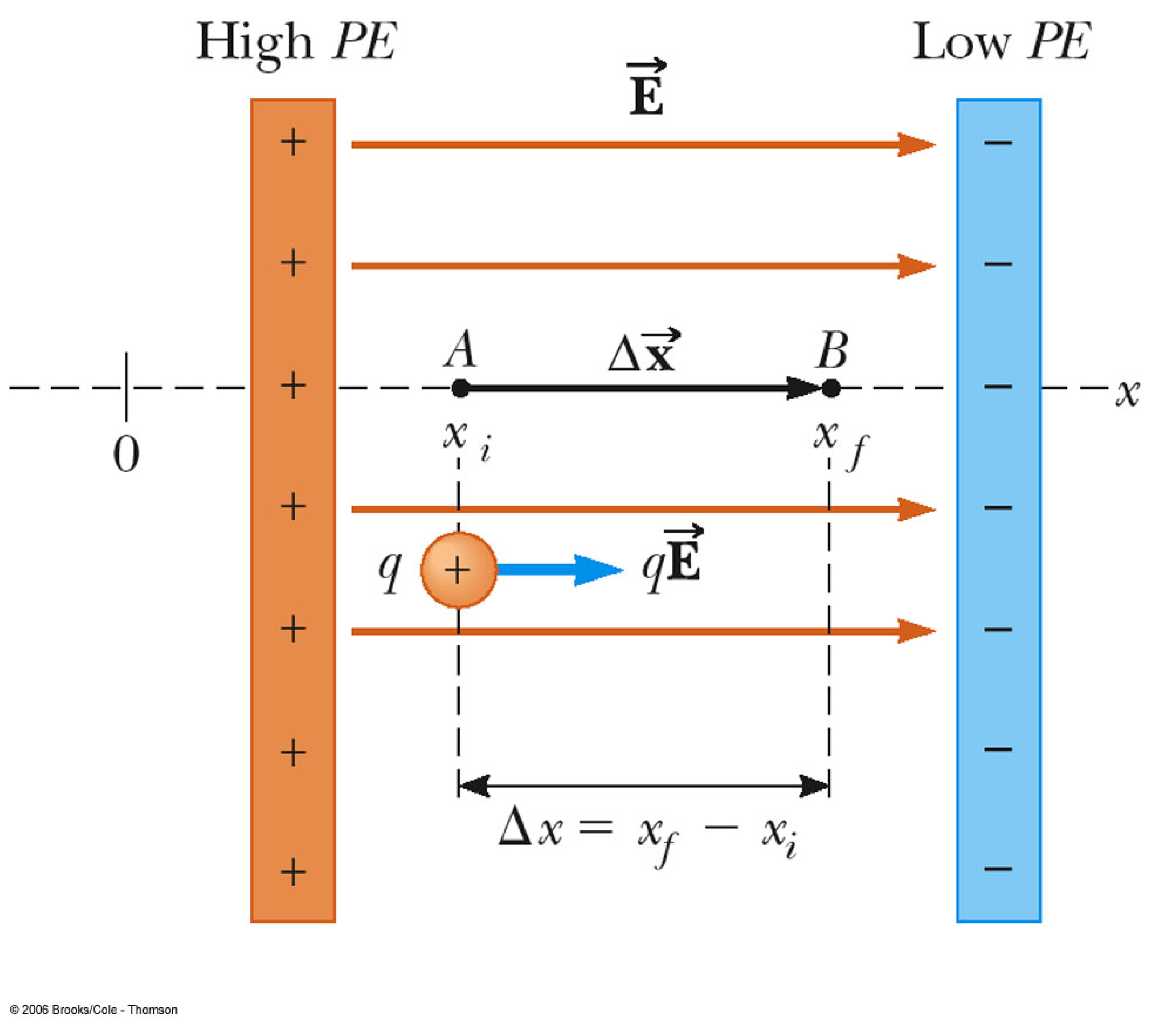
When z is large?

Parallel Plates

- Place a charge, +q, in between oppositely charged plates and then release it, what would happen?
- The positive charge would move away from the positive plate (high PE, high V) toward the negative plate (low PE, low V).
- The work done by the electric field would be:

$$W = \vec{F}_{elec} \cdot \Delta\vec{x}$$

$$W = q\vec{E} \cdot \Delta\vec{x} = -\Delta PE$$



Electric Potential

- If we were to then turn to electric potential, we can say that:

