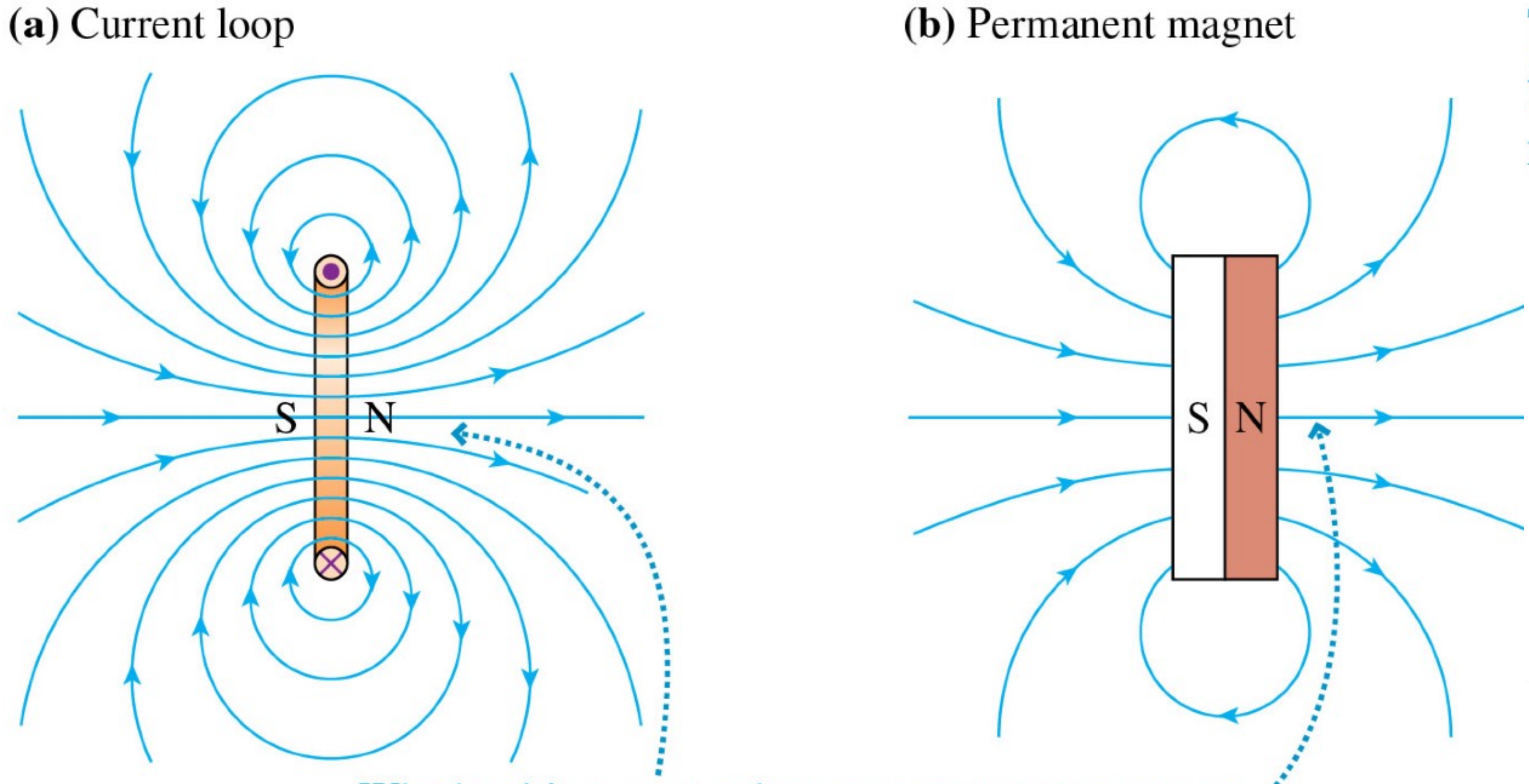
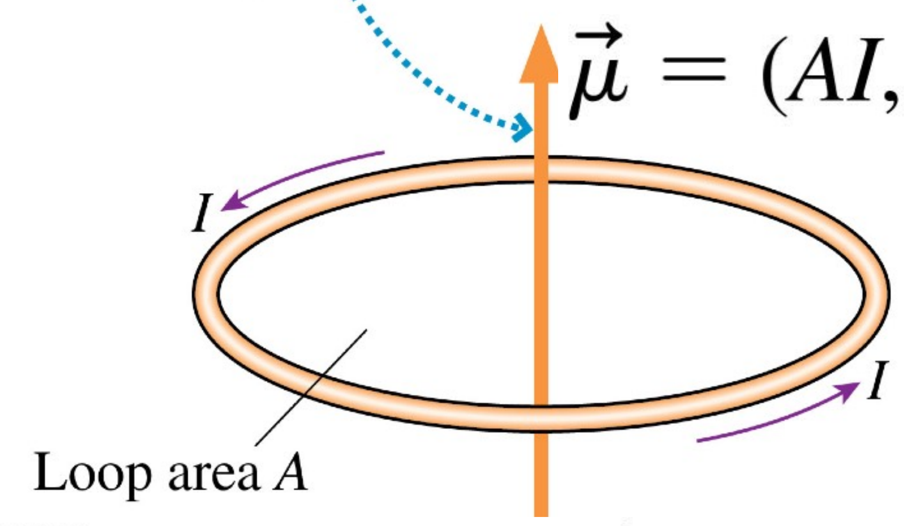
Last time: Magnetic Dipole Moment, Ampere's Law



Whether it's a current loop or a permanent magnet, the magnetic field emerges from the north pole.

The magnetic dipole moment is perpendicular to the loop, in the direction of the right-hand rule. The magnitude of $\vec{\mu}$ is AI.

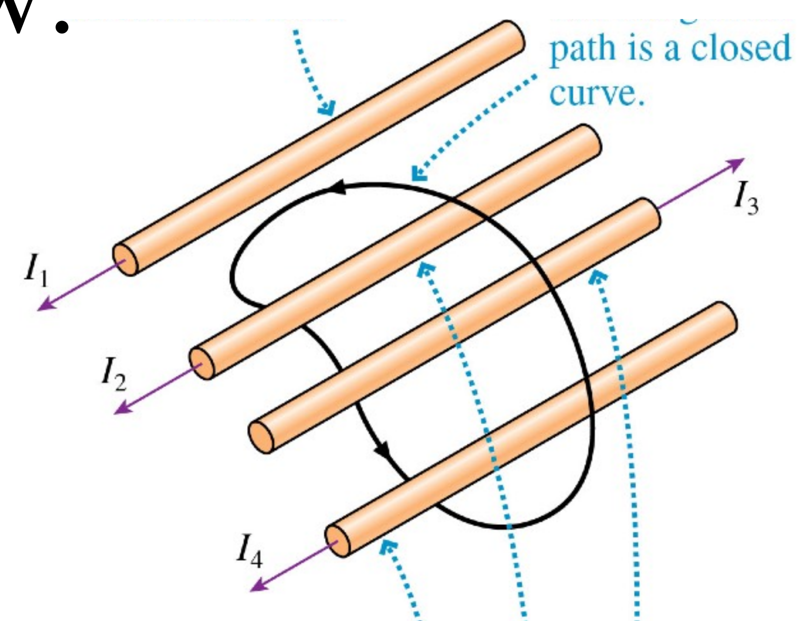


$\vec{\mu} = (AI,$ **S to N**

$$\vec{B}_{\text{dipole}} = \frac{\mu_0}{4\pi}\frac{2\vec{\mu}}{z^3}$$ **along axis**

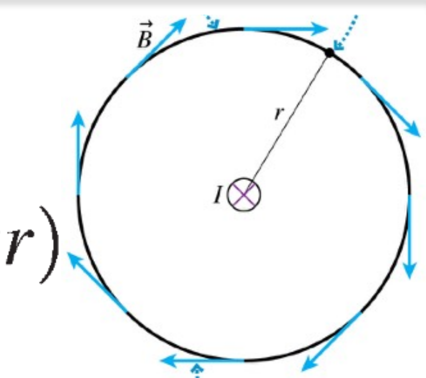
Ampère's Law:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{through}}$$



1. Straight wire with current:

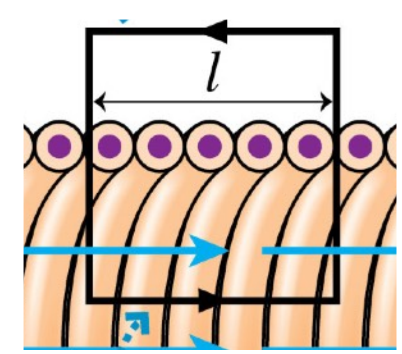
$$\oint \vec{B} \cdot d\vec{s} = Bl = B(2\pi r)$$



