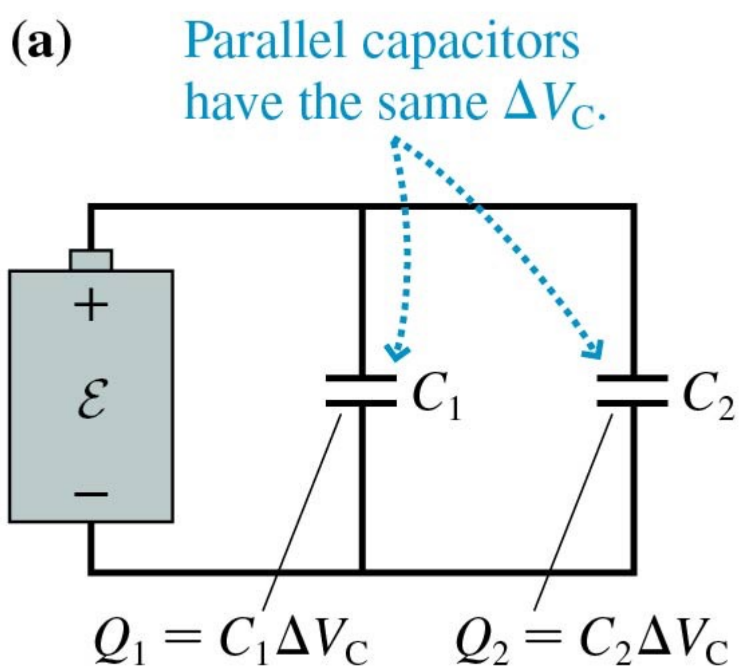
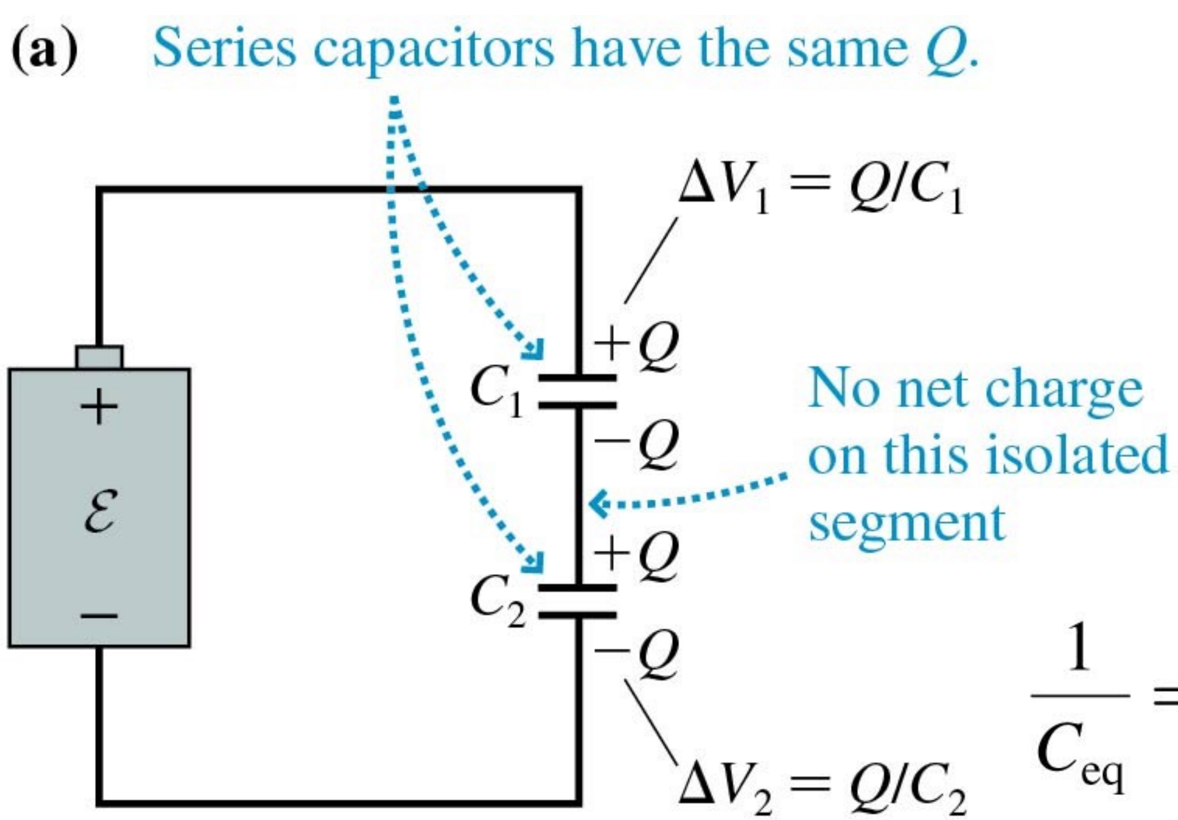
Capacitors in parallel



- Charge adds up:
  $Q = Q_1 + Q_2$
- Voltage is the same:
  $\Delta V_C = \Delta V_1 = \Delta V_2$

$$C_{eq} = \frac{Q}{\Delta V_C} = \frac{Q_1 + Q_2}{\Delta V_C} = \frac{Q_1}{\Delta V_C} + \frac{Q_2}{\Delta V_C} = C_1 + C_2$$

Capacitors in series



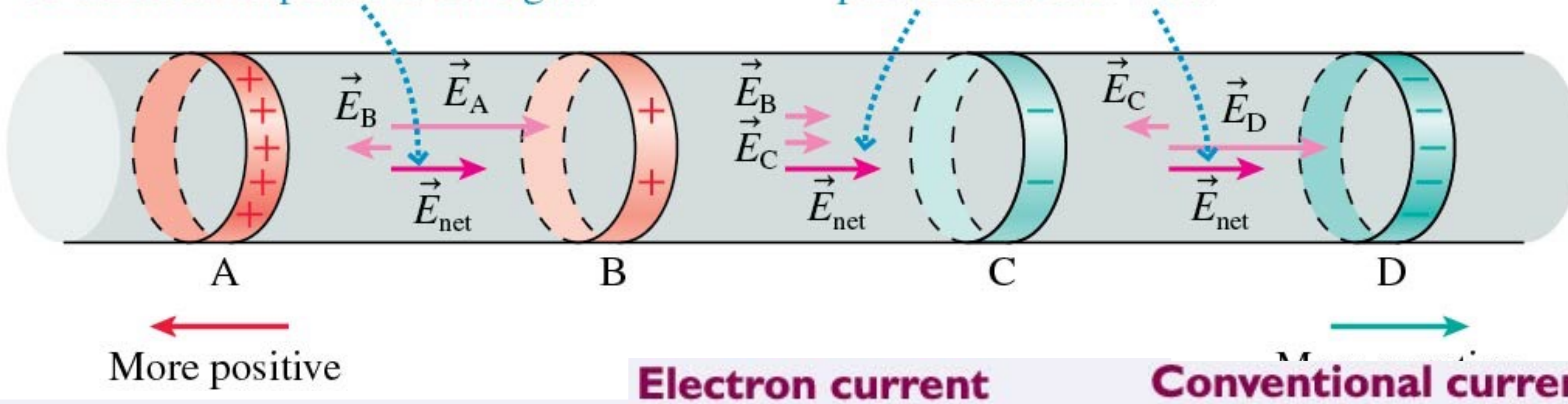
- Charge is the same:
  $Q = Q_1 = Q_2$
- Voltage adds up:
  $\Delta V_C = \Delta V_1 + \Delta V_2$

$$\frac{1}{C_{eq}} = \frac{\Delta V_C}{Q} = \frac{\Delta V_1 + \Delta V_2}{Q} = \frac{\Delta V_1}{Q} + \frac{\Delta V_2}{Q} = \frac{1}{C_1} + \frac{1}{C_2}$$

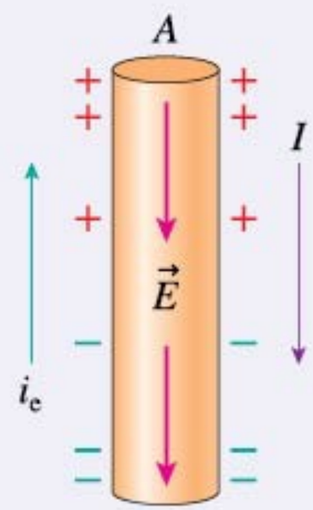
Establishing the Electric Field in a Wire

$\vec{E}_A$ points away from A and $\vec{E}_B$ points away from B, but A has more charge so the net field points to the right.

The nonuniform charge distribution creates a net field to the right at all points inside the wire.



Current is a nonequilibrium motion of charges sustained by an electric field. Nonuniform surface charge density creates an electric field in a wire. The electric field pushes the electron current i_e in a direction opposite to $\vec{E}$. The conventional current I is in the direction in which positive charge *seems* to move.



Electron current

i_e = rate of electron flow

$N_e = i_e \Delta t$

Conventional current

I = rate of charge flow = ei_e

$Q = I\,\Delta t$

The **drift speed** is $v_d = \dfrac{e\tau}{m}E$, where τ is the mean time between collisions.

The electron current is related to the drift speed by

$$i_e = n_e A v_d$$

where n_e is the electron density.

Current density

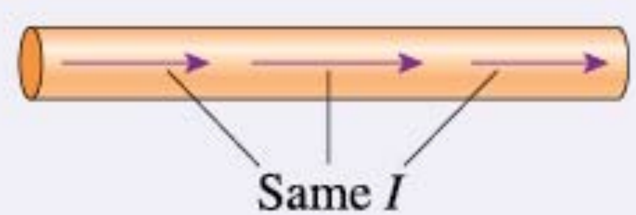
$$J = I/A$$

Conservation of Charge

The current is the same at any two points in a wire.
At a junction,

$$\sum I_{in} = \sum I_{out}$$

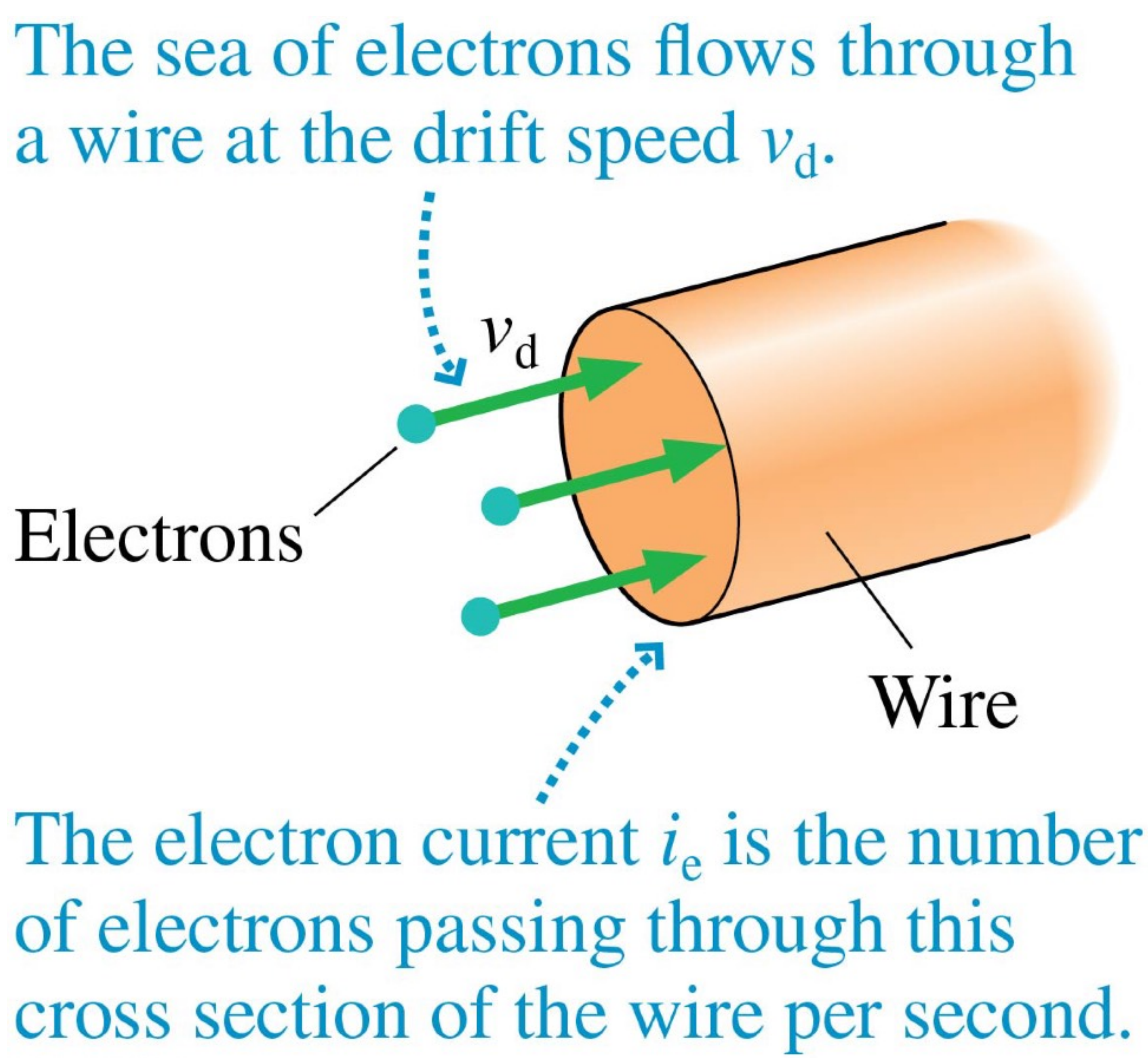
This is **Kirchhoff's junction law.**



The Electron Current

- We define the **electron current** i_e to be the number of electrons per second that pass through a cross section of the conductor.
- The number N_e of electrons that pass through the cross section during the time interval Δt is

$$N_e = i_e \Delta t$$



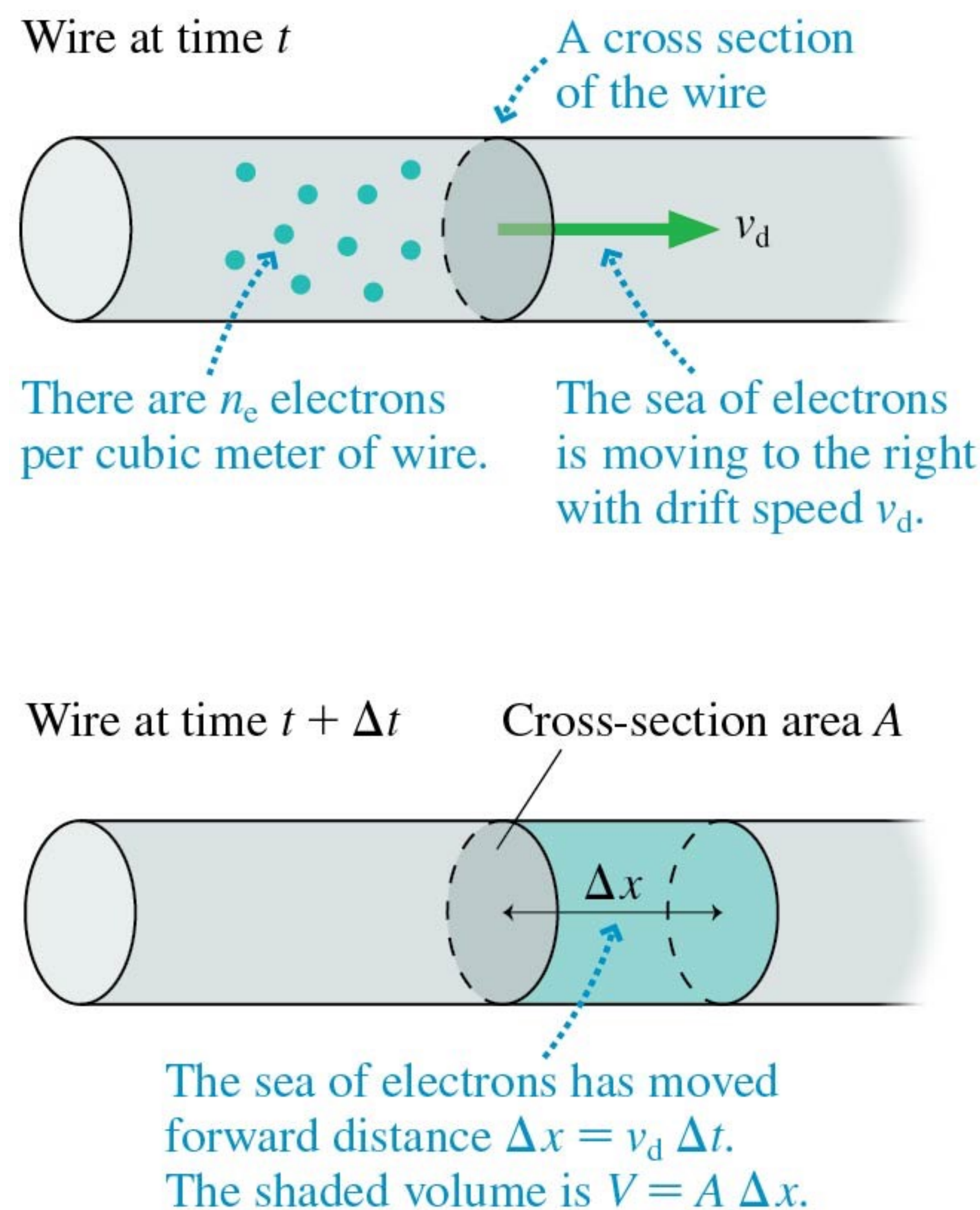
The Electron Current

- If the number density of conduction electrons is n_e, then the total number of electrons in the shaded cylinder is