$$\Delta V = \frac{\Delta PE}{q} = \frac{-q\left|\vec{E}\right|\Delta x}{q}$$

$$\Delta V = -\left|\vec{E}\right|\Delta x$$

$$\left|\vec{E}\right| = \frac{-\Delta V}{\Delta x}$$

The Electric Potential Inside a Parallel-Plate Capacitor

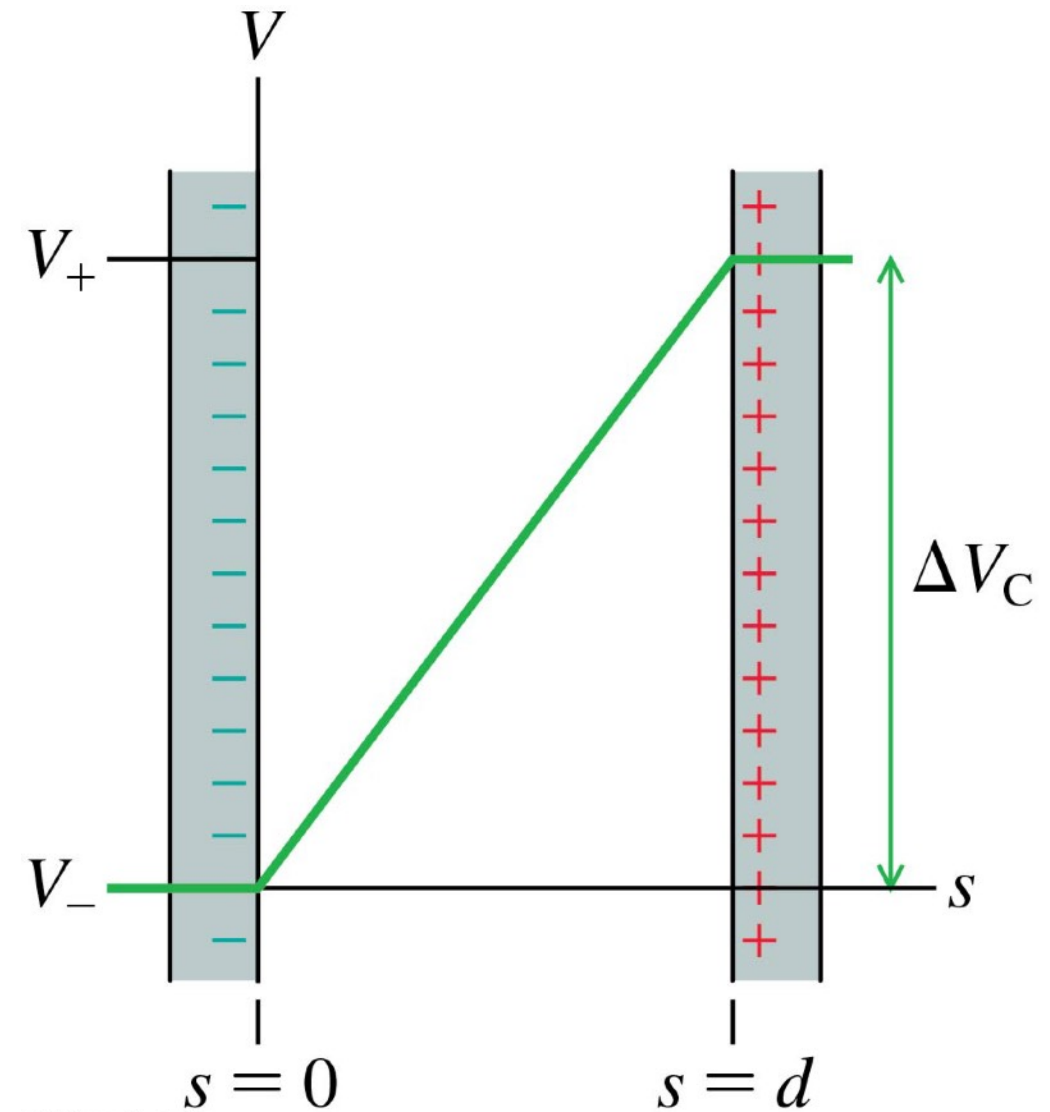
- The electric potential inside a parallel-plate capacitor is

$$V = Es \qquad \text{(electric potential inside a parallel-plate capacitor)}$$

where s **is the distance from the *negative* electrode**.

- The *potential difference* ΔV_C, or "voltage" between the two capacitor plates is

$$\Delta V_C = V_+ - V_- = Ed$$



Units of Electric Field

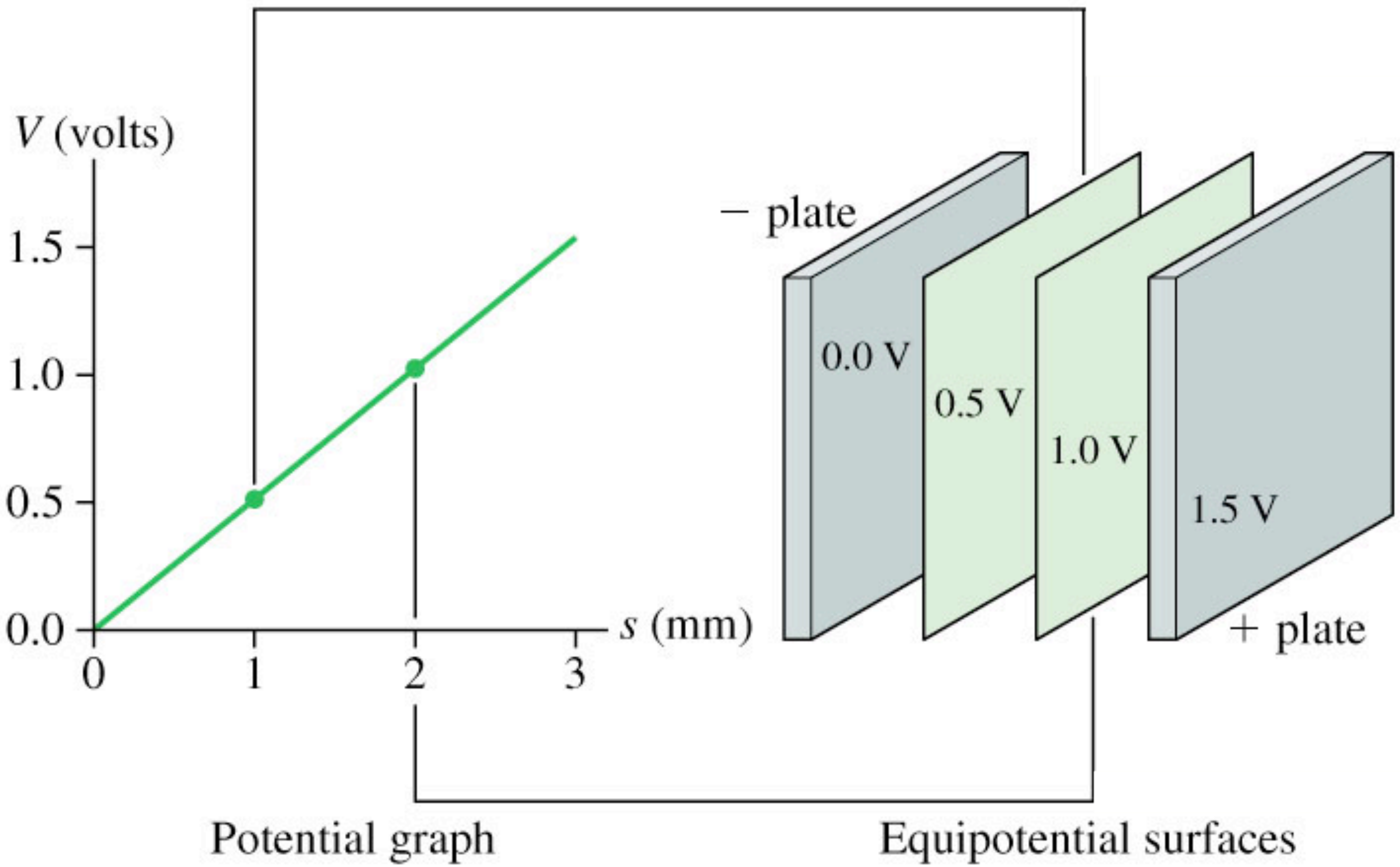
- If we know a capacitor's voltage ΔV and the distance between the plates d, then the electric field strength within the capacitor is