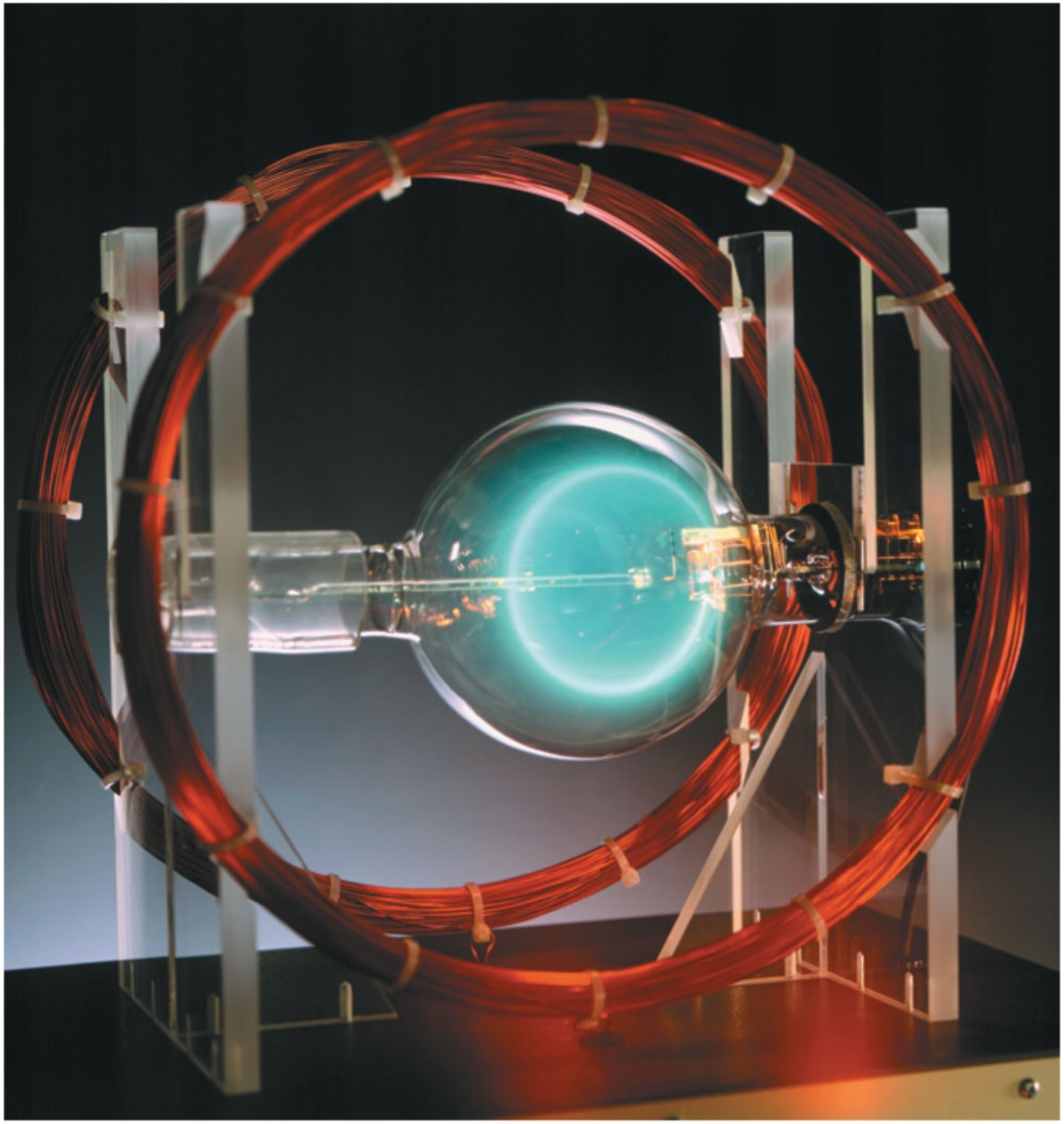
2. & inside wire

3. Solenoid:

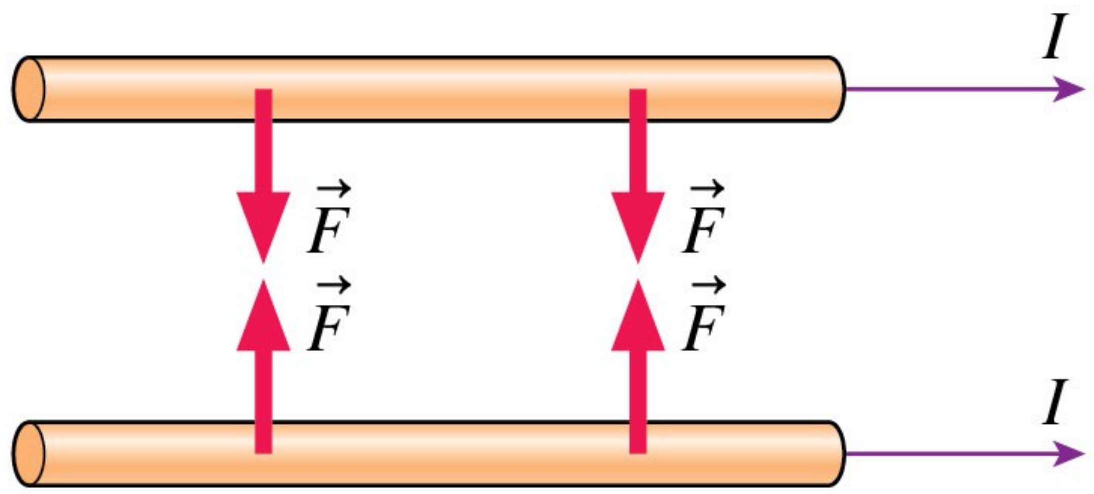
$$\oint \vec{B} \cdot d\vec{s} = Bl = \mu_0 NI$$



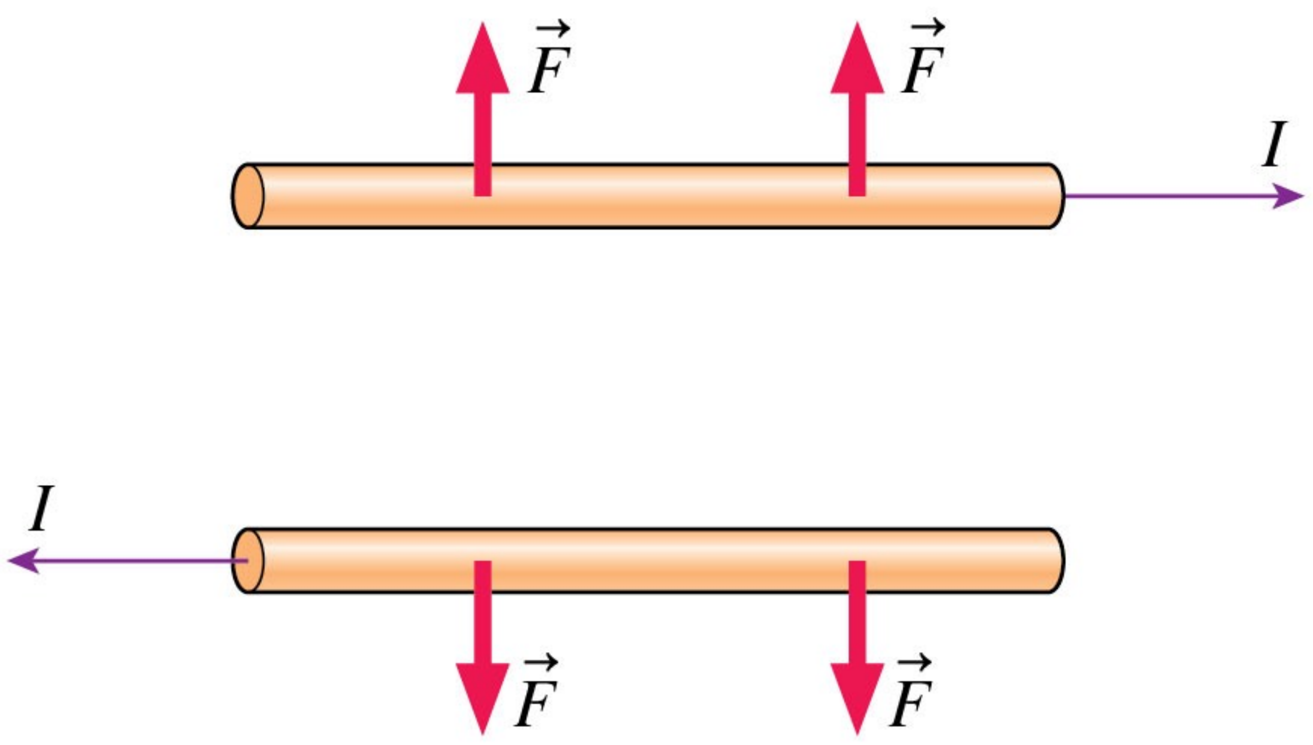
Observation 1: Charged particles in magnetic fields



Observation 2: Ampère's Experiment



"Like" currents attract.



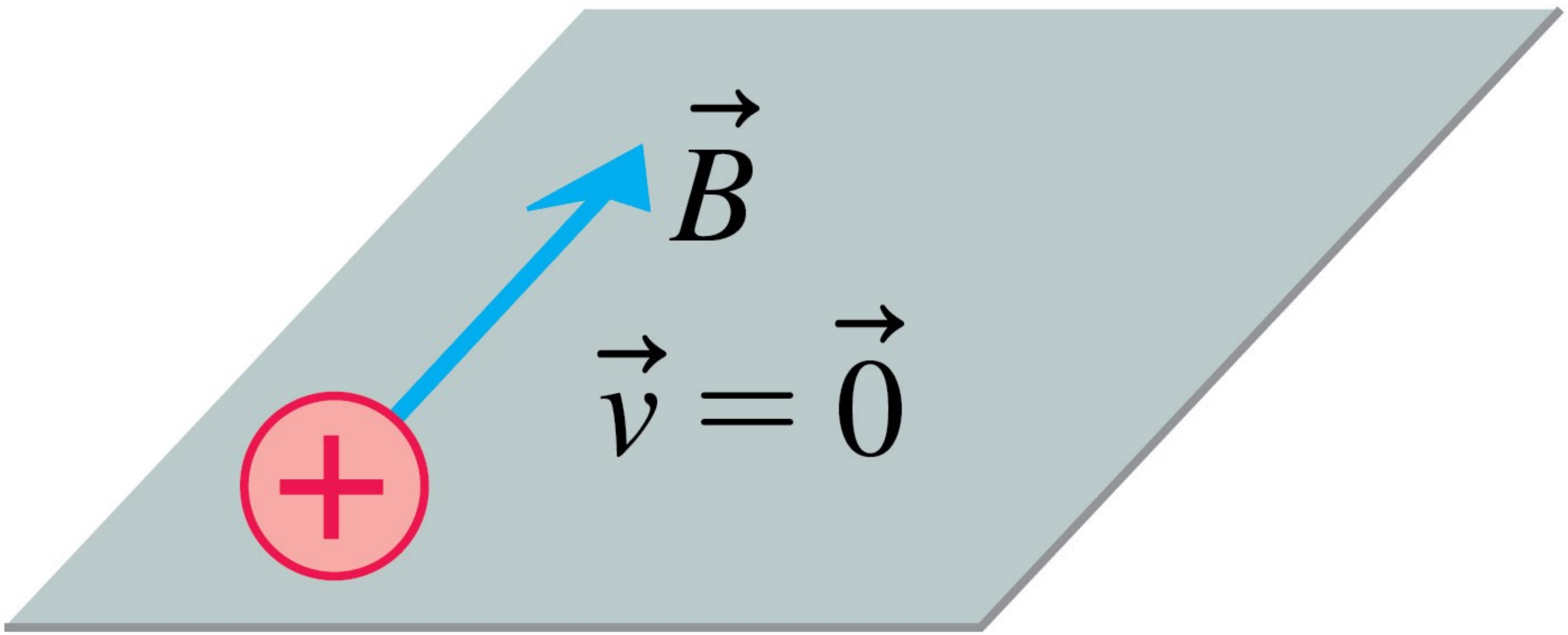
"Opposite" currents repel.

- After the discovery that electric current produces a magnetic field, Ampère set up two parallel wires that could carry large currents either in the same direction or in opposite directions.

