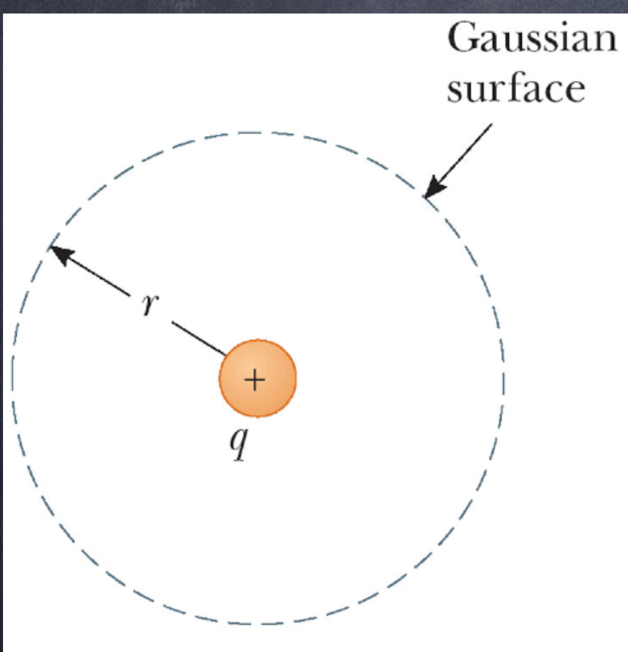
Lecture 6
PHYS 2B

Gauss' Law

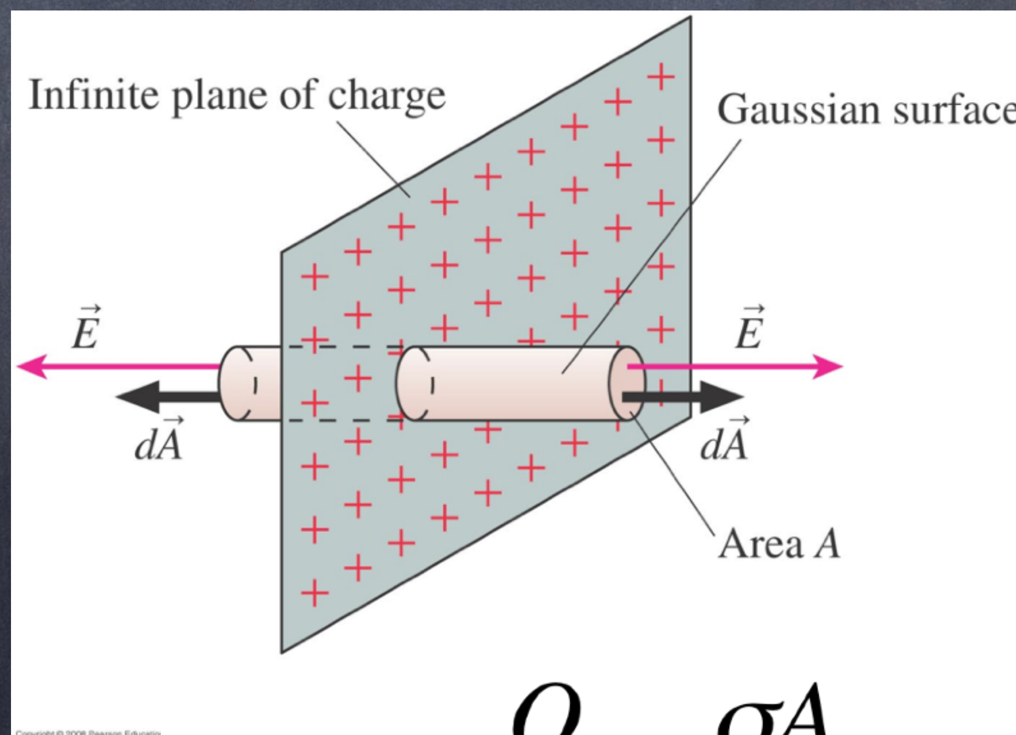
$$\Phi_E = \oint \vec{E} \cdot d\vec{A}$$

$$\Phi_E = \frac{Q_{inside}}{\varepsilon_o}$$

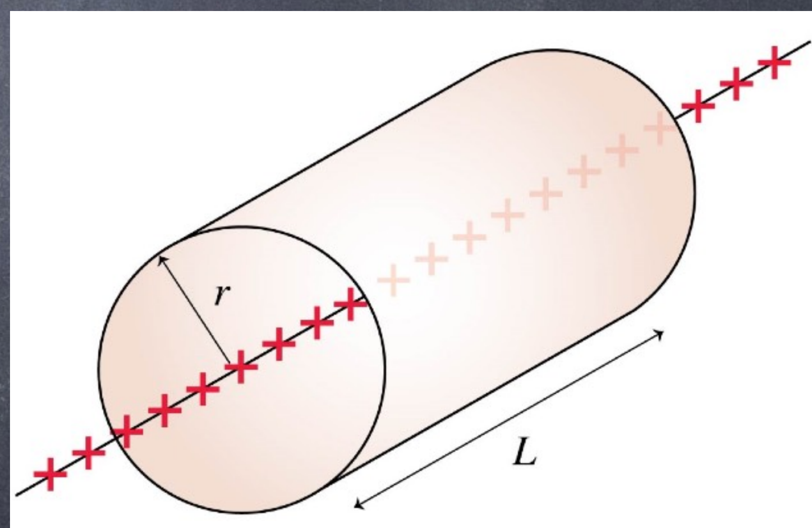
integral of E over surface = charge enclosed



$$E\left(4\pi r^2\right) = \frac{+q}{\varepsilon_o}$$



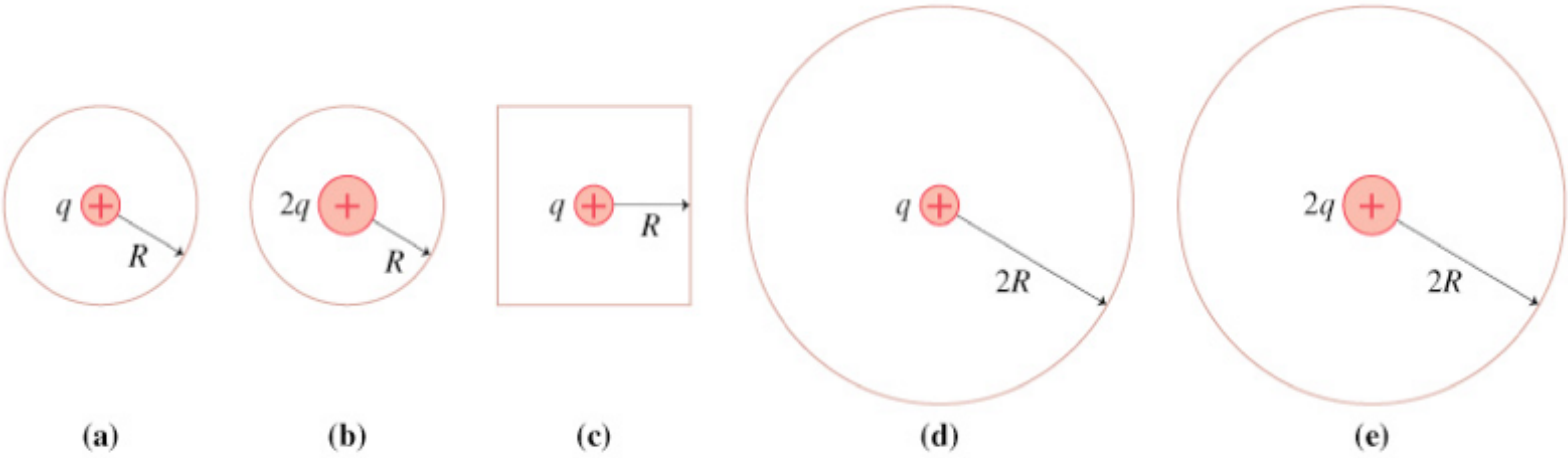
$$2EA = \frac{Q}{\varepsilon_o} = \frac{\sigma A}{\varepsilon_o}$$



$$E\left(2\pi zL\right) = \frac{\lambda L}{\varepsilon_o}$$

iClicker Question 6-1

- These are two-dimensional cross sections through three dimensional closed spheres and a cube. Rank the order, from largest to smallest, the electric fluxes Φ_a to Φ_e through surfaces a to e.



A) $\Phi_a > \Phi_c > \Phi_b > \Phi_d > \Phi_e$.

B) $\Phi_b = \Phi_e > \Phi_a = \Phi_c = \Phi_d$.

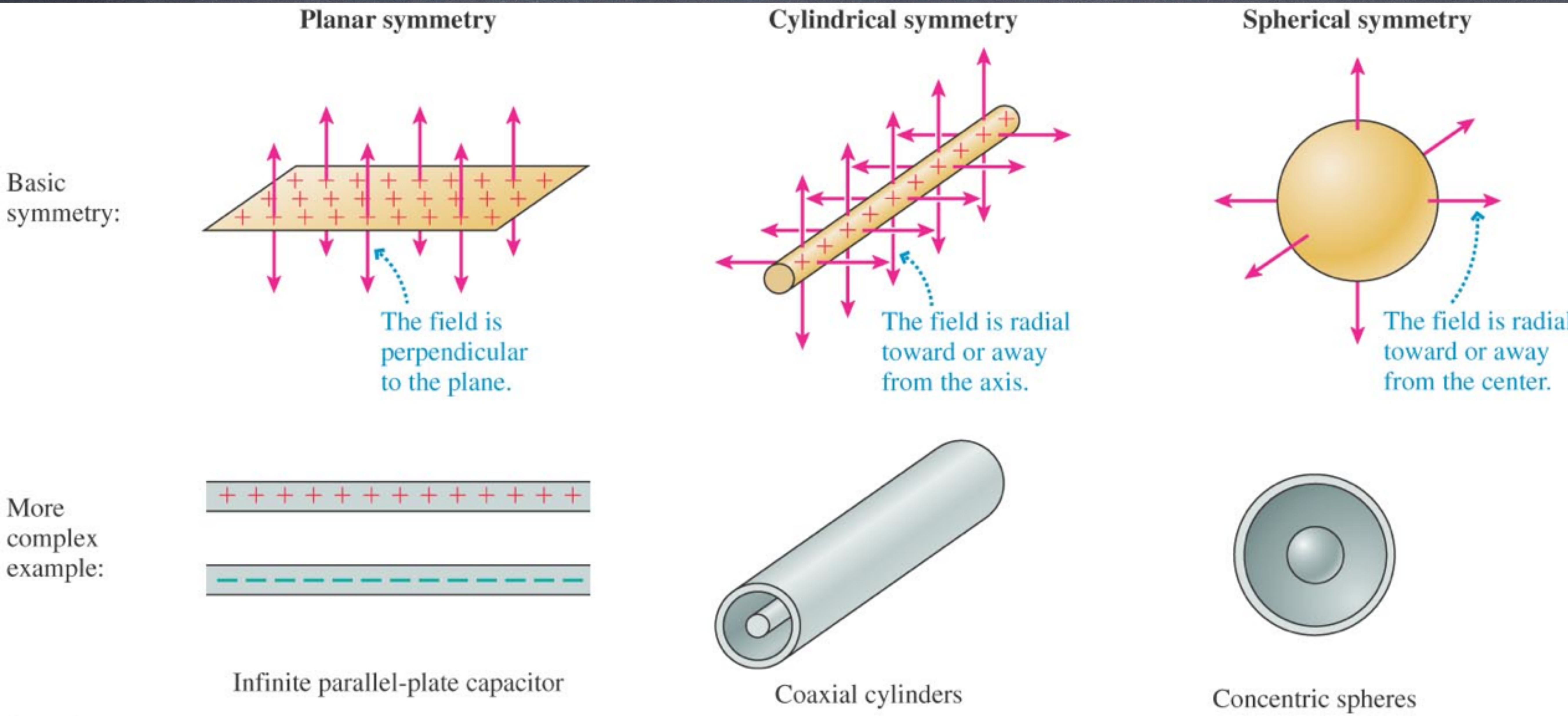
C) $\Phi_e > \Phi_d > \Phi_b > \Phi_c > \Phi_a$.

D) $\Phi_b > \Phi_a > \Phi_c > \Phi_e > \Phi_d$.

E) $\Phi_d = \Phi_e > \Phi_c > \Phi_a = \Phi_b$.

Gauss' Law

- Remember: There are three fundamental symmetries that you exploit in order to use Gauss' Law to solve for a given electric field: Planar, Cylindrical, and Spherical.



Gauss' Law

- Gauss' Law affords us the opportunity to solve very complex situations by exploiting symmetry.
- Before Gauss' Law, we had a hard time solving situations that had distributions of charges.
- After Gauss' Law, we can examine a situation for symmetry and then fully describe the electric field for that situation.
- Nowhere is this more evident, then when we try and describe real world applications, as with charged conductors.

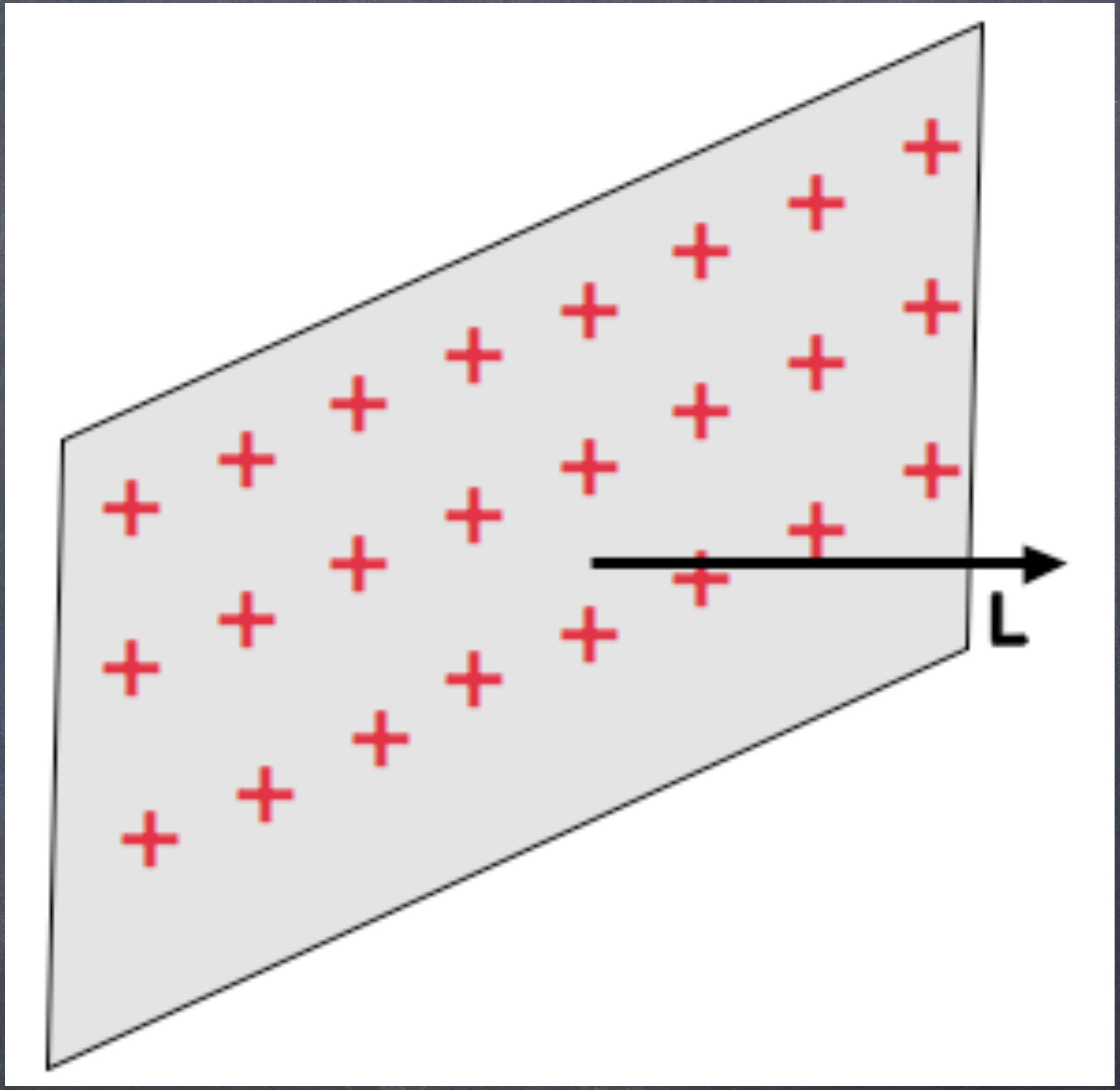
$$\Phi_E = \oint \vec{E} \cdot d\vec{A} \quad = \quad \Phi_E = \frac{Q_{inside}}{\varepsilon_o}$$

integral of E over surface

charge enclosed

Gauss' Law

- Example
- What is the electric field, E, a distance L above an infinite thin sheet of charge with surface density, σ.



