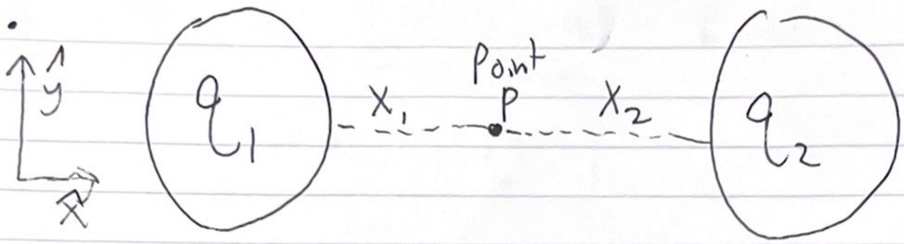
Discussion 10/1

1.



Q: what is the electric field at point p?

A: Electric fields add linearly,
The electric field from q_1 is:

$$\frac{kq_1}{x_1^2}\hat{x} \quad \text{(positive x direction)}$$

The electric field from q_2 is:

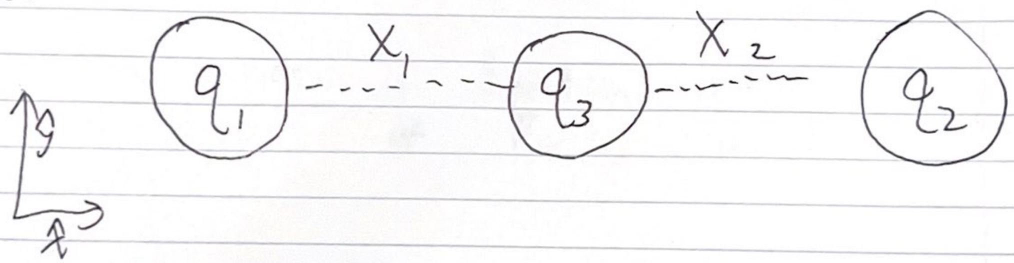
$$\frac{kq_2}{x_2^2}(-\hat{x})$$

The field can be thought of as being $\frac{kq}{r^2}$ going away from the source charge. Notice that if q is negative, it will be toward the source charge.

So the E field at p is

$$\boxed{\left(\frac{kq_1}{x_1^2} - \frac{kq_2}{x_2^2}\right)\hat{x}}$$

2.



Q: If the net force on q_3 is 0, what is the relationship between x_1 and x_2?

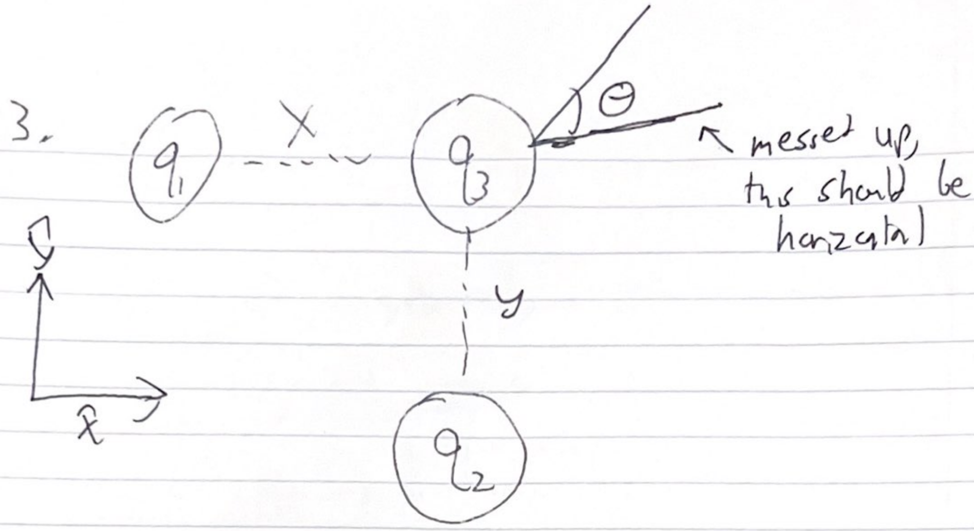
A: Again, forces add up linearly.

$$\vec{F}_{q_1} = \frac{kq_1q_3}{x_1^2}\hat{x}$$

$$\vec{F}_{q_2} = \frac{kq_2q_3}{x_2^2}(-\hat{x})$$

$$\vec{F}_{q_1} + \vec{F}_{q_2} = \vec{0} \overset{\text{algebra}}{\Rightarrow} \boxed{\frac{x_1}{x_2} = \sqrt{\frac{q_1}{q_2}}}$$

3.