Magnetic Force on a Charged Particle

- Existing magnetic field B

$$\vec{F} = \vec{0}$$

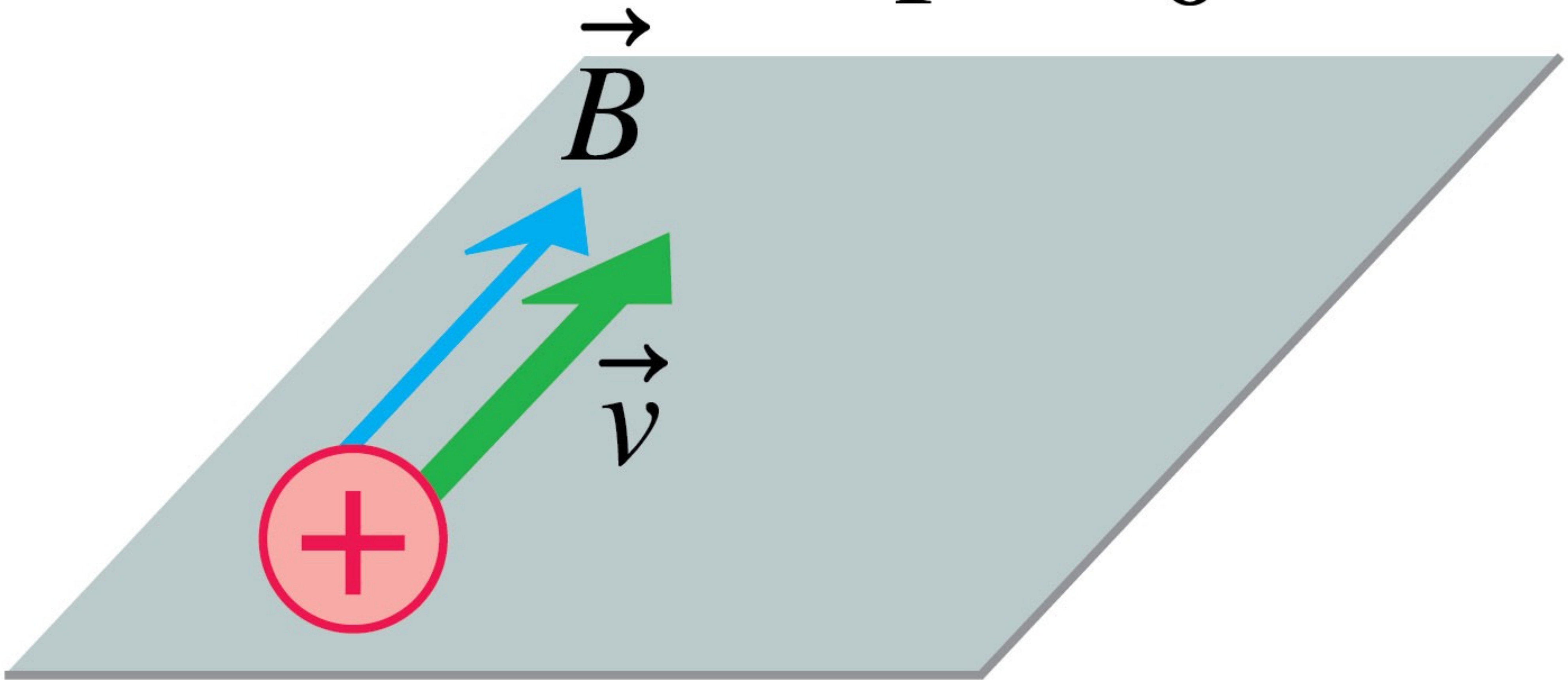


- There is no magnetic force on a charged particle at rest.

Magnetic Force on a Charged Particle

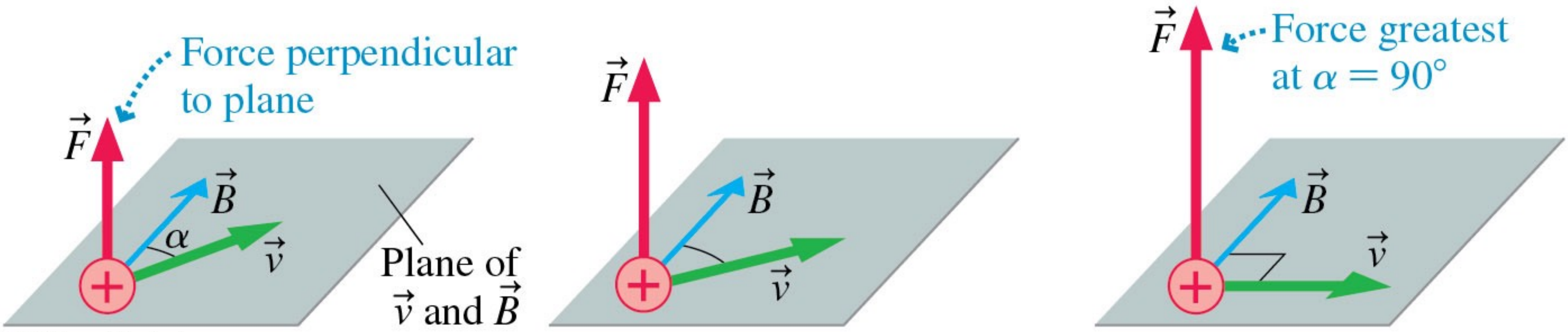
- Existing magnetic field B

$$\vec{F} = \vec{0}$$



- There is no magnetic force on a charged particle moving *parallel* to a magnetic field.

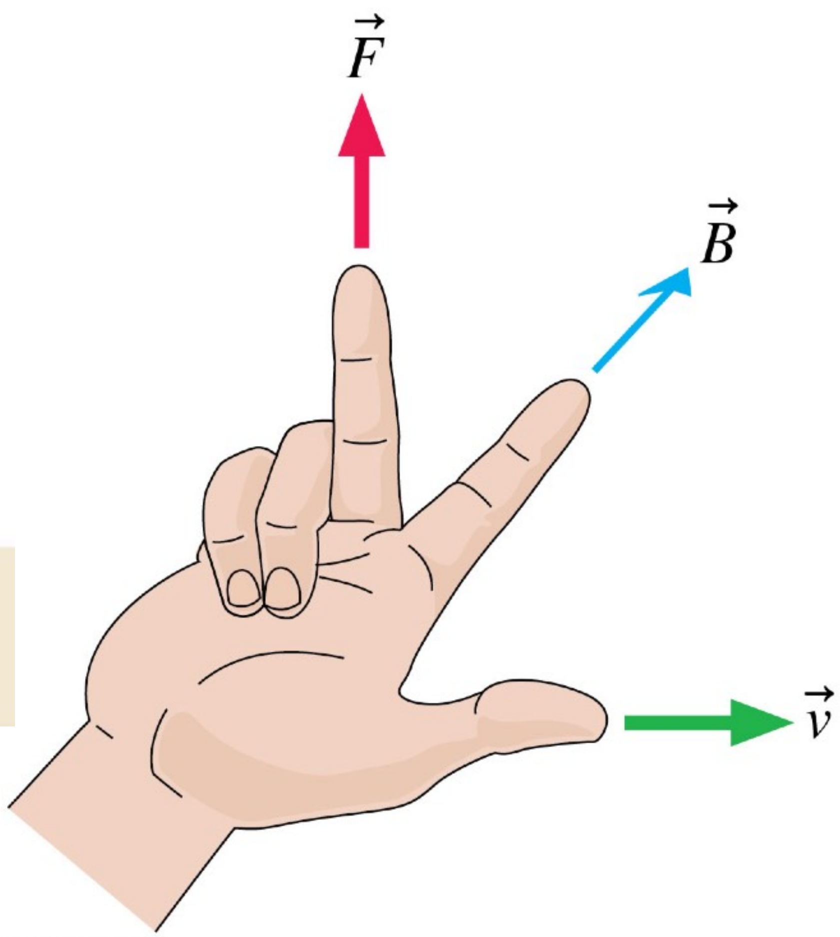
The Magnetic Force on a Moving Charge



- As the angle α between velocity and magnetic field increases, the magnetic force also increases.
- The force is greatest when the angle is 90º.
- The magnetic force is always perpendicular to the plane containing $\boldsymbol{v}$ and $\boldsymbol{B}$.

$$\vec{F}_{\text{on } q} = q\vec{v} \times \vec{B}$$

$$= (qvB \sin \alpha, \text{ direction of right-hand rule})$$

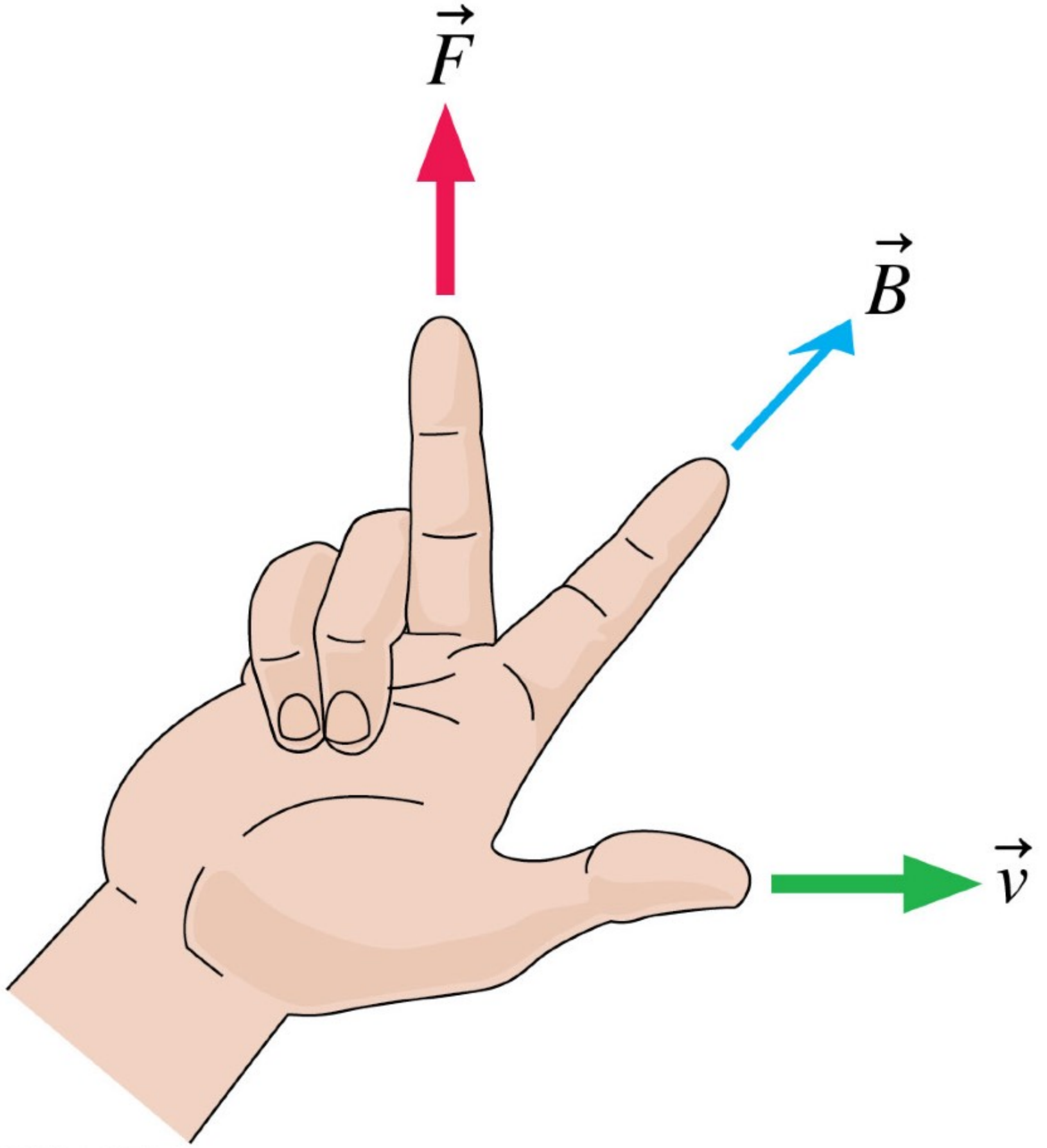


The Magnetic Force on a Moving Charge

- The magnetic force on a charge q as it moves through a magnetic field $\vec{B}$ with velocity $\vec{v}$ is