- Answer
- Choose an appropriate Gaussian surface.
- Take a cylindrical surface with length 2L and faces of area A.

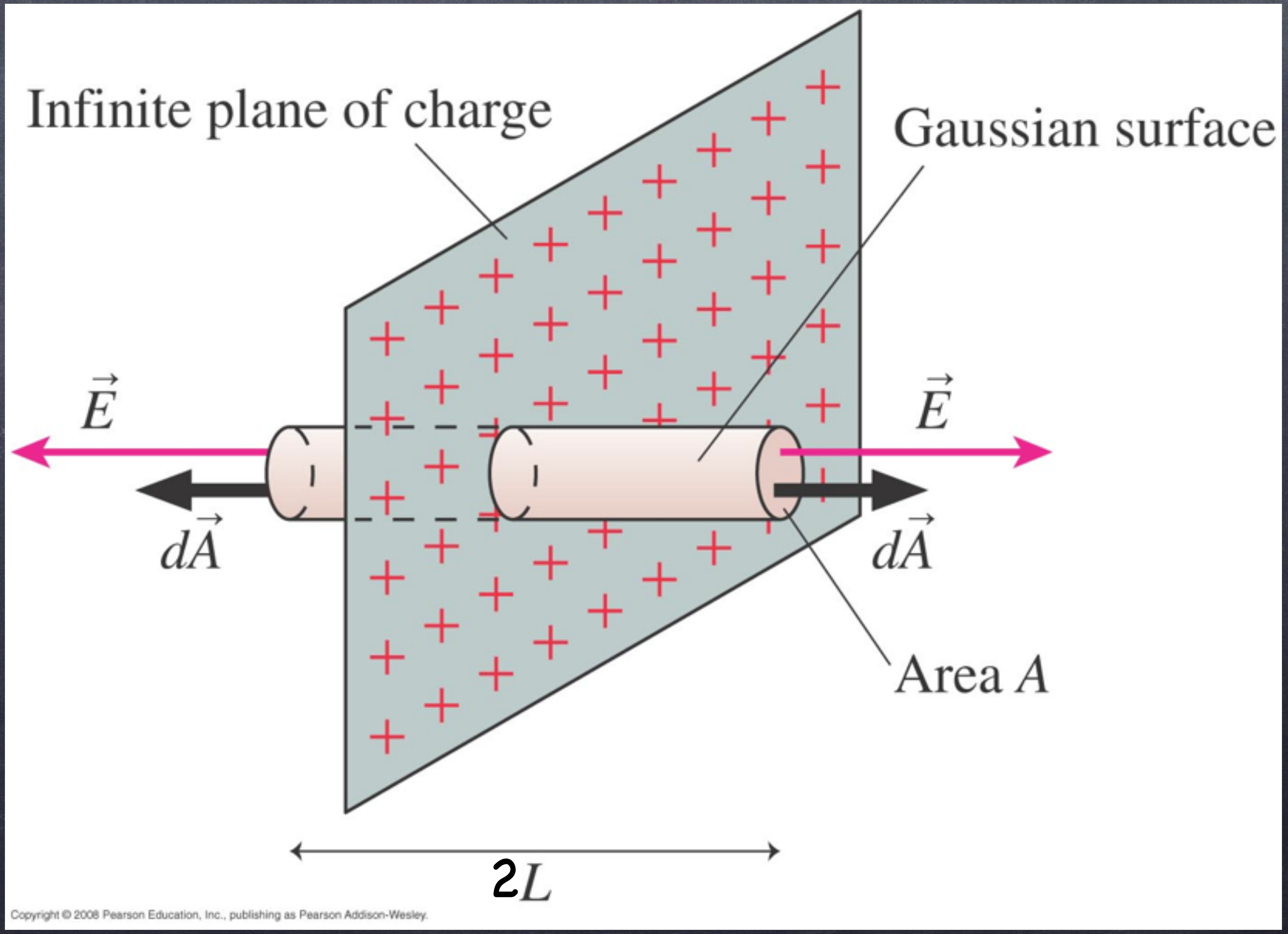
Gauss' Law

- Answer

- The electric field is perpendicular to both faces of the cylinder.
- So the total flux through both faces is: $$\Phi_E = 2EA$$

- There is no flux through the wall of the cylinder because the field vectors are tangent to the wall.
- Leaving the net flux to be:

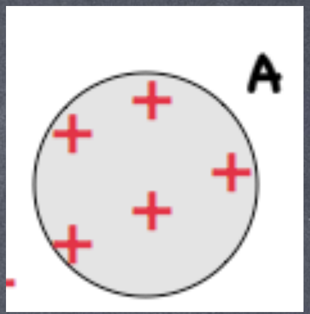
$$\Phi_E = 2EA$$



Gauss' Law

- Answer

- Next, we should find the amount of charge contained in the Gaussian surface.
- The surface charge density will be:



$$\sigma = \frac{Q}{A}$$

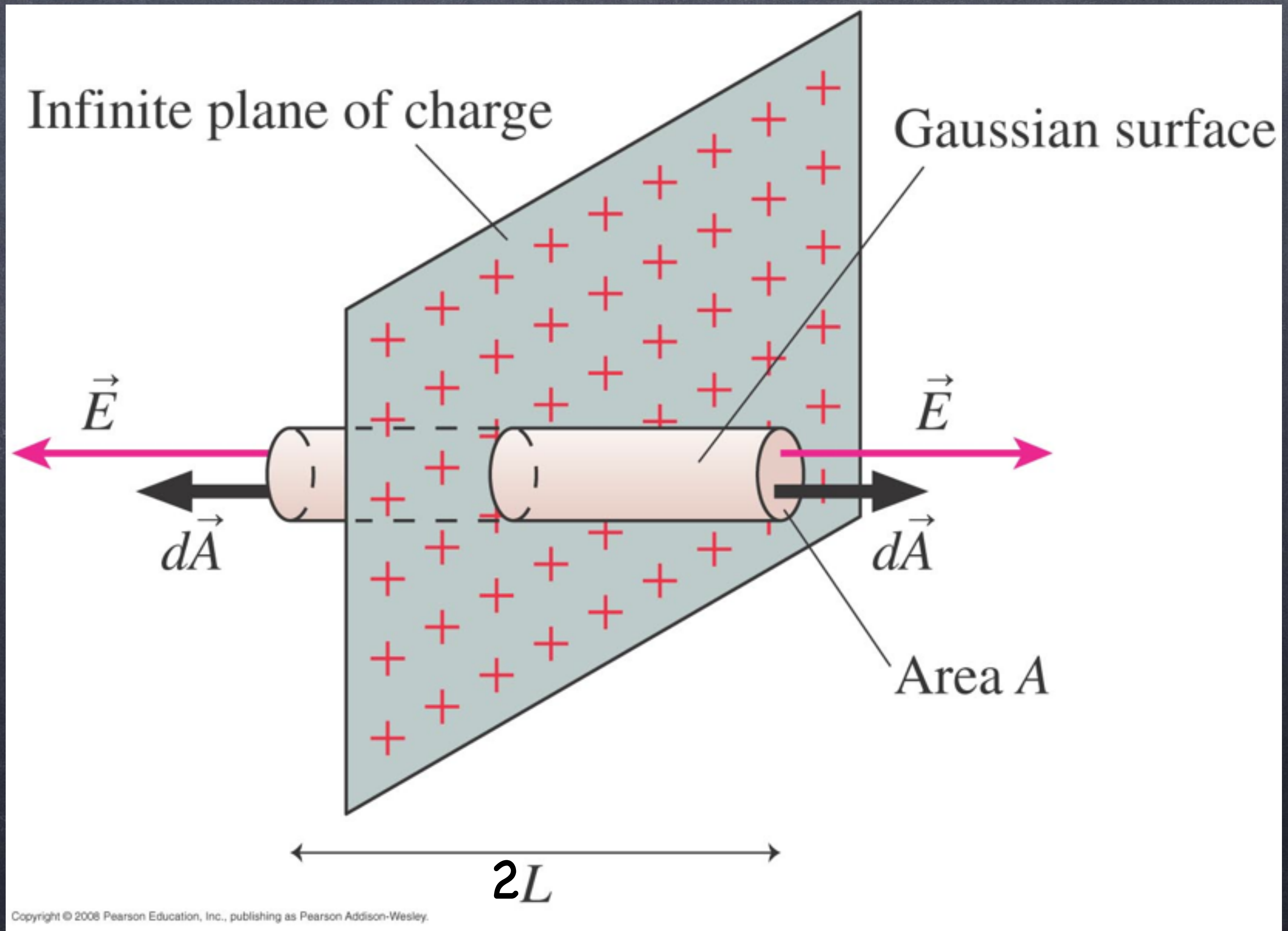
$$Q = \sigma A$$

- That is the charge enclosed by the Gaussian surface.

$$\Phi_E = \Phi_E$$

$$2EA = \frac{Q}{\varepsilon_o} = \frac{\sigma A}{\varepsilon_o}$$

$$E = \frac{\sigma}{2\varepsilon_o}$$



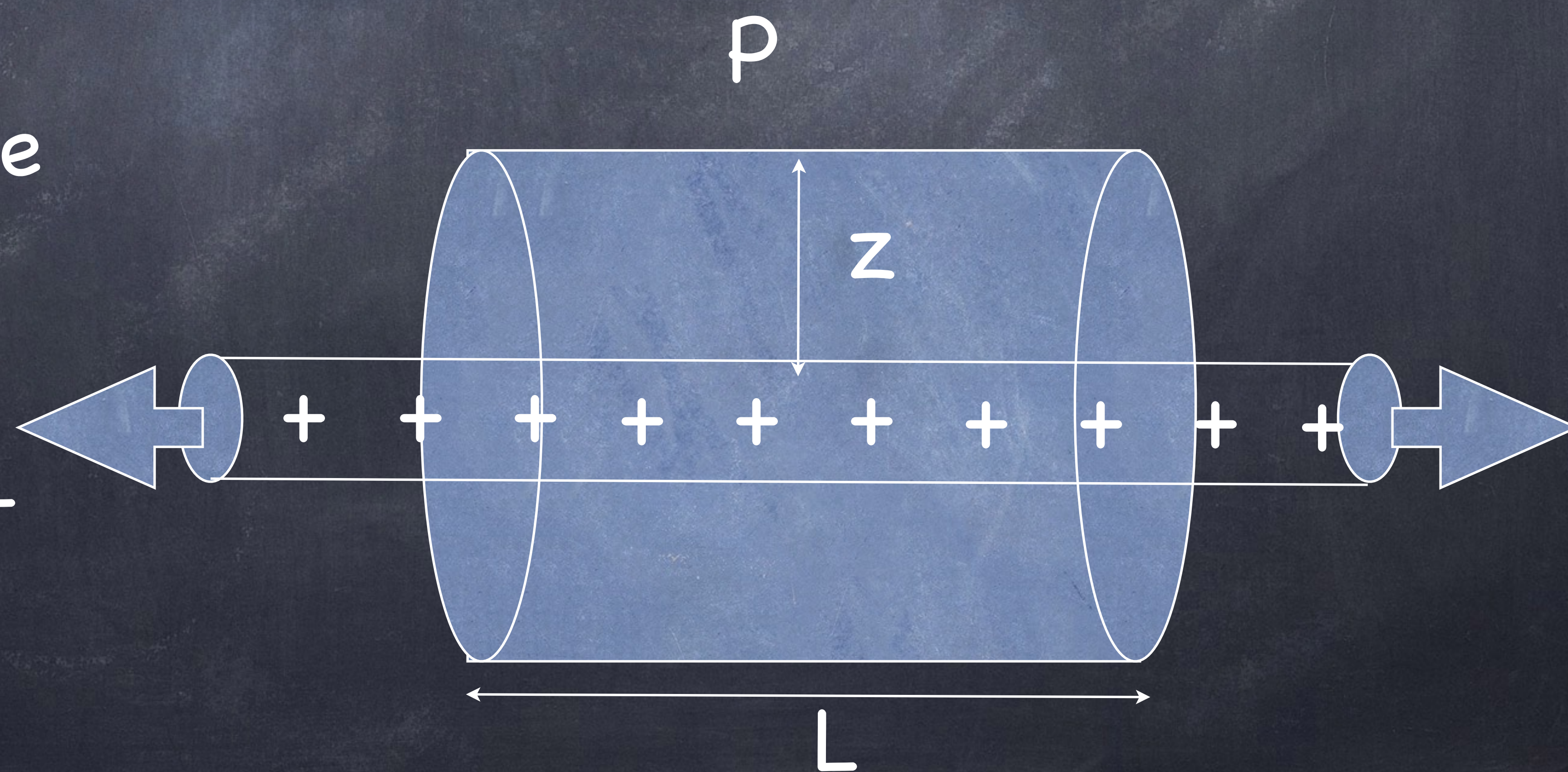
Gauss' Law

- Again let's determine the magnitude of the electric field at a point P which is at a distance z from a very long (nearly infinite) wire of uniformly distributed charge. Assume z is much smaller than the length of the wire and let ∂ be the charge per unit length of the wire.

- <u>Solution</u>:

Choose an appropriate Gaussian surface.

Take a cylindrical surface with length L and radius z.



Gauss' Law

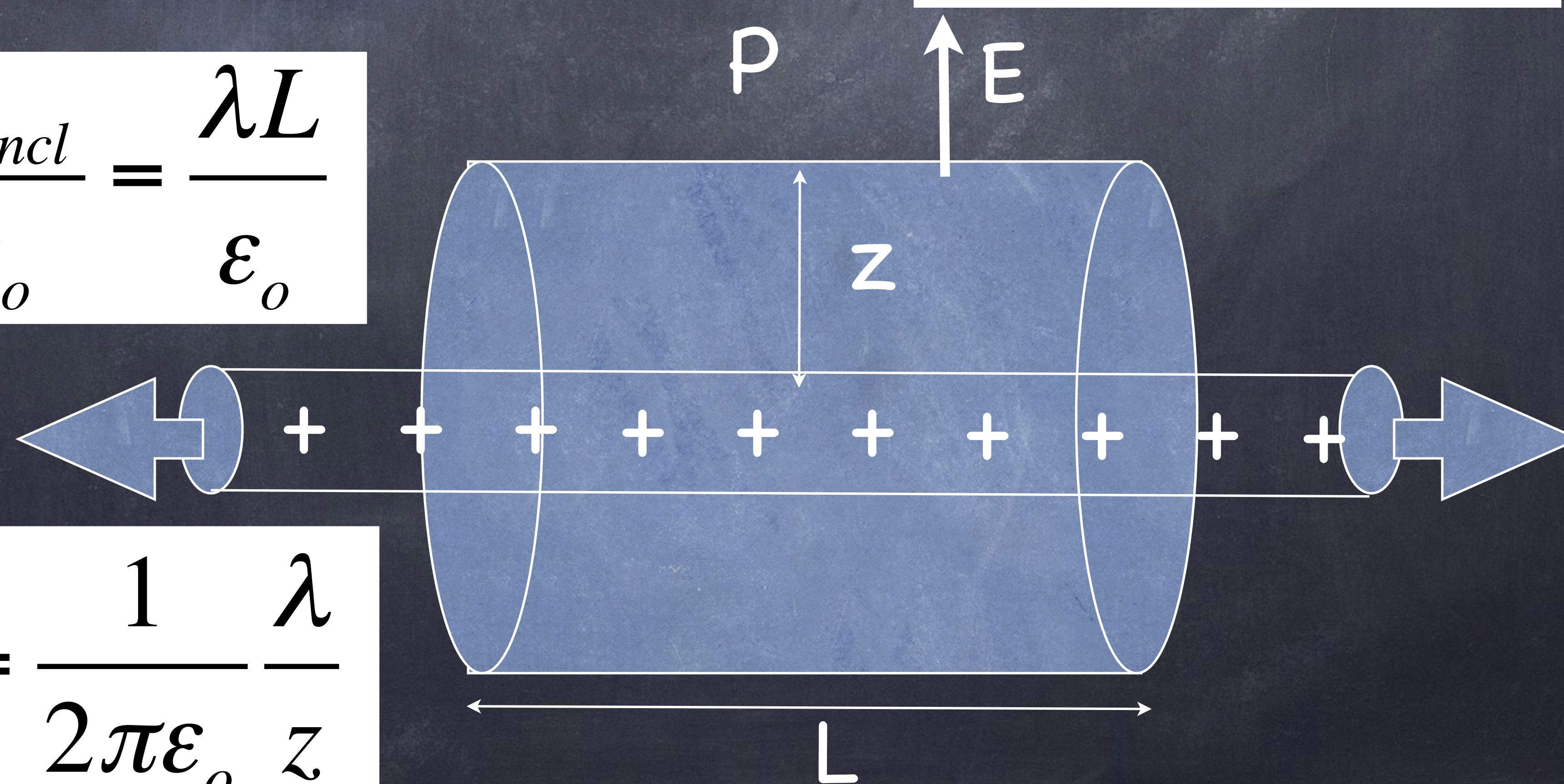
- The electric field is perpendicular to the entire cylindrical part of the surface (radially outward with the same magnitude at each point).
- The electric field is parallel to both faces of the cylinder.
- So the total flux through the cylindrical surface is:

$$\Phi_E = EA_{cylindrical\ part}\cos\theta = E\left(2\pi zL\right)\cos 0$$

$$\Phi_E = E\left(2\pi zL\right)$$

Via Gauss' Law:

$$\Phi_E = \frac{q_{encl}}{\varepsilon_o} = \frac{\lambda L}{\varepsilon_o}$$



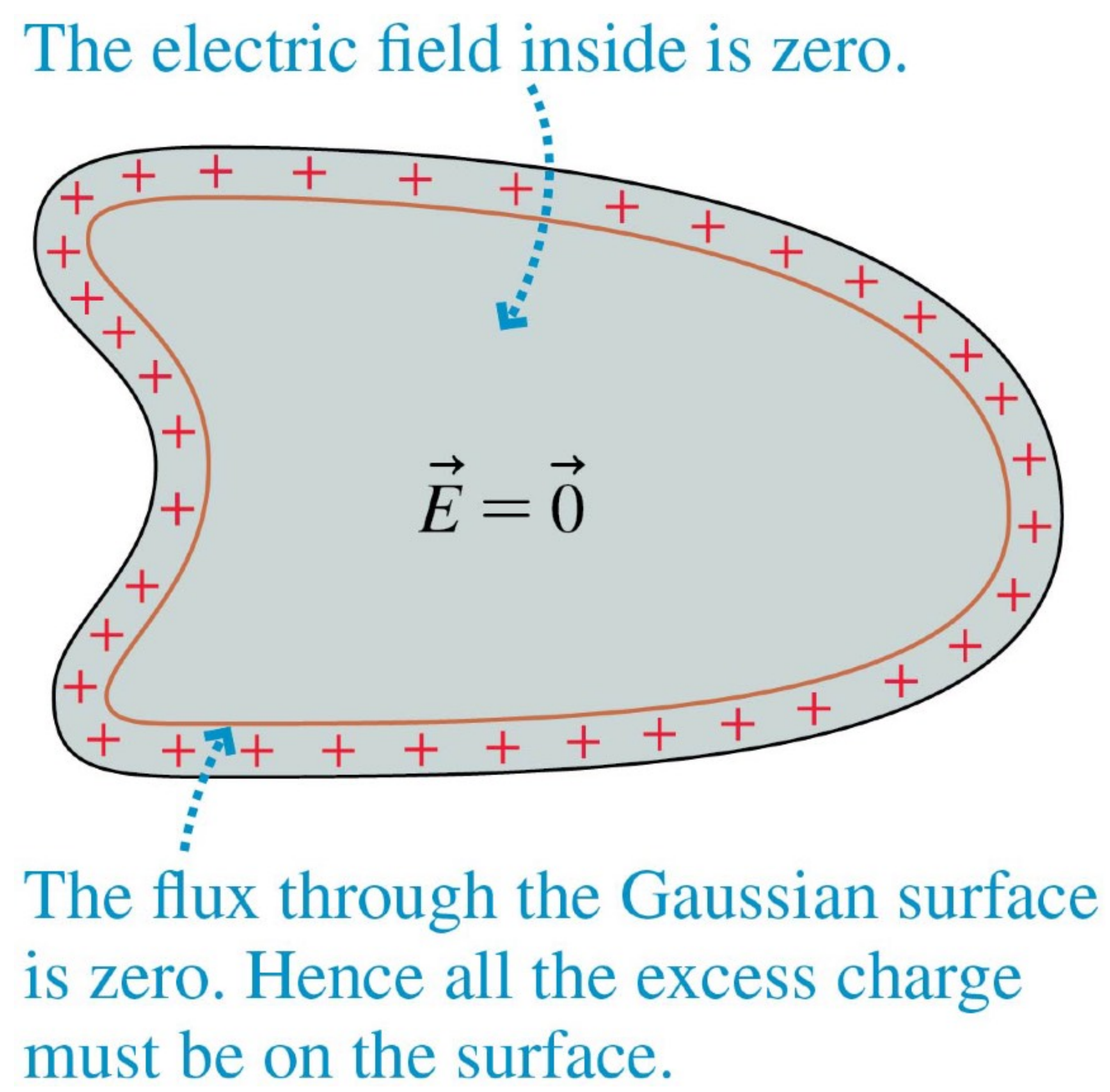
Equating the two:

$$E\left(2\pi zL\right) = \frac{\lambda L}{\varepsilon_o}$$

$$E = \frac{1}{2\pi\varepsilon_o}\frac{\lambda}{z}$$

Conductors in Electrostatic Equilibrium

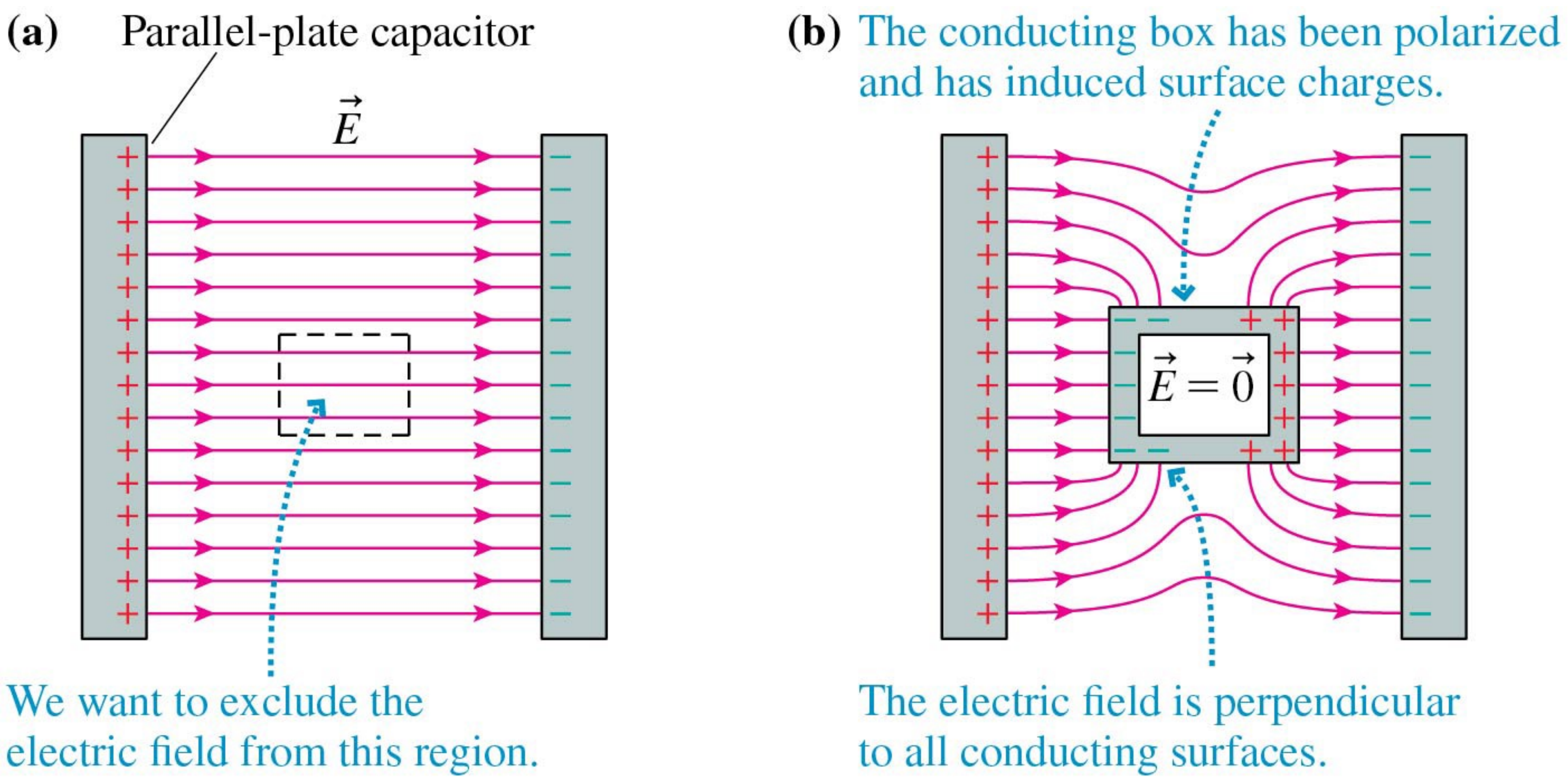
- The figure shows a Gaussian surface just inside a conductor’s surface.



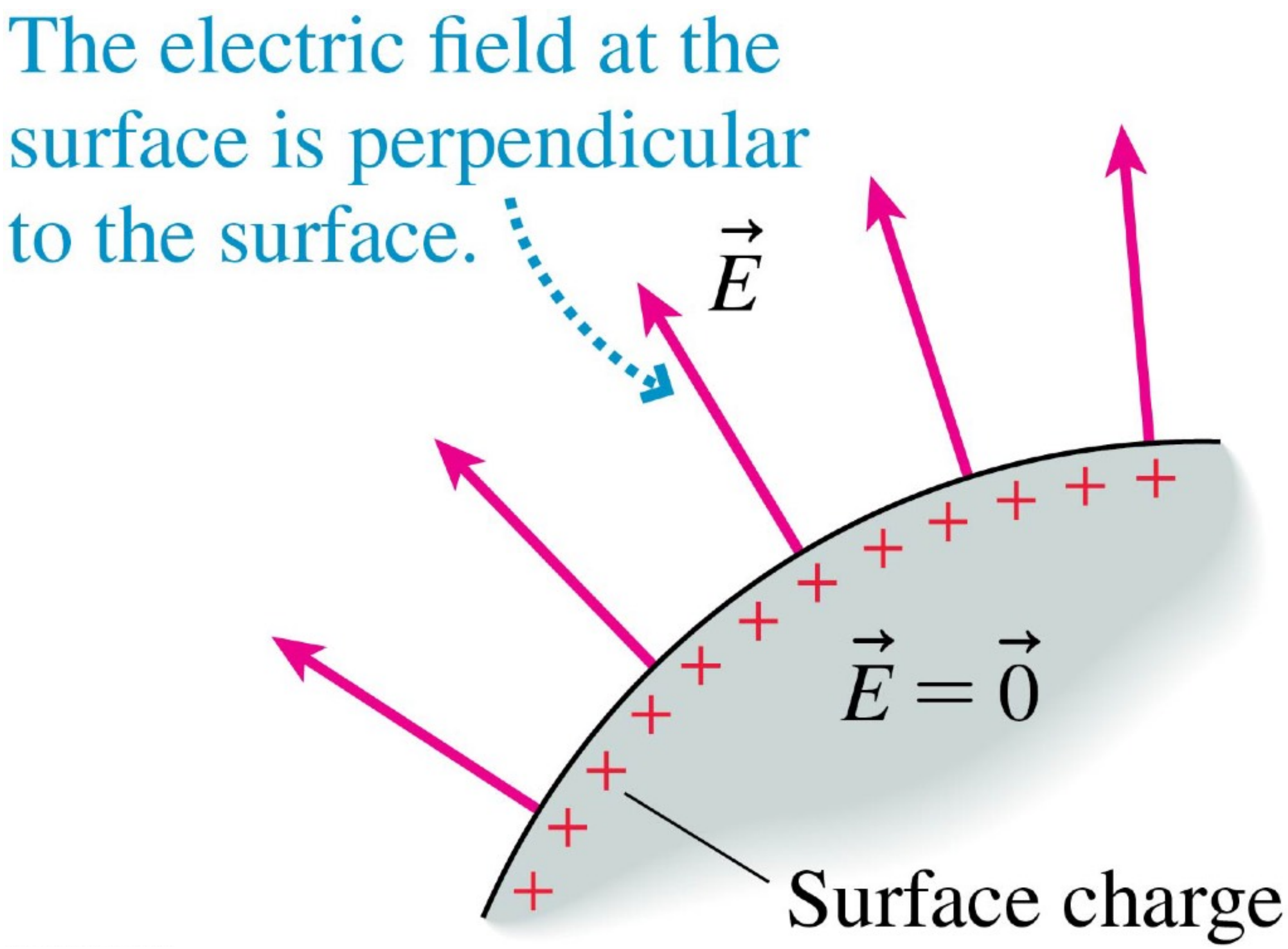
- The <u>electric field must be zero at all points within the conductor</u>, or else the electric field would cause the charge carriers to move and it wouldn’t be in equilibrium.
- By Gauss’s Law, $Q_{\text{in}} = 0$

Faraday Cages

- The use of a conducting box, or *Faraday cage*, to exclude electric fields from a region of space is called **screening**.



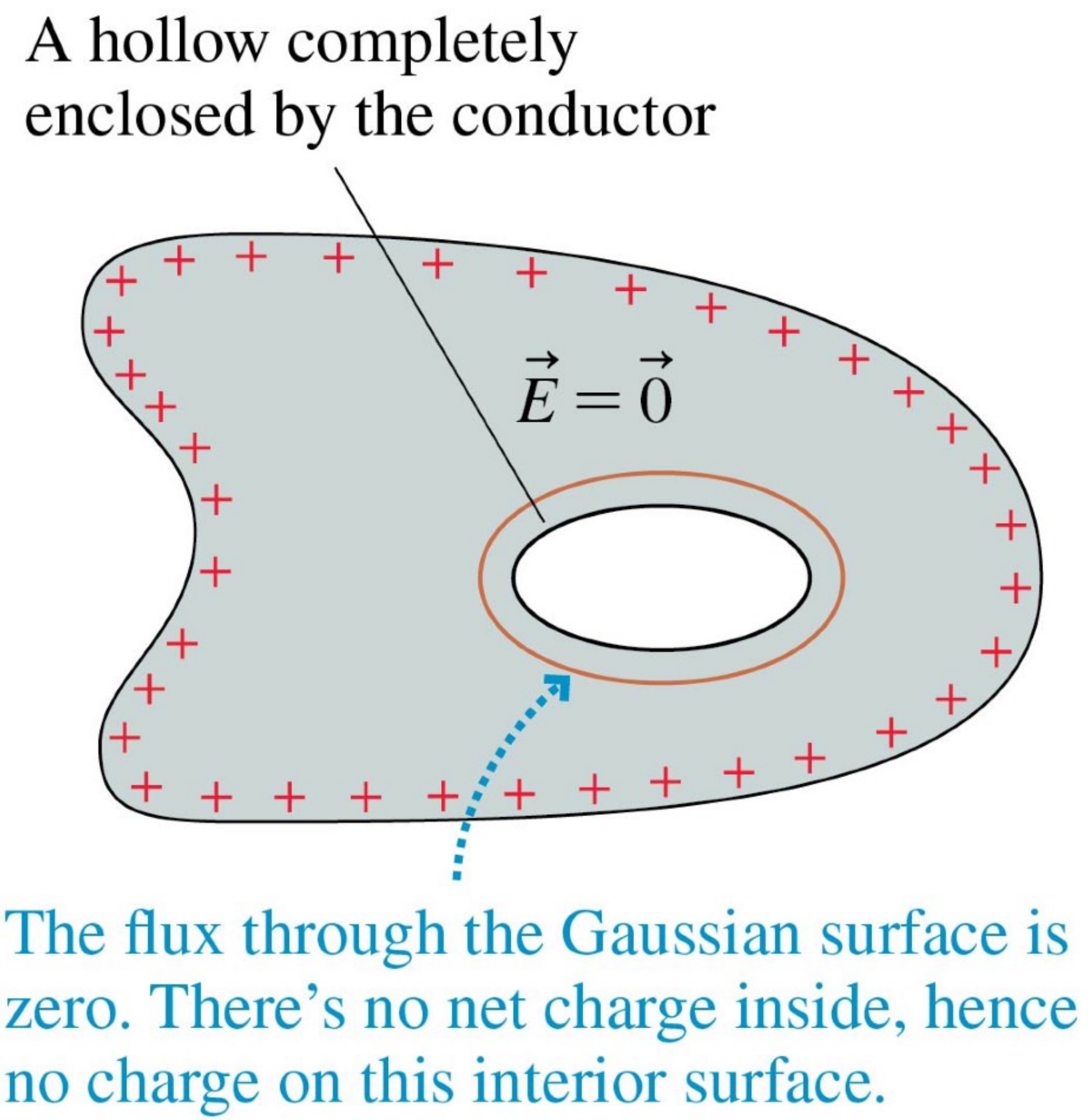
Conductors in Electrostatic Equilibrium



- **The external electric field right at the surface of a conductor must be perpendicular to that surface.**
- If it were to have a tangential component,
  it would exert a force on the surface charges and cause a surface current, and the conductor would not be in electrostatic equilibrium.