$$N_e = n_e V$$
$$= n_e A \Delta x$$
$$= n_e A v_d \Delta t$$

- So the electron current is

$$i_e = n_e A v_d$$



The Electron Density

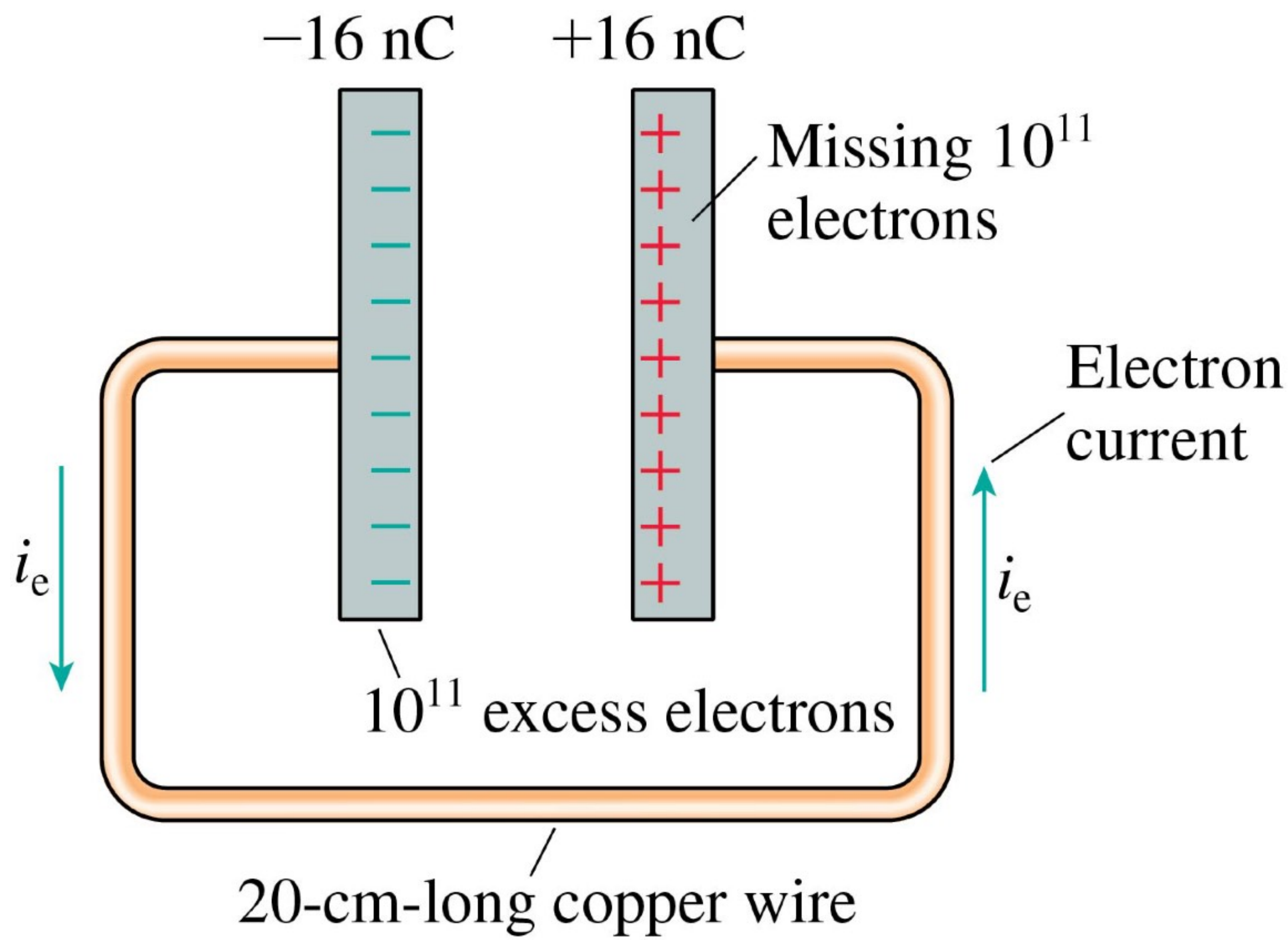
- In most metals, each atom contributes one valence electron to the sea of electrons.
- Thus the number of conduction electrons n_e is the same as the number of atoms per cubic meter.

TABLE 27.1 Conduction-electron density in metals

Metal	Electron density (m^{-3})
Aluminum	6.0×10^{28}
Copper	8.5×10^{28}
Iron	8.5×10^{28}
Gold	5.9×10^{28}
Silver	5.8×10^{28}

Discharging a Capacitor

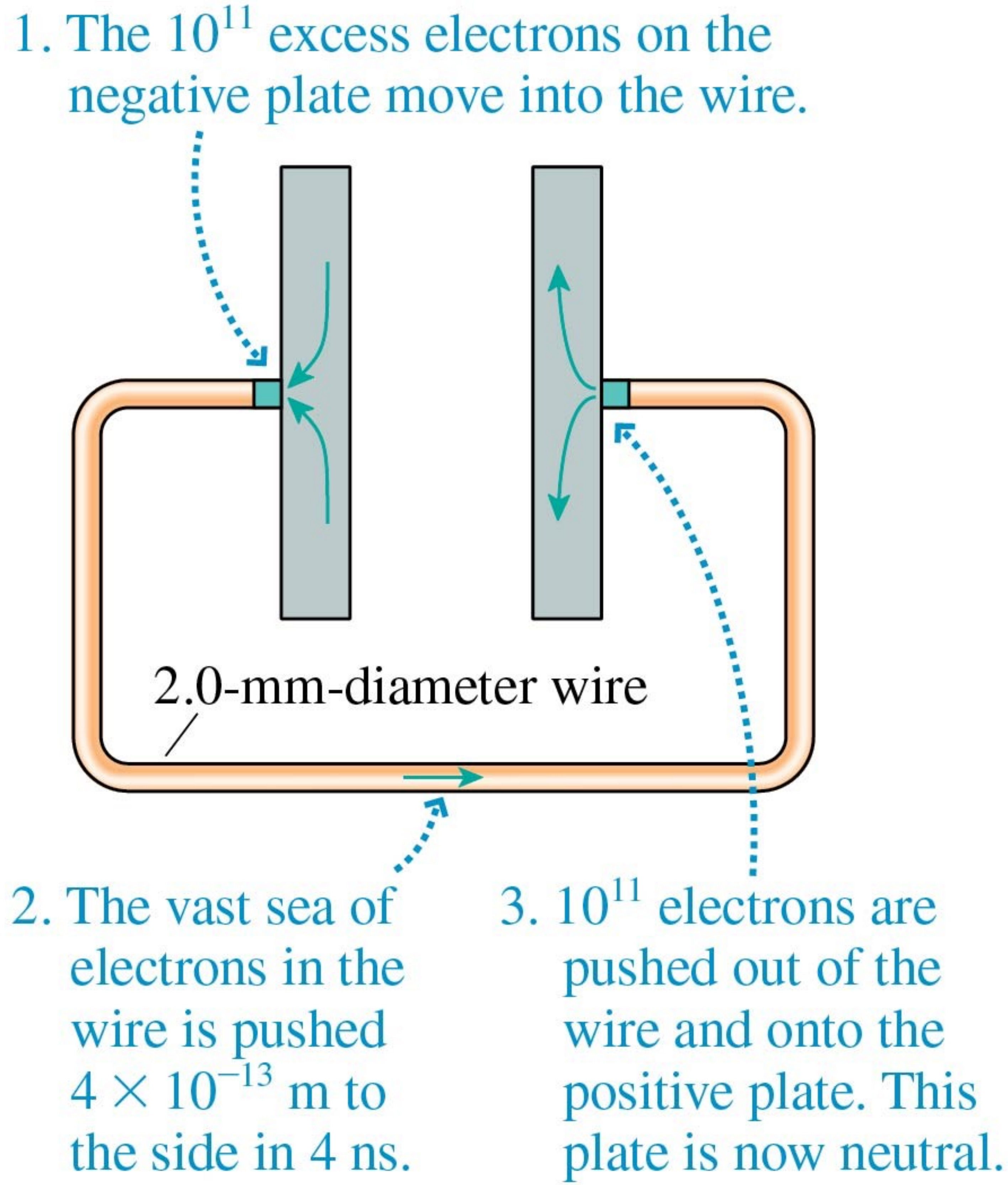
- How long should it take to discharge this capacitor?
- A typical drift speed of electron current through a wire is $v_d \approx 10^{-4}$ m/s.
- At this rate, it would take an electron about 2000 s (over half an hour) to travel 20 cm.



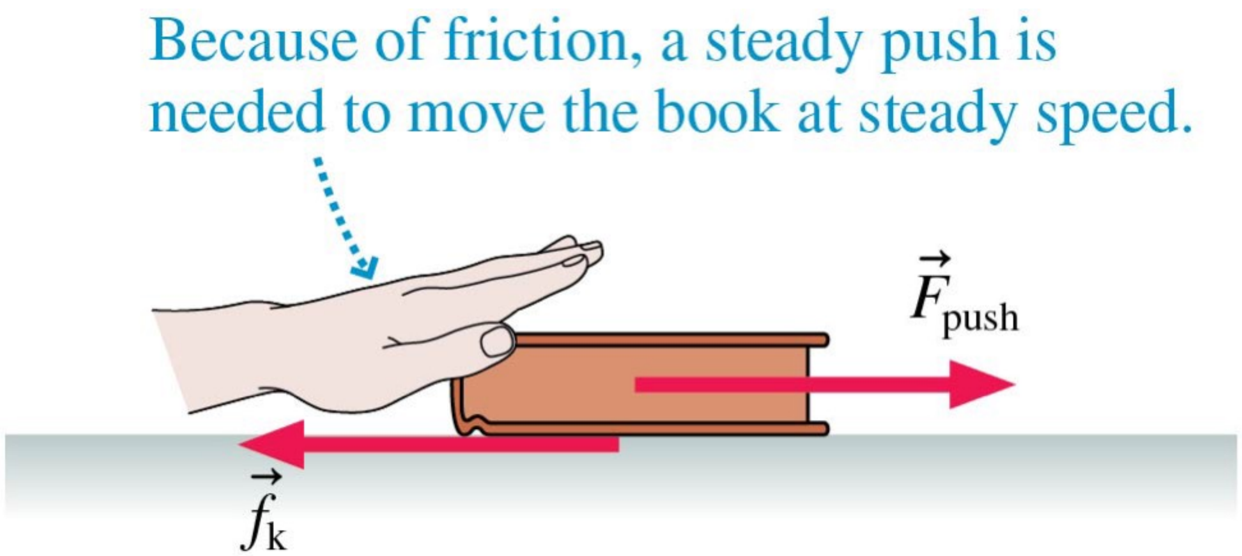
- But real capacitors discharge almost instantaneously!
- What’s wrong with our calculation?

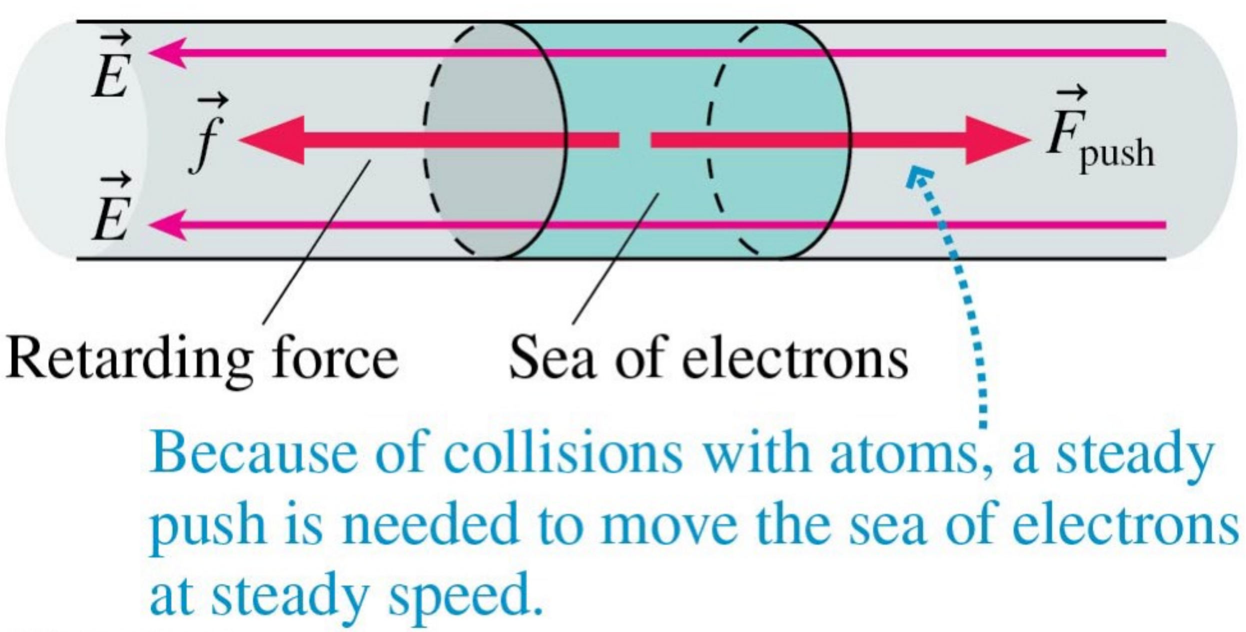
Discharging a Capacitor

- The wire is *already full* of electrons!
- We don't have to wait for electrons to move all the way through the wire from one plate to another.
- We just need to slightly rearrange the charges on the plates *and* in the wire.



Creating a Current

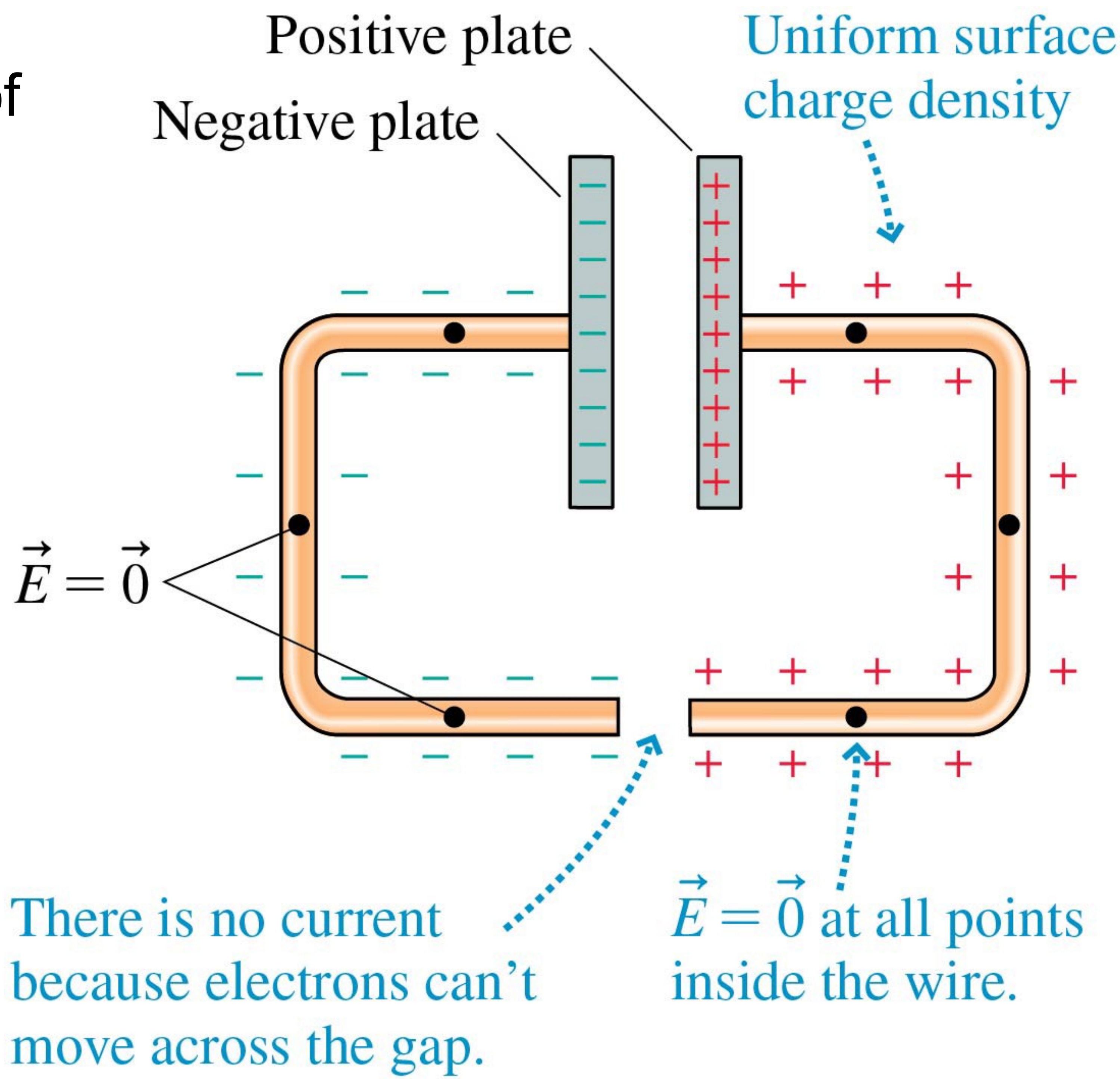




- A book on a table will slow down and stop unless you continue pushing.
- Analogously, the sea of electrons will slow down and stop unless you continue pushing with an electric field.