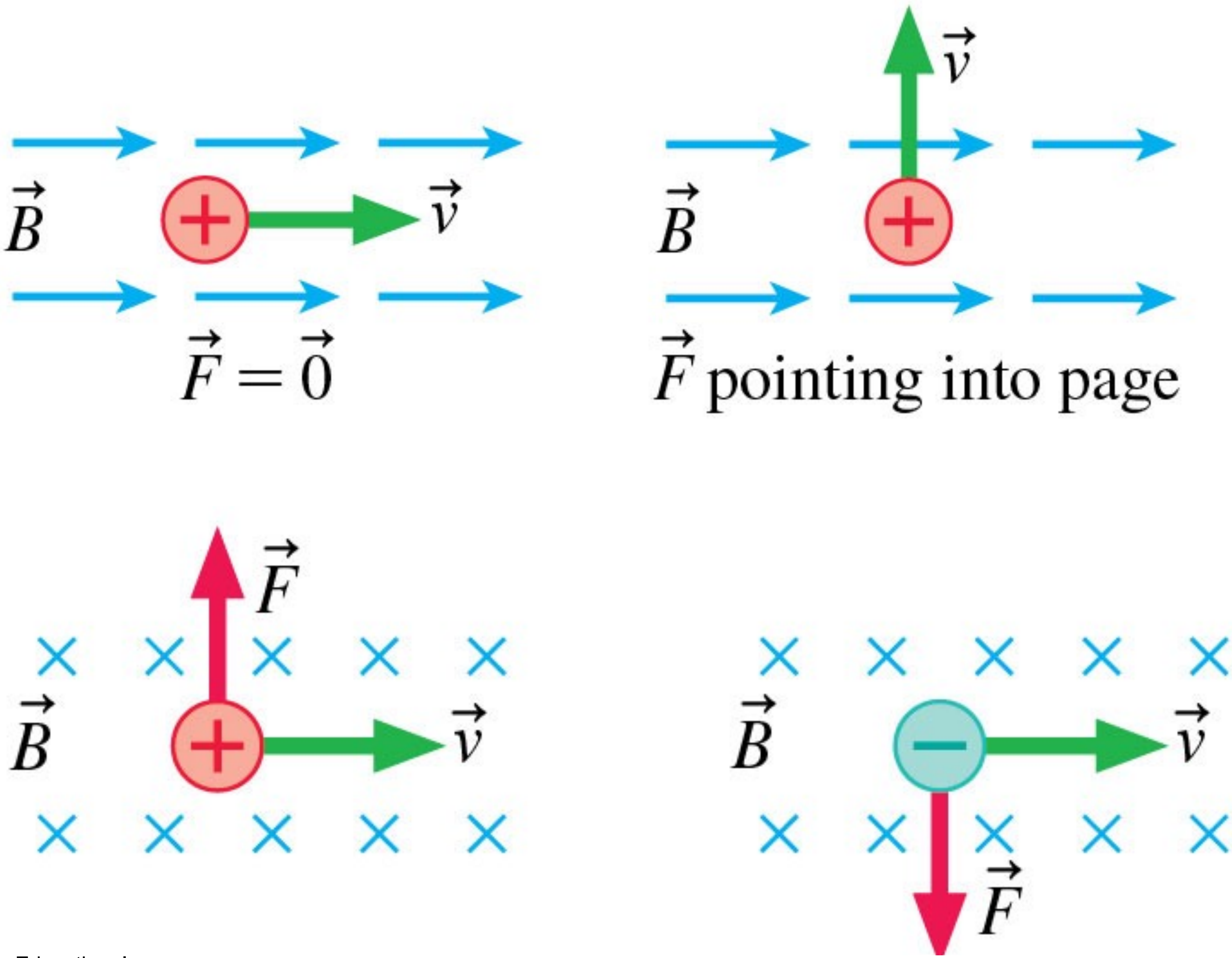
$$\vec{F}_{\text{on } q} = q\vec{v} \times \vec{B} = (qvB\sin\alpha, \text{ direction of right-hand rule})$$

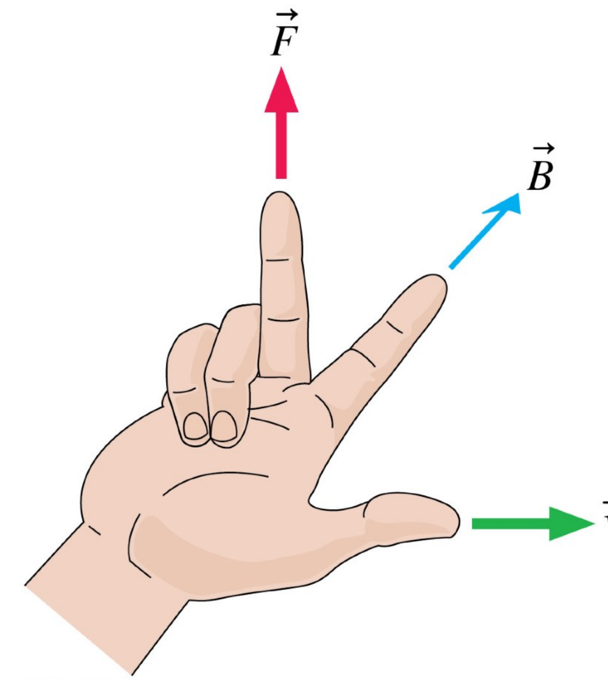
where α is the angle between $\vec{v}$ and $\vec{B}$.



The Magnetic Force on a Moving Charge

$$\vec{F}_{\text{on } q} = q\vec{v} \times \vec{B}$$

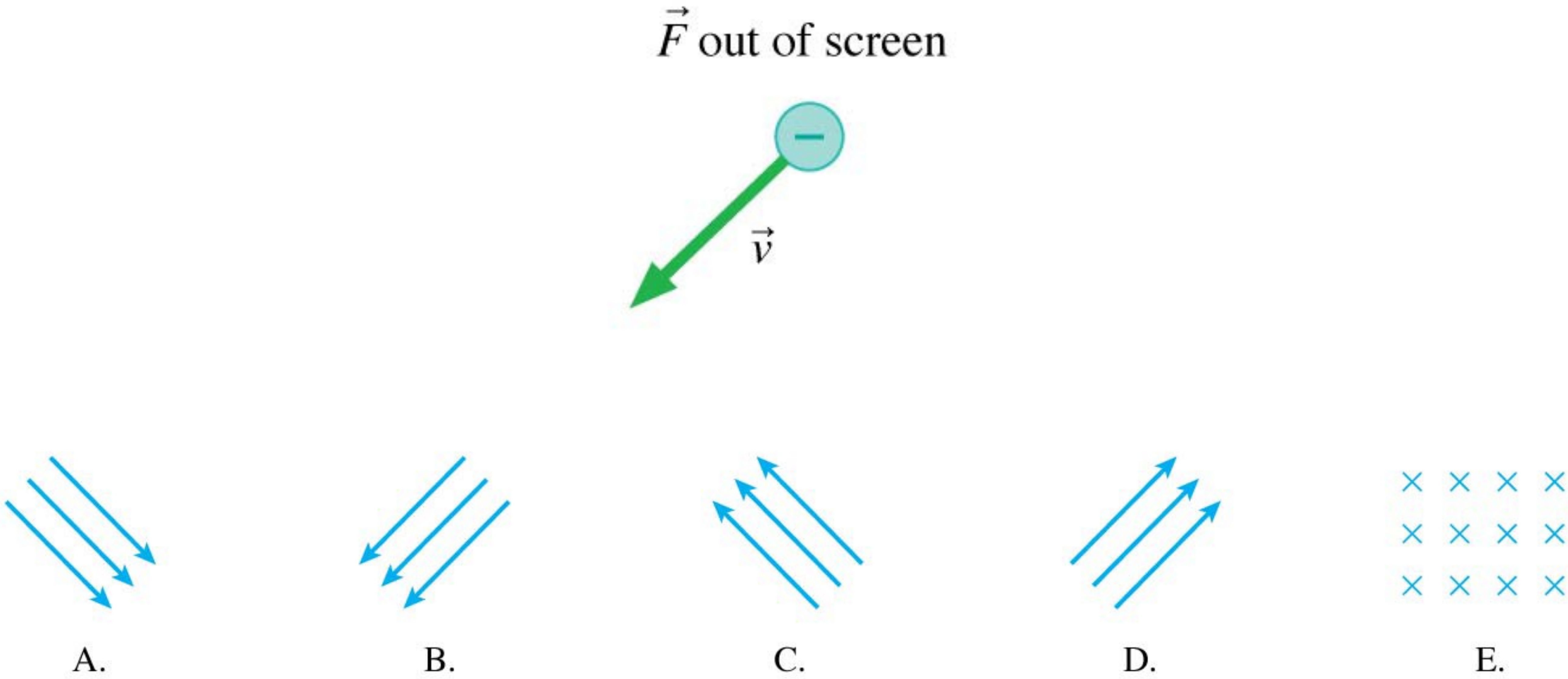




iClicker question #15-1

$$\vec{F}_{\text{on } q} = q\vec{v} \times \vec{B}$$

Which **magnetic field B** causes the observed force?