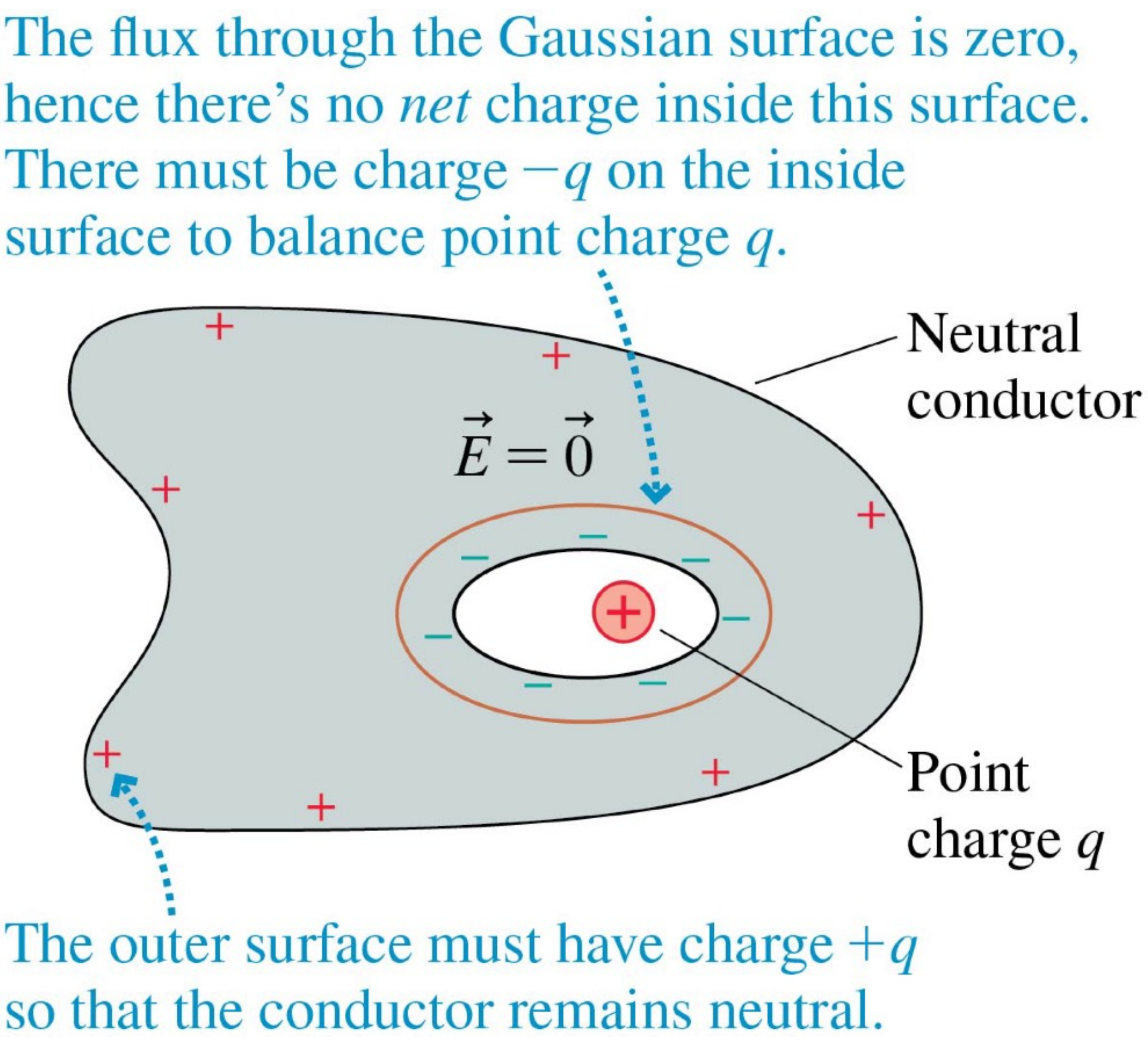
- The figure shows a charged conductor with a hole inside.

- Since the electric field is zero inside the conductor, we must conclude that $Q_{\text{in}} = 0$ for the interior surface.
- Furthermore, since there's no electric field inside the conductor and no charge inside the hole, the electric field in the hole must be zero.



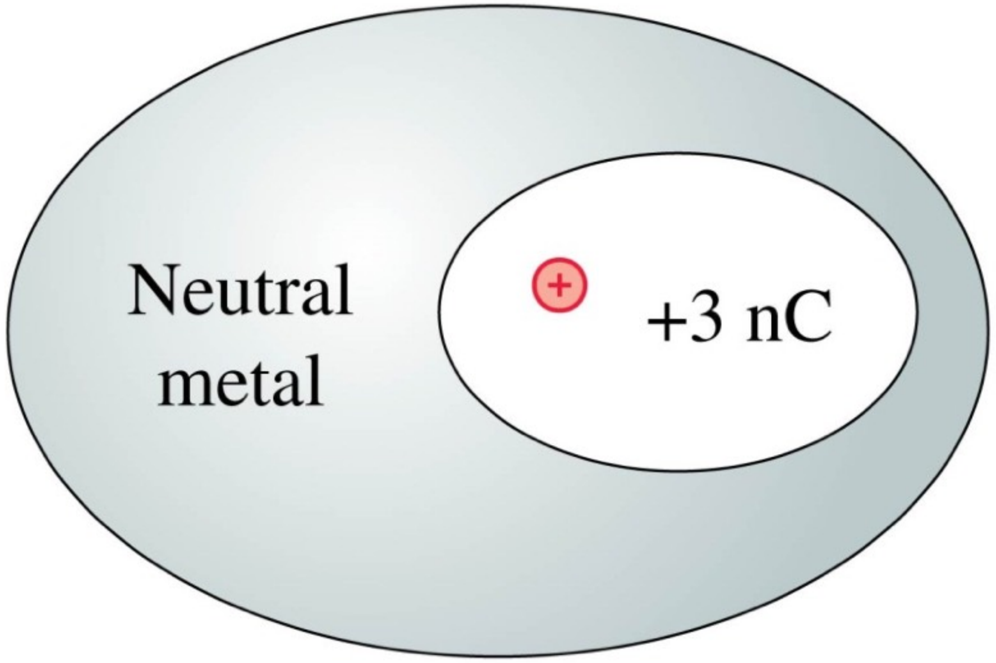
Conductors in Electrostatic Equilibrium

- The figure shows a charge q inside a hole within a neutral conductor.
- Net charge $-q$ moves to the inner surface and net charge $+q$ is left behind on the exterior surface.



iClicker question 6-2

Charge $+3\ \text{nC}$ is in a hollow cavity inside a large chunk of metal that is electrically neutral. The total charge on the exterior surface of the metal is



A. $0\ \text{nC}$

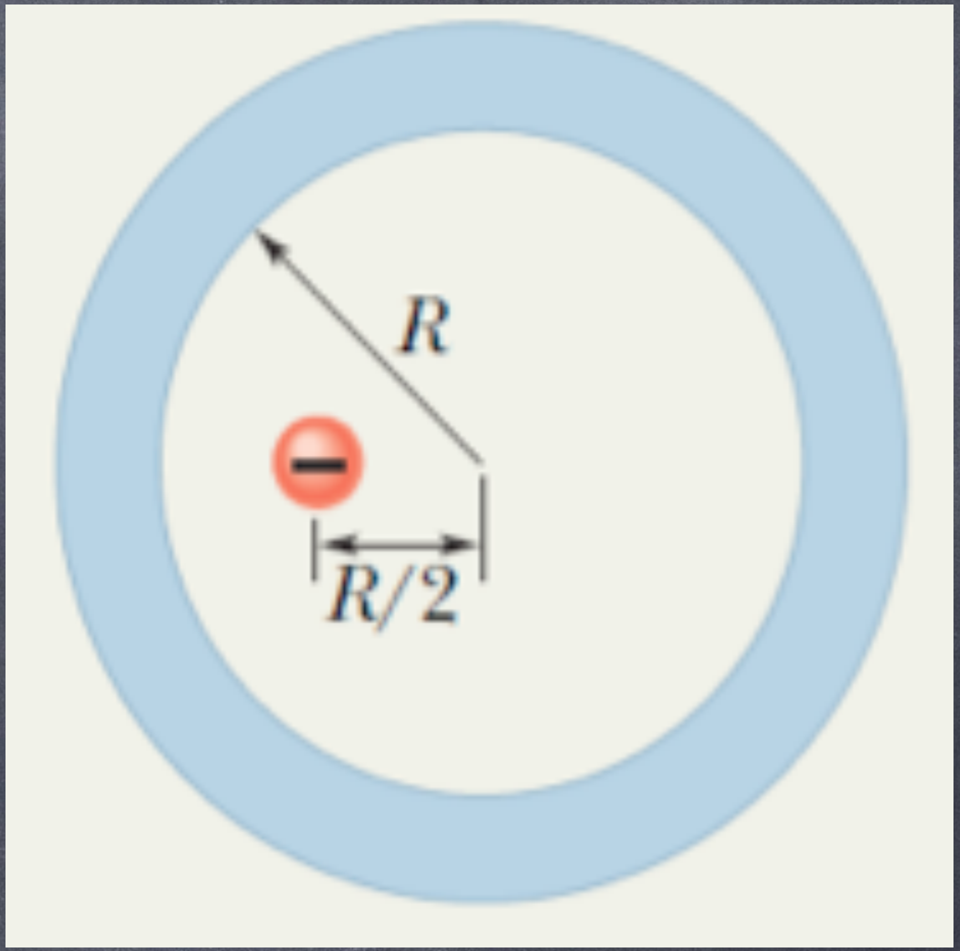
B. $+3\ \text{nC}$

C. $-3\ \text{nC}$

D. Can't say without knowing the shape and location of the hollow cavity.

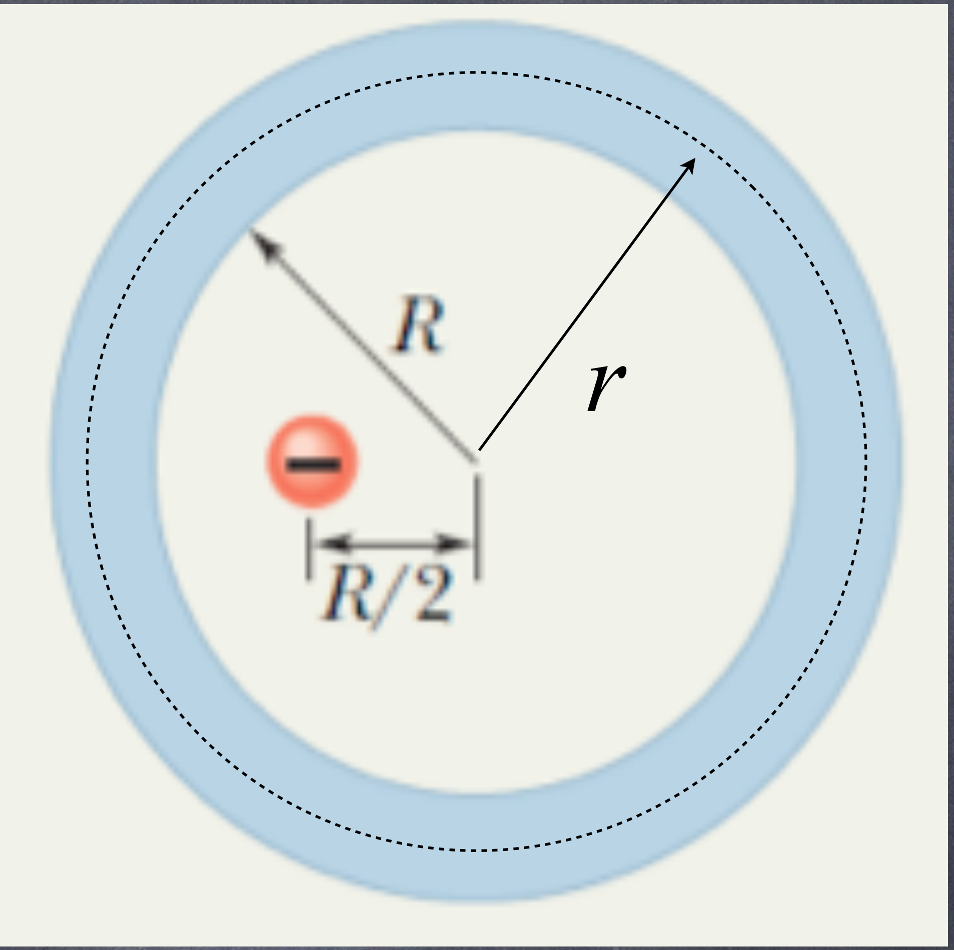
Gauss' Law

- Example
- The figure to the right shows a cross section of an electrically neutral spherical conducting shell of inner radius R. A point charge of $-5.0\,\mu C$ is located at a distance R/2 from the center of the shell. What are the induced charges on the outer and inner surfaces? Are those charges uniformly distributed on the surface?



Gauss' Law

- Answer
- Choose an appropriate Gaussian surface.
- Take a spherical surface with radius r > R, slightly (so it is inside the conductor).

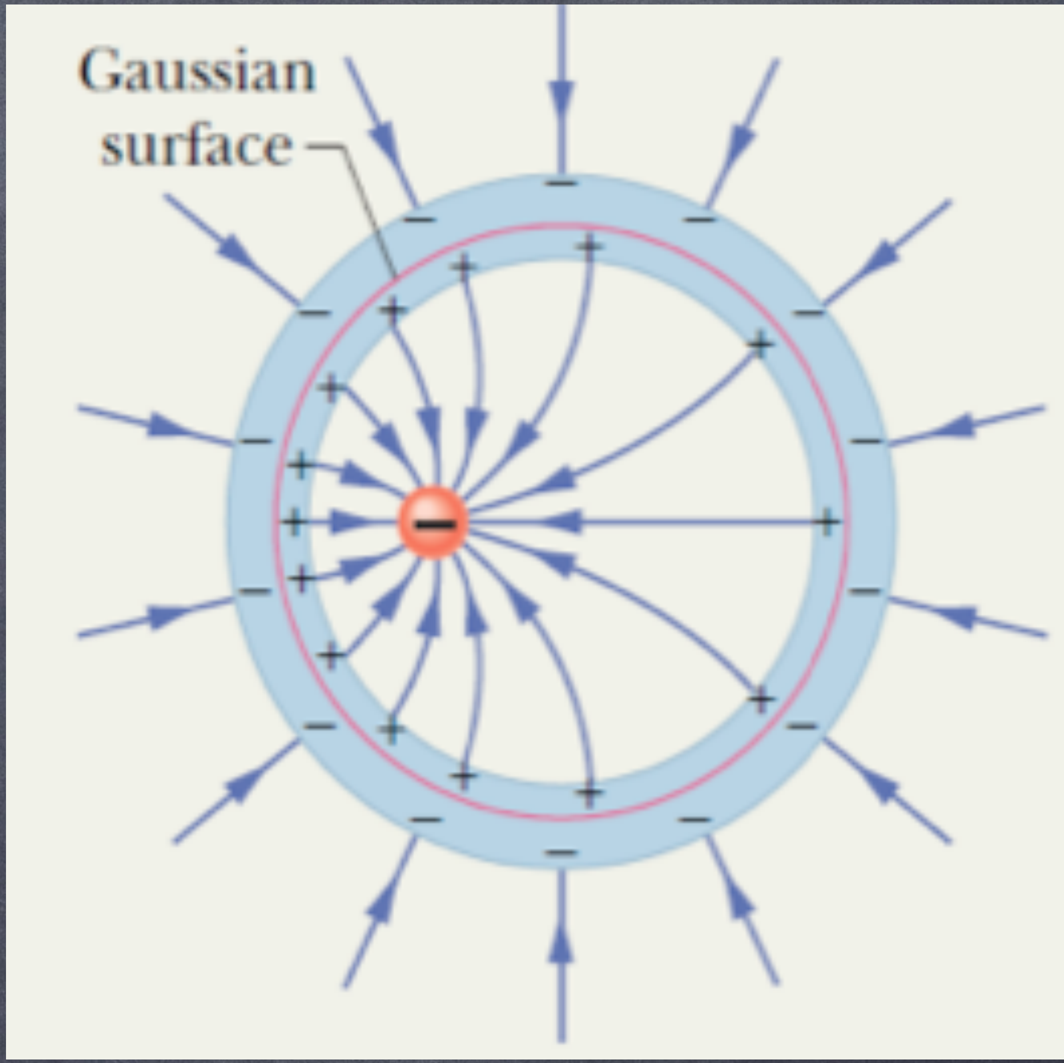


- The electric field inside any conductor is zero (assume equilibrium).
- Thus, by Gauss' Law: $\Phi_E = EA\cos\theta = (0)A\cos\theta = 0$