Establishing the Electric Field in a Wire

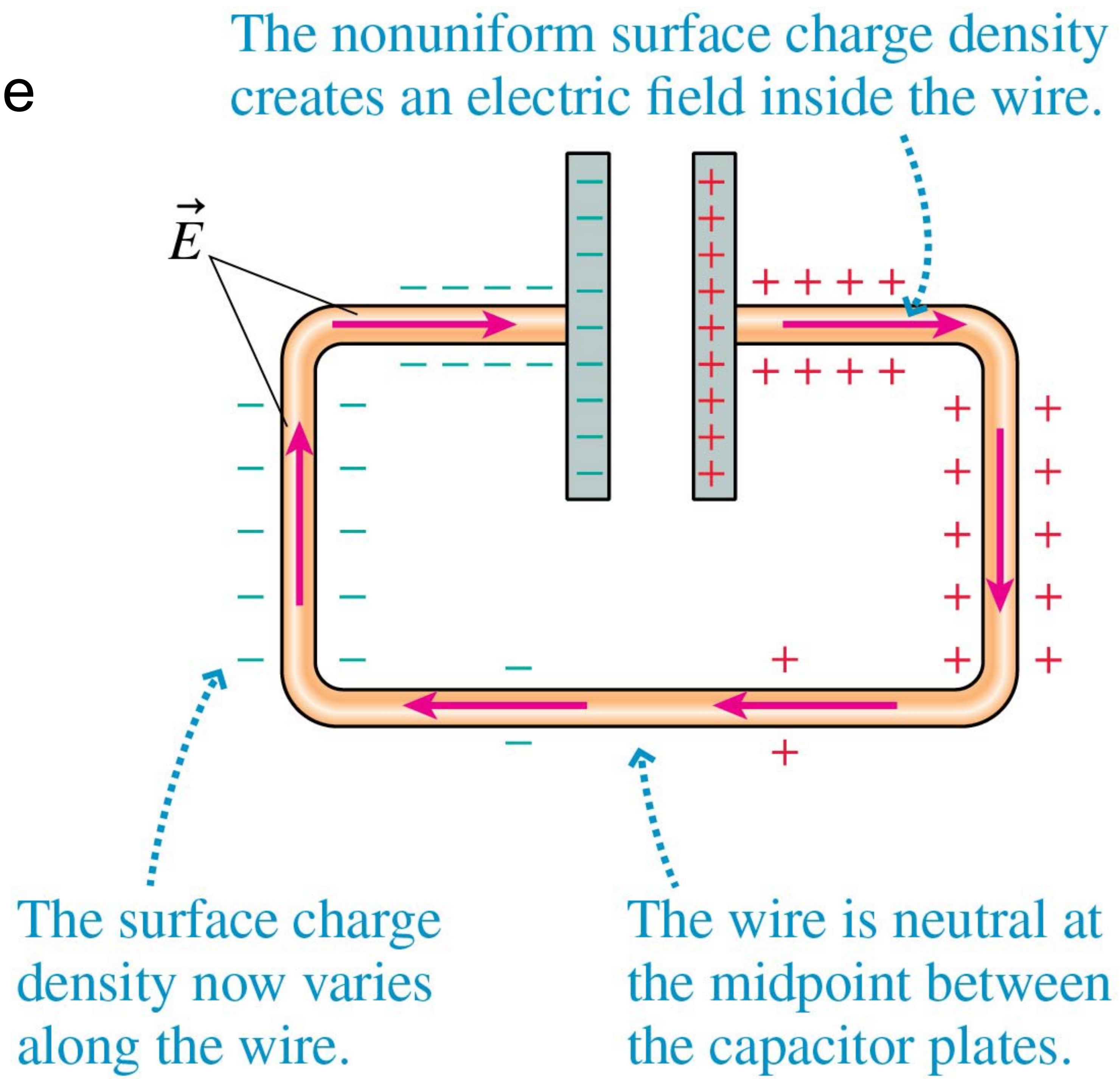
- The figure shows two metal wires attached to the plates of a charged capacitor.
- This is an electrostatic situation.
- What will happen if we connect the bottom ends of the wires together?



Establishing the Electric Field in a Wire

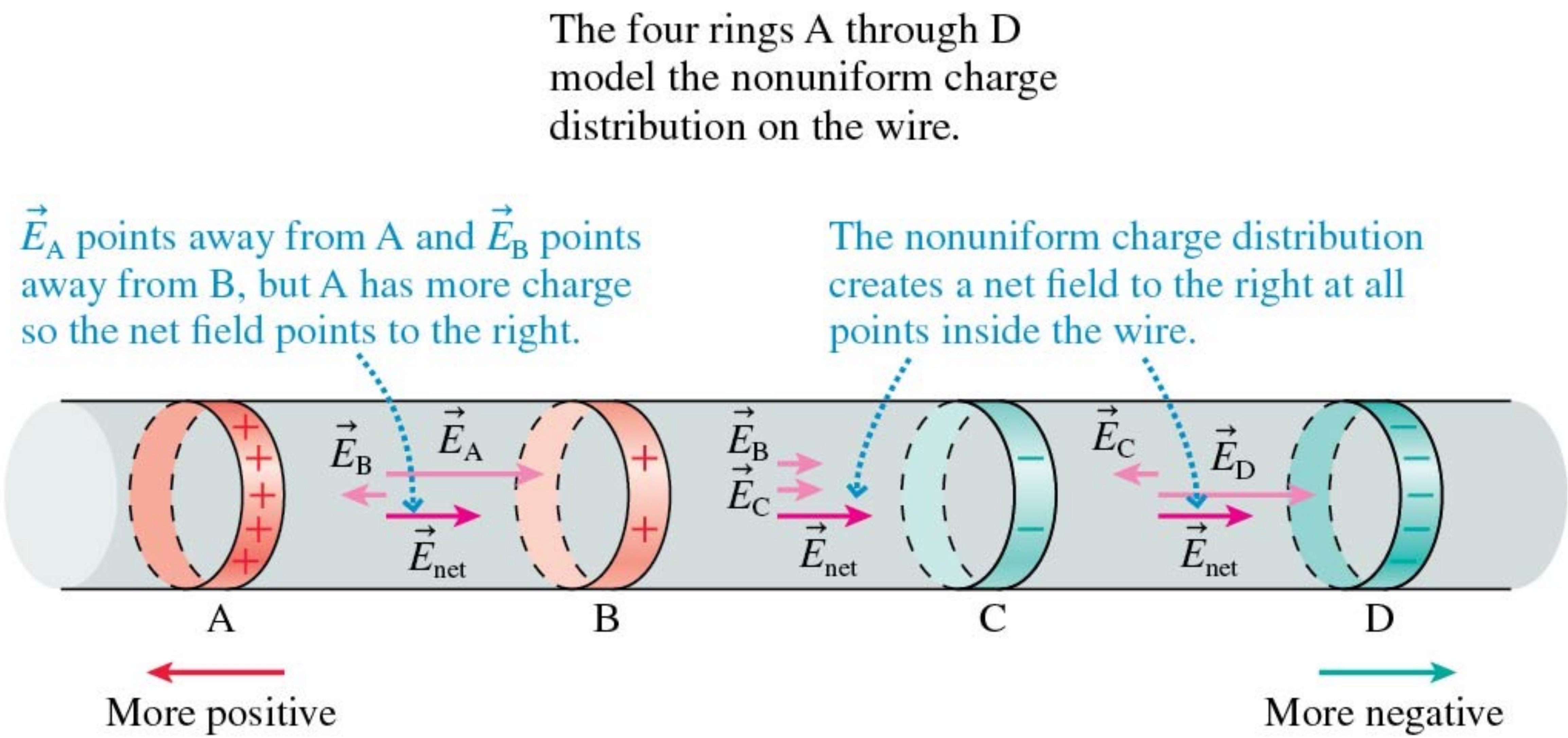
- Within a *very* brief interval of time ($\approx 10^{-9}$ s) of connecting the wires, the sea of electrons shifts slightly.

- The surface charge is rearranged into a *nonuniform* distribution, as shown in the figure.



Establishing the Electric Field in a Wire

- The *nonuniform* distribution of surface charges along a wire creates a net electric field *inside* the wire that points from the more positive end toward the more negative end of the wire.
- This is the internal electric field that pushes the electron current through the wire.



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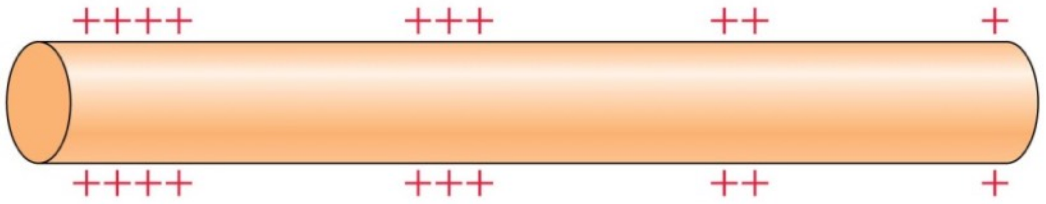
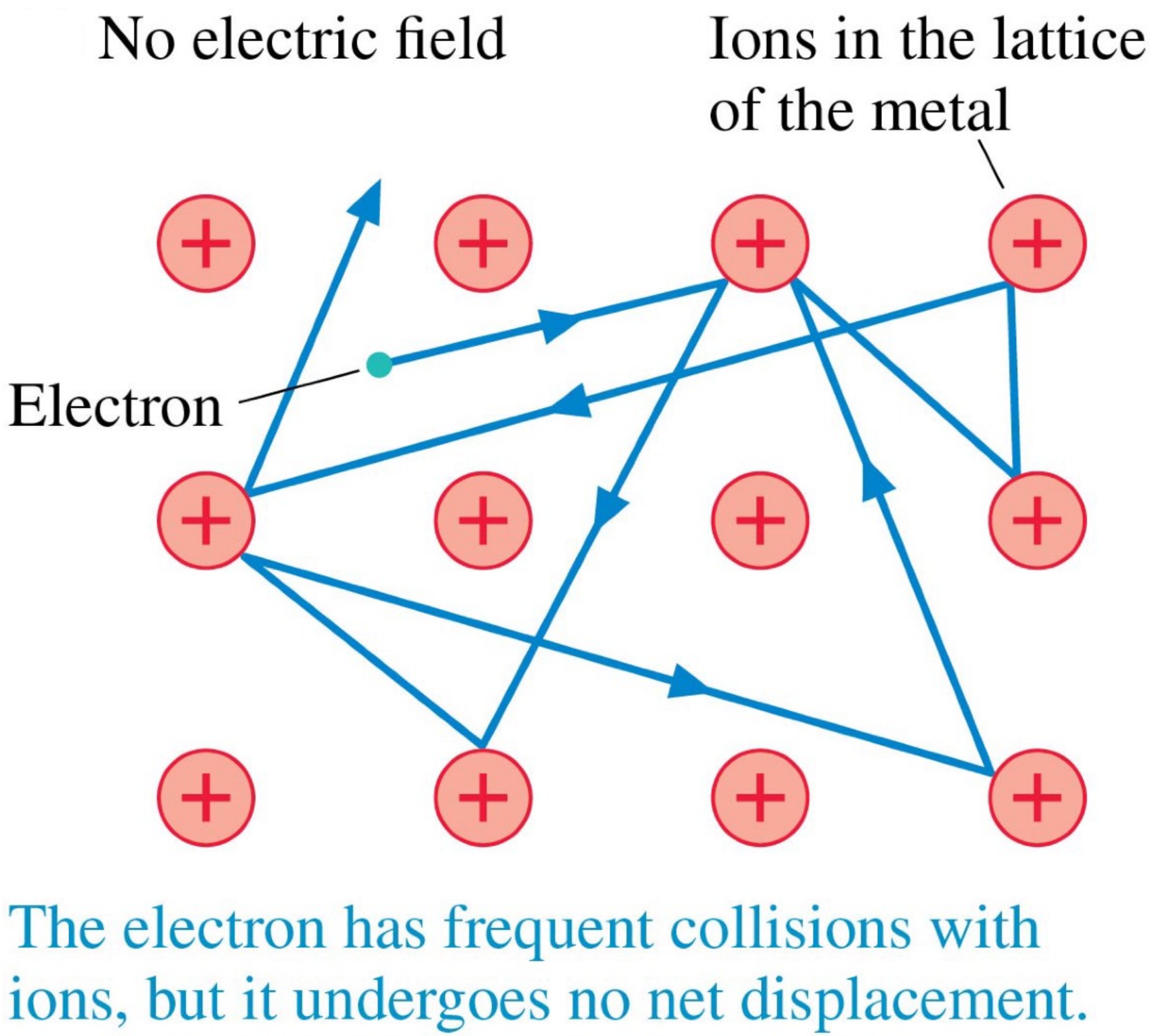
Surface charge is distributed on a wire as shown. **Electrons** in the wire



A. Drift to the right.
B. Drift to the left.
C. Move upward.
D. Move downward.
E. On average, remain at rest.

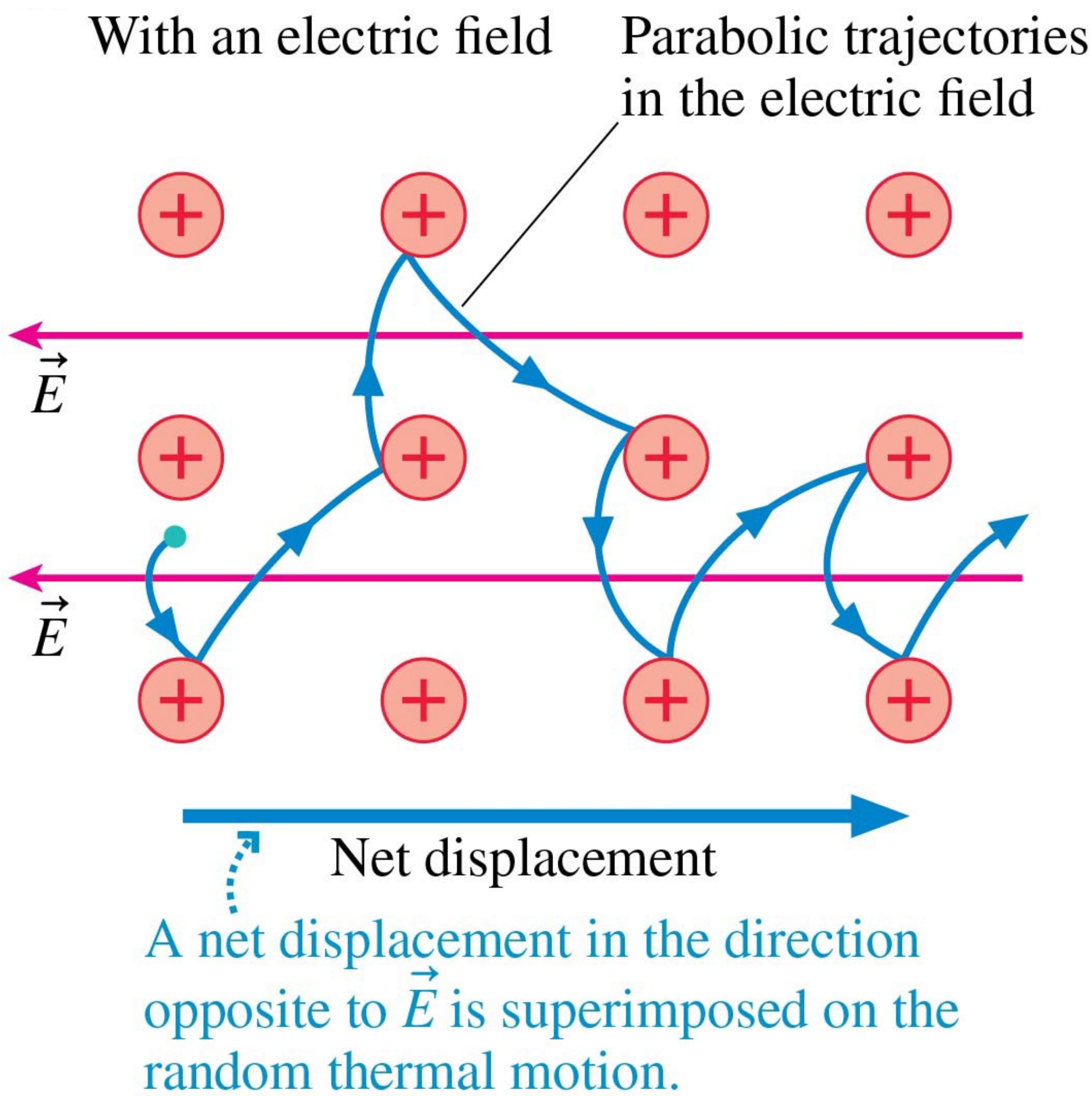
A Model of Conduction

- Within a conductor in electrostatic equilibrium, there is no electric field.
- In this case, an electron bounces back and forth between collisions, but its *average* velocity is zero.