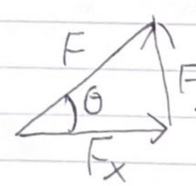


Q: what is direction and magnitude of Forces on q_3?

A: Forces are vectors, and add like vectors.

$$\vec{F}_x = \frac{kq_1q_3}{x^2}\hat{x}$$

$$\vec{F}_y = \frac{kq_2q_3}{y^2}\hat{y}$$



$$|\vec{F}| = |\vec{F}_y + \vec{F}_x|$$

$$\boxed{= \sqrt{\left(\frac{kq_1q_3}{x^2}\right)^2 + \left(\frac{kq_2q_3}{y^2}\right)^2}}$$

$$\tan\theta = \frac{F_y}{F_x}$$

To get θ generally, you would use the two argument inverse;

$$\boxed{\Theta = \operatorname{atan2}(F_y, F_x)}$$

This will give you correct direction with correct sign.

I misspoke during discussion;

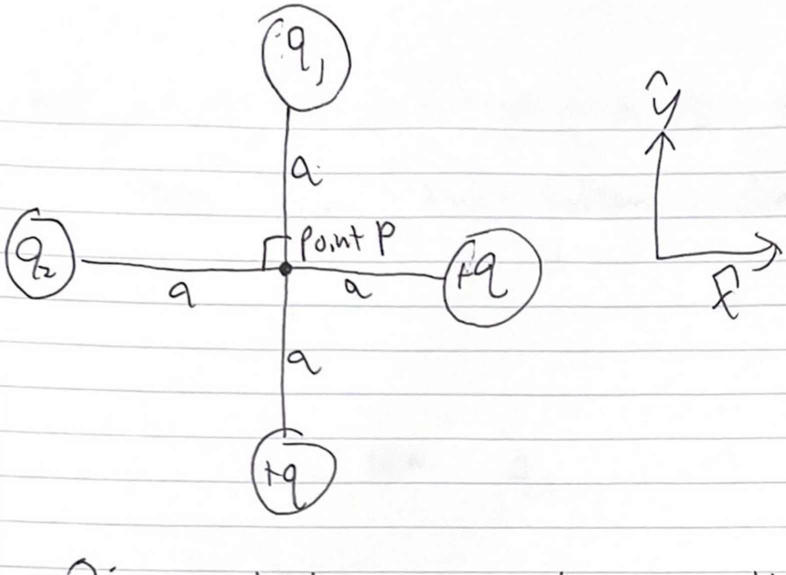
$$\Theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$ is only correct up to $n\pi$,

I said this was answer

because;

$$\tan^{-1}(\tan(\theta)) = \theta \pm n\pi, \quad n \text{ integer}$$

However, by looking at the signs of F_x, F_y, we can determine the correct quadrant for $\tan^{-1}\left(\frac{F_y}{F_x}\right)$



Q: What is electric field at point P? if:

a) $q_1 = q_2 = +q$

b) $q_1 = q_2 = -q$

c) $q_1 = -q \quad q_2 = +q$

A: This is a problem where symmetry is helpful.

a) The system is the same if you rotate it $90°$, so there can not be any E field in the x,y plane (and hopefully its obvious why there is none in the z-direction)

If there was, then $\vec{E}$ and $\vec{E}$ rotated $90°$ would both have to be correct, which is impossible unless $\vec{E} = 0$.

b) Lets first look in y direction:
E field
from the bottom charge is

$$\frac{kq}{a^2}\hat{y}$$

and from ~~the~~ q_1 it is

$$\frac{kq_1}{q^2}(-\hat{y}) = \frac{k(-q)}{a^2}(-\hat{y}) =$$

$\frac{kq}{a^2}\hat{y}$ = same as the other one!

q_1 is on the opposite side, so the direction of field flips, but it is opposite charge so sign of field flips again.

By noticing this, you could avoid doing any more math for q_1.

In this case, the math was easy anyway, but it's good to be able to spot symmetries to help simplify problems.

So the total E-Field is

$$\boxed{2\frac{kq}{q^2}\left(\hat{y} + -\hat{x}\right)}$$

$\theta = \frac{3\pi}{4}$ turn

magnitude $= 2\frac{kq}{a^2}\sqrt{2}$

c) ~~x~~ x direction field cancel.

y direction field from ~~each~~

vertical particles are the same.

$$\boxed{\vec{E} = \frac{2kq}{a^2}\,\hat{y}}$$