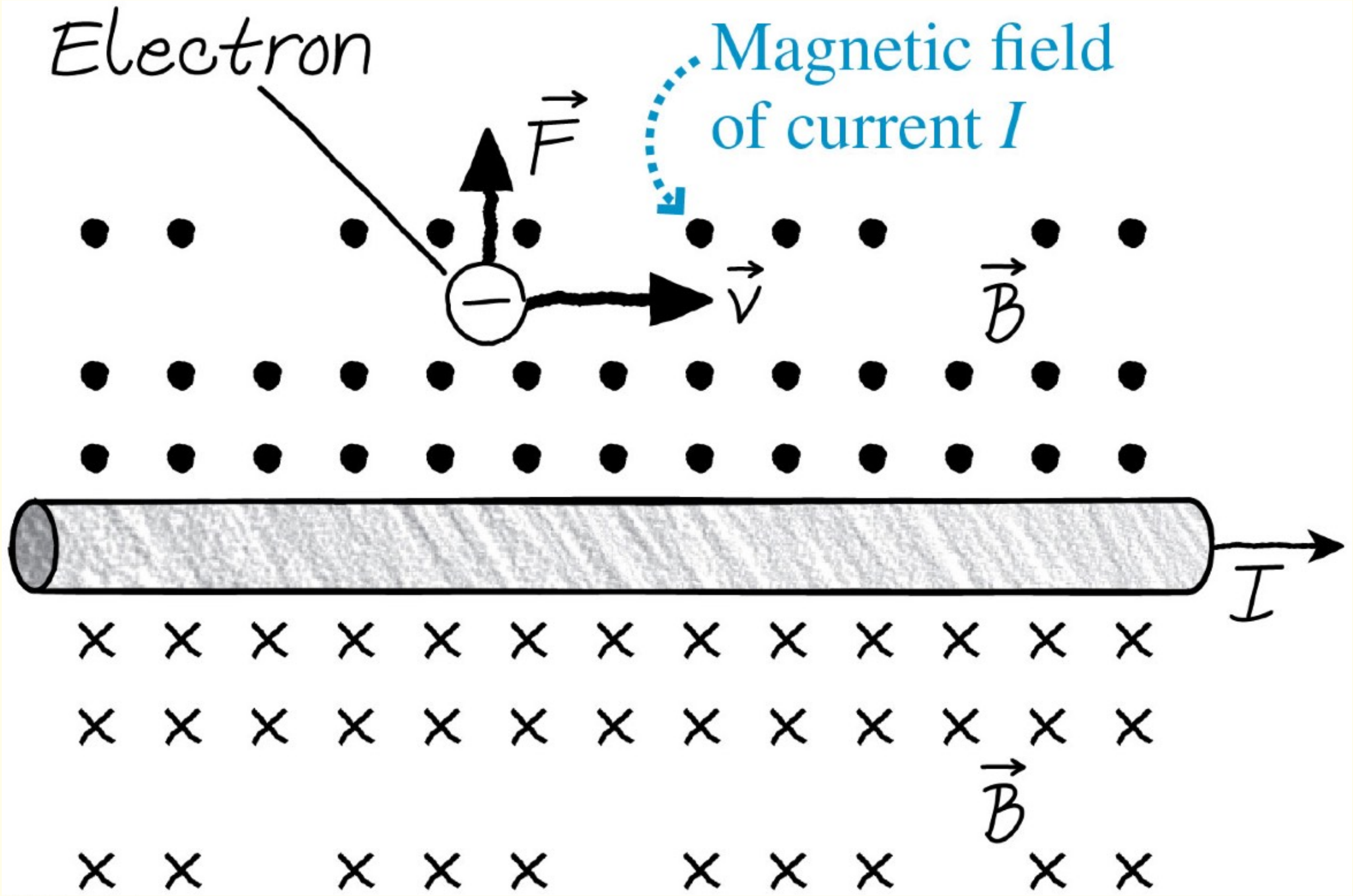


Example: The Magnetic Force on an Electron

A long wire carries a **10 A** current from left to right.
An electron **1.0 cm** above the wire is traveling to the right at a speed of **1.0 x 10^7 m/s**.

What are the magnitude and direction of the magnetic force on the electron?

Model:



Example: The Magnetic Force on an Electron

$$\vec{F}_{\text{on } q} = q\vec{v} \times \vec{B}$$

but for a straight conductor: $B = \dfrac{\mu_0 I}{2\pi r} = 2.0 \times 10^{-4}\ \mathrm{T}$

$$F = evB = (1.60 \times 10^{-19}\ \mathrm{C})(1.0 \times 10^{7}\ \mathrm{m/s})(2.0 \times 10^{-4}\ \mathrm{T})$$
$$= 3.2 \times 10^{-16}\ \mathrm{N}$$

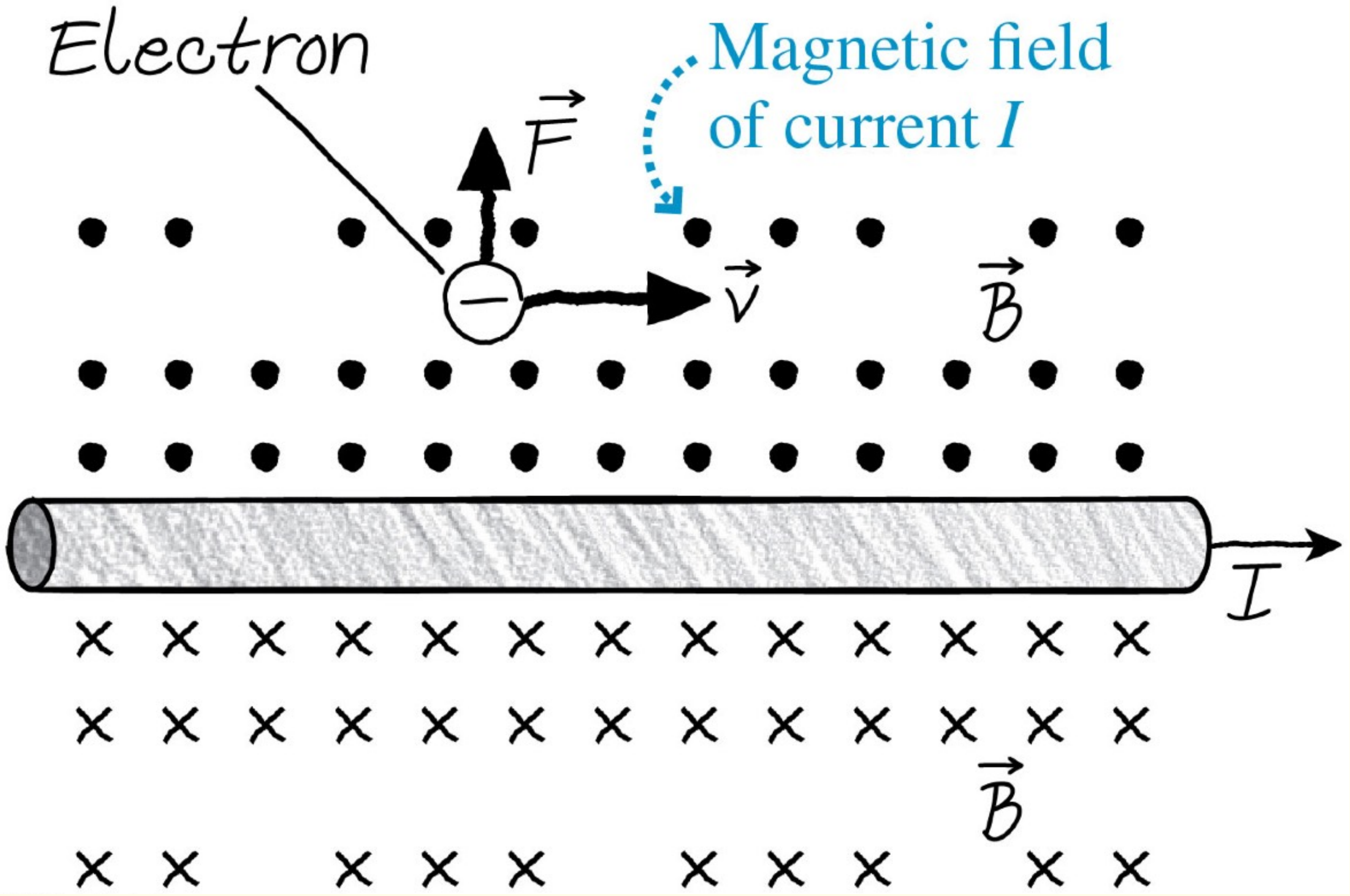
r = **1.0 cm**

I = **10 A**

v = **1.0 x 10^7 m/s**

The force on the electron is $\vec{F} = (3.2 \times 10^{-16}\ \mathrm{N},\ \mathrm{up})$.

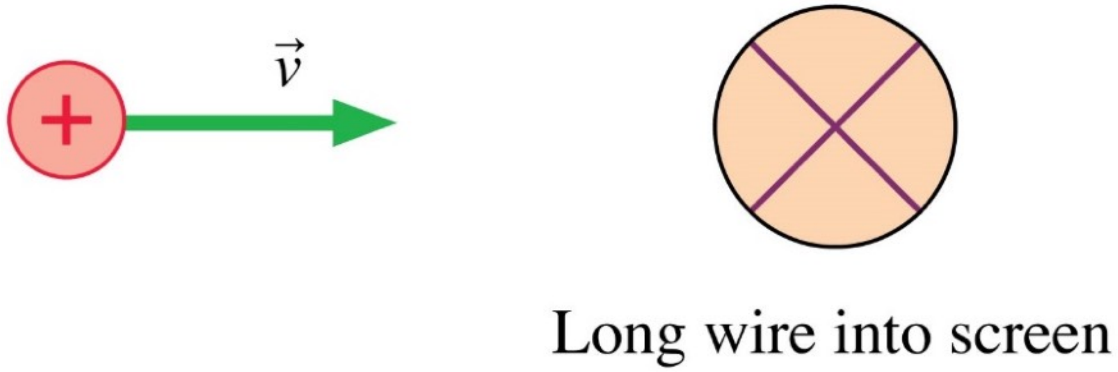
This force will cause the electron to curve away from the wire!



iClicker question #15-2

$\vec{F}_{\text{on } q} = q\vec{v} \times \vec{B}$

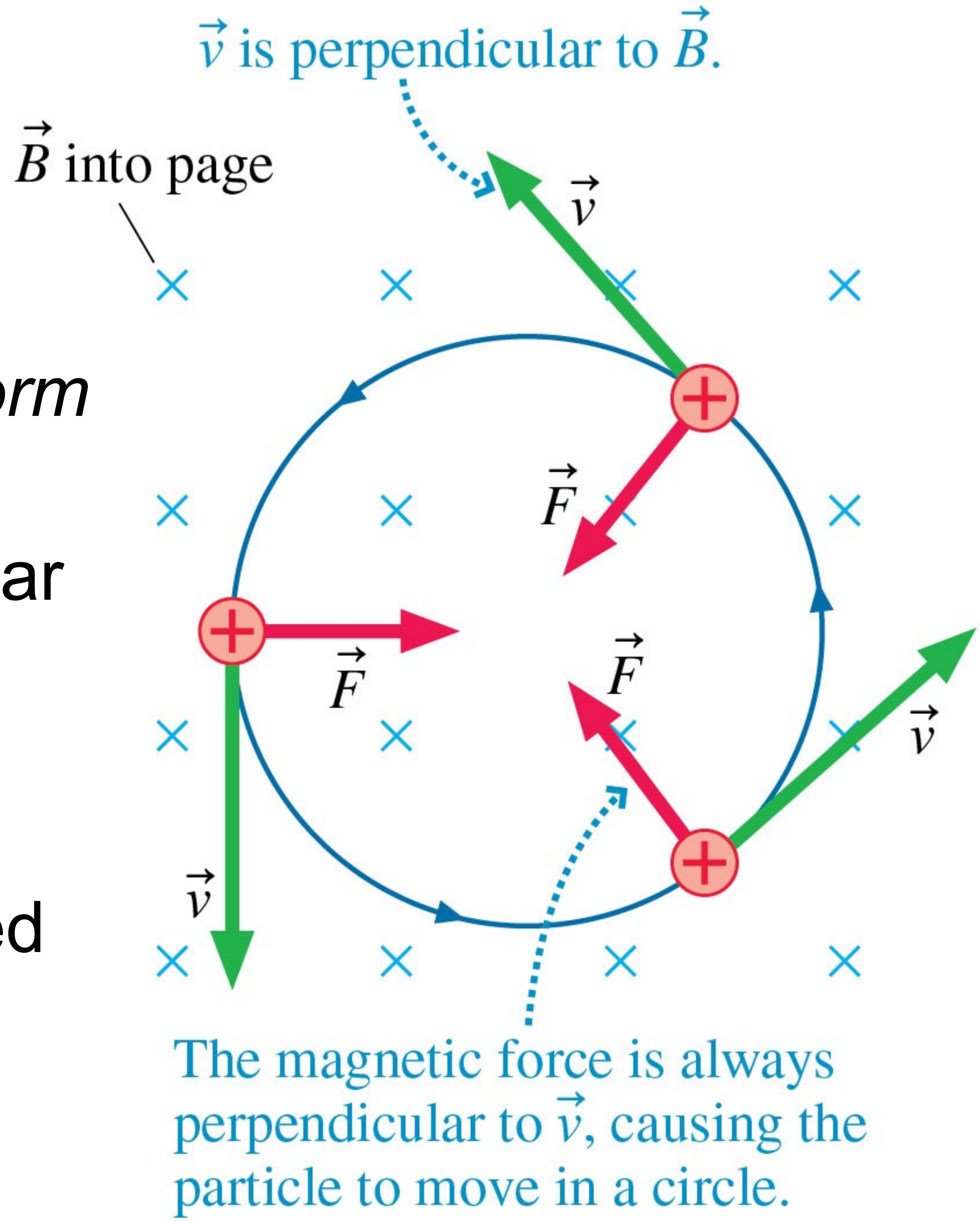
A proton is shot straight at the center of a long, straight wire carrying current into the screen.
The proton will