A Model of Conduction

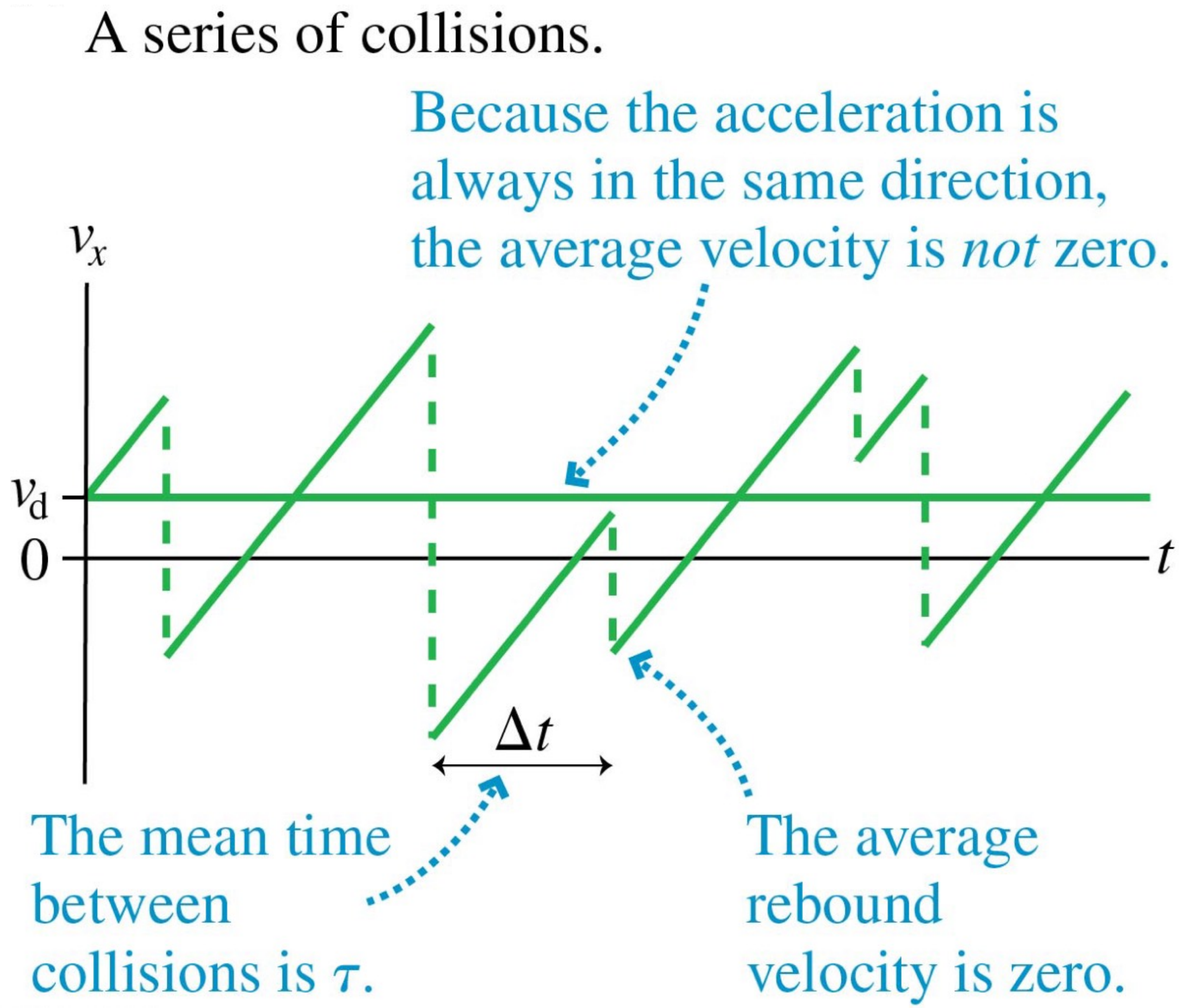
- In the presence of an electric field, the electric force causes electrons to move along parabolic trajectories between collisions.
- Because of the curvature of the trajectories, there is a slow net motion in the "downhill" direction.



A Model of Conduction

- The graph shows the speed of an electron during multiple collisions.
- The average drift speed is

$$v_d = \frac{e\tau}{m}E$$



Electron Current

- The electric field strength E in a wire of cross-section A causes an electron current:

$$i_e = \frac{n_e e \tau A}{m} E$$

- The electron density n_e and the mean time between collisions τ are properties of the metal.
- The electron current is directly proportional to the electric field strength.

Current

- If Q is the total amount of charge that has moved past a point in a wire, we define the current I in the wire to be the rate of charge flow:

$$I \equiv \frac{dQ}{dt}$$

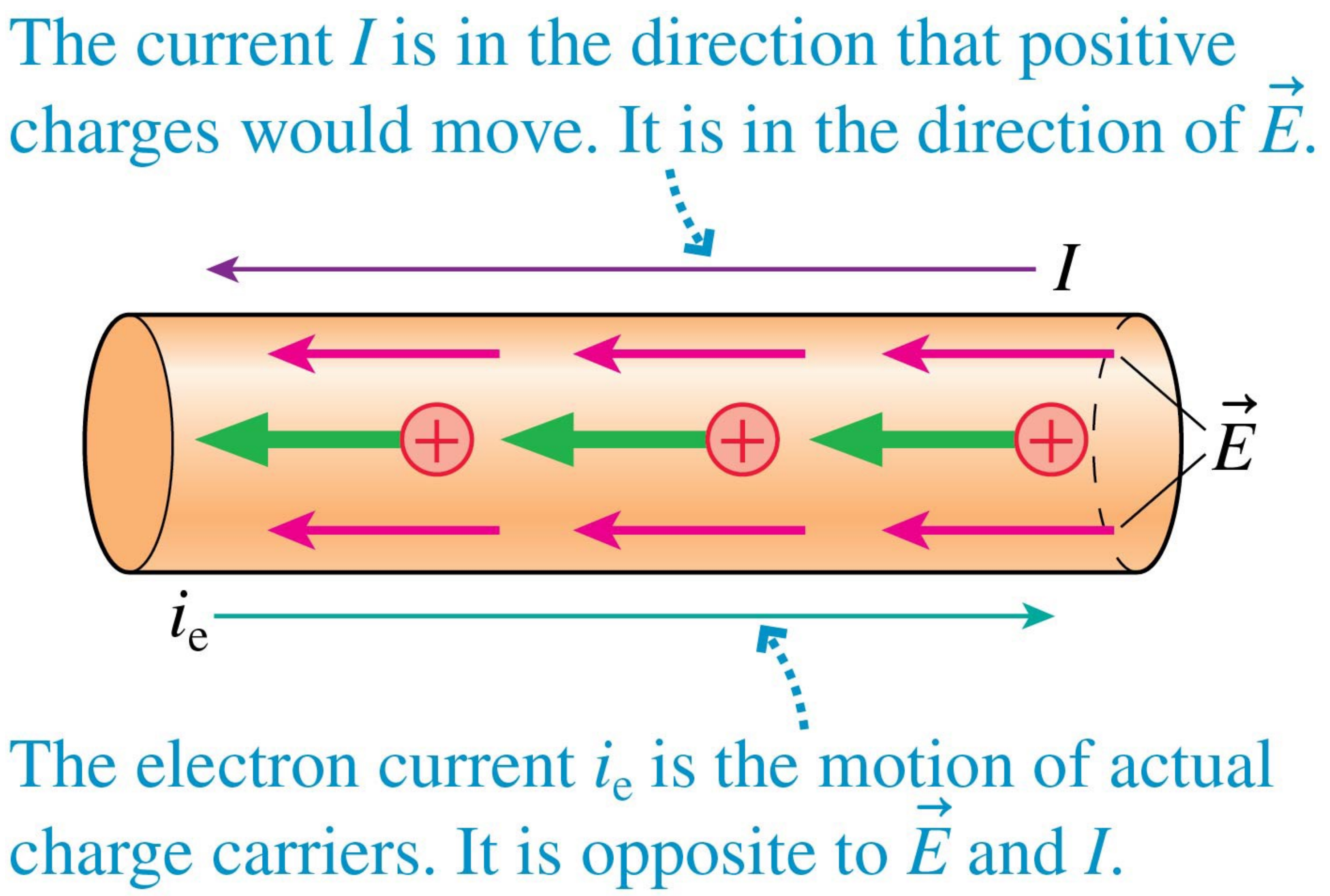
current is the *rate* at which charge flows

- The SI unit for current is the coulomb per second, which is called the **ampere**.
- $1 \text{ ampere} = 1 \text{ A} = 1 \text{ C/s}$
- The conventional current I and the electron current i_{e} are related by

$$I = \frac{Q}{\Delta t} = \frac{eN_{\mathrm{e}}}{\Delta t} = ei_{\mathrm{e}}$$

Current

- Note that the direction of the current I in a metal is opposite to the direction of the electron current i_e.



The Current Density in a Wire

- The **current density** J in a wire is the current per square meter of cross section:

$$J = \text{current density} \equiv \frac{I}{A} = n_e e v_d$$

- The current density has units of $\mathrm{A/m^2}$.

The electron drift speed is small!

EXAMPLE 27.4 **Finding the electron drift speed**

A 1.0 A current passes through a 1.0-mm-diameter aluminum wire. What are the current density and the drift speed of the electrons in the wire?

SOLVE We can find the drift speed from the current density. The current density is

$$J = \frac{I}{A} = \frac{I}{\pi r^2} = \frac{1.0\ \text{A}}{\pi(0.00050\ \text{m})^2} = 1.3 \times 10^6\ \text{A/m}^2$$

The electron drift speed is thus

$$v_{\text{d}} = \frac{J}{n_{\text{e}} e} = 1.3 \times 10^{-4}\ \text{m/s} = 0.13\ \text{mm/s}$$

where the conduction-electron density for aluminum was taken from Table 27.1.

ASSESS We earlier used 1.0×10^{-4} m/s as a typical electron drift speed. This example shows where that value comes from.

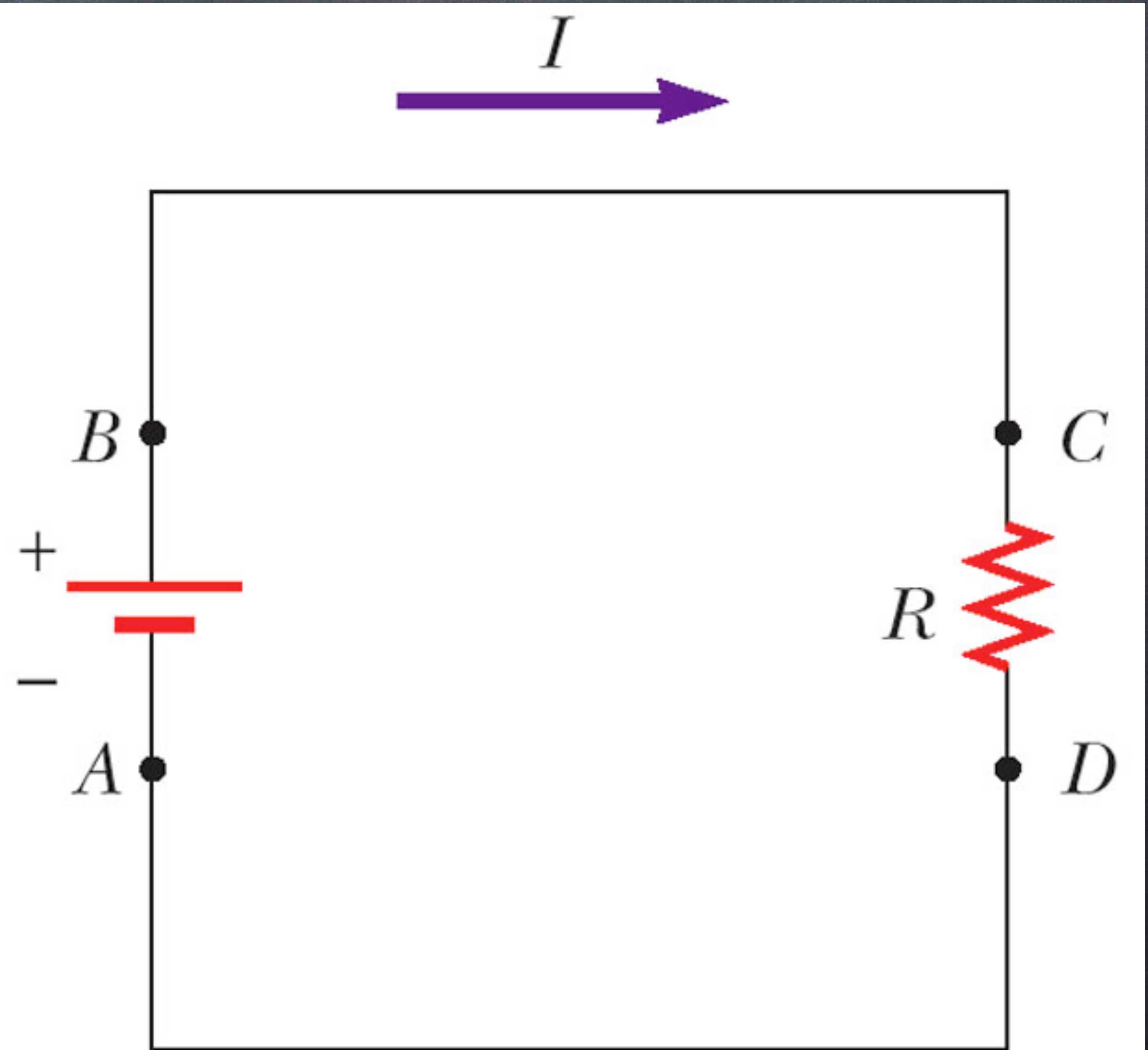
Current

- When a battery is placed in a closed loop an electric field is established inside the conducting wire.

- This electric field starts on the positive terminal and ends on the negative terminal.

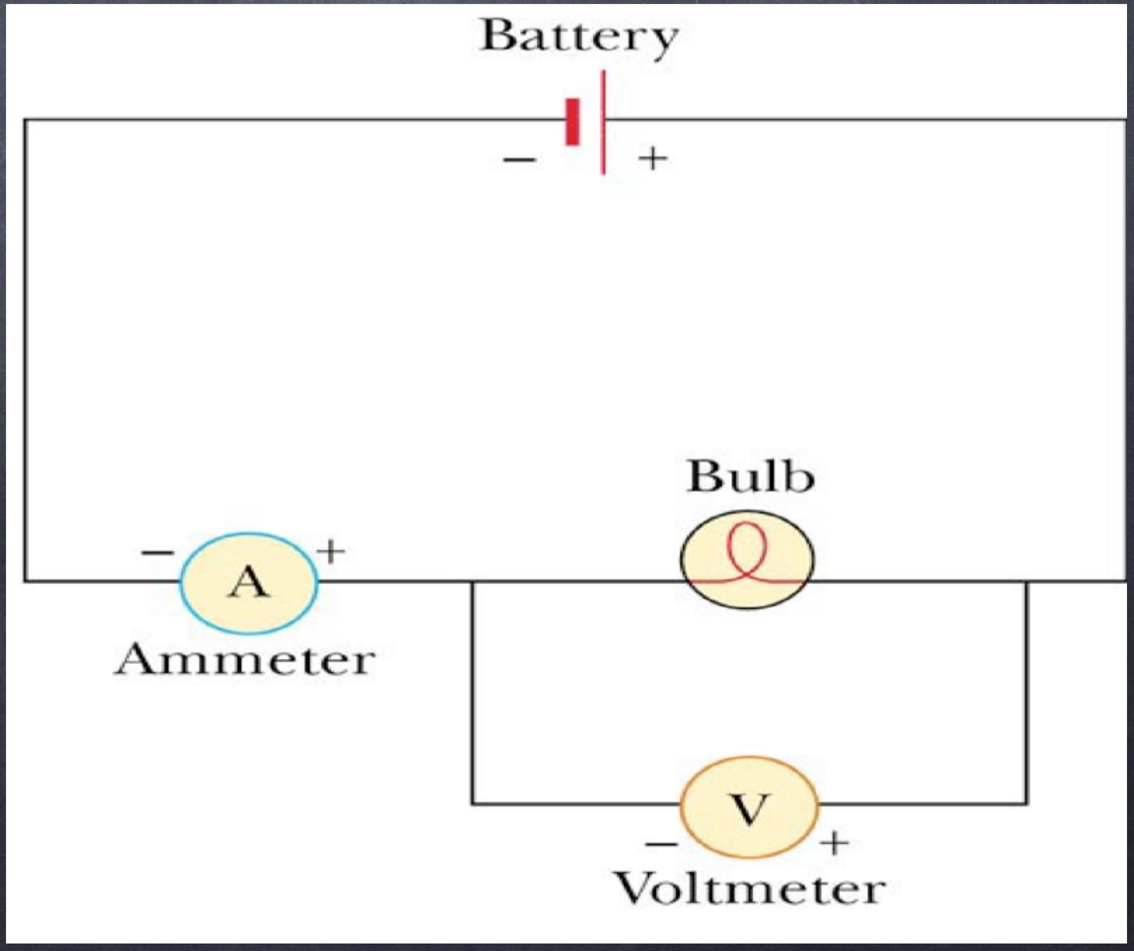
- Positive charges will move with the electric field toward the negative terminal.

- When the circuit is open, the positive charges don't see a negative terminal and thus, no current.