$$\Phi_E = 0 = \frac{q_{encl}}{\varepsilon_o}$$

Gauss' Law

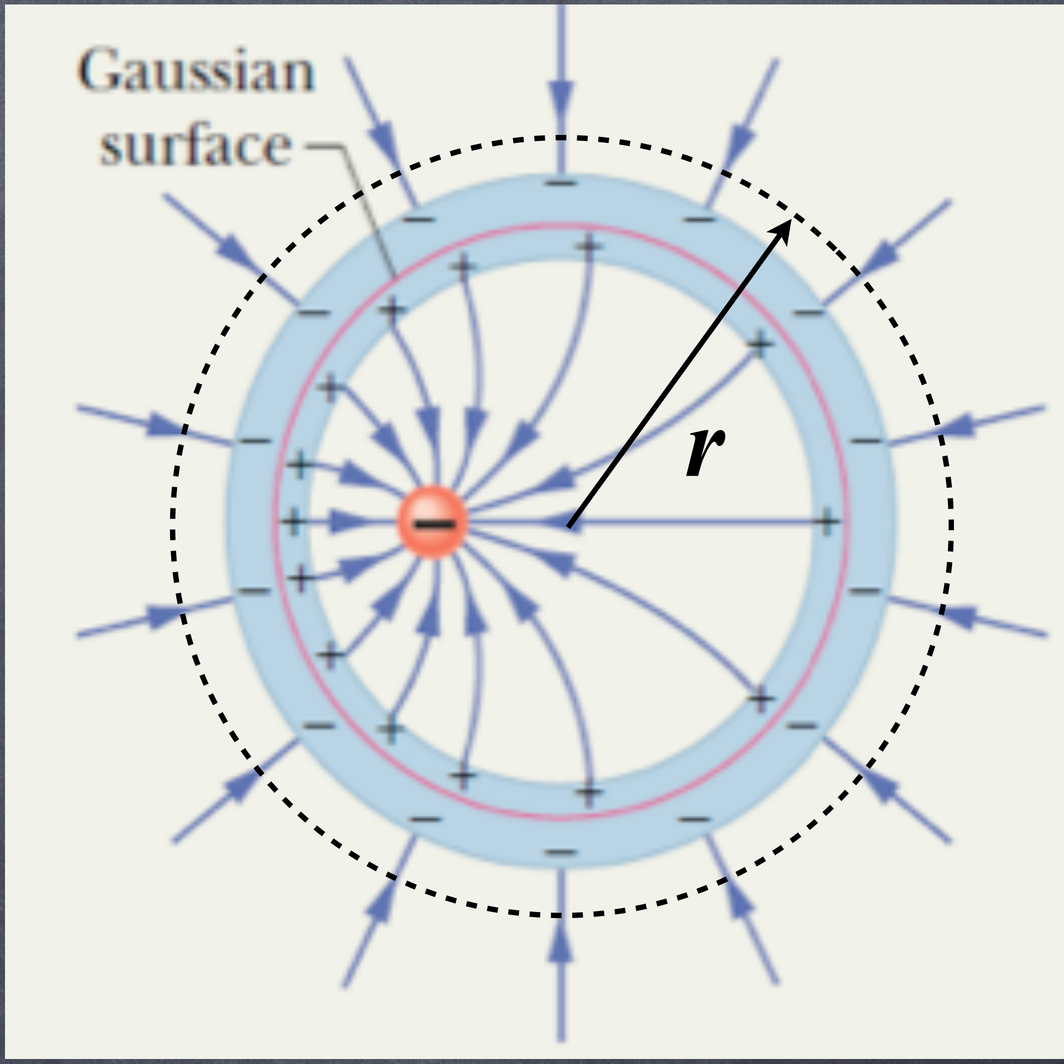
- Answer
- Since the amount of charge enclosed within the Gaussian surface is zero, this means that $+5.0\,\mu C$ must be on the inner surface (to cancel out the $-5.0\,\mu C$ point charge).



- And, since the conducting shell is neutral overall, then $-5.0\,\mu C$ must be located on the outer surface.
- The inner surface charges will be skewed since the center charge is off-center.

Gauss' Law

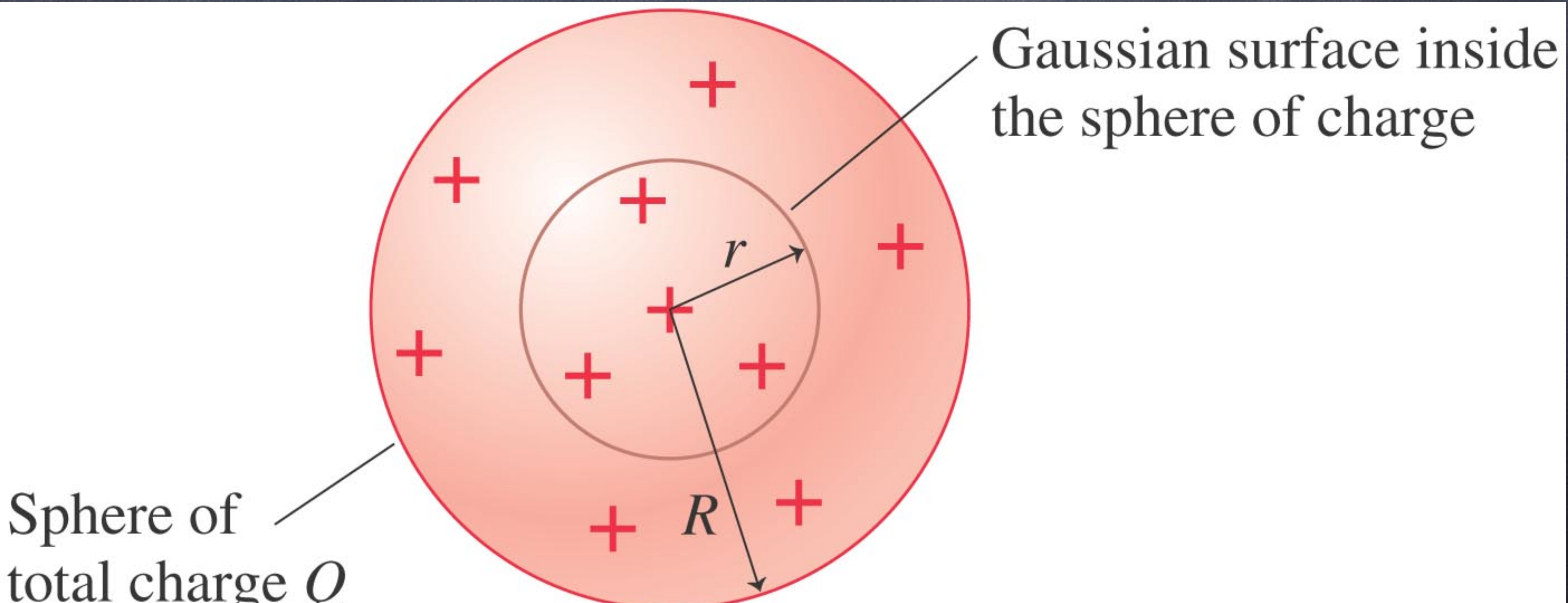
- Answer
- Yet the outer surface charges will be uniformly distributed.
- This can be confirmed by taking another Gaussian surface with a larger radius, r.



- The charge enclosed by this larger Gaussian surface encloses -5.0μC of charge (the point charge).
    - Note the zero electric field due to the inner charges do not affect anything on the outside of the conducting shell.

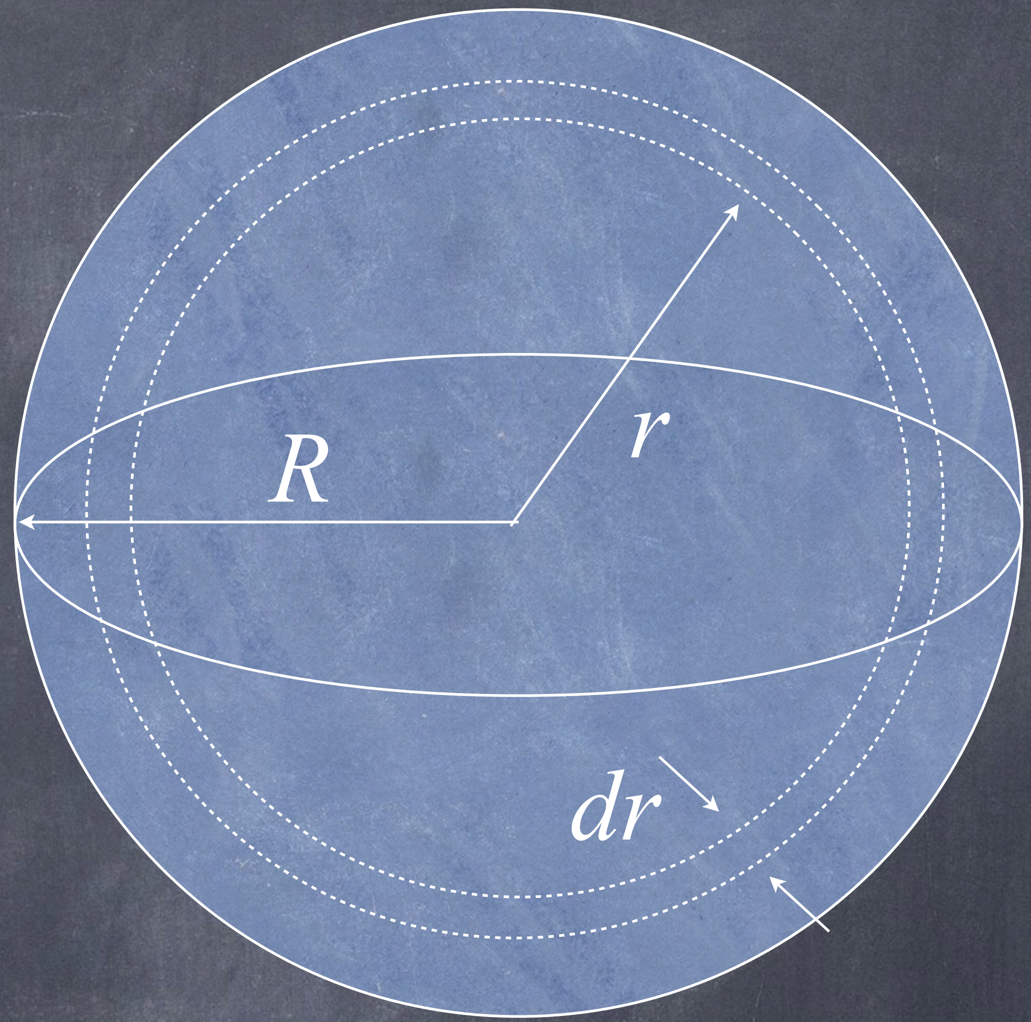
Gauss' Law

- Example
- Suppose the charge density of the solid sphere of radius R below is given by $\rho = \alpha r^2$, where α is a constant.
  (a) Find α in terms of the total charge Q on the sphere and its radius R.
  (b) Find electric field as a function of r inside the sphere.



Gauss' Law

- Answer
- We can divide the sphere up into concentric thin shells of thickness, dr, and volume dV.
- In order to get the total charge Q integrate over all the thin shells from 0 to R.



- Note that every shell will have a volume dV: $$dV = 4\pi r^2 dr$$

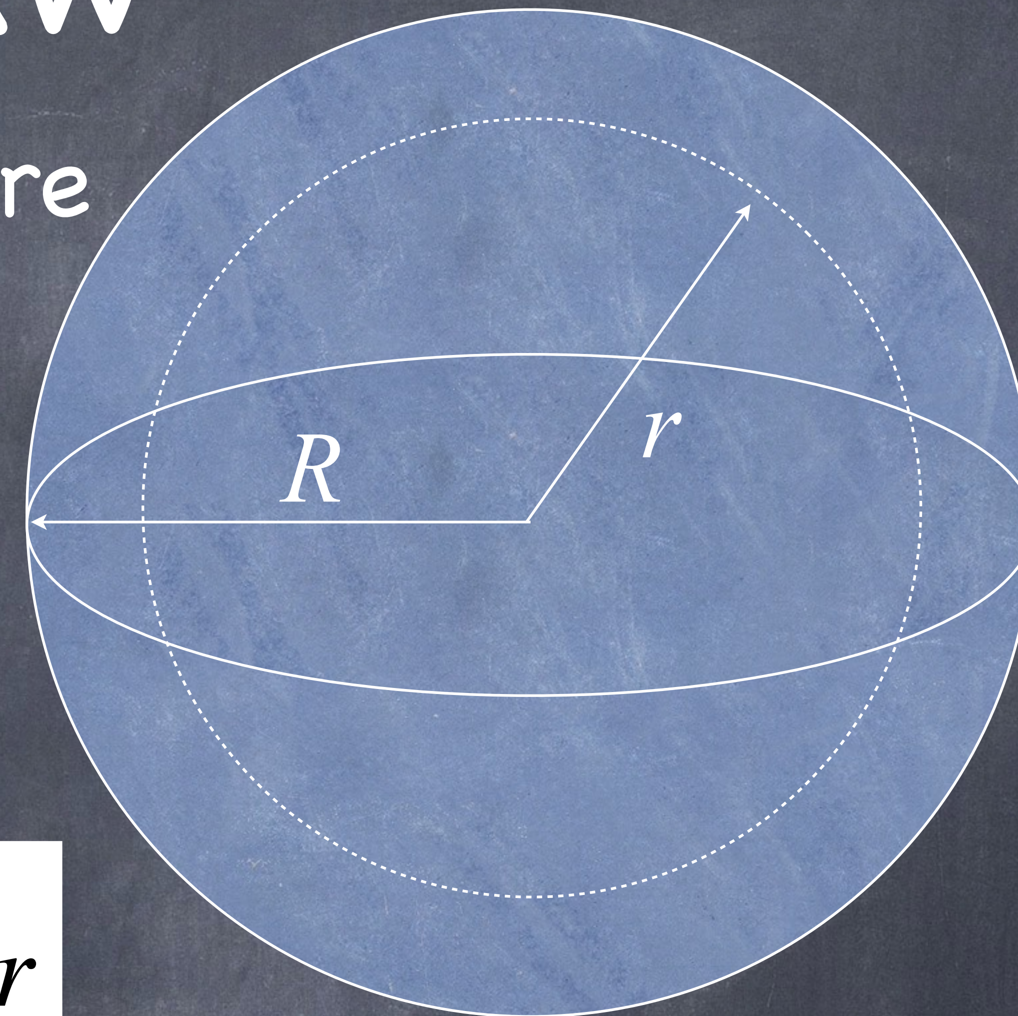
$$Q = \int \rho \, dV \qquad Q = \int_0^R \left(\alpha r^2\right) 4\pi r^2 dr \qquad Q = 4\pi\alpha \int_0^R r^4 dr$$

$$Q = 4\pi\alpha \frac{R^5}{5} \qquad \alpha = \frac{5Q}{4\pi R^5}$$

Gauss' Law

- Answer

- For (b) we need a Gaussian sphere of radius r ($r < R$).
- We need to find how much charge we are enclosing with our Gaussian sphere.