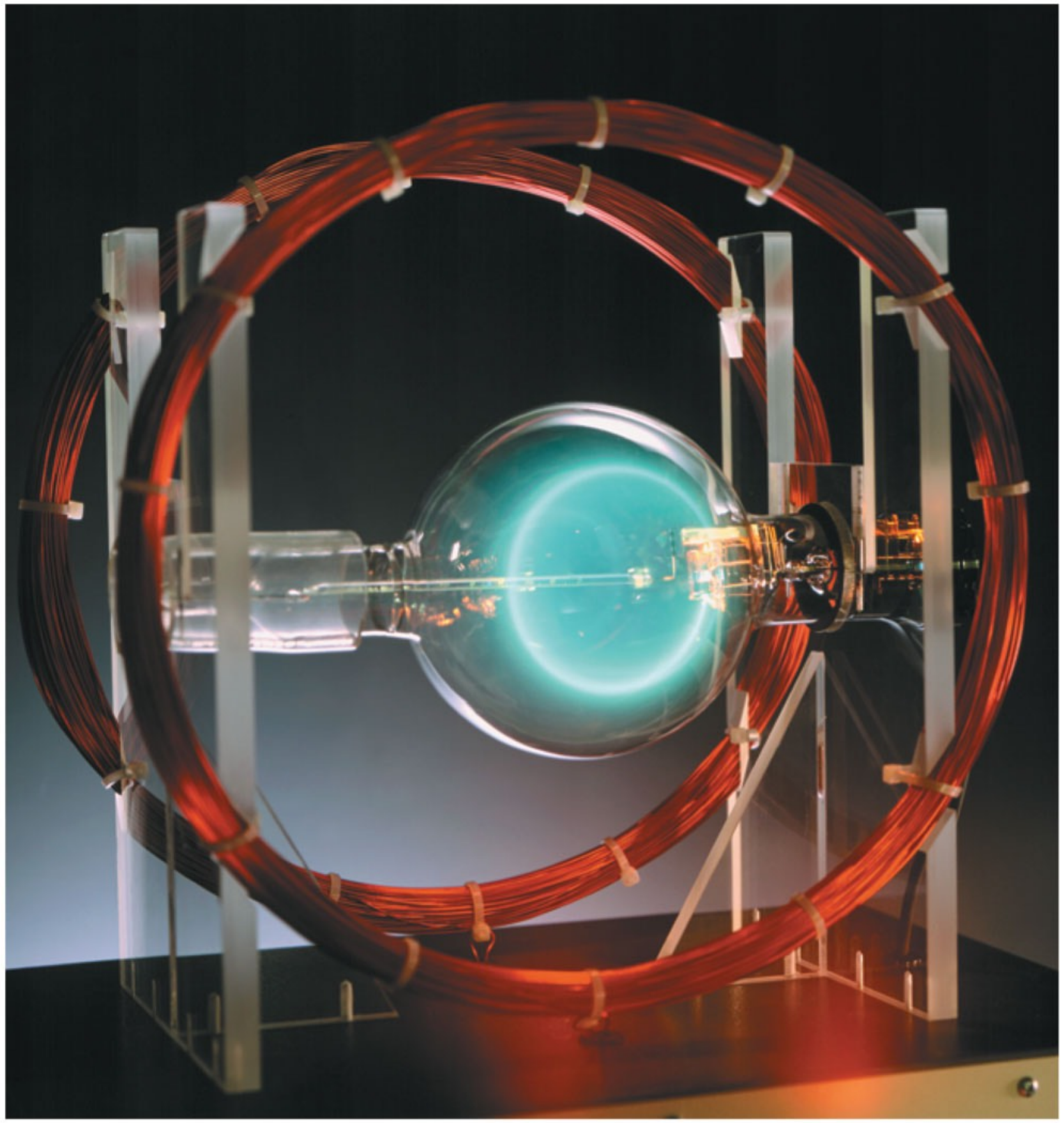
A. Go straight into the wire.
B. Hit the wire in front of the screen.
C. Hit the wire behind the screen.
D. Be deflected over the wire.
E. Be deflected under the wire.

Cyclotron Motion

- The figure shows a positive charge moving in a plane that is perpendicular to a *uniform* magnetic field.
- Since $\vec{F}$ is always perpendicular to $\vec{v}$, the charge undergoes **uniform circular motion**.
- This motion is called the **cyclotron motion** of a charged particle in a magnetic field.



Cyclotron Motion



- Electrons undergoing circular **cyclotron motion** in a magnetic field. Above, you can see the electrons' path because they collide with a low density gas that then emits light.

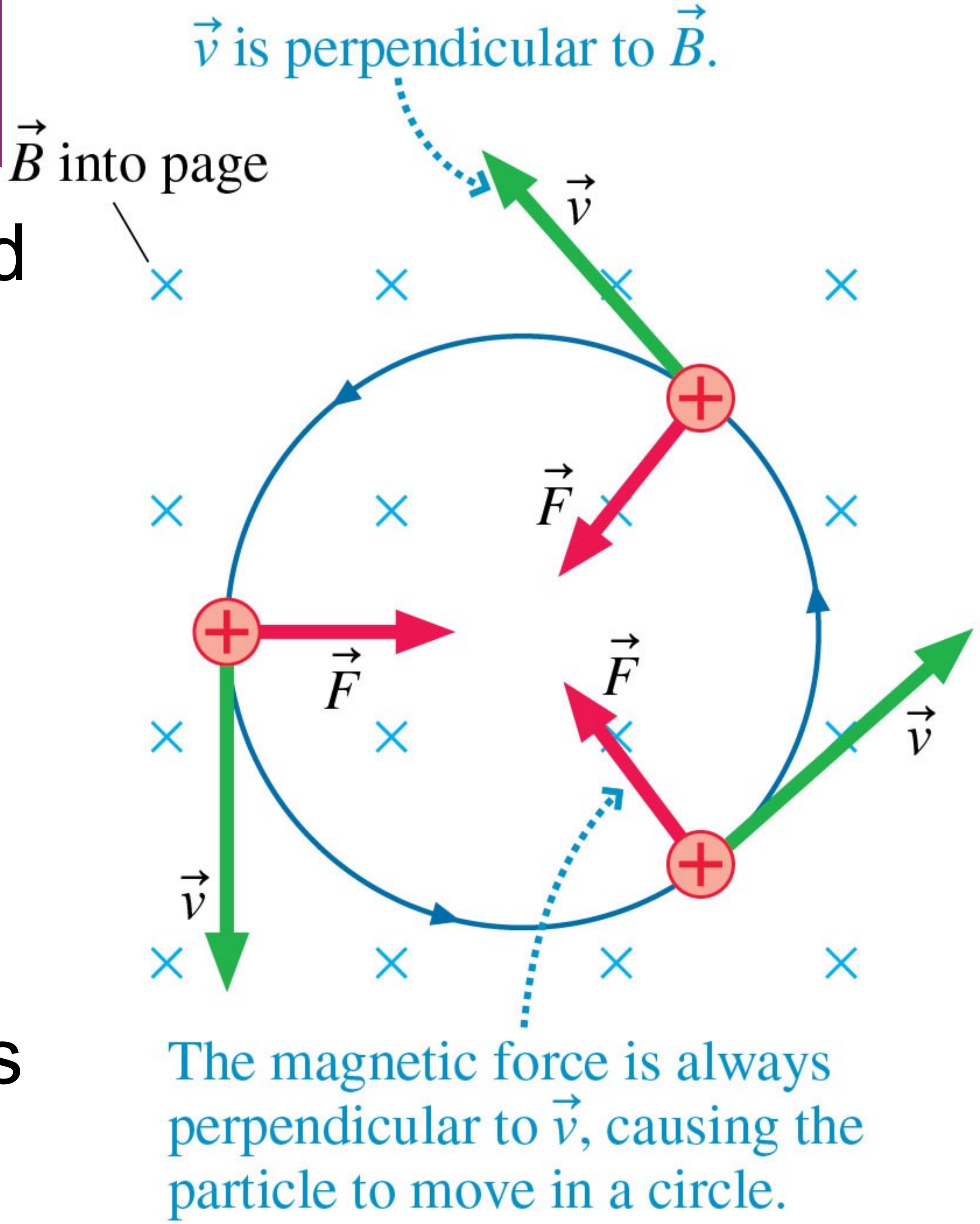
Cyclotron Motion

- Consider a particle of mass m and charge q moving at speed v in a plane perpendicular to a uniform magnetic field of strength B.
- Newton's second law for circular motion (PHYS 2A) is