$$E = \frac{\Delta V_{\mathrm{C}}}{d}$$

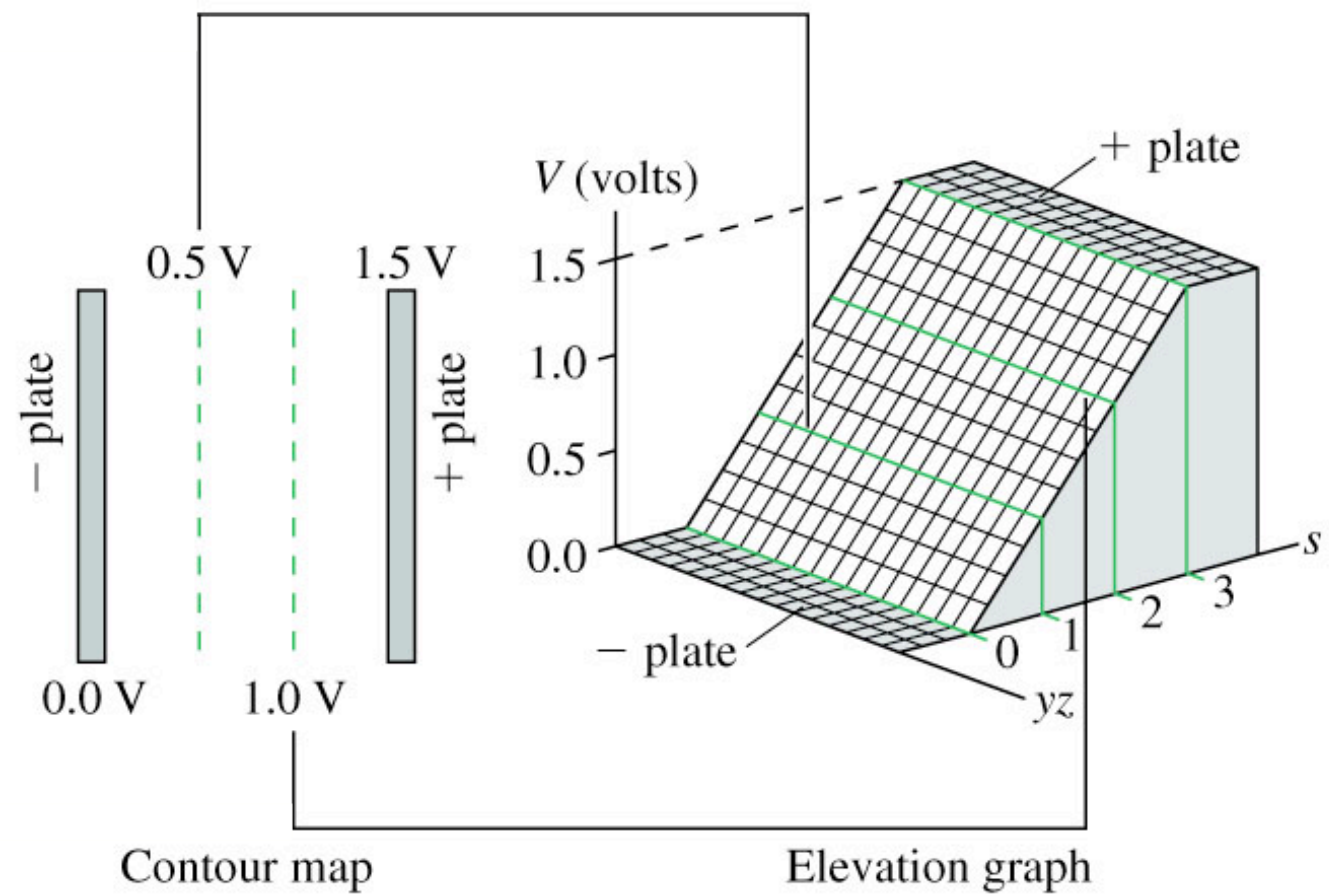
- This implies that the units of electric field are volts per meter, or $\mathrm{V/m}$.
- Previously, we have been using electric field units of newtons per coulomb.
- In fact, as you can show as a homework problem, these units are equivalent to each other:

$$1\ \mathrm{N/C} = 1\ \mathrm{V/m}$$

The Electric Potential Inside a Parallel-Plate Capacitor

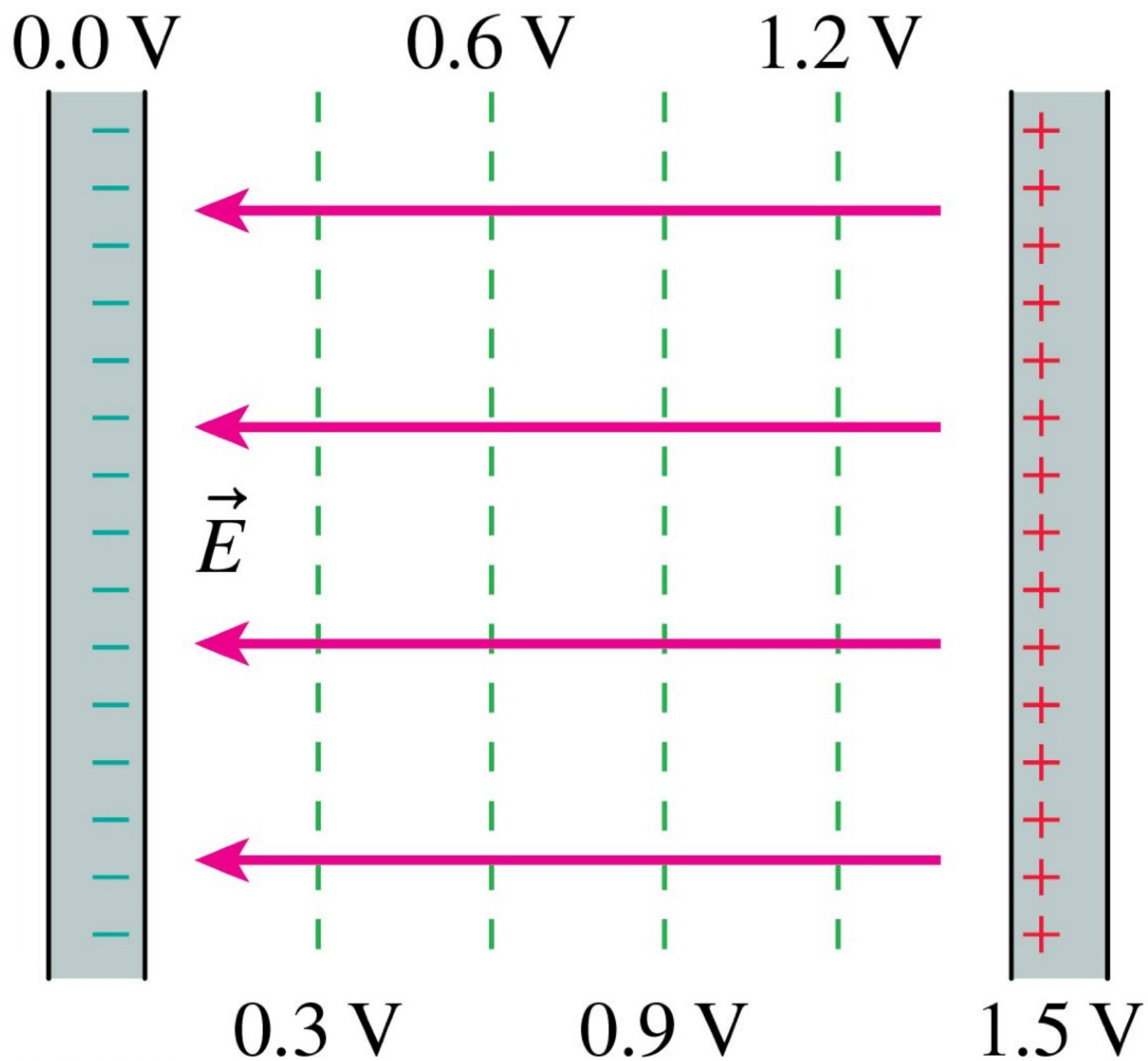


The Electric Potential Inside a Parallel-Plate Capacitor



The Parallel-Plate Capacitor

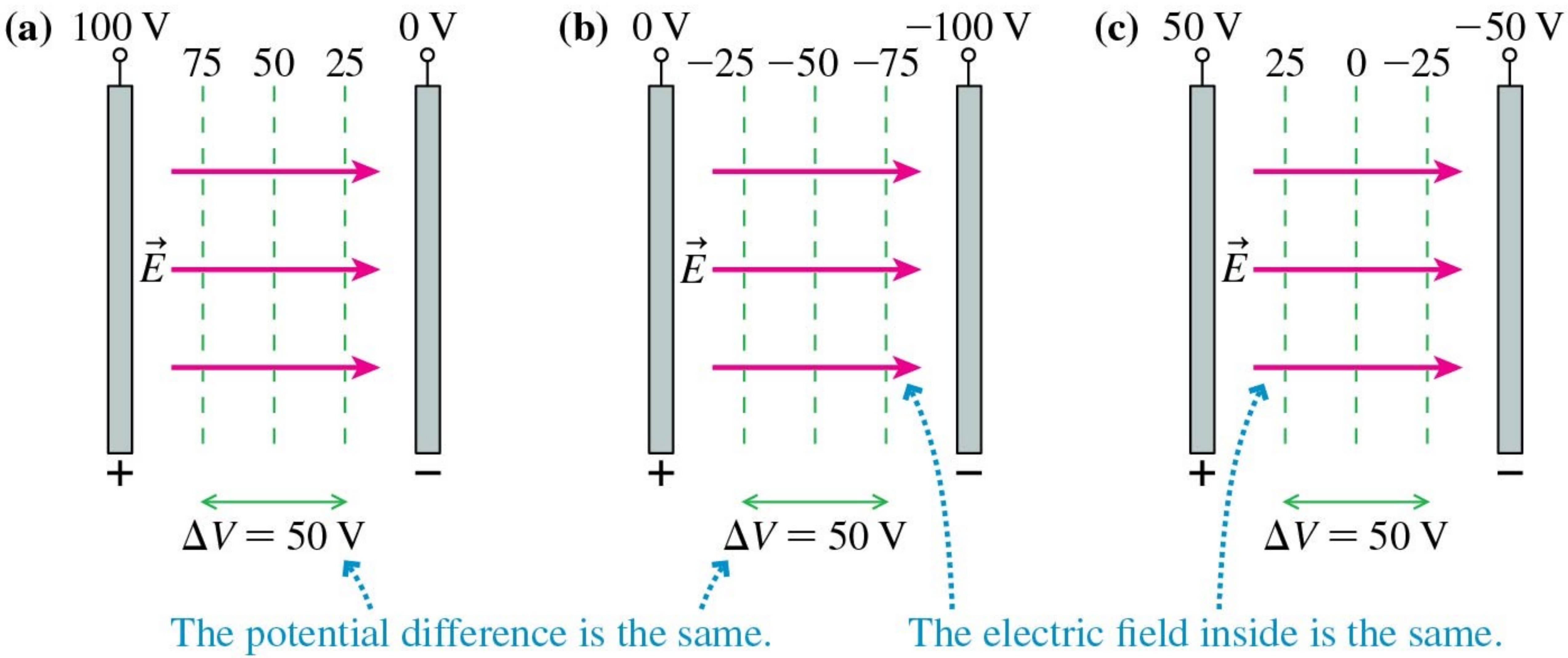
- The figure shows the contour lines of the electric potential and the electric field vectors inside a parallel-plate capacitor.