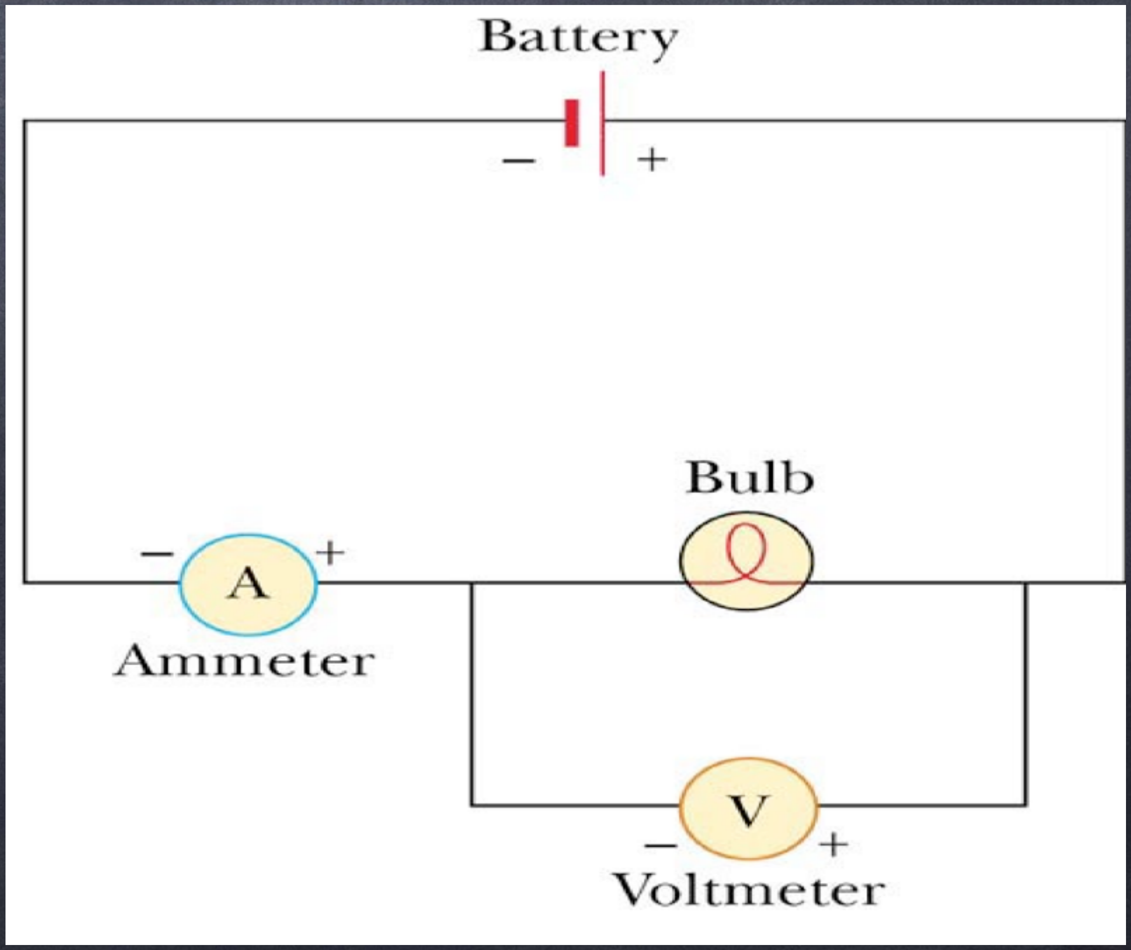
Current

- Remember: The potential difference created by the battery terminal is also called EMF (ElectroMotive Force).
- This is a silly name as EMF is measured in Volts.
- We physically measure the amount of current in a circuit with an ammeter.
- You connect the ammeter in the circuit in series with the element you are trying to measure the current of.

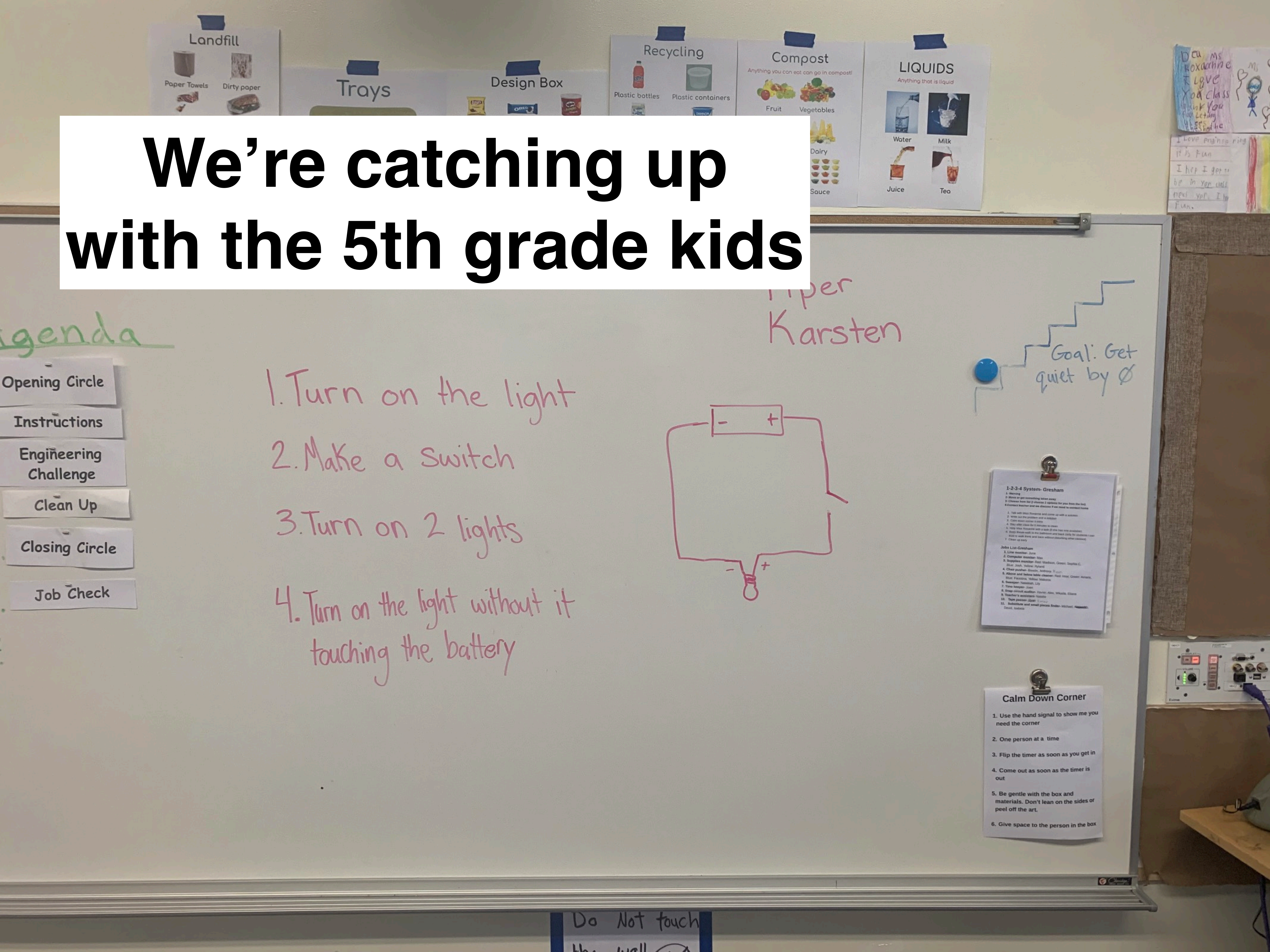


Current

- A voltmeter is used to measure electric potential difference (or voltage).
- You connect a voltmeter in the circuit in parallel to the element you are trying to measure the potential difference of.
  - You basically want to measure what the potential was before the element and what the potential is after the element.

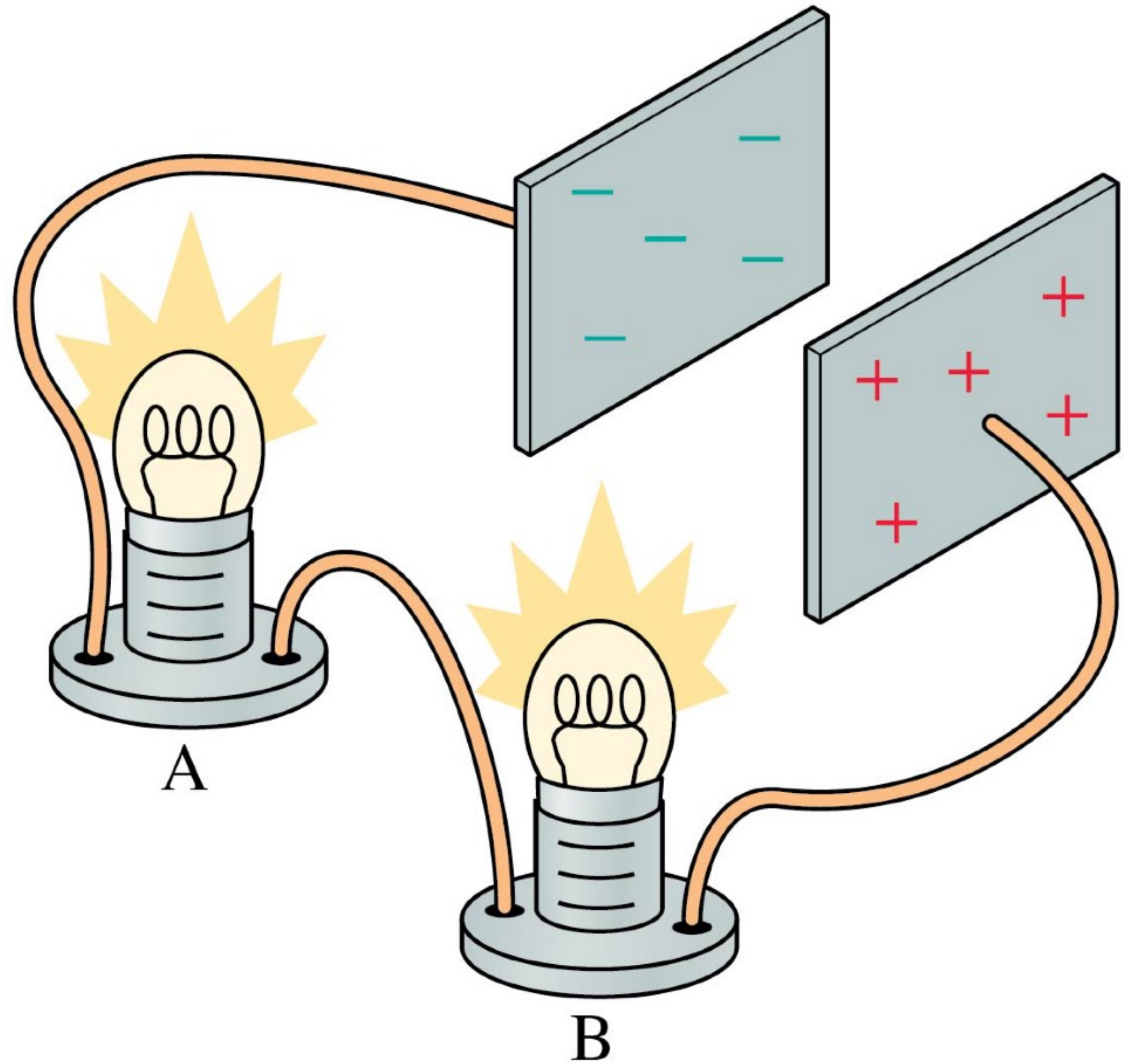


We’re catching up with the 5th grade kids



Conservation of Current

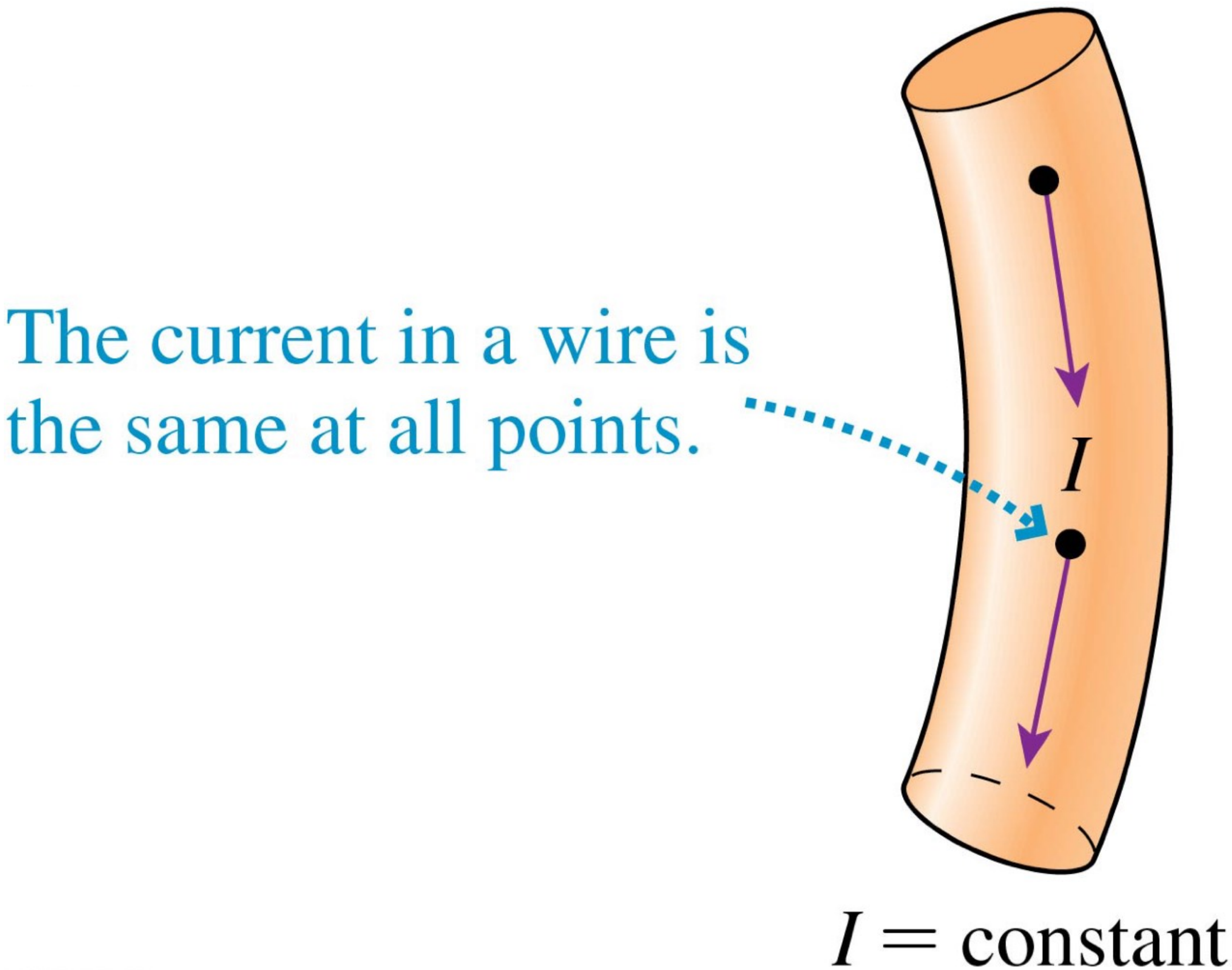
- The figure shows two lightbulbs in the wire connecting two charged capacitor plates.
- As the capacitor discharges, the current through both bulbs is *exactly the same!*
- The rate of electrons *leaving* a lightbulb (or any other device) is exactly the same as the rate of electrons *entering* the lightbulb.



The bulbs are equally bright.

Charge Conservation and Current

- Due to conservation of charge, the current must be the same at all points in a current-carrying wire.



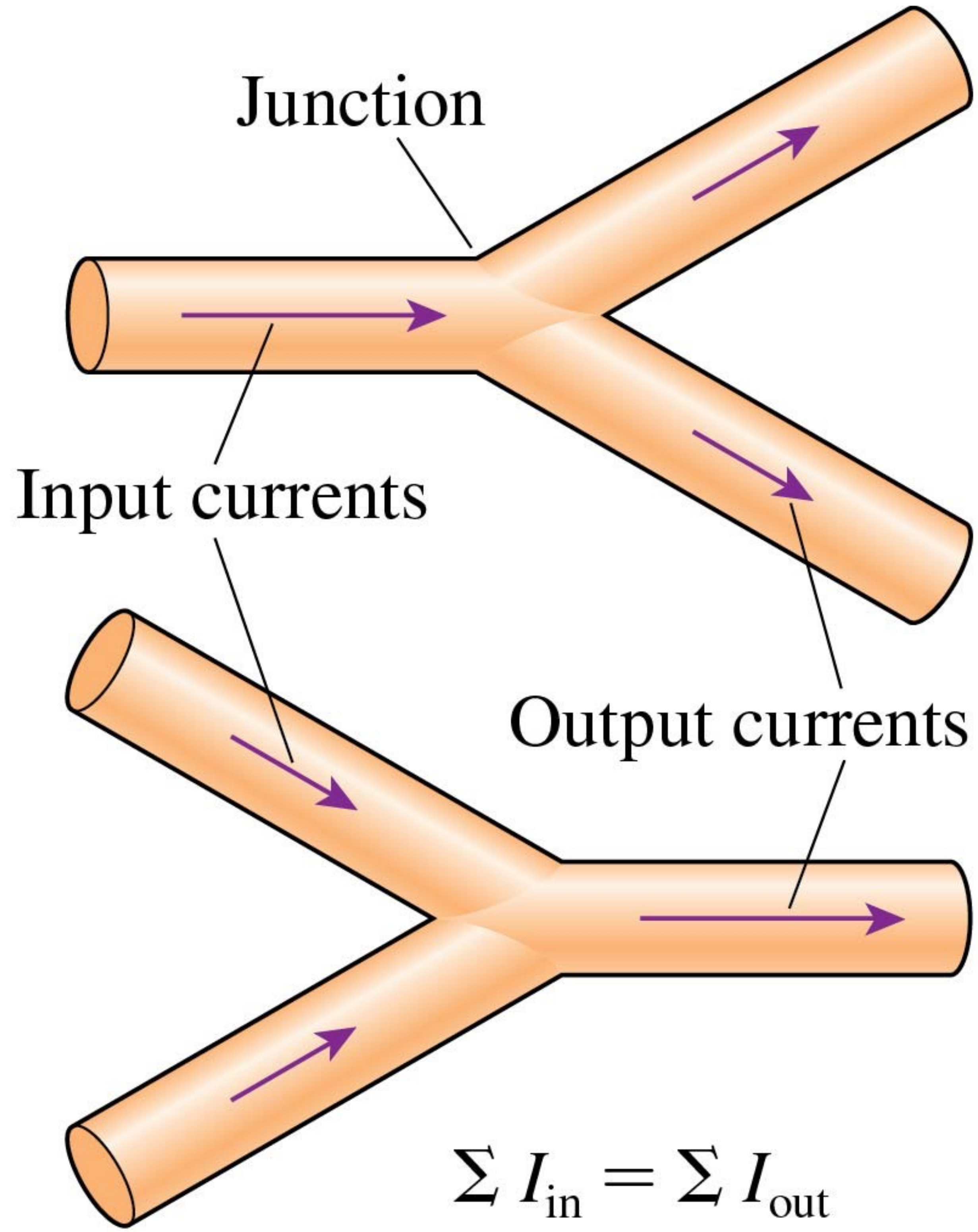
Kirchhoff's Junction Law

- For a *junction*, the law of conservation of current requires that

$$\sum I_{in} = \sum I_{out}$$

where the Σ symbol means summation.