$$F = qvB = ma_r = \frac{mv^2}{r}$$

- The radius of the cyclotron orbit is

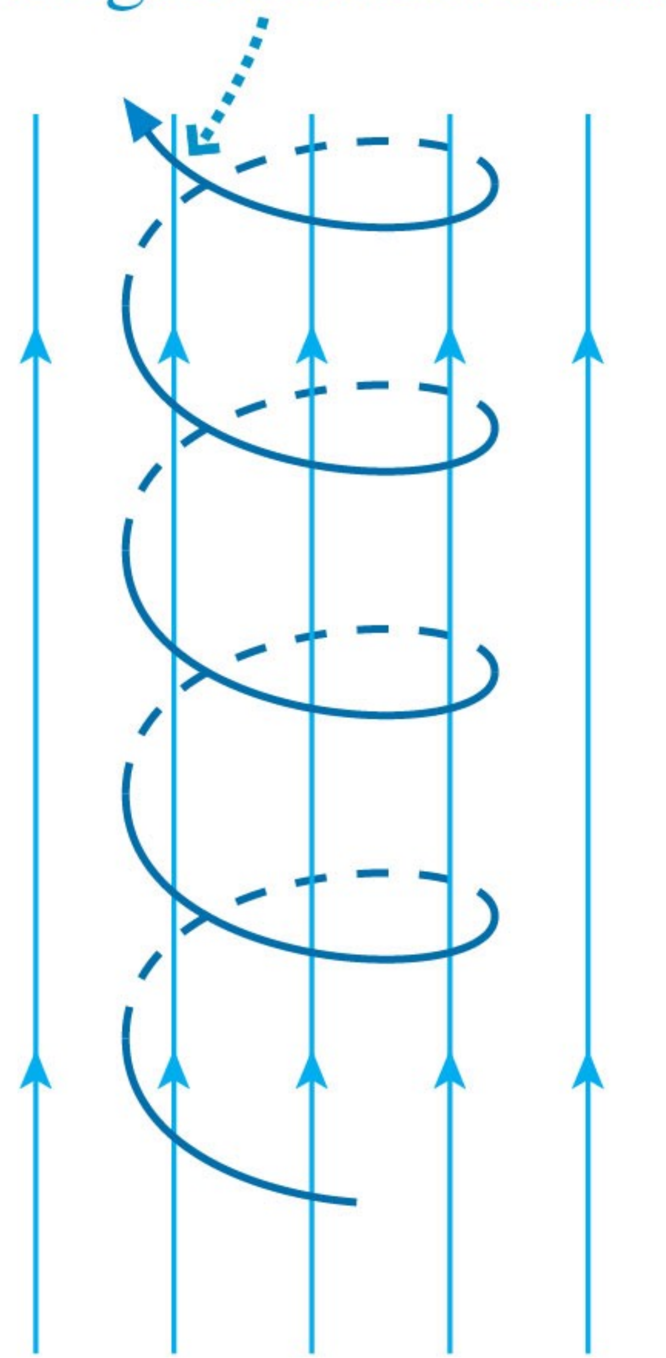
$$r_{\text{cyc}} = \frac{mv}{qB}$$



- Recall that the frequency of revolution of circular motion is $f = v/2\pi r$, so the cyclotron frequency is

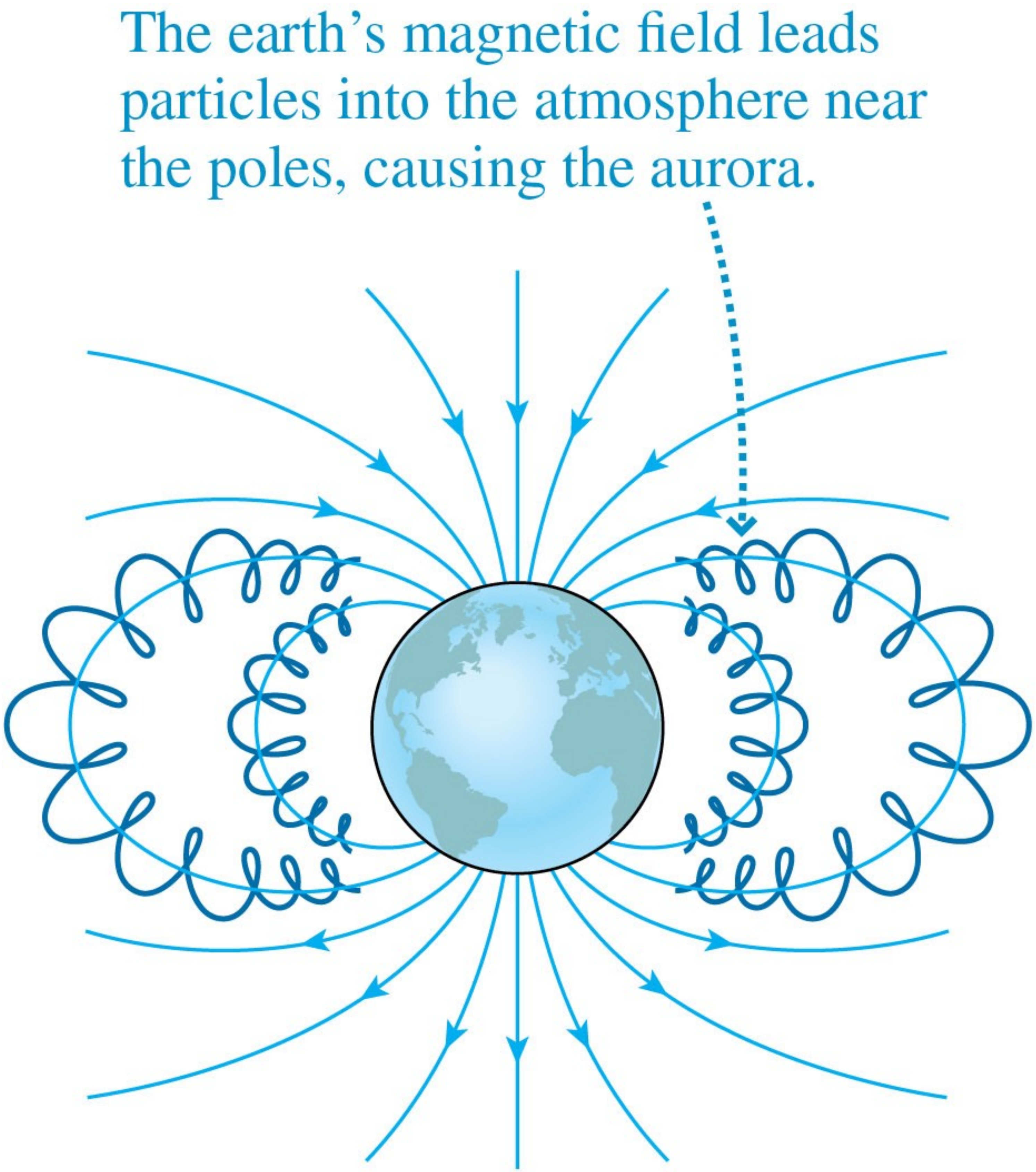
$$\boxed{f_{\text{cyc}} = \frac{qB}{2\pi m}}$$

Cyclotron Motion



- The figure shows a more general situation in which the charged particle's velocity is not exactly perpendicular to $\vec{B}$.
- The component of $\vec{v}$ parallel to $\vec{B}$ is not affected by the field, so the charged particle spirals around the magnetic field lines in a helical trajectory.
- The radius of the helix is determined by $v_{\perp}$, the component of $\vec{v}$ perpendicular to $\vec{B}$.