$$Q_{encl} = \int_0^r \rho \, dV$$

$$Q_{encl} = \int_0^r \left(\alpha r^2\right) 4\pi r^2 \, dr$$

$$Q_{encl} = \int_0^r \left(\frac{5Q}{4\pi R^5} r^2\right) 4\pi r^2 \, dr$$

$$Q_{encl} = \frac{5Q}{R^5} \int_0^r r^4 \, dr$$

$$Q_{encl} = \frac{5Q}{R^5} \frac{r^5}{5} = Q \frac{r^5}{R^5}$$

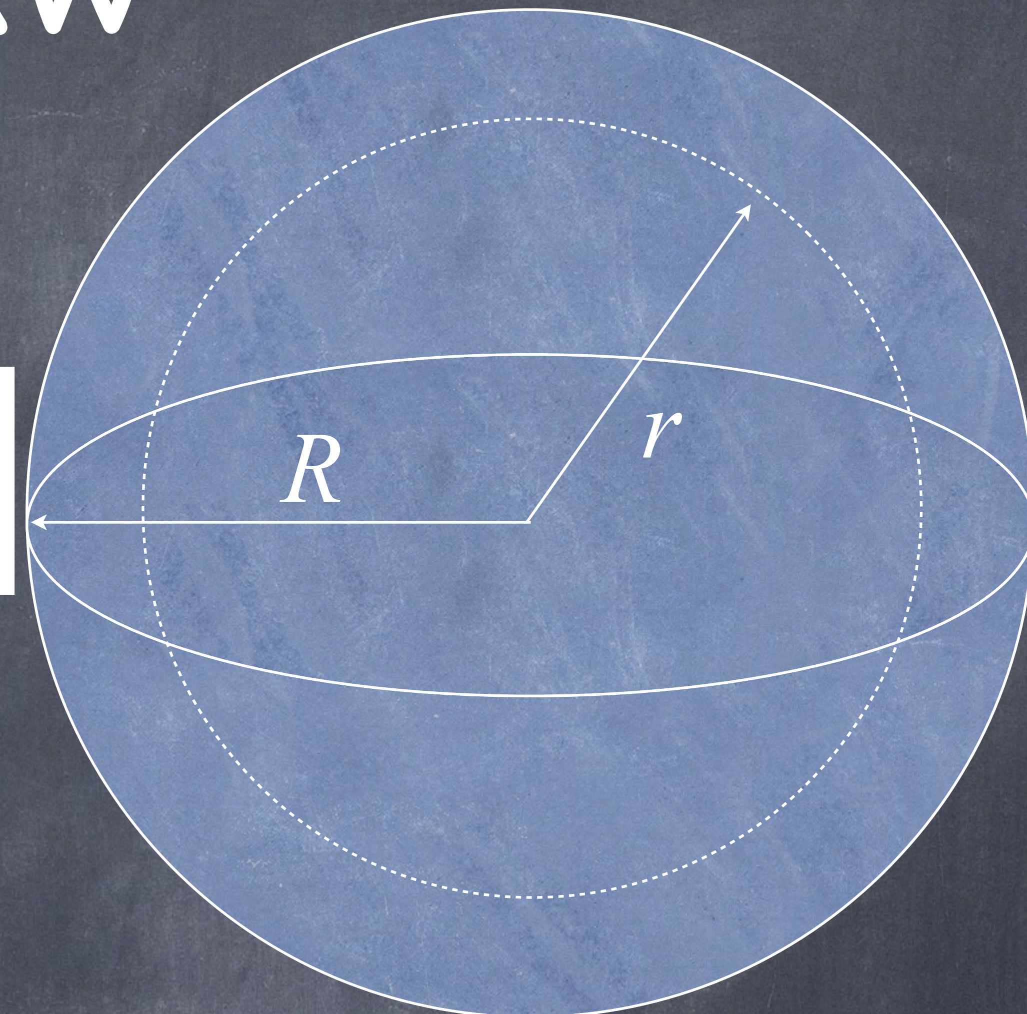
- Answer

Gauss' Law

- Next to find the E field, take the surface integral:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A}$$

$$\oint dA = 4\pi r^2$$



$$\Phi_E = E\left(4\pi r^2\right)$$

- Equating the two sides of Gauss' Law:

$$\Phi_E = \Phi_E$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_o}$$

$$E\left(4\pi r^2\right) = \frac{Q}{\varepsilon_o}\frac{r^5}{R^5}$$

$$E = \frac{1}{4\pi r^2}\frac{Qr^5}{\varepsilon_o R^5}$$

$$E = \frac{1}{4\pi\varepsilon_o}\frac{Qr^3}{R^5}$$

Chapter 25 The Electric Potential

You will learn to use the electric potential and electric potential energy.

Gravitational Potential Energy

- If I were to lift a mass up off of the floor and put it over my head, what have I done to that mass?
- I have increased the gravitational potential energy of the mass. If I let go of the mass it has the potential to move towards the floor.
- In order to increase the energy of the mass, I had to perform work on the mass.
- Since the gravitational force is a conservative force, the work I put into the mass is equivalent to the increase in its gravitational potential energy.

Electric Potential Energy

- If I were to move a negatively charged sphere away from a positively charged wall, what have I done to that sphere?
- I have increased the electric potential energy of the sphere. If I let go of the sphere it has the potential to move towards the wall.
- In order to increase the energy of the sphere, I had to perform work on the sphere.
- Since the electric force is a conservative force, the work I put into the sphere is equivalent to the increase in its electrical potential energy.

Electric Potential Energy

- Recall that gravitational potential energy, U_{grav} or PE_{grav}, **for point masses** (or spherically symmetric masses) was:

$$PE_{grav} = -G\frac{m_1 m_2}{r}$$

- where zero potential energy was defined as having a separation distance of infinity.

- Similarly, the electric potential energy, U_{elec} or PE_{elec}, for point charges (or spherically symmetric charge distributions) is:

$$PE_{elec} = k_e \frac{q_1 q_2}{r}$$

Electric Potential Energy

- As with all potential energy, it is far more useful to look at changes in electric potential energy as opposed to absolute electric potential energy.

- For a point charge, the change in potential energy between points A and B is given by:

$$\Delta PE_{A \to B} = PE_B - PE_A = k_e \frac{q_1 q_2}{r_B} - k_e \frac{q_1 q_2}{r_A}$$

$$\Delta PE_{A \to B} = k_e q_1 q_2 \left(\frac{1}{r_B} - \frac{1}{r_A} \right)$$

- Note the lack of absolute value signs. The sign of the charge must be taken into account.

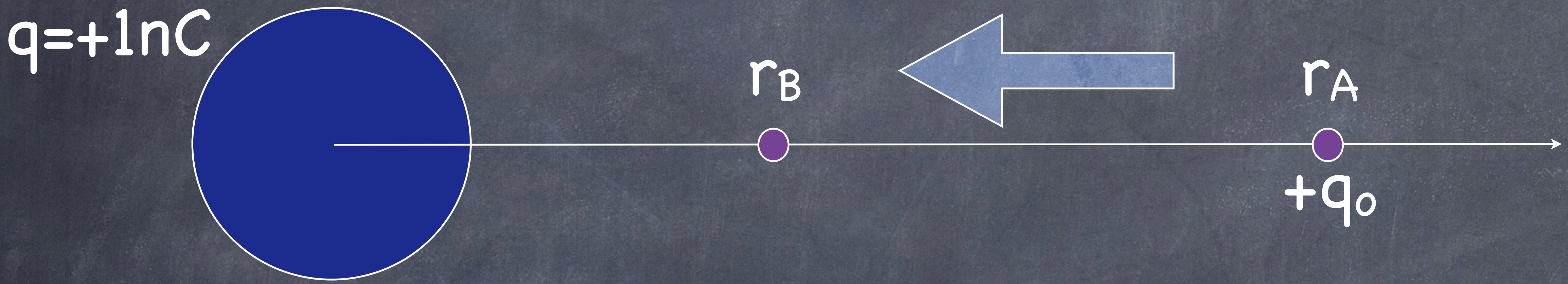
Electric Potential Energy

- Example
- A proton is placed 3.0cm to the right of a +1.0nC charged sphere. It is then brought in to a distance of 1.0cm from the sphere.
  What is the change in potential energy of the proton?

  - Answer
  - Define a coordinate system.
  - Choose the sphere as r = 0. Any outward distance would be positive.

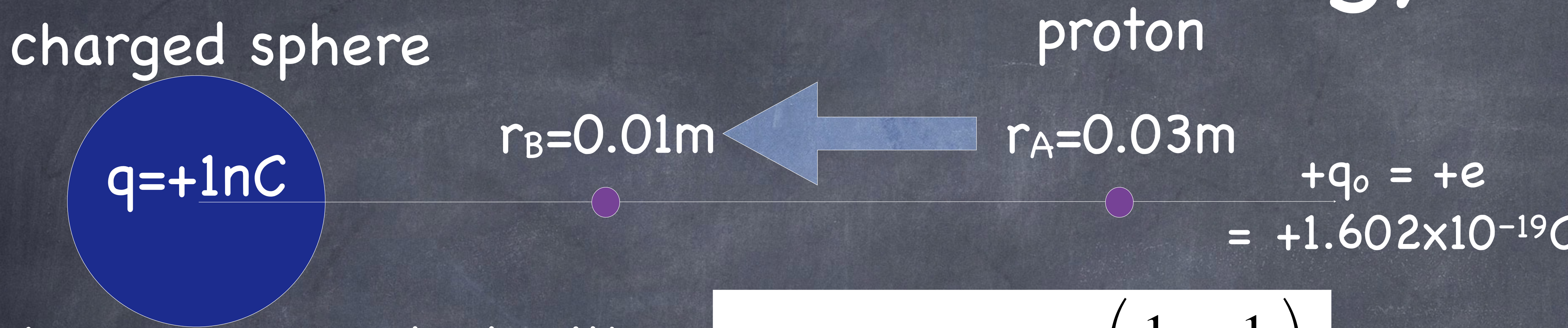
Electric Potential Energy

- <u>Answer</u>
- Next, draw a picture of the situation:



- List what quantities we know:
- $q = +1.0\times10^{-9}C$
- $q_o = +e = +1.602\times10^{-19}C$
- $r_A = 0.03m$
- $r_B = 0.01m$
- Let's turn to the equation for change in electric potential energy.

Electric Potential Energy



- Answer
- Start with:

$$\Delta PE_{A\to B} = k_e q_1 q_2 \left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

$$\Delta PE_{AB} = \left(8.99\times10^{+9}\ \mathrm{Nm^2/C^2}\right)\left(+1.602\times10^{-19}\mathrm{C}\right)\left(+1.0\times10^{-9}\mathrm{C}\right)\left(\frac{1}{0.01\mathrm{m}} - \frac{1}{0.03\mathrm{m}}\right)$$

$$\Delta PE_{AB} = 9.60\times10^{-17}\ \mathrm{J}$$

- This is a positive value so potential energy increased.
- What if we substituted in an electron for the proton?
- The value would be the same, but the potential energy would have decreased instead of increased.

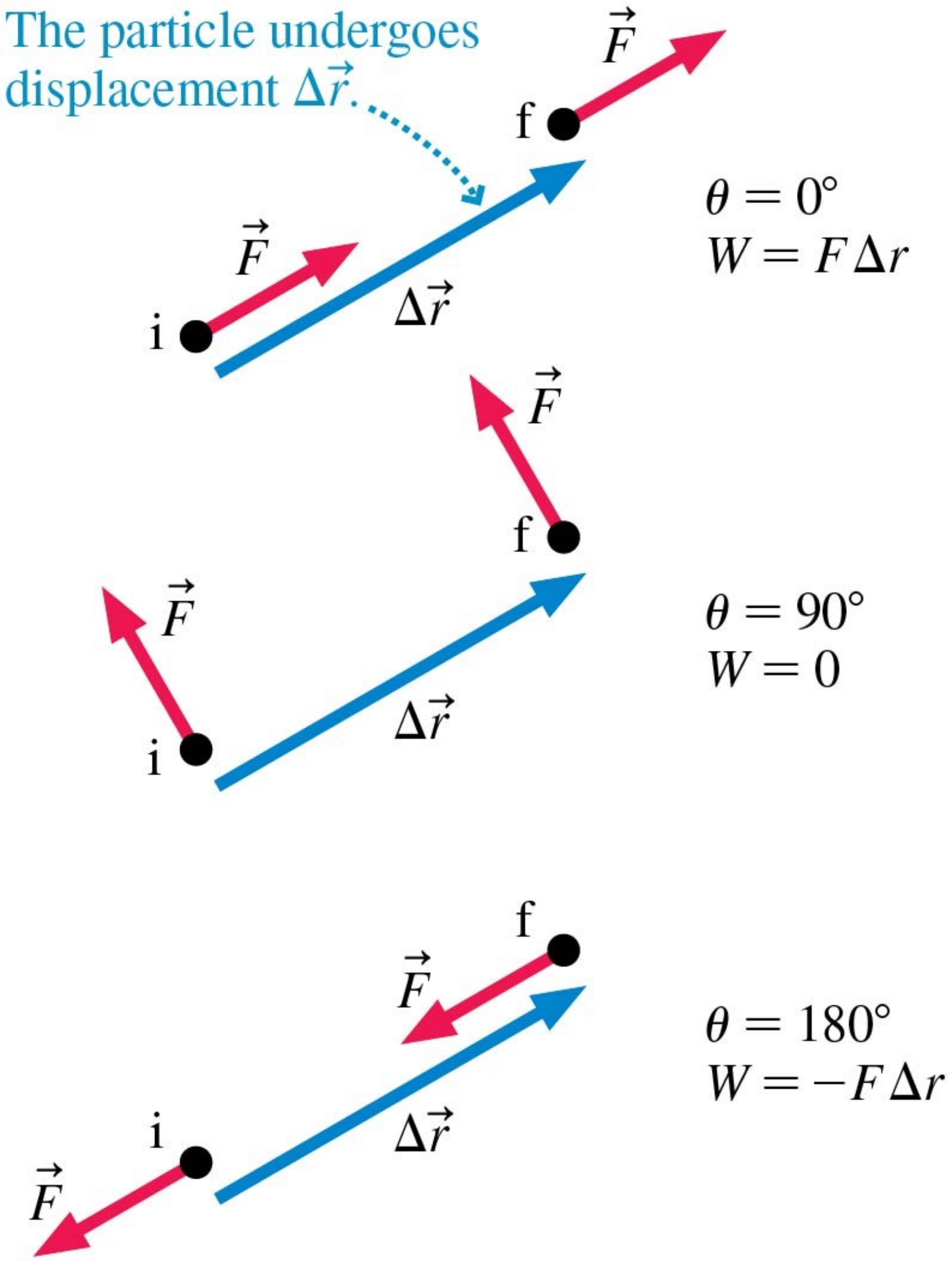
Work Done by a Constant Force

- Let's motivate the equation of electric potential energy above …

- Recall that the work done by a constant force depends on the angle θ between the force F and the displacement Δr:

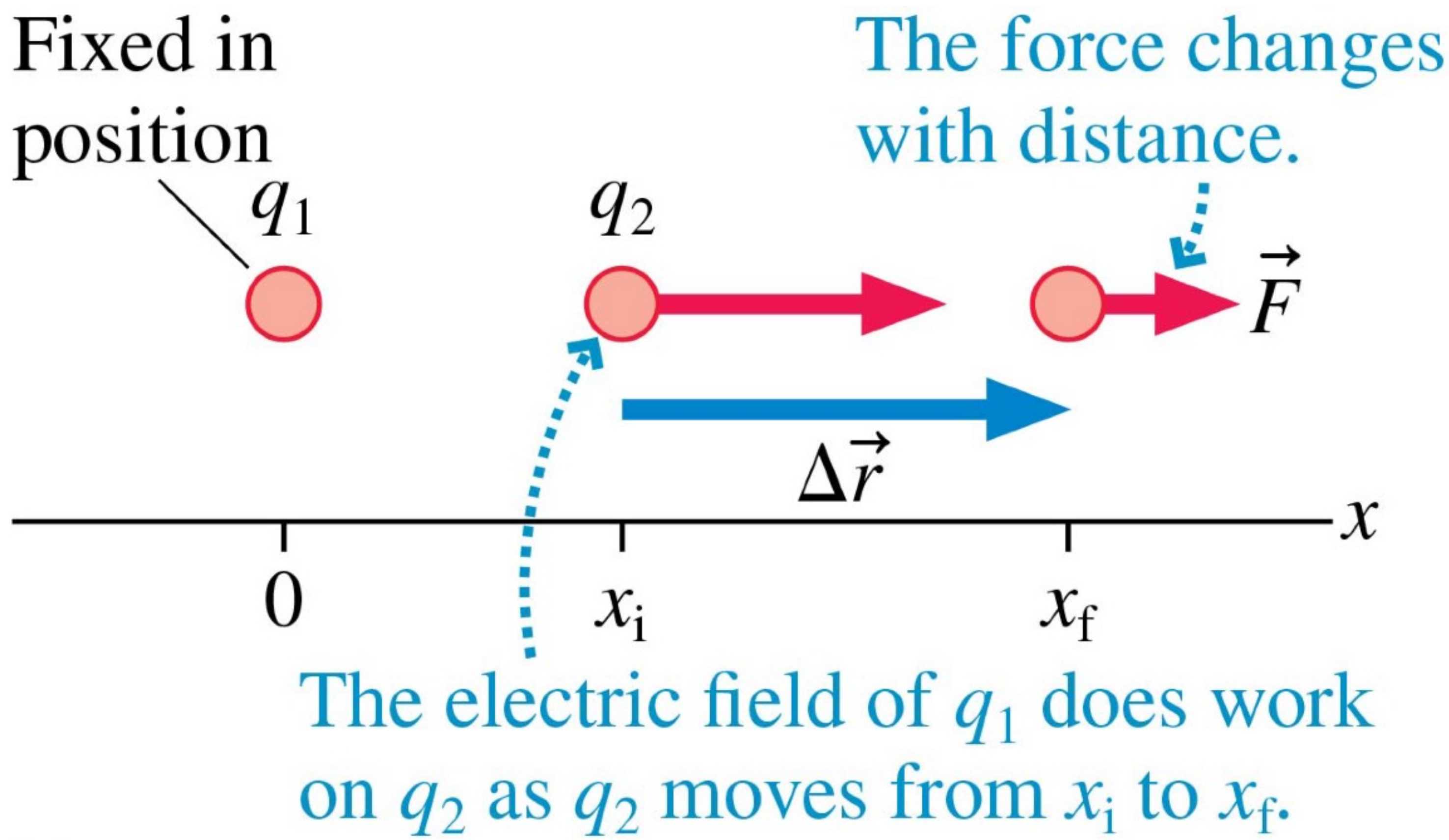
$$W = \vec{F} \cdot \Delta\vec{r} = F\,\Delta r \cos\theta$$

In general, an integral over the path taken by the particle.