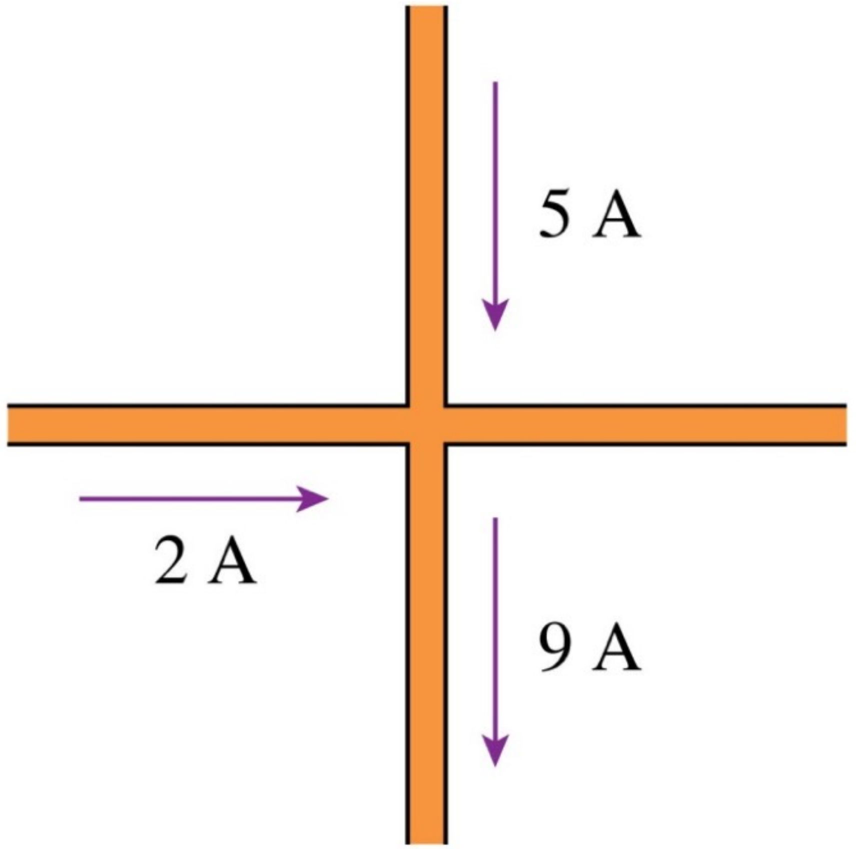
- This basic conservation statement is called **Kirchhoff's junction law**.



iClicker question 10-2

The current in the fourth wire is

A. 16 A to the right.
B. 4 A to the left.
C. 2 A to the right.
D. 2 A to the left.
E. Not enough information to tell.



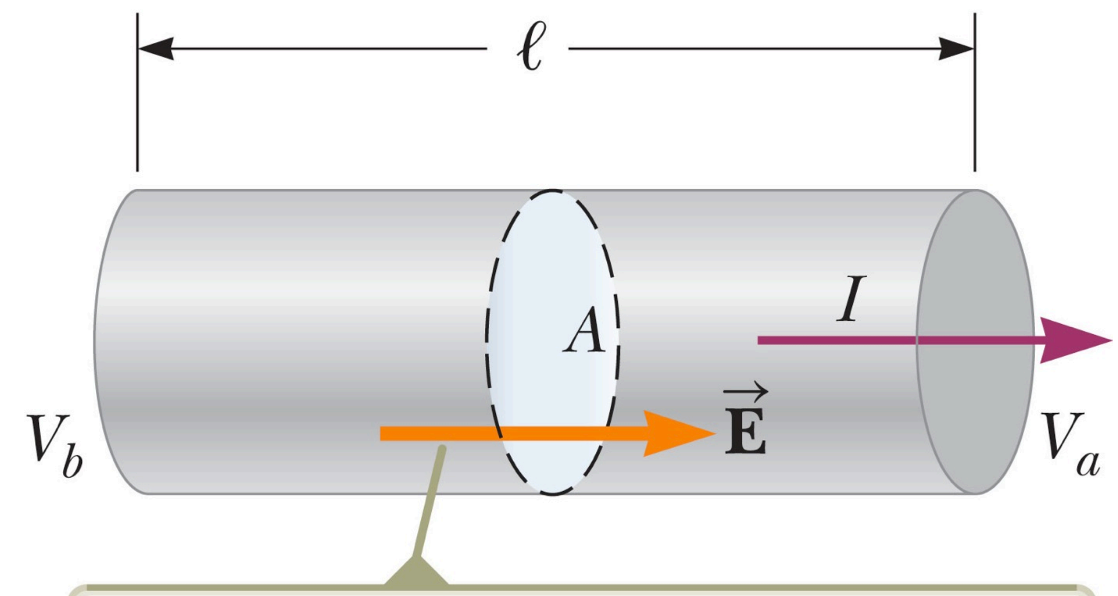
Ohm's Law

- In a conductor, the voltage applied across the ends of the conductor is proportional to the current through the conductor.
  - The constant of proportionality is the **resistance** of the conductor:

$$R \equiv \frac{\Delta V}{I}$$

  - SI units of resistance are *ohms* (Ω)

    1 Ω = 1 V / A



A potential difference $\Delta V = V_b - V_a$ maintained across the conductor sets up an electric field $\vec{\mathbf{E}}$, and this field produces a current I that is proportional to the potential difference.

Resistivity

- If I were constructing a wire, for a given battery how could I increase the current flow in the wire?
  - Use a good conductor.
  - Increase its cross-sectional area.
  - Shorten its length.
- The resistance, R, of a conductor is given by:

$$R = \rho \frac{L}{A}$$

- ρ is resistivity (depends on the type of material of the conductor).
- L is the length of the conductor and A is the cross-sectional area of the conductor.

Resistivity

- Resistivity is measured in units of: [Ω m]
- Metals typically have very low resistivity:

$$\text{Silver} \Rightarrow \rho = 1.59 \times 10^{-8}\,\Omega \cdot \text{m}$$

$$\text{Tungsten} \Rightarrow \rho = 5.25 \times 10^{-8}\,\Omega \cdot \text{m}$$

- Insulators have very high resistivity:

$$\text{Glass} \Rightarrow \rho \approx 10^{12}\,\Omega \cdot \text{m}$$

- We can also talk about how easily it is for charge to flow through a material.
- We define conductivity, σ, this way:

$$\sigma = \frac{1}{\rho}$$

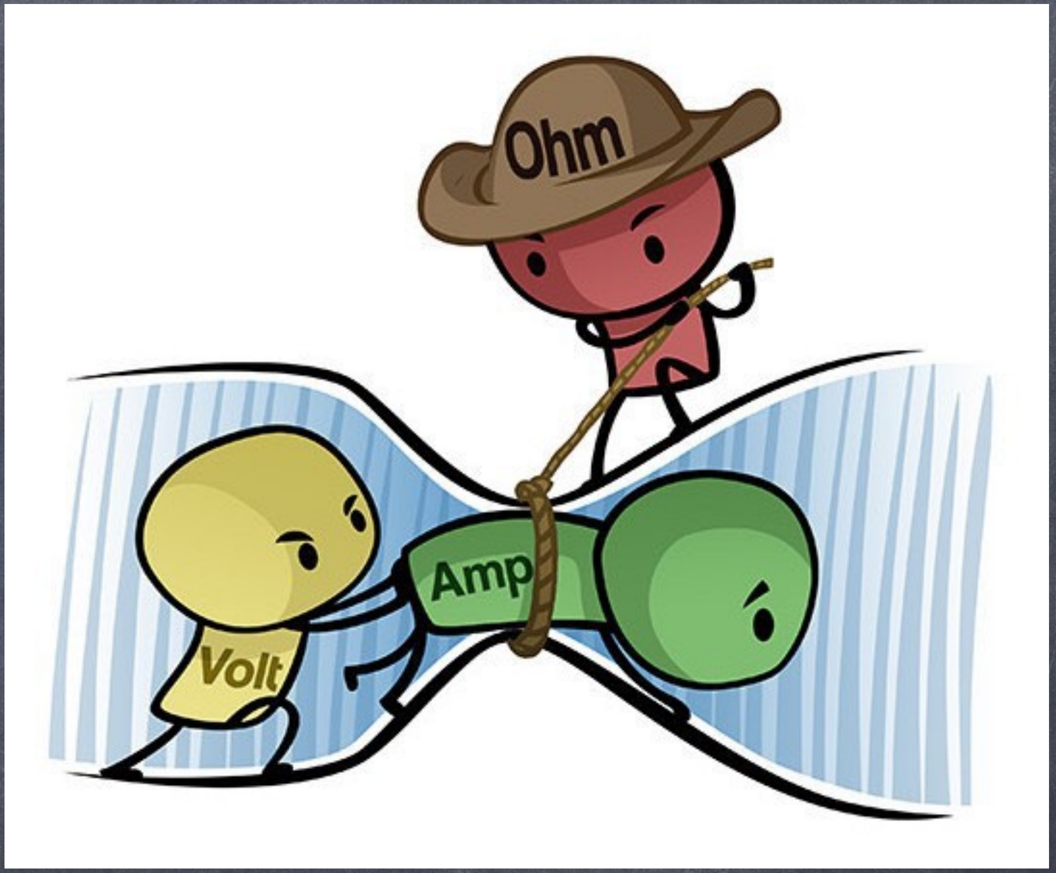
- Conductivity is measured in units of: 1/[Ω m]

Ohm's Law

- Ohm's Law quantifies the ability of a given material to resist the flow of charge for a given electric potential difference.

$$I = \frac{\Delta V}{R}$$

$$\Delta V = I(R)$$

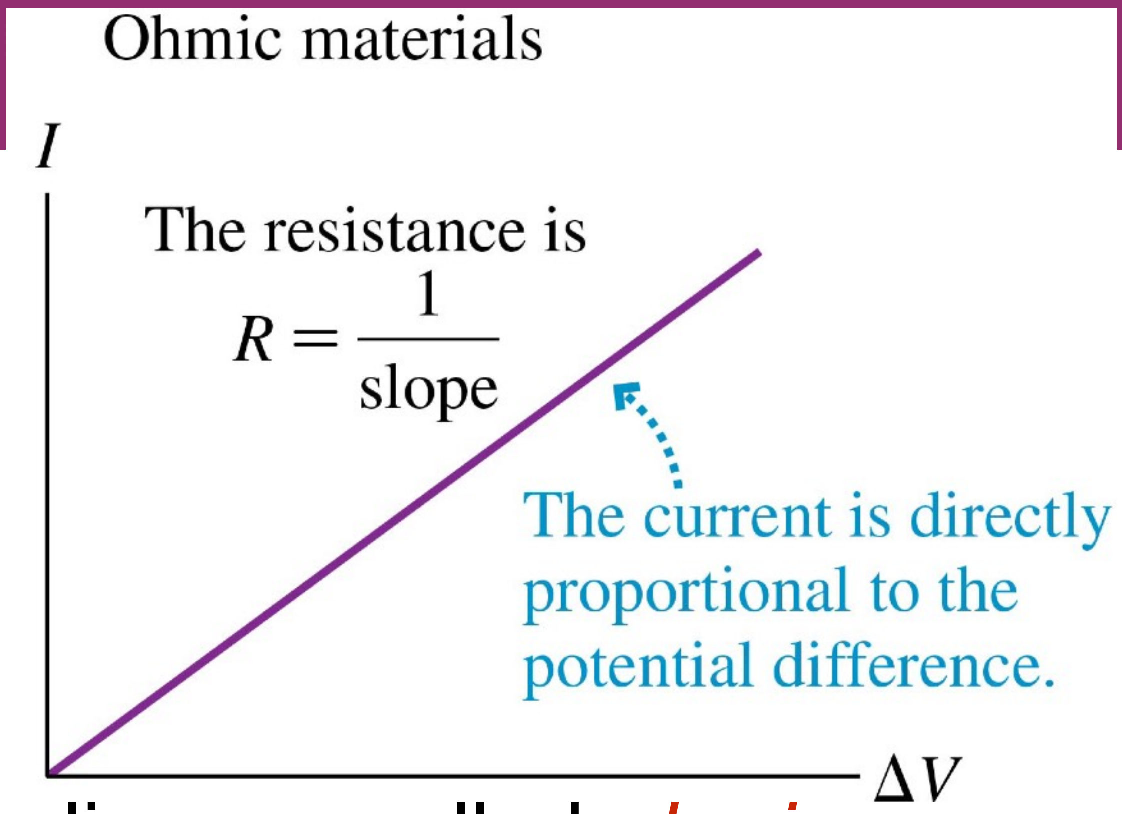


- where R is the resistance and is measured in Ω (Ohm's).

Ohm's Law

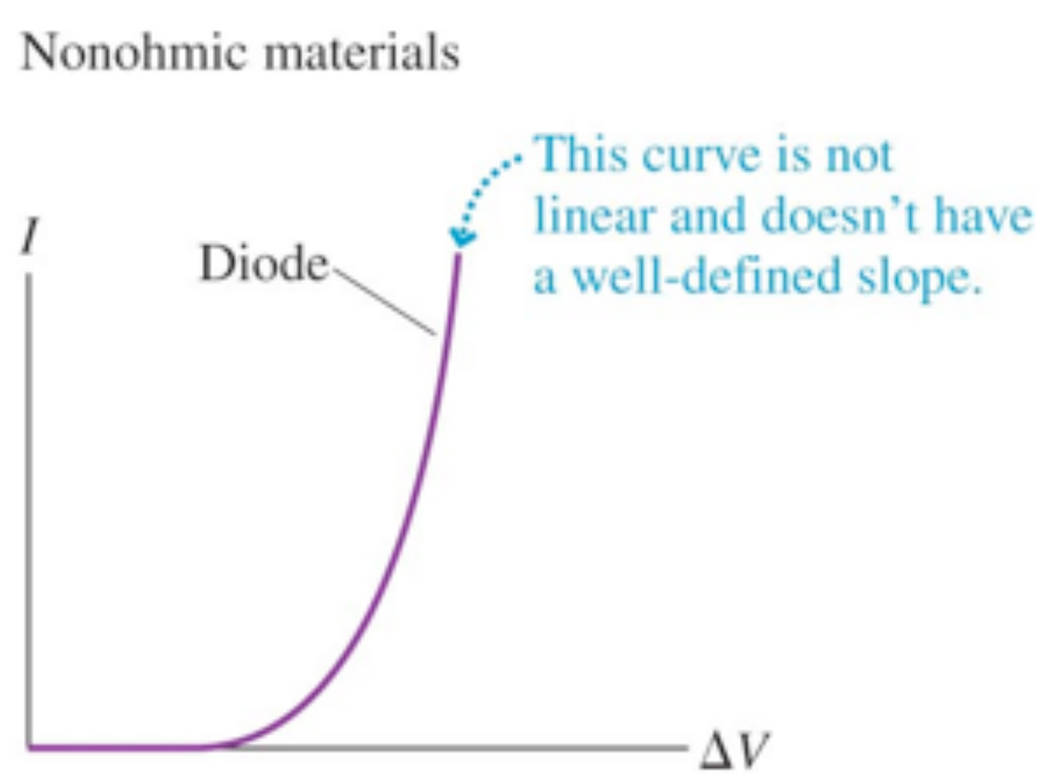
- A good conductor will have a low resistance, this means that current will easily flow through it.
- A good insulator will have a high resistance, this means that current will have a very hard time flowing through it.
- With Ohm's Law, as you increase electric potential ($\mathscr{D}$V), then current will increase linearly.
- The resistance in a circuit arises due to collisions between the electrons moving against the electric field and the electrons that are fixed in the atoms.

Ohm's Law



- Ohm's law is limited to those materials whose resistance R remains constant—or very nearly so—during use.
- The materials to which Ohm's law applies are called *ohmic*.
- The current through an ohmic material is proportional to the potential difference; doubling the potential difference doubles the current.
- Metal and other conductors are ohmic devices.