- The electric field vectors are *perpendicular* to the equipotential surfaces.
- The electric field points in the direction of *decreasing* potential.

The Zero Point of Electric Potential

- Where you choose $V = 0$ is arbitrary. The three contour maps below represent the *same physical situation*.



Electric Potential

- If we were to then turn to electric potential, we can say that:

$$\Delta V = \frac{\Delta PE}{q} = \frac{-q\left|\vec{E}\right|\Delta x}{q}$$

$$\Delta V = -\left|\vec{E}\right|\Delta x$$

$$\left|\vec{E}\right| = \frac{-\Delta V}{\Delta x}$$

- This is true for a uniform electric field.

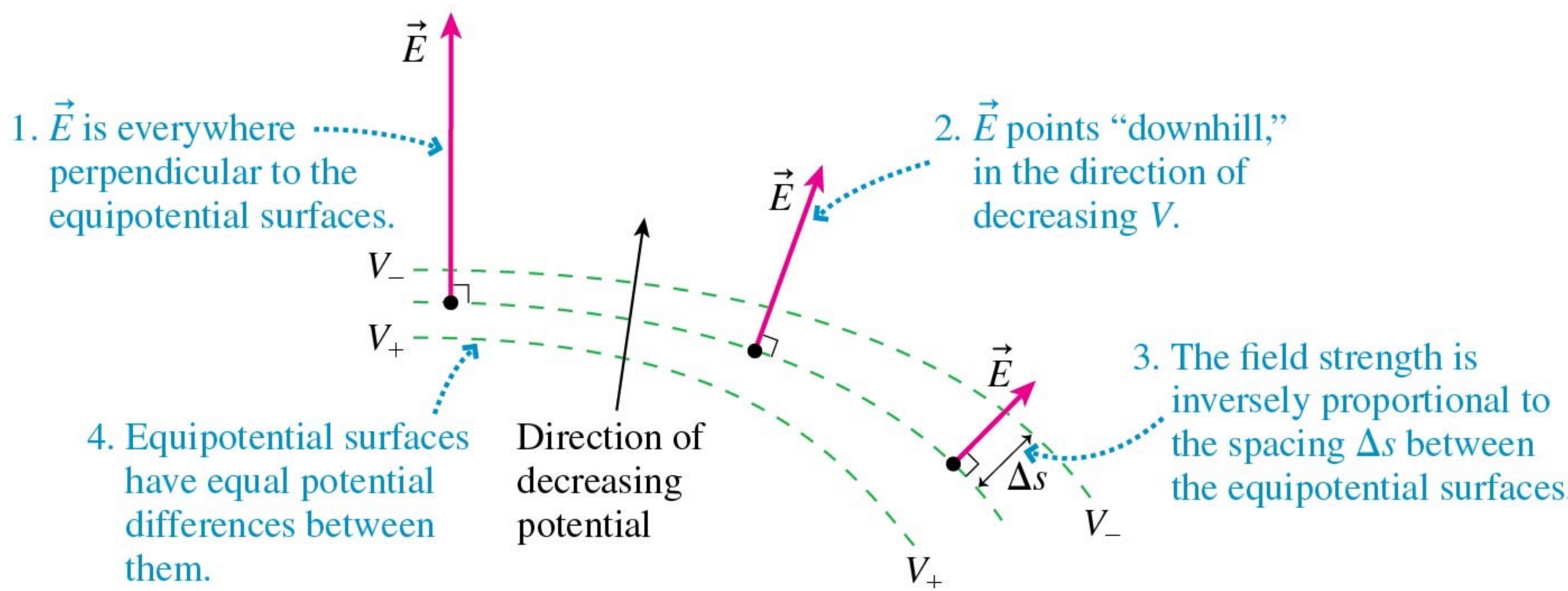
- But, in general, you can say that:

$$\left|\vec{E}\right| = \frac{-dV}{dx}$$

- These equations demonstrate that if you move with the electric field, your electric potential will decrease.

- In three dimensions, we can find the electric field from the electric potential as

$$\vec{E} = E_x\hat{\imath} + E_y\hat{\jmath} + E_z\hat{k} = -\left(\frac{\partial V}{\partial x}\hat{\imath} + \frac{\partial V}{\partial y}\hat{\jmath} + \frac{\partial V}{\partial z}\hat{k}\right)$$