The Potential Energy of Two Point Charges

- Consider two like charges q_1 and q_2.
- The electric field of q_1 *pushes* q_2 as it moves from x_i to x_f.



- The work done is

$$W_{\text{elec}} = \int_{x_i}^{x_f} F_{1\text{ on }2}\, dx = \int_{x_i}^{x_f} \frac{Kq_1q_2}{x^2}\, dx = Kq_1q_2 \frac{-1}{x}\bigg|_{x_i}^{x_f} = -\frac{Kq_1q_2}{x_f} + \frac{Kq_1q_2}{x_i}$$

- The change in electric potential energy of the system is $\Delta U_{\text{elec}} = -W_{\text{elec}}$ if

$$U_{\text{elec}} = \frac{Kq_1q_2}{x}$$

Direct Approaches

Source q_1 $\boxed{\text{exerts } \vec{F} \text{ on}}$ test q_2

Source q_1 $\boxed{\text{stores } U_E \text{ w/}}$ test q_2

Electric Potential Energy

Source q_1 $\boxed{\text{stores } U_E \text{ w/}}$ test q_2

$$\Delta U_E = \begin{cases} (+) \text{ increases} & \begin{Bmatrix} \rightarrow \pm \quad \pm \leftarrow \\ \leftarrow \pm \quad \mp \rightarrow \end{Bmatrix} \\ (-) \text{ decreases} & \begin{Bmatrix} \leftarrow \pm \quad \pm \rightarrow \\ \rightarrow \pm \quad \mp \leftarrow \end{Bmatrix} \end{cases}$$

Units of Electric Potential Energy

$[\mathrm{N\cdot m}] = [\mathrm{J}]$

$$\boxed{U_E = k\,\frac{q_1\,q_2}{r}}$$

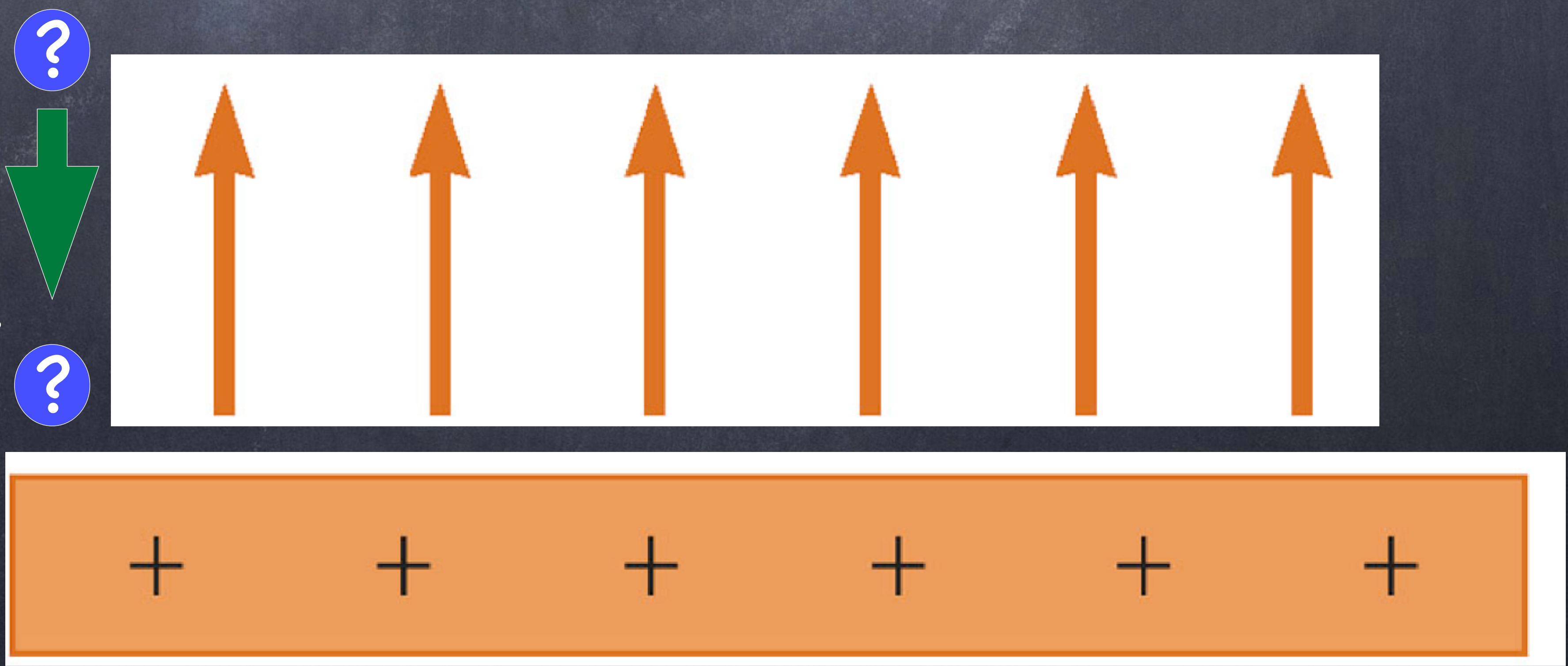
$$\frac{[\cancel{\mathrm{C}}]\,[\cancel{\mathrm{C}}]}{[\cancel{\mathrm{m}}]}$$

$$8.99 \times 10^{9}\ \frac{\mathrm{N\cdot m}^{\cancel{2}}}{\cancel{\mathrm{C}^2}}$$

Electric Potential Energy, increase/decrease?

- Let's say we have a positively charged floor. What will the electric field look like?
  - Now let's say that an unknown charged object that is originally far away from the floor is brought in closer to the floor.
  - Has the electric potential energy <u>of the object</u> increased or decreased?

- We are not sure. It is dependent on the sign of the test charge.



Electric Potential Energy, increase/decrease?

- To solve this problem, we introduce a new variable, the electric potential, V.
- We define electric potential as:

$$V \equiv \frac{PE_{elec}}{q_2}$$

- Electric potential is measured in Volts. Where: 1 Volt = 1 Joule/Coul. = 1 (Nm)/Coul
- DO NOT CONFUSE electric potential energy, PE_{elec}, and electric potential, V.
- Many times electric potential will be referred to as voltage just to distinguish it further from electric potential energy.

change in Electric Potential

- Again the far more useful quantity will be the change in electric potential or what is referred to as the electric potential difference.

$$\Delta V_{A\to B} = V_B - V_A = \frac{\Delta PE_{elec}}{q_2}$$

- This equation is always valid.
- This means that for a point charge we can say:

$$\Delta V_{A\to B} = \frac{\Delta PE_{elec}}{q_2} = k_e q_1 \left(\frac{1}{r_B} - \frac{1}{r_A} \right)$$

- Note that the electric potential difference is independent of the test charge, q_2.

Electric Potential or Voltage

- The electric potential at a location is the electric potential energy that a +1C charge would have due to outside charges if it were placed at that location.
- It is similar to the relationship that electric force and electric field have.
- Electric potential is a scalar quantity.
- Note that it really only depends on the outside charges, q, and the separation distance, r.
- If the outside charges are a distribution then, the electric potential will depend on the electric field, E, and the separation distance, r.

Field Model

Source Q $\boxed{\text{creates}}$ $\vec{E}$ field

$\vec{E}$ field $\boxed{\text{exerts } \vec{F} \text{ on}}$ test q_2

Potential Model

Source Q $\boxed{\text{creates}}$ V potential

V potential $\boxed{\text{stores } U_E \text{ w/}}$ test q_2

“Fire in the Disco! Fire in the Taco Bell!”
(Source: Caffeinatrix Au Lait)
http://www.flickr.com/photos/caffeina/2622453274/

1.5 V

"...and runs and runs and..."
(Source: Demion)
http://www.flickr.com/photos/7989285@N07/2184022784/

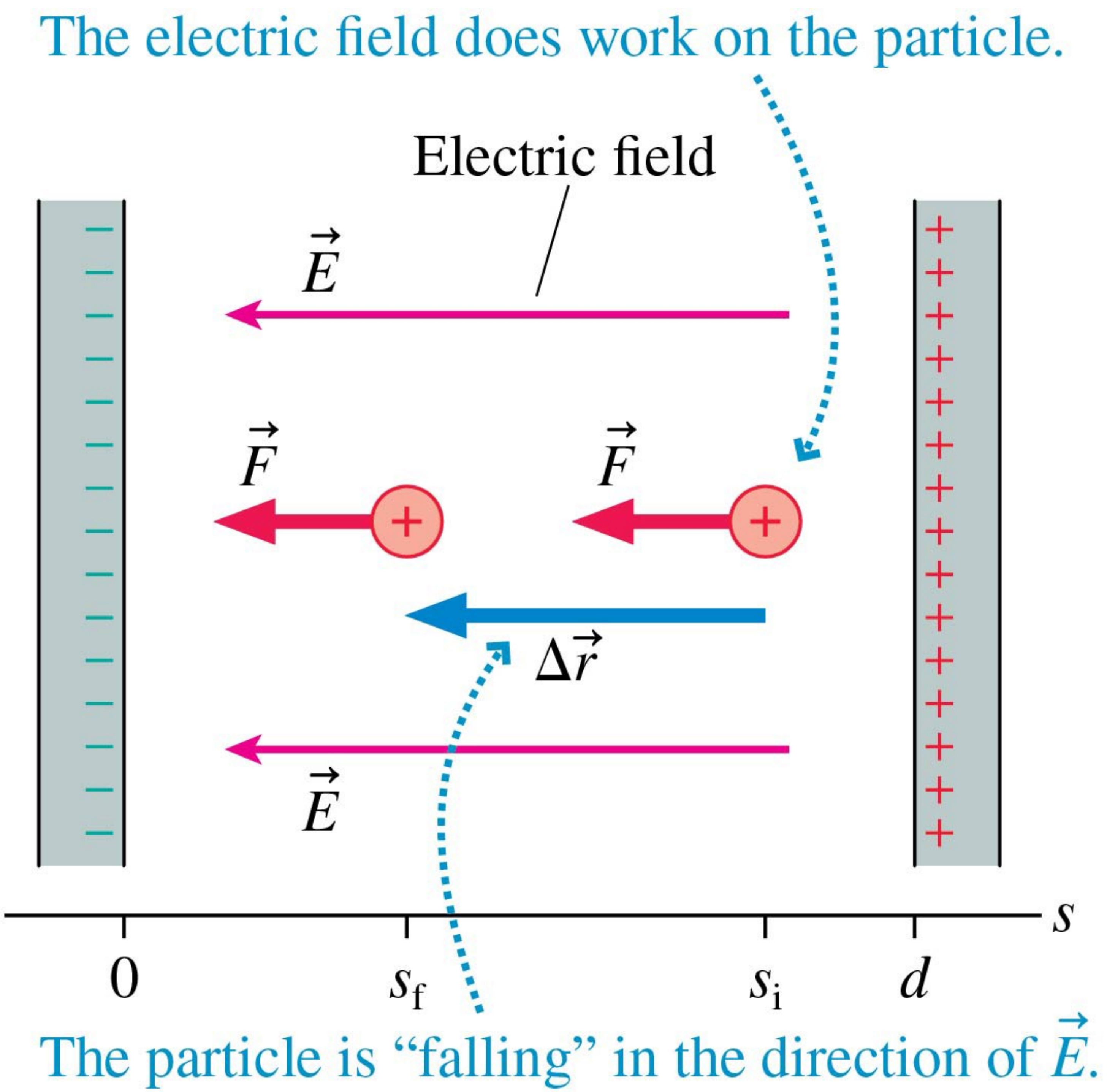
!!!!!

"Danger 27,600 Volts"
(Source: Jeremy Burgin)
http://www.flickr.com/photos/jburgin/4279941141/

Electric Potential Energy in a Uniform Field

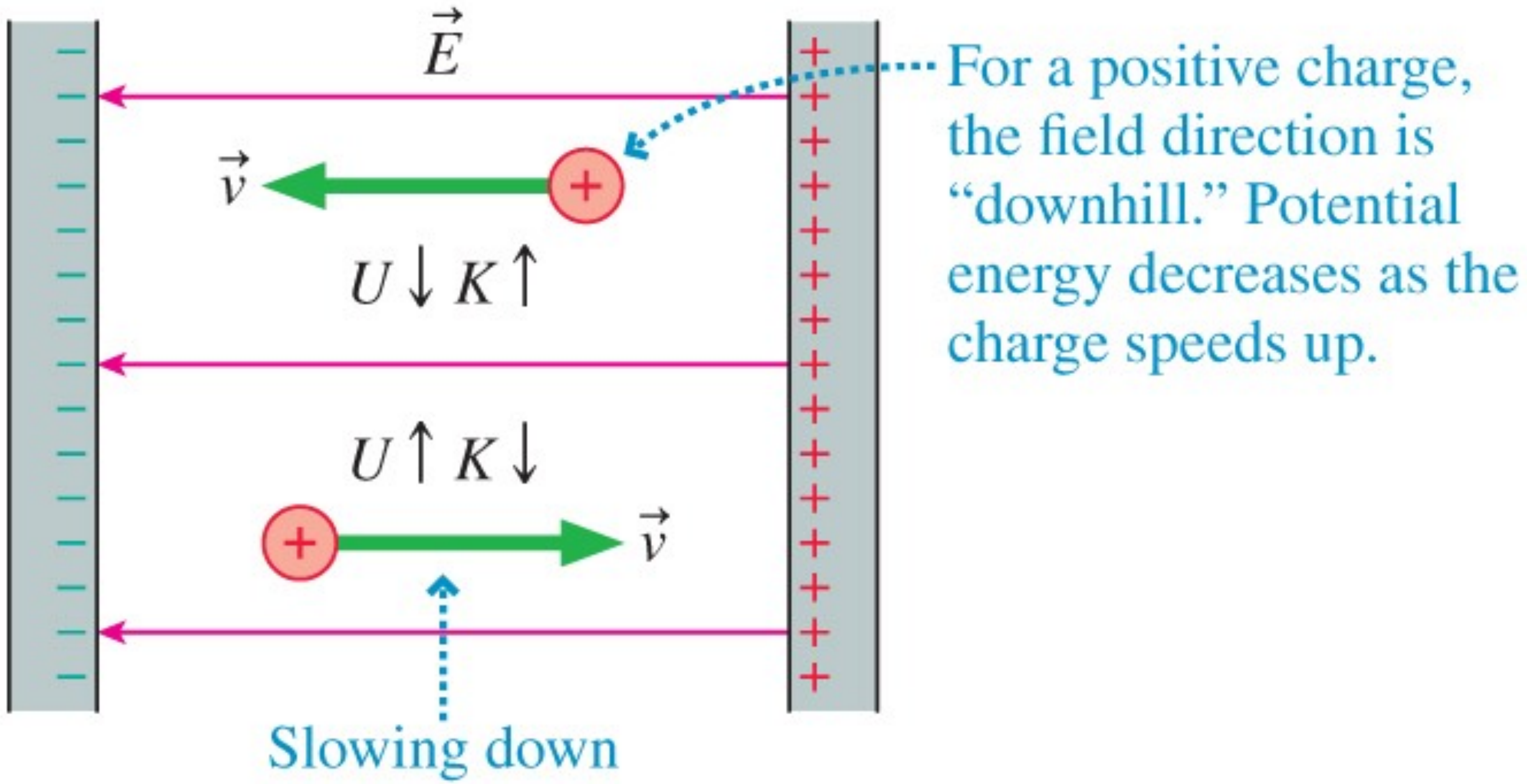
- A positive charge inside a capacitor speeds up and gains kinetic energy as it “falls” toward the negative plate.
- The charge is losing potential energy as it gains kinetic energy.