Certain materials do not obey Ohm's Law, these are called nonohmic materials



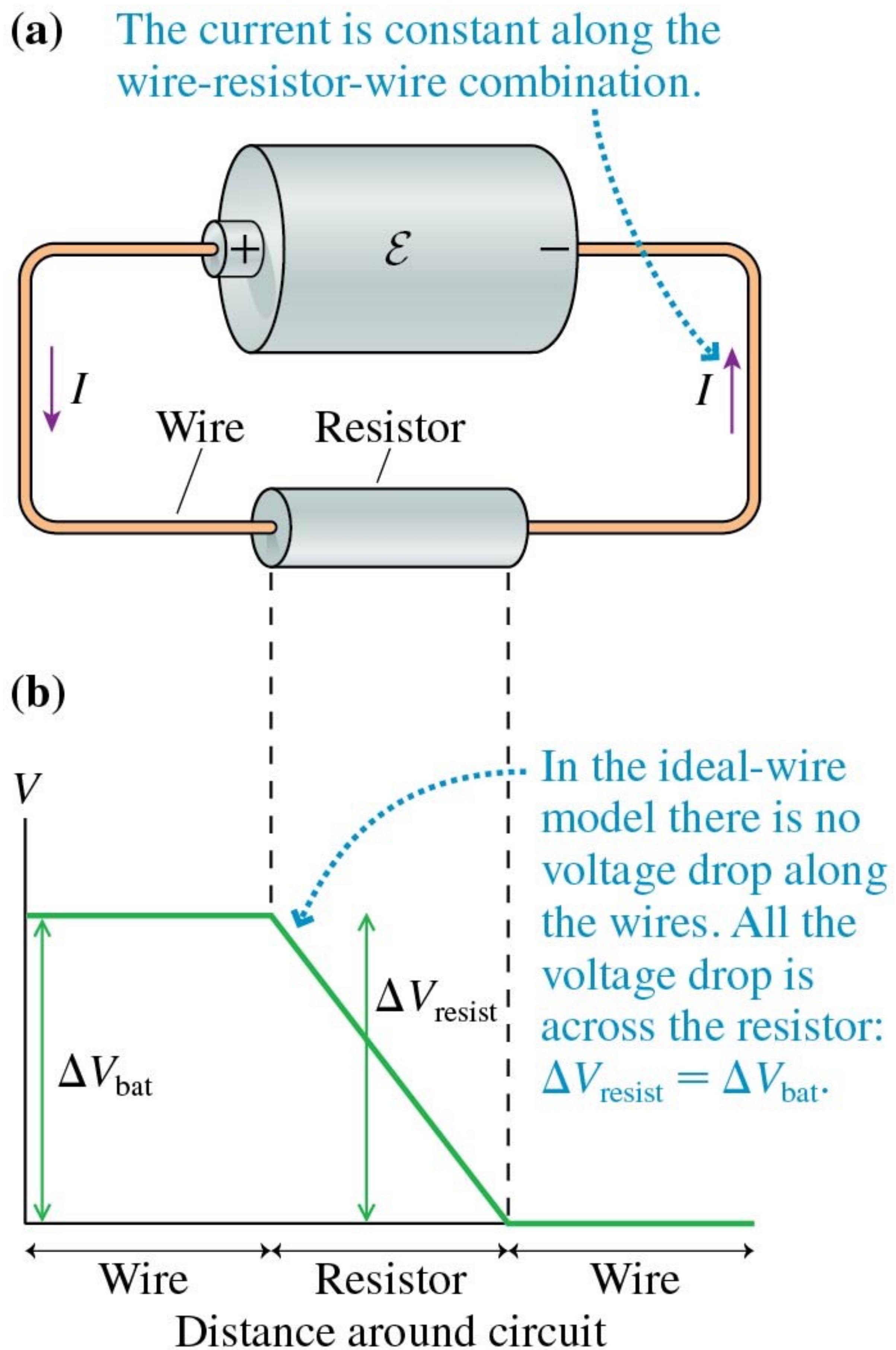
Resistivity

- For most metals, as you increase the temperature the resistivity of the metal increases.
- The following equation takes temperature into account for resistivity:

$$\rho = \rho_o[1 + \alpha(T - T_o)]$$

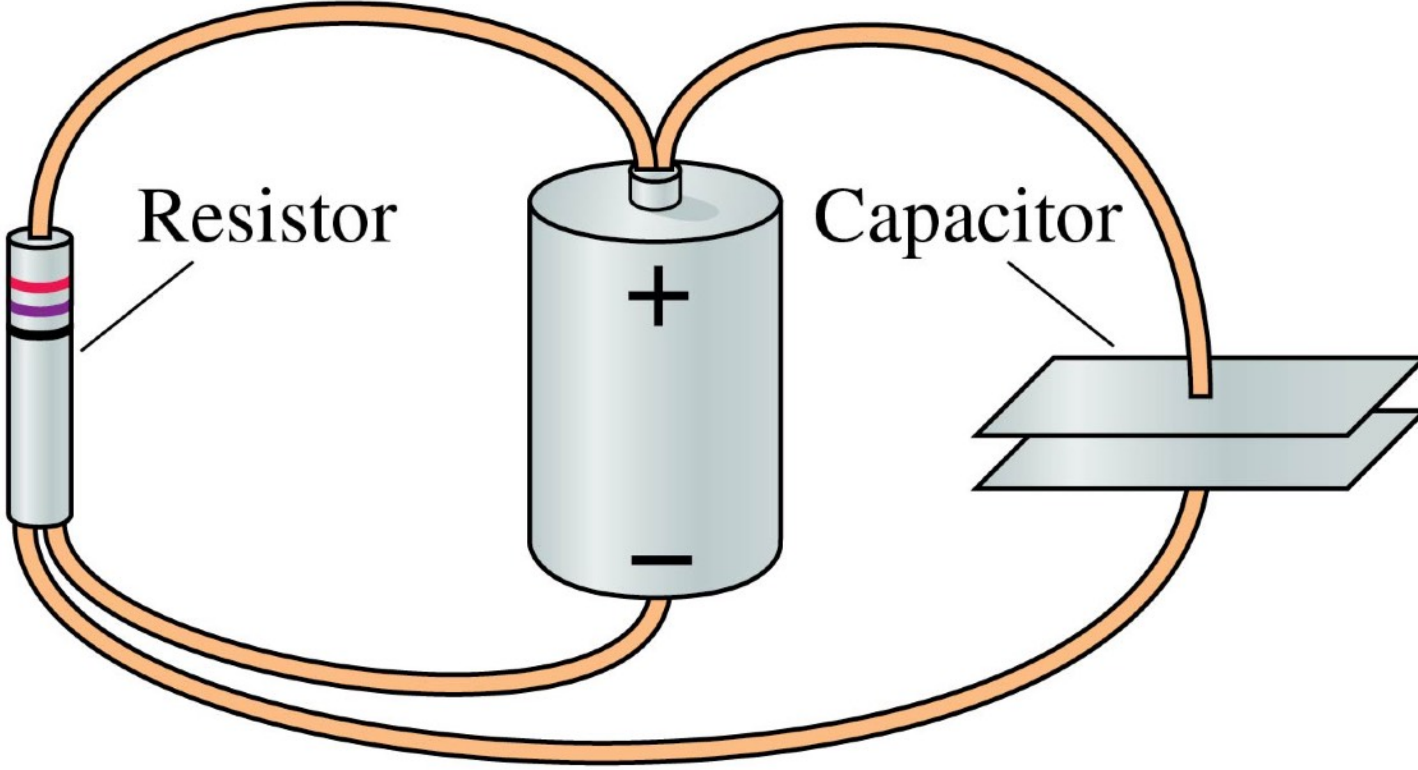
- where ρ is the resistivity at some temperature T.
- ρ_o is the resistivity at some reference temperature T_o.
- α is the temperature coefficient of resistivity.
- T_o is usually taken to be 20°C.

Battery-Wire-Resistor-Wire Circuit

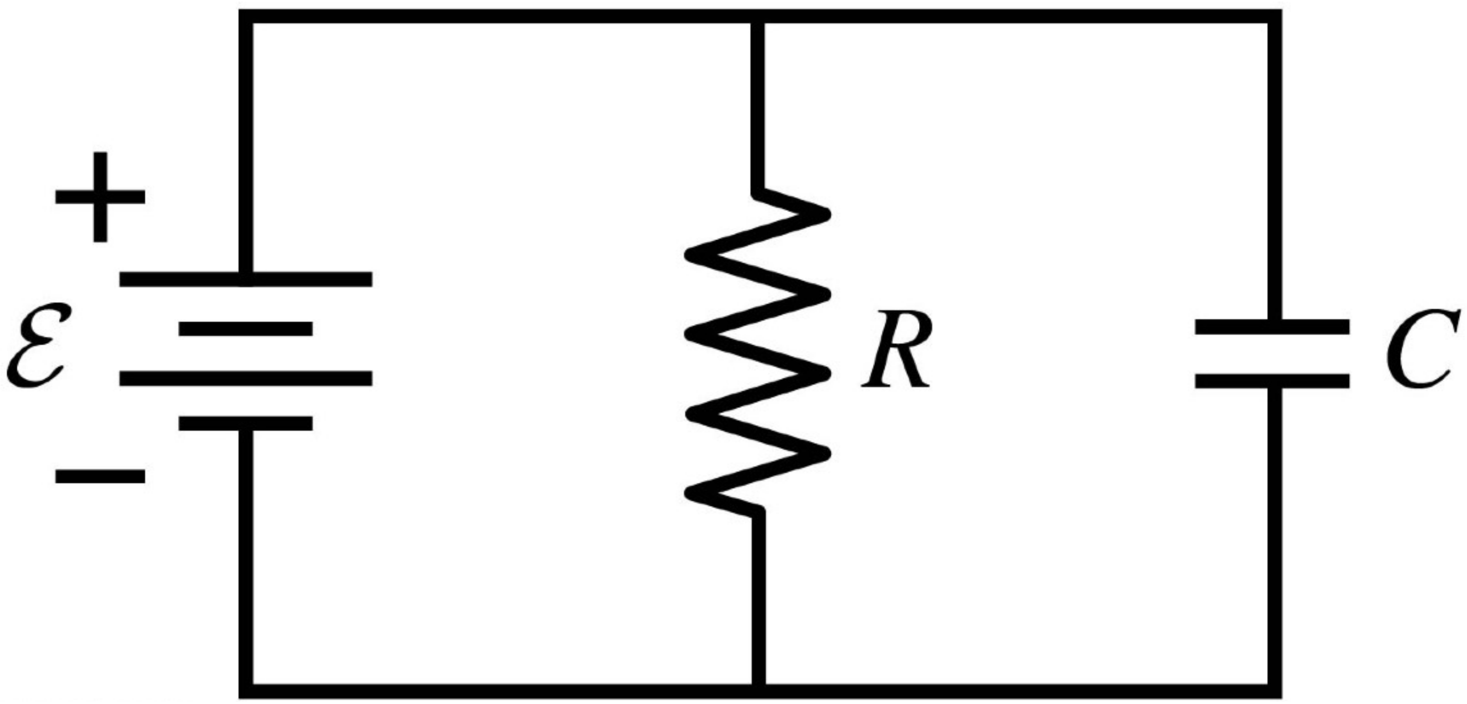


Circuit Diagrams

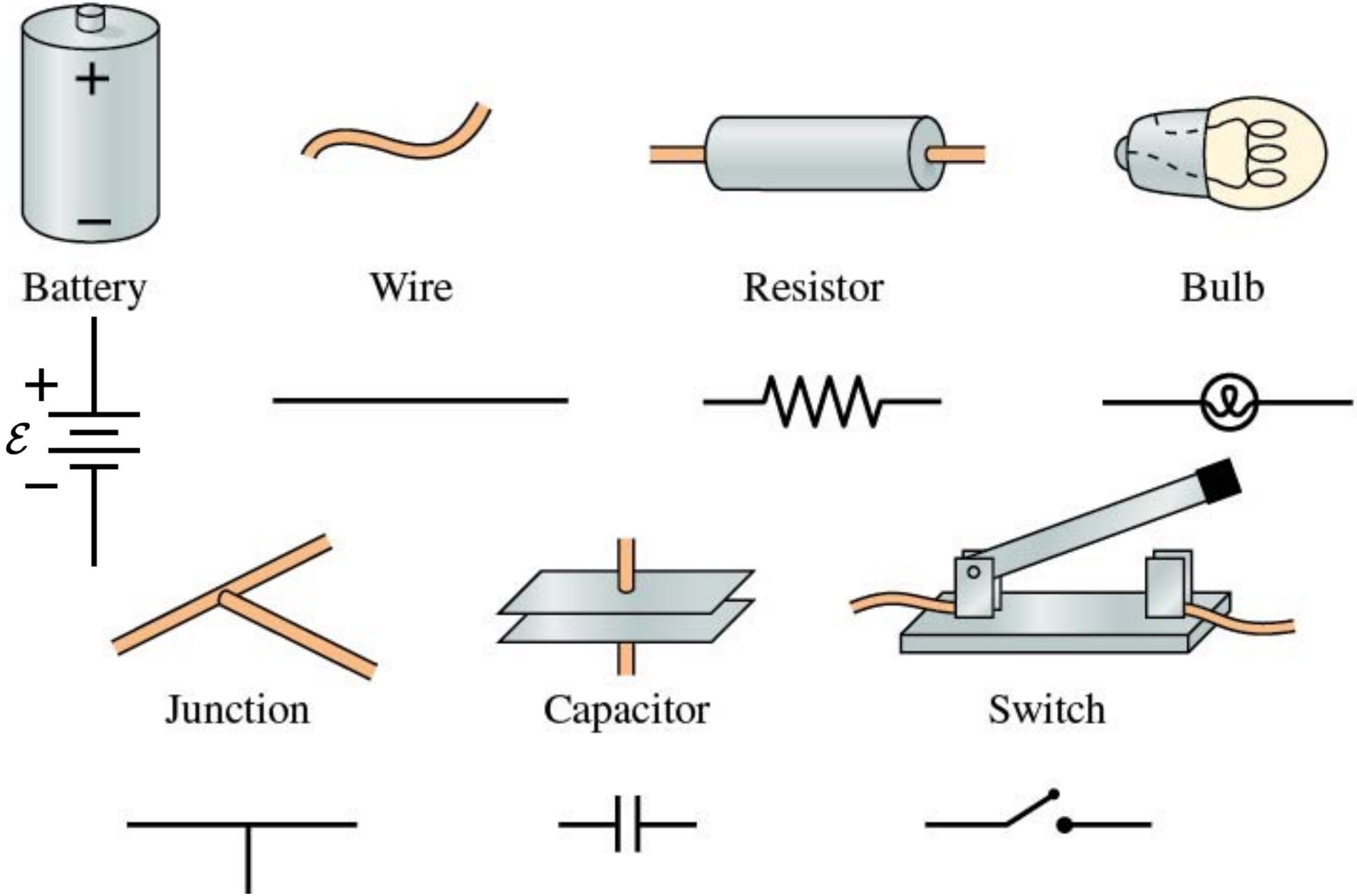
A literal picture of a resistor and a capacitor connected by wires to a battery:



- A circuit diagram replaces pictures of the circuit elements with symbols.
- A circuit diagram is a *logical* picture of what is connected to what.
- A **circuit diagram** of the same circuit above:



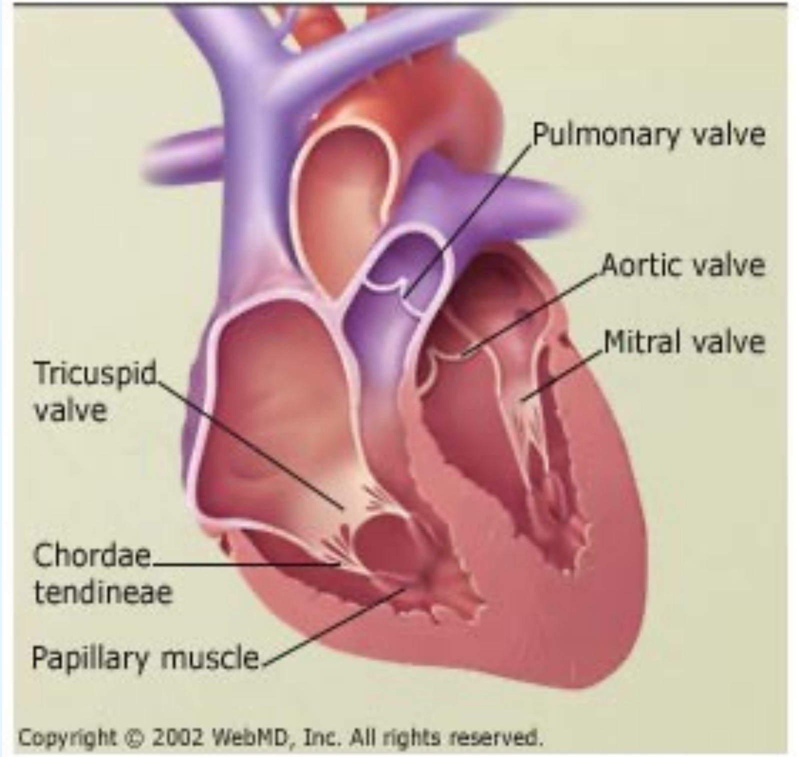
Circuit Elements

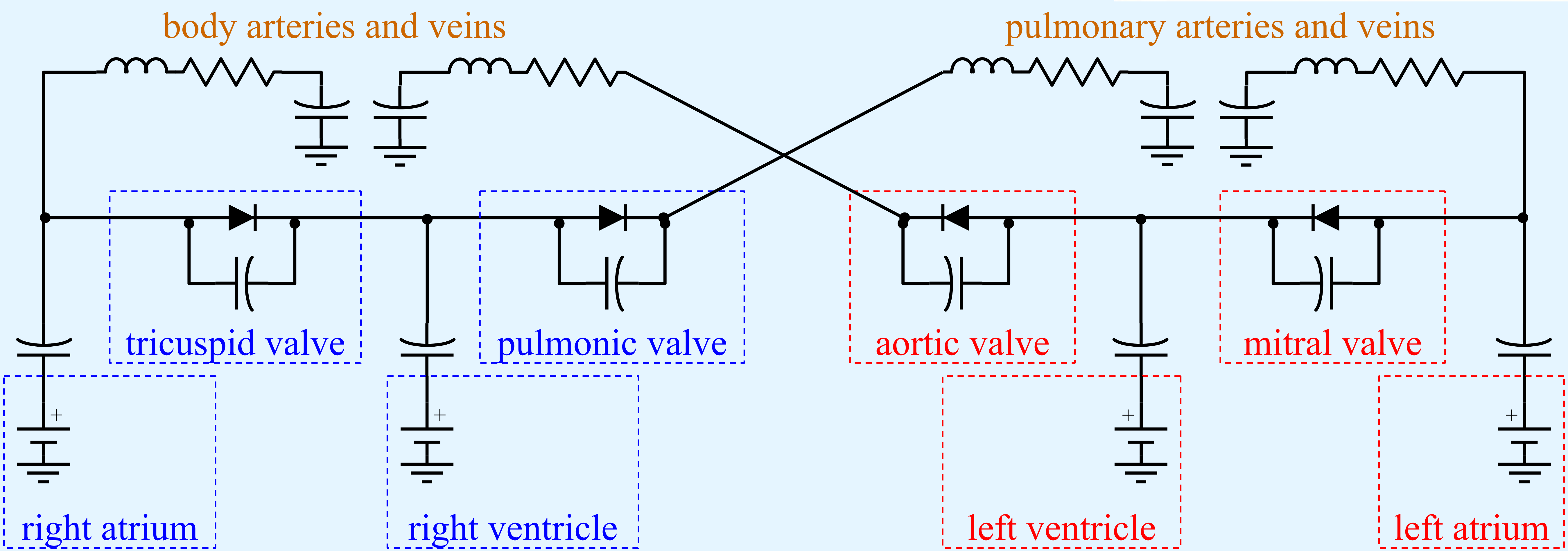


- Use these elements to understand circuits, determine I, V, P, Q

Electricity, with all my heart

- Electrical model allows localizing performance of individual muscles or small groups from multiple measurements on the skin