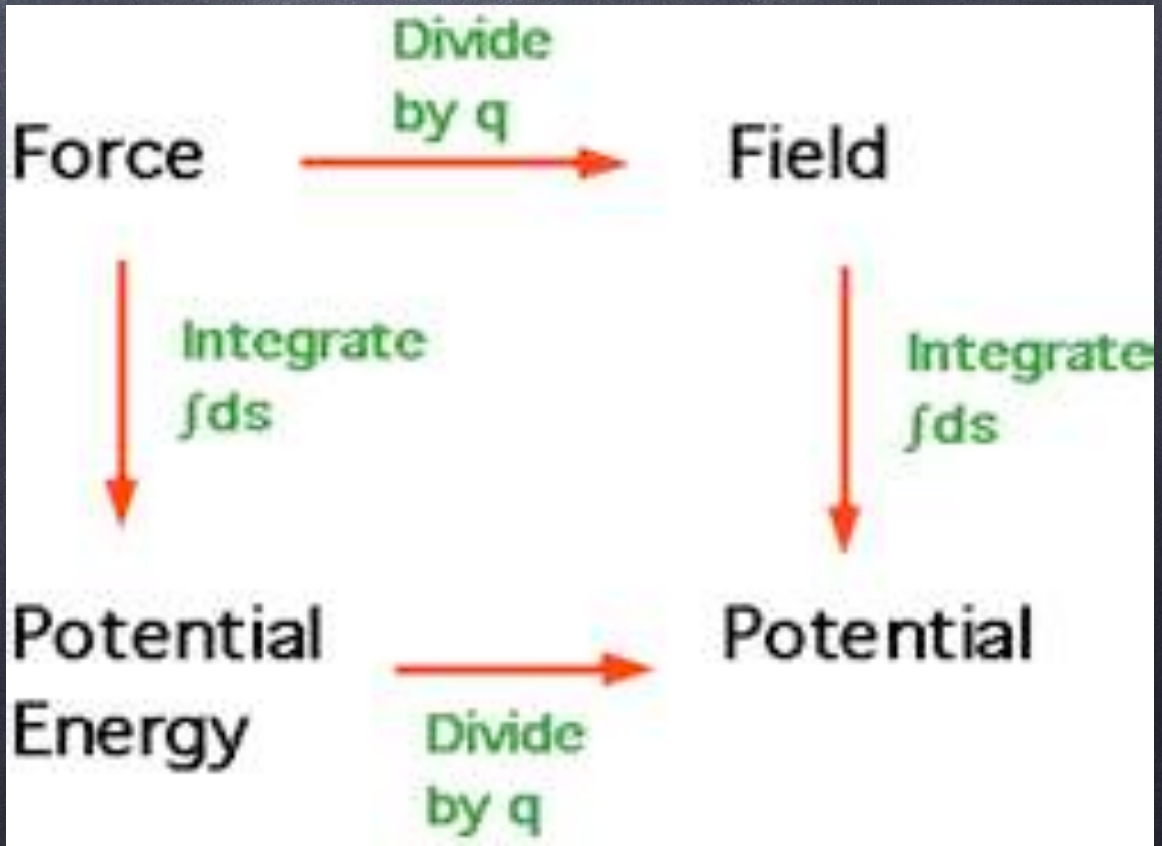
Electric Potential

- Conversely, you can also calculate voltage from the electric field by integrating over a distance (two points: initial and final):

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$$

- This gives us the following relationships:

- Many times you will be asked to find one variable given the expression of one of the others, just recall how they are related.



Finding the Potential from the Electric Field

- The potential difference between two points in space is

$$\Delta V = V_f - V_i = -\int_{s_i}^{s_f} E_s \, ds = -\int_i^f \vec{E} \cdot d\vec{s}$$

where s is the position along a line from point i to point f.

- We can find the potential difference between two points if we know the electric field.
- Thus a graphical interpretation of the equation above is

$$V_f = V_i - (\text{area under the } E_s\text{-versus-}s \text{ curve between } s_i \text{ and } s_f)$$

Which set of equipotential surfaces matches this electric field?

A.

B.

C.

D.

E.

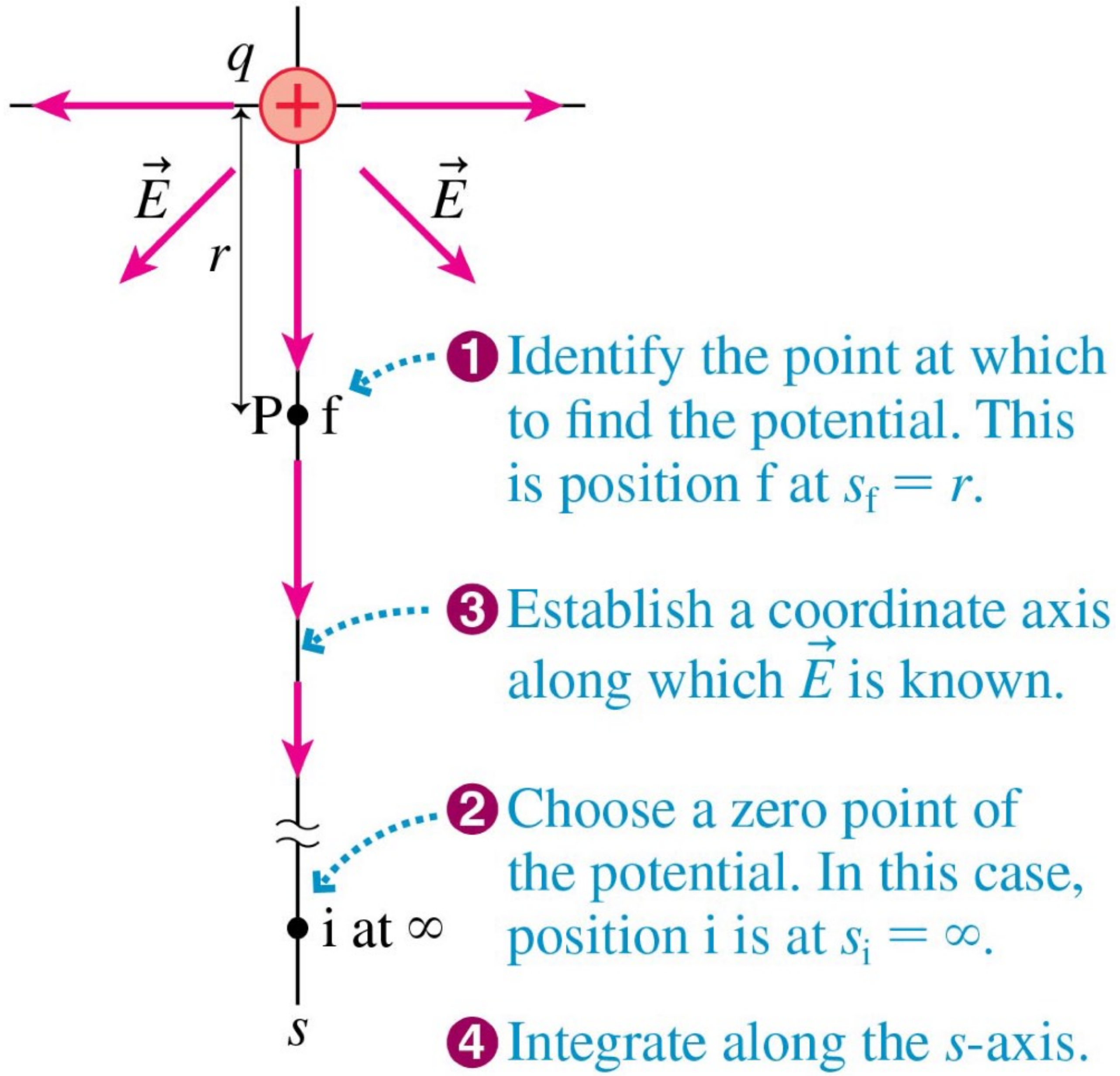
F.