$$U_{\text{elec}} = U_0 + qEs$$



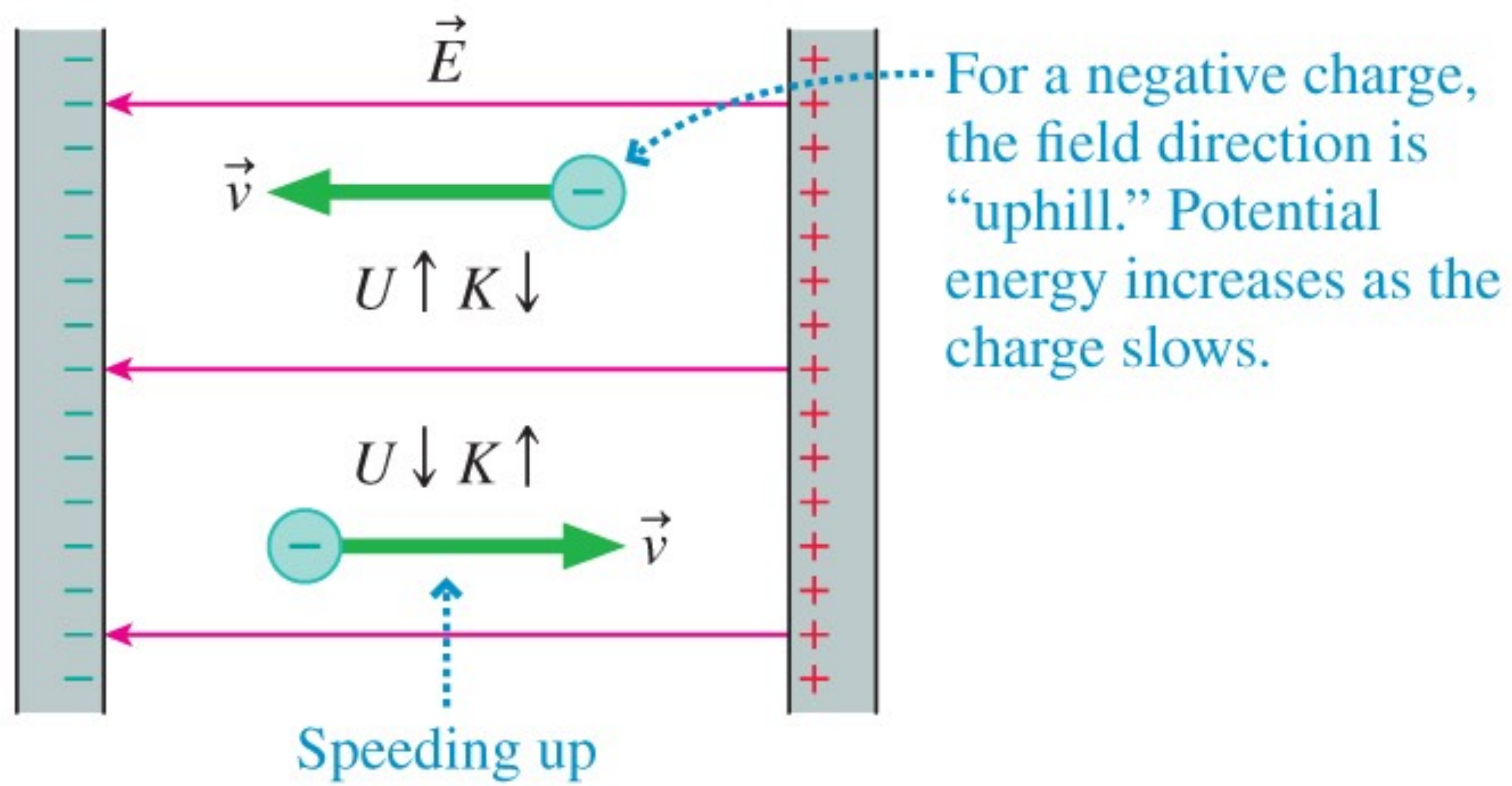
Electric Potential Energy in a Uniform Field

- For a positive (+) charge, U decreases and K increases as the charge moves toward the negative plate.
- A positive charge moving opposite the field direction is going "uphill," slowing as it transforms kinetic energy into electric potential energy.



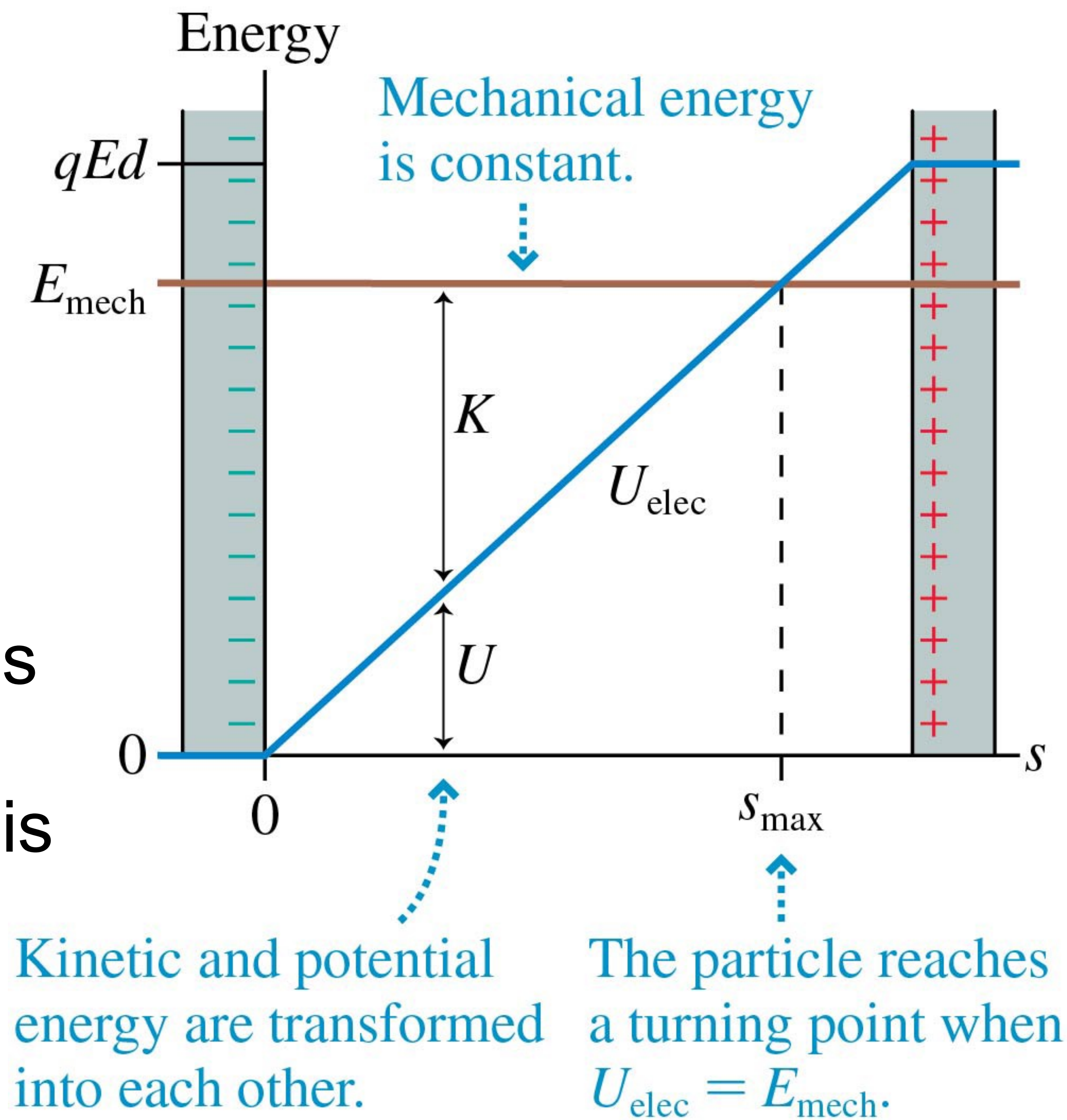
Electric Potential Energy in a Uniform Field

- A negative (-) charged particle has *negative* potential energy.
- U increases (becomes less negative) as the negative charge moves toward the negative plate.
- A negative charge moving in the field direction is going "uphill," transforming $K \rightarrow U$ as it slows.



Electric Potential Energy in a Uniform Field

- The figure shows the **energy diagram** for a positively charged particle in a uniform electric field.



The potential energy increases linearly with distance, but the total mechanical energy E_{mech} is fixed.

Electric Potential

- The lessons we have learned from examining electric potential energy and electric potential are that:
- **Electric potential** only depends on your position in the electric field.
- **Electric potential <u>energy</u>** will **also** depend on what your test charge is.
- No matter what, if a charge is free to move in an applied electric field, it will move in the direction that lowers its potential energy.

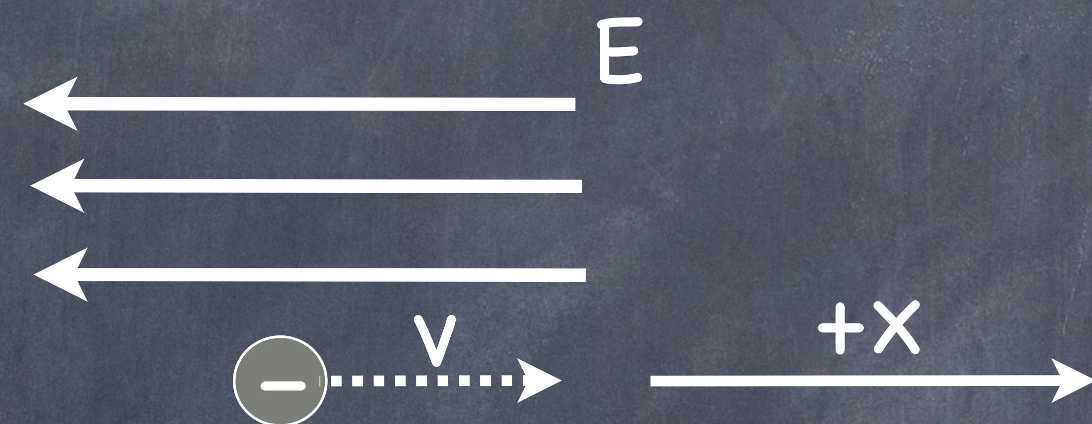
Electric Potential, example:

- An electron in the picture tube of an older TV set is accelerated from rest through a potential difference ΔV = 5000V by a uniform electric field.
  - ➡ What is the change in potential energy of the electron?
  - ➡ What is the speed of the electron as a result of this acceleration (assume it started from rest)?

- Answer
- Define a coordinate system.
- Choose where the electron starts it motion as x = 0 and its direction of motion as +x.

Electric Potential, example:

- <u>Answer</u>
- Draw a quick picture of the situation:



- Let's turn to the definition of electric potential:
- $$\mathscr{D}V = \mathscr{D}PE/q_2$$

Electric Potential

- <u>Answer</u>

$$\Delta PE = q_2\left(\Delta V\right)$$

$$\Delta PE = -1.602 \times 10^{-19}\mathrm{C}\left(+5000V\right) = -8.0 \times 10^{-16}\mathrm{J}$$

- So, the electron lost potential energy by moving in this electric field.
- Where did this energy go?
- It went into the kinetic energy of the system, since the electric force is a conservative force.
- We can then turn to conservation of energy to solve for the final velocity of the electron.
- Did any energy leave the system?
- No, it stays with electron.

Electric Potential

- <u>Answer</u>

$$0 = \Delta PE + \Delta KE$$

$$\Delta KE = -\Delta PE$$

$$\frac{1}{2} m v^2 = -\Delta PE$$

$$v^2 = \frac{-2(\Delta PE)}{m}$$

$$v = \sqrt{\frac{-2(\Delta PE)}{m}} = \sqrt{\frac{-2(-8.0\times10^{-16}\,\mathrm{J})}{9.11\times10^{-31}\,\mathrm{kg}}} = 4.2\times10^{7}\,\mathrm{m/s}$$

- Wow! The electron got almost close to the speed of light, with just a 5000 V difference.

electron Volt

- It becomes very inconvenient to work with Joules when you are dealing with electrons or protons.
- We then introduce a new unit, the electron Volt.
- The electron Volt (eV) is defined as the energy that an electron gains when accelerated through a potential difference of 1Volt.
- $$1\text{eV} = 1.602\times10^{-19}\text{J}$$
- An electron in a normal atom has about 10 eV while gamma rays (light) may have millions of eV.

Voltage

- Voltage is 1.5V
  electron gains 1.5 eV

- High Voltage TV: 15,000V
  electron gains 15 keV

Voltage

- Voltage is 1,000,000 V
  a million volts!
  (or 1MV)

  1 electron gains
  potential energy
  1 MeV

Large Electron Accelerators

- Advanced Photon Source, Argonne

(near Chicago)

- Electrons are accelerated to 7 Billion Volts

7,000,000,000 V

or 7GV

- Electron Energy is 7 GeV

Large Electron Accelerators

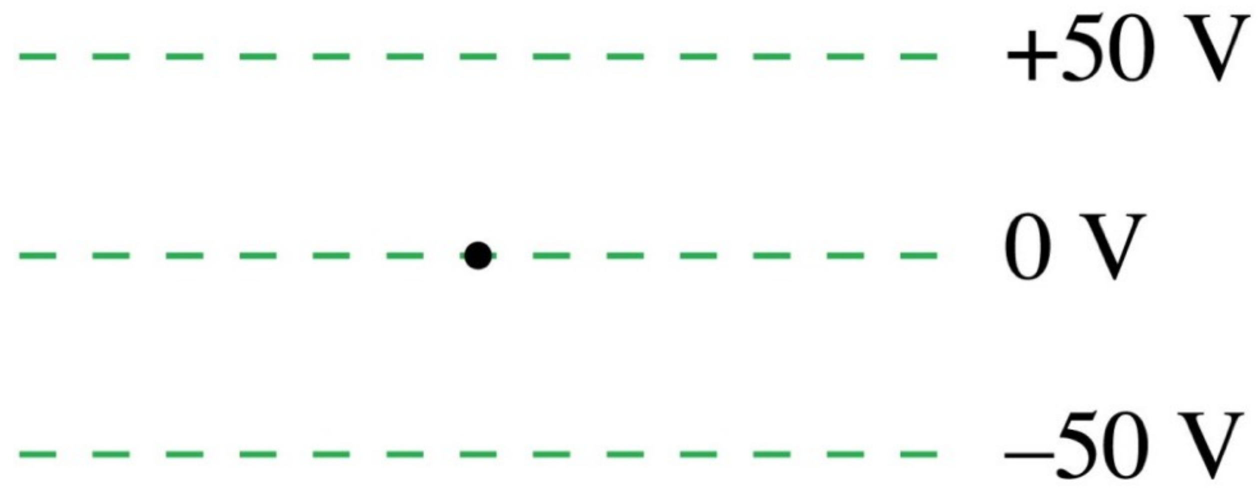
- Electrons are accelerated to 14 Billion Volts
- Electron Energy is 14 GeV

- Stanford's Linear Coherent Light Source

(world's first X-ray Free Electron Laser)

iClicker question 6-3

A proton is released from rest at the dot. Afterward, the proton



A. Remains at the dot.
B. Moves upward with steady speed.
C. Moves upward with an increasing speed.
D. Moves downward with a steady speed.
E. Moves downward with an increasing speed.