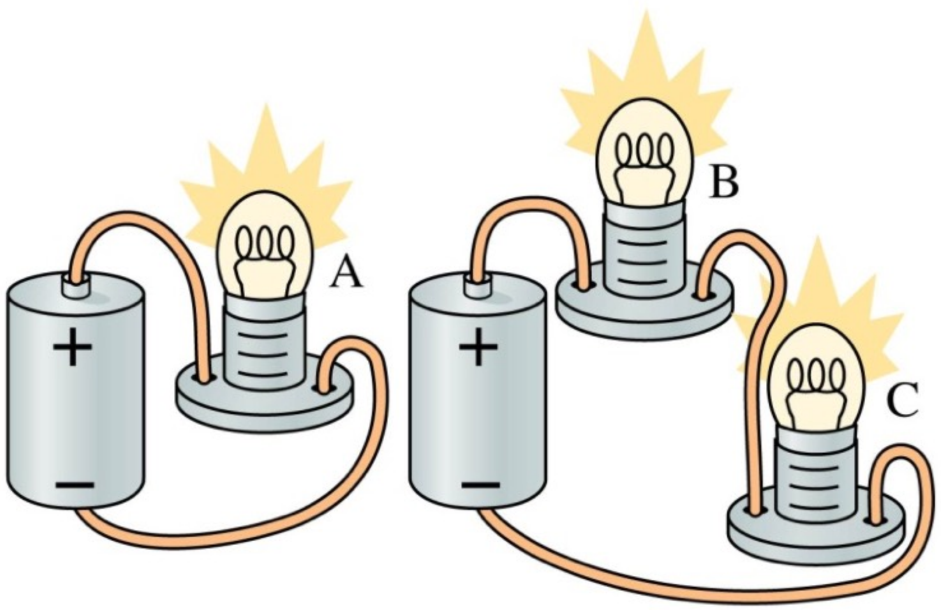




Jose Bohorquez, *An Integrated-Circuit Switched-Capacitor Model and Implementation of the Heart*, Proceedings of the First International Symposium on Applied Sciences on Biomedical and Communication Technologies, 25-28 October 2008. DOI 10.1109/ ISABEL.2008.4712624

iClicker conceptual/intuition question 10-3

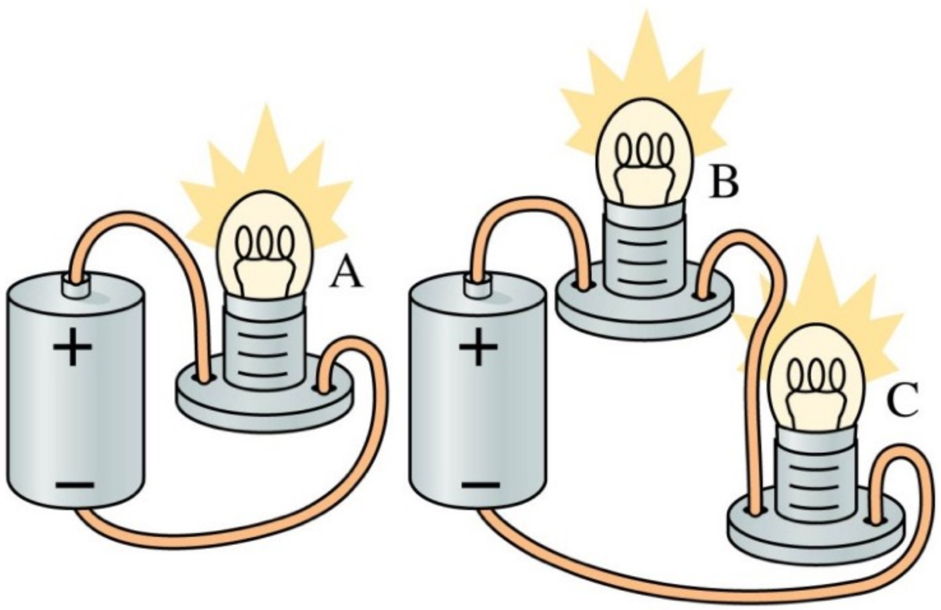
The three bulbs are identical and the two batteries are identical. Compare the brightnesses of the bulbs.



A. $A > B > C$
B. $A > C > B$
C. $A > B = C$
D. $A < B = C$
E. $A = B = C$

iClicker conceptual/intuition question 10-3

The three bulbs are identical and the two batteries are identical. Compare the brightnesses of the bulbs.



A. $A > B > C$

B. $A > C > B$

C. $A > B = C$

D. $A < B = C$

E. $A = B = C$

This question is checking your initial intuition.
(only participation credit, not graded)
We'll return to it later.

iClicker conceptual/intuition question 10-4

The three bulbs are identical and the two batteries are identical. Compare the brightnesses of the bulbs.

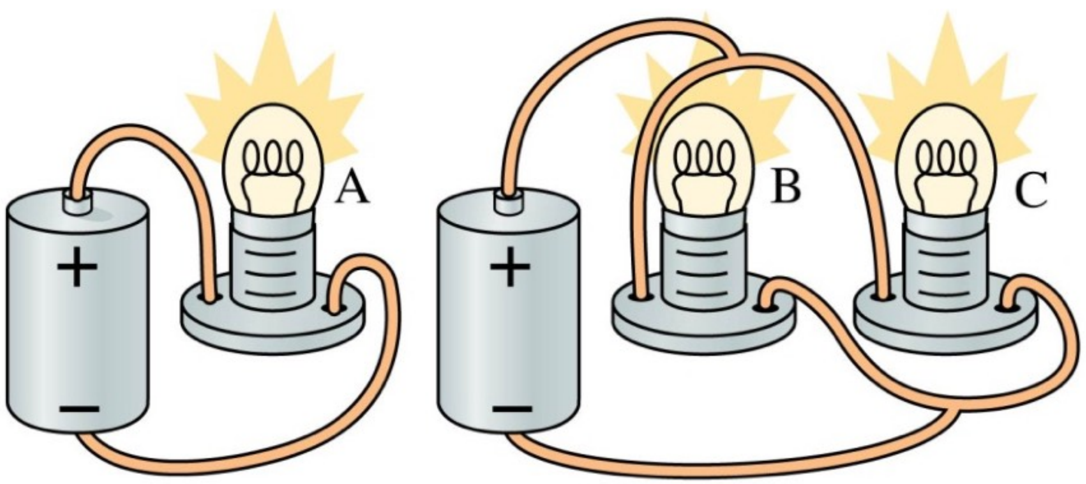
A. A > B > C

B. A > C > B

C. A > B = C

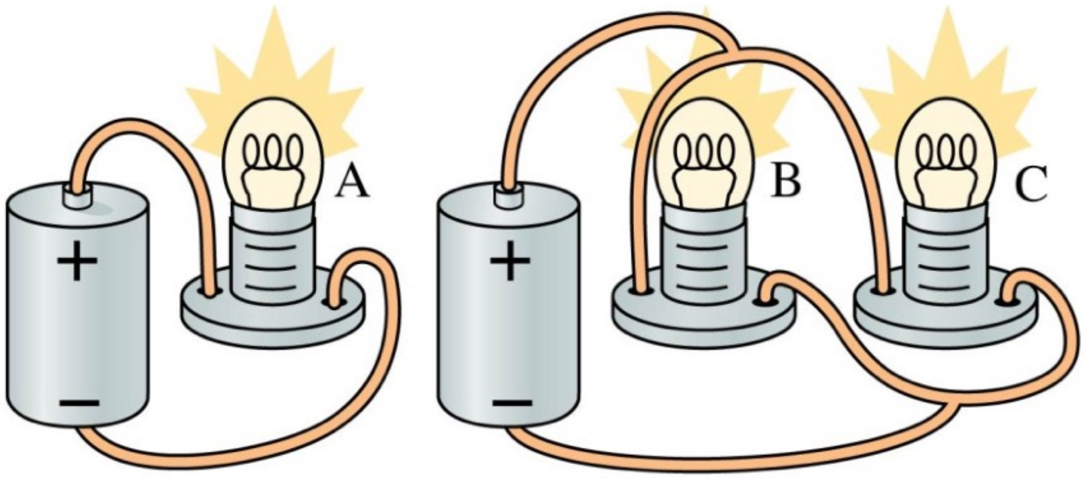
D. A < B = C

E. A = B = C



iClicker conceptual/intuition question 10-4

The three bulbs are identical and the two batteries are identical. Compare the brightnesses of the bulbs.



A. A > B > C

B. A > C > B

C. A > B = C

D. A < B = C

E. A = B = C

This question is checking your initial intuition.
(only participation credit, not graded)
We'll return to it later.

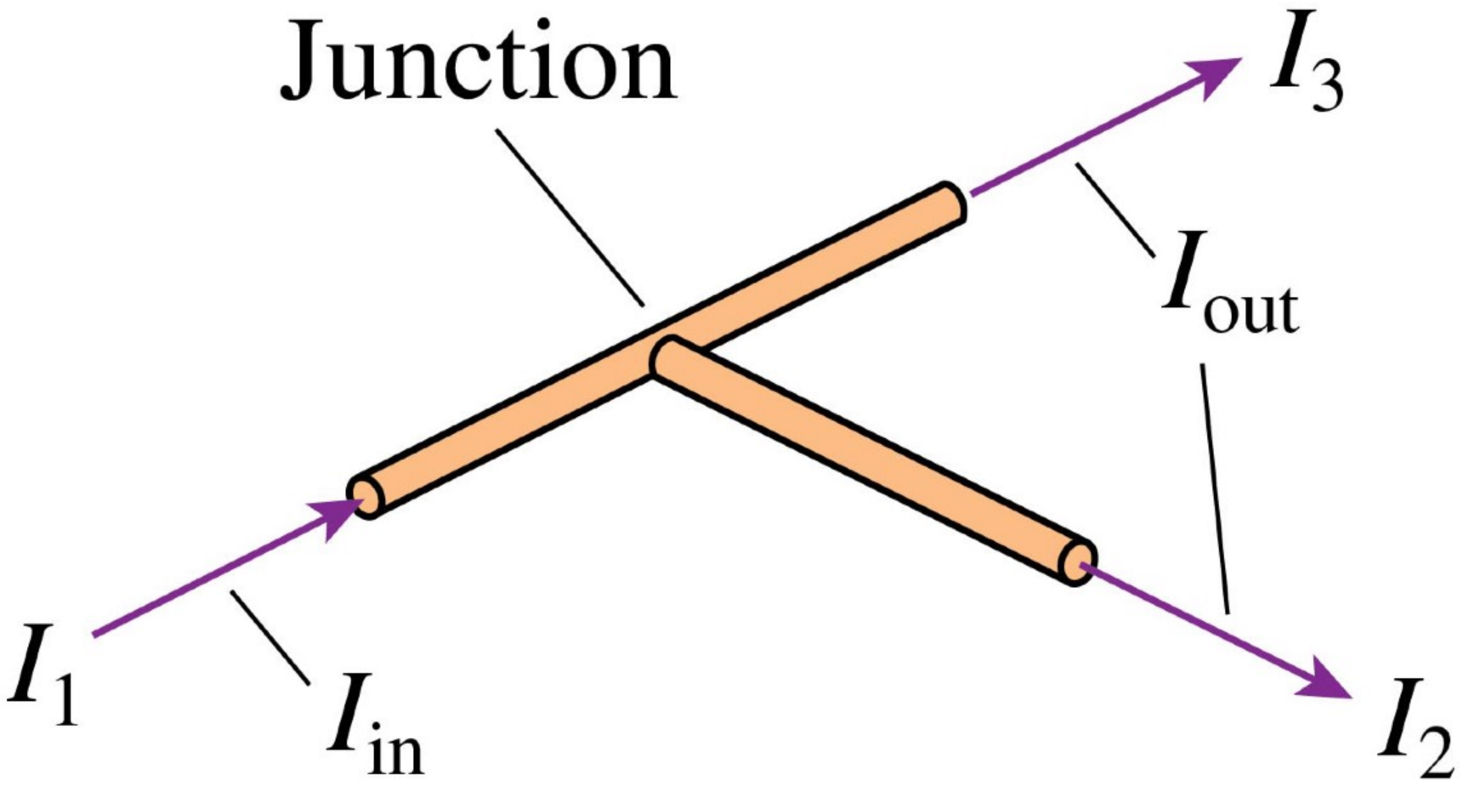
Kirchhoff's Junction Law

- For a *junction*, the law of conservation of current requires that:

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

where the Σ symbol means summation.

- This basic conservation statement is called **Kirchhoff's junction law**.



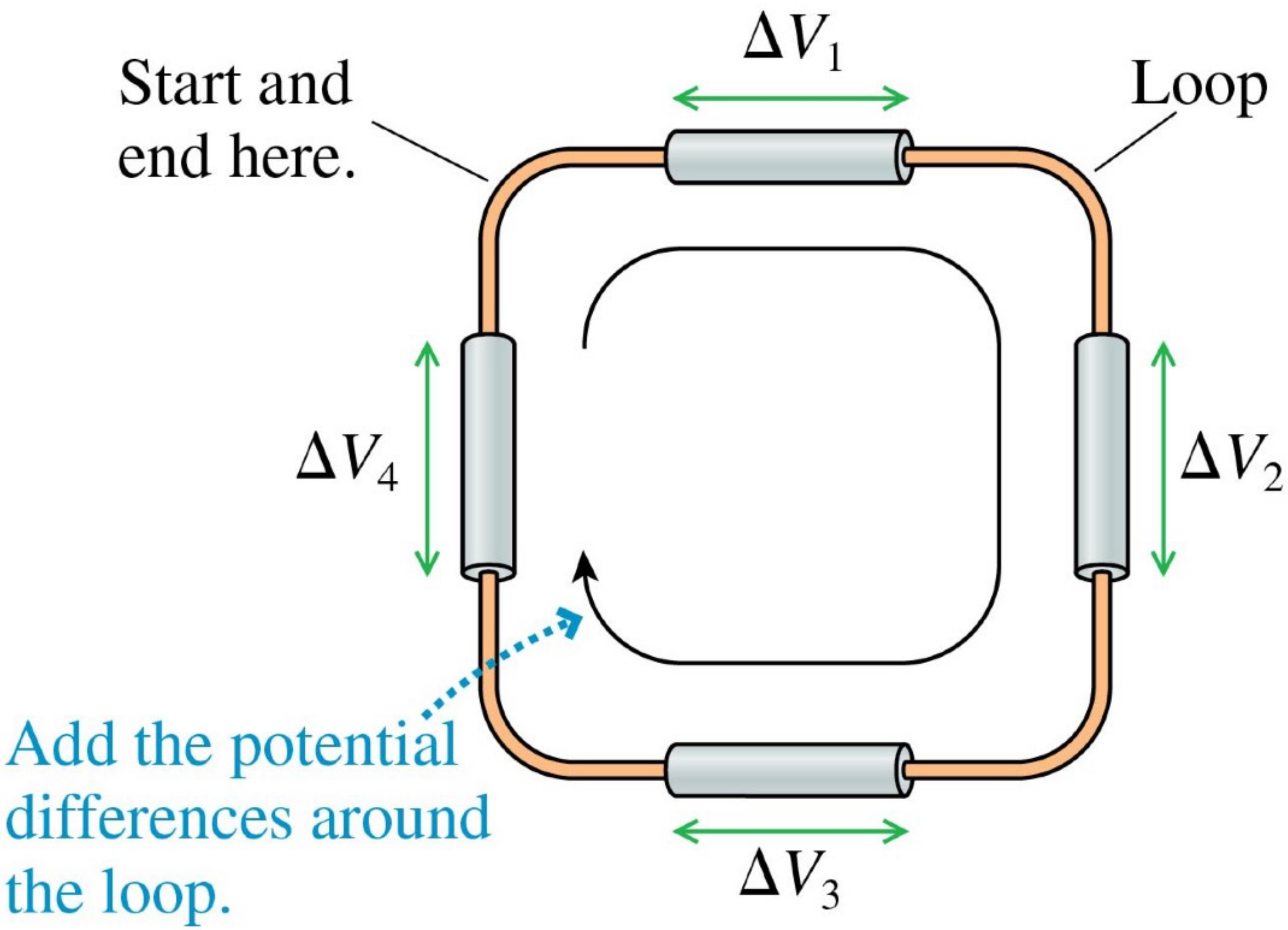
Junction law: $I_1 = I_2 + I_3$

Kirchhoff's Loop Law

- For any path that starts and ends at the same point,

$$\Delta V_{\text{loop}} = \sum(\Delta V)_i = 0$$

  - The sum of all the potential differences encountered while moving around a loop or closed path is zero.
  - This statement is known as **Kirchhoff's loop law**.



Loop law: $\Delta V_1 + \Delta V_2 + \Delta V_3 + \Delta V_4 = 0$

How do you solve this training quiz problem?

What is the current through the $3\ \Omega$ resistor?