Example: Finding the Potential of a Point Charge

$$E_s = \frac{1}{4\pi\epsilon_0}\frac{q}{s^2}$$

$$V(r) = V(\infty) + \frac{q}{4\pi\epsilon_0}\int_r^{\infty}\frac{ds}{s^2}$$

$$V_{\text{point charge}} = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$



Example: potential of charged disk

- For example, in Chapter 23 we found that the electric field on the z-axis from a charged disk was:

$$E_{disk} = \frac{\sigma}{2\varepsilon_o}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$

- If we wanted to find the electric potential we can turn to:

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$$

- Evaluating the right side we get:

$$\int_i^f \vec{E} \cdot d\vec{s} = \frac{\sigma}{2\varepsilon_o}\int_\infty^z \left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right) dz$$

Example: potential of charged disk

- The first term is easy, but the second term needs a u-substitution, let $u = (z^2+R^2)$

$$\int_i^f \vec{E}\cdot d\vec{s} = \frac{\sigma}{2\varepsilon_o}\left(z-\sqrt{z^2+R^2}\right)_\infty^z$$

$$\int_i^f \vec{E}\cdot d\vec{s} = \frac{\sigma}{2\varepsilon_o}\left(z-\sqrt{z^2+R^2}\right)$$

Turning back to our equation for potential:

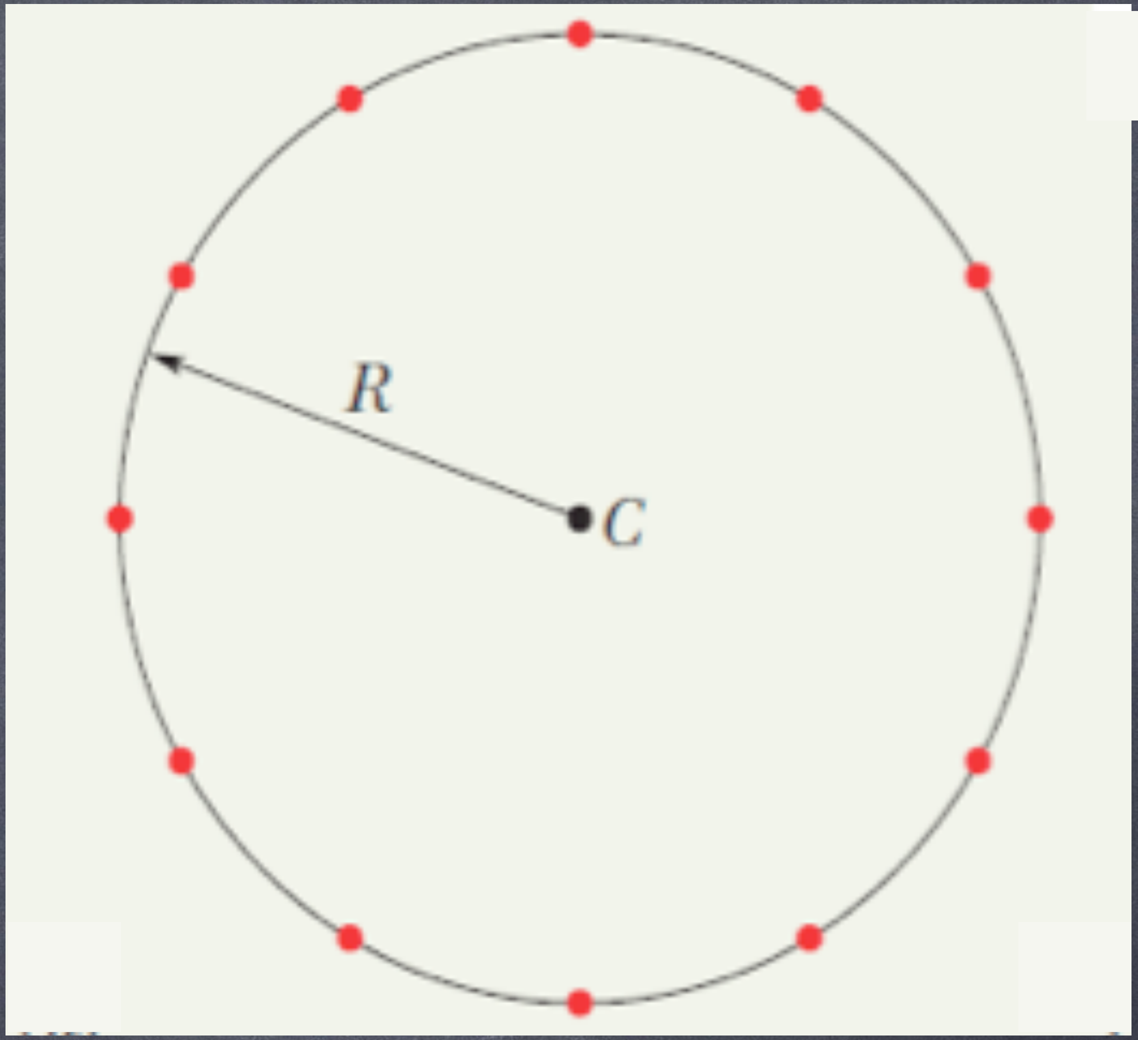
$$V_f - V_i = -\frac{\sigma}{2\varepsilon_o}\left(z-\sqrt{z^2+R^2}\right)$$

- We usually define a place (like V_i to be zero at infinite distance), yielding:

$$V = \frac{\sigma}{2\varepsilon_o}\left(\sqrt{z^2+R^2}-z\right)$$

Remember: Electric Potential

- Example
- Twelve protons are uniformly placed in a circle of radius R = 1.5cm around point C. What is the electric potential at point C?



- Answer
- Direction doesn't matter when it comes to potential (and all the charges are equidistant from C).

Remember: Electric Potential

- Answer
- We can then use:

$$V_{Tot} = \sum_{i=1}^{12} V_i = 12V_1$$

$$V_1 = k\frac{+e}{R}$$

$$V_1 = \left(8.99\times10^{9}\ \mathrm{N\cdot m^2}/\mathrm{C^2}\right)\frac{\left(1.60\times10^{-19}\ \mathrm{C}\right)}{\left(1.5\times10^{-2}\ \mathrm{m}\right)} = 9.6\times10^{-8}\ \mathrm{V}$$

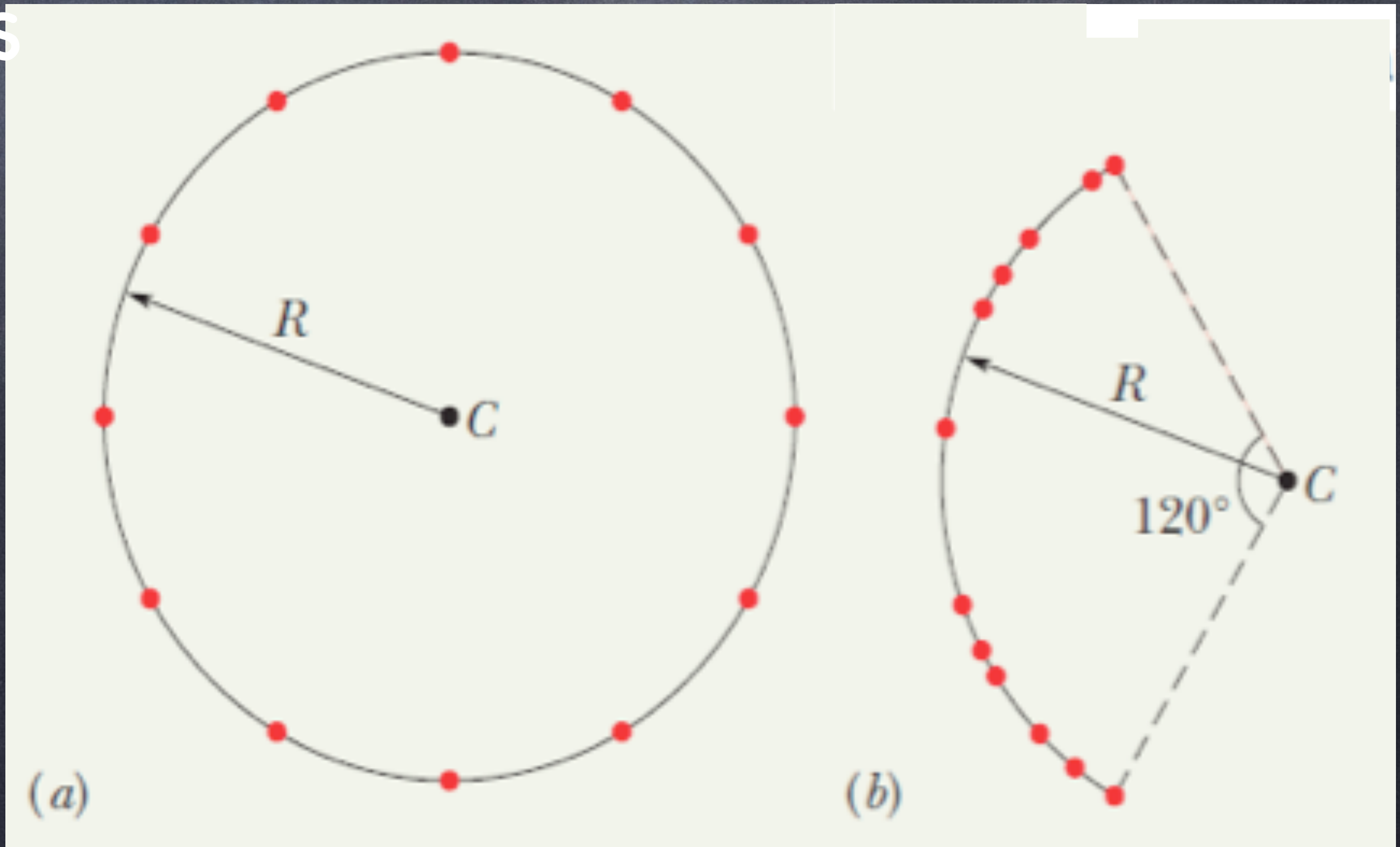
$$V_{Tot} = 12\left(9.6\times10^{-8}\ \mathrm{V}\right) = 1.2\times10^{-6}\ \mathrm{V}$$

- Much easier to calculate than electric field.
- But this result tells only one piece of information and thus it would be incomplete if we wanted to do something useful with it.

E-Field from Potential at one point?

- Because many configurations (with different electric fields) can give you the same result.
- You would have to find the electric potential at another place in order to find the electric field.
- Because what matters is how the electric field is changing with respect to location.

$$\left|\vec{E}\right| = \frac{-dV}{dx}$$



iClicker question 7-5

The electric field at the dot is

A. $10\hat{\imath}$ V/m
B. $-10\hat{\imath}$ V/m
C. $20\hat{\imath}$ V/m
D. $30\hat{\imath}$ V/m
E. $-30\hat{\imath}$ V/m