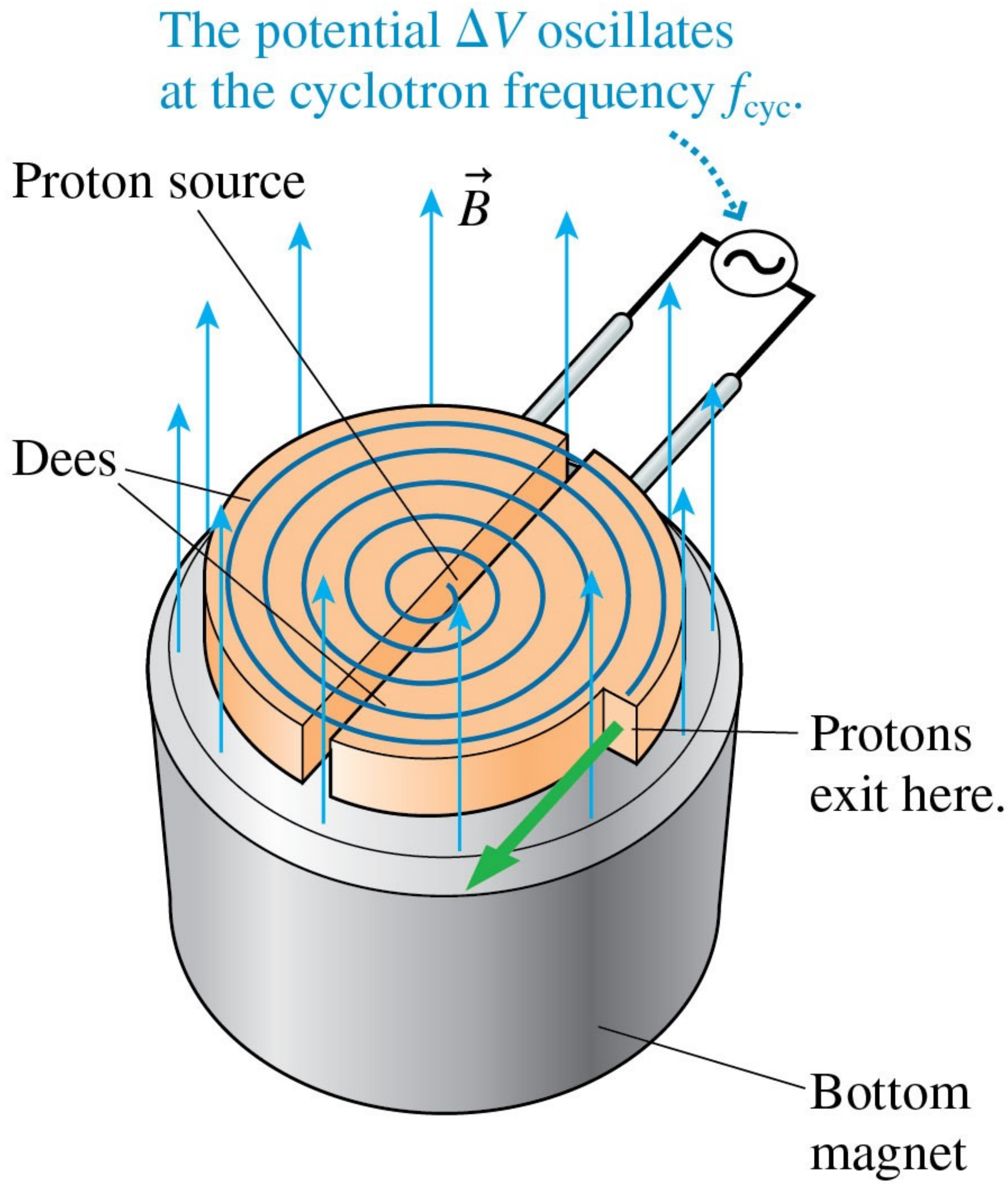
In Nature: Aurora



The aurora

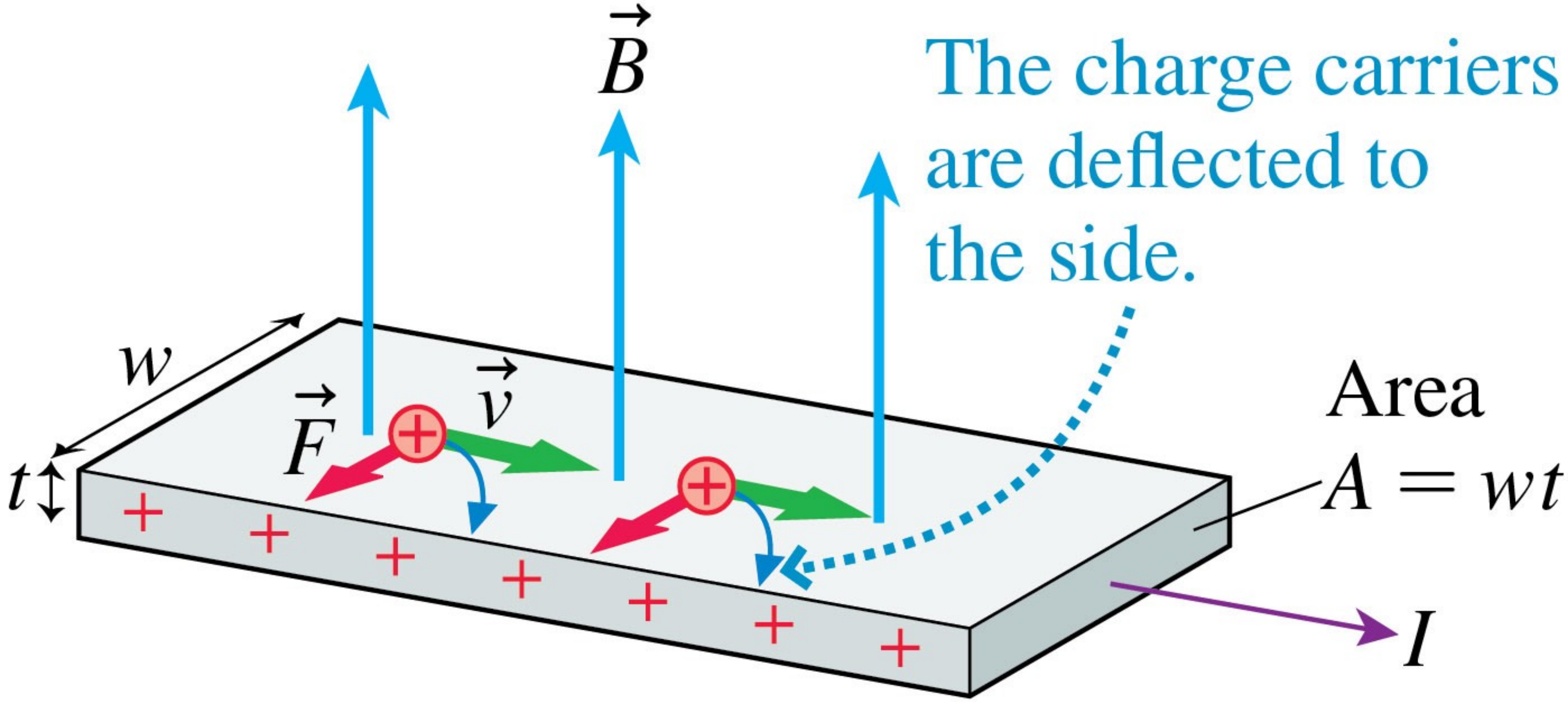
Application: The Cyclotron

- The first practical particle accelerator, invented in the 1930s, was the **cyclotron**.
- Cyclotrons remain important for many applications of nuclear physics, such as the creation of radioisotopes for medicine.



The Hall Effect

(See whiteboard on table)

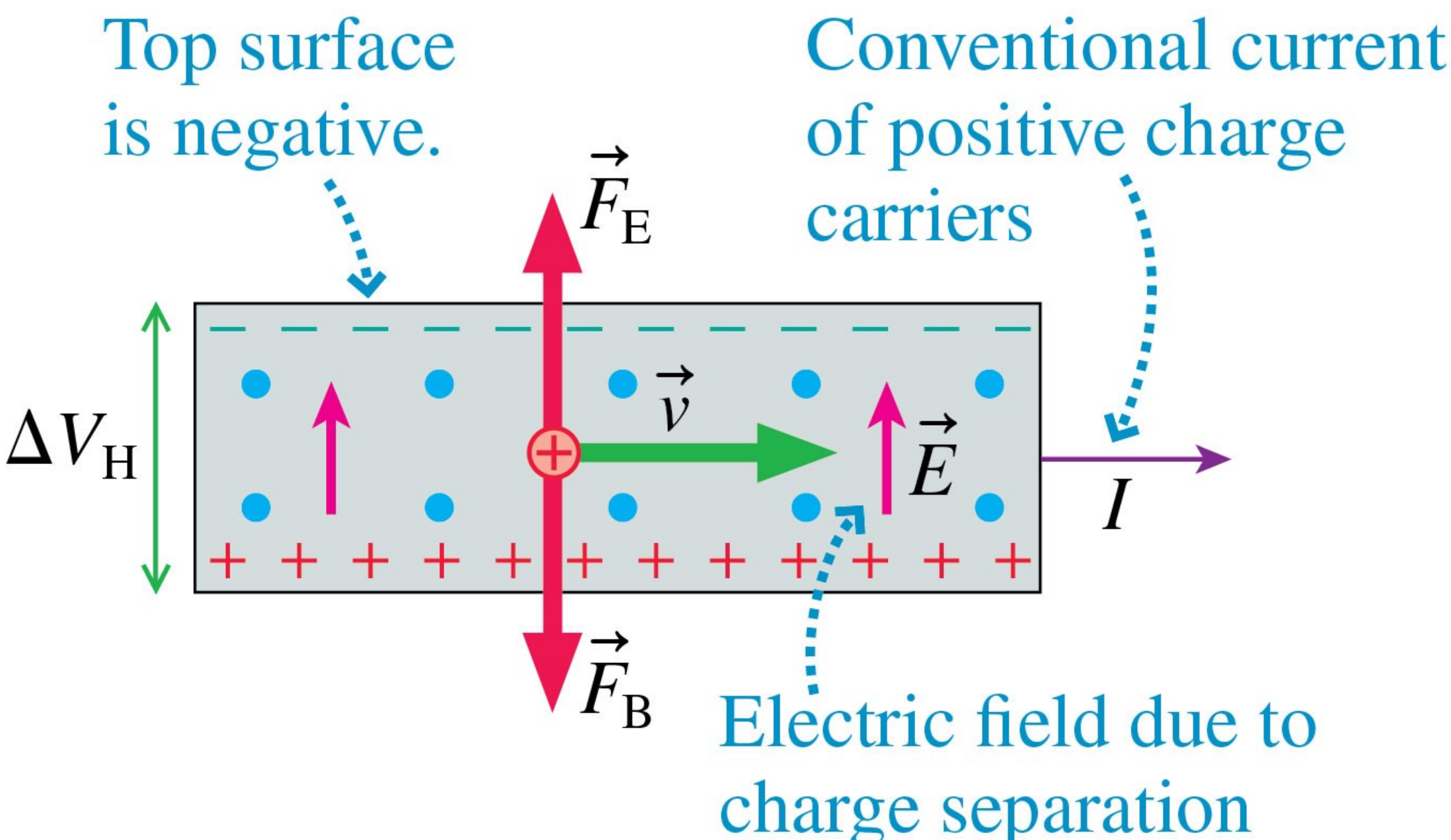


Magnetic field perpendicular to a flat, current-carrying conductor.

The Hall Effect

- If the charge carriers are ***positive***,
  the magnetic force pushes these positive charges *down*,
  creating an excess positive charge on the bottom surface,
  and leaving negative charge on the top.

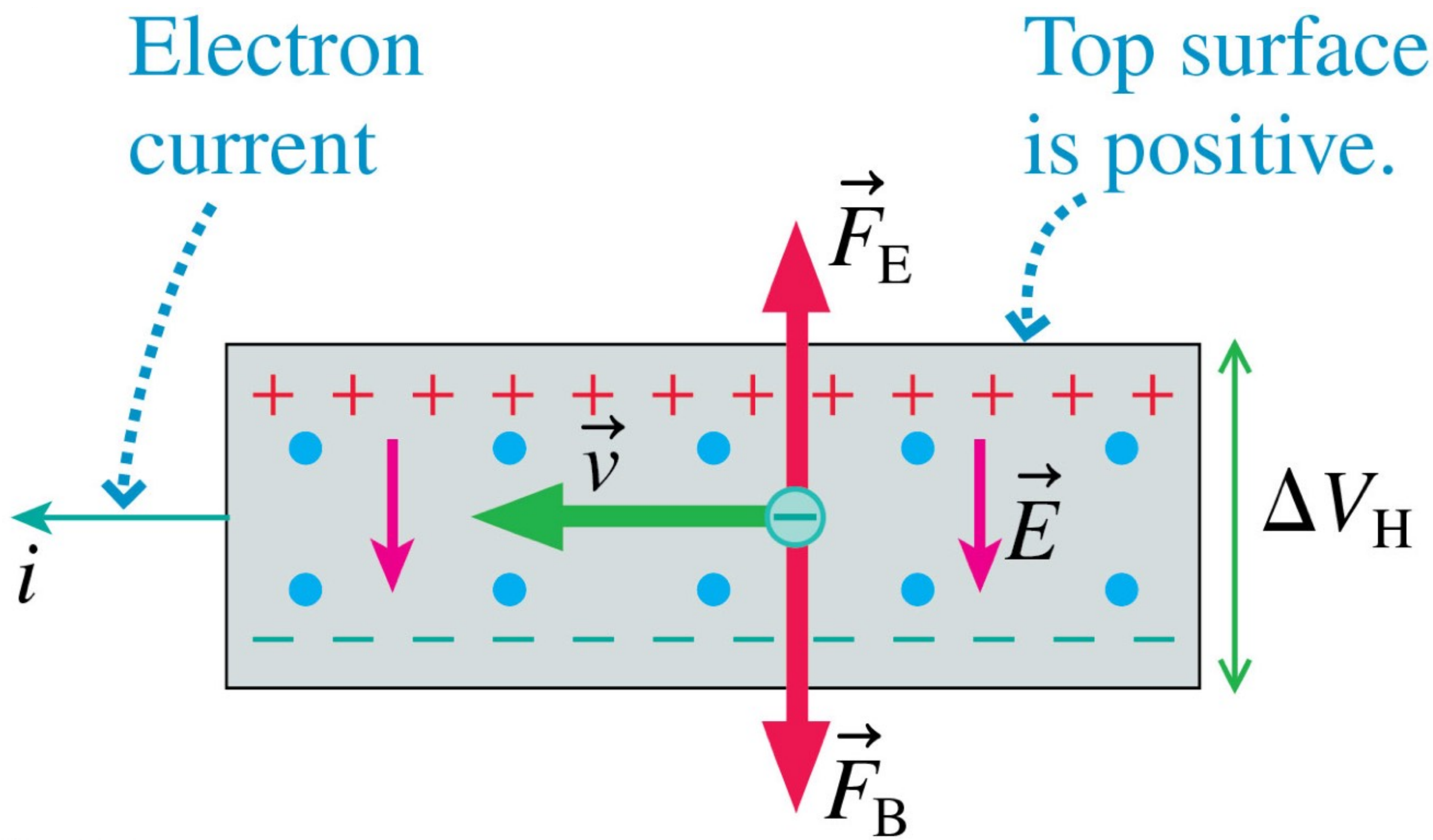
- This creates a measurable Hall voltage ΔV_{H}
  higher on the *bottom* surface.



The Hall Effect

- If the charge carriers are ***negative***, the magnetic force pushes these negative charges *down*, creating an excess negative charge on the bottom surface, and leaving positive charge on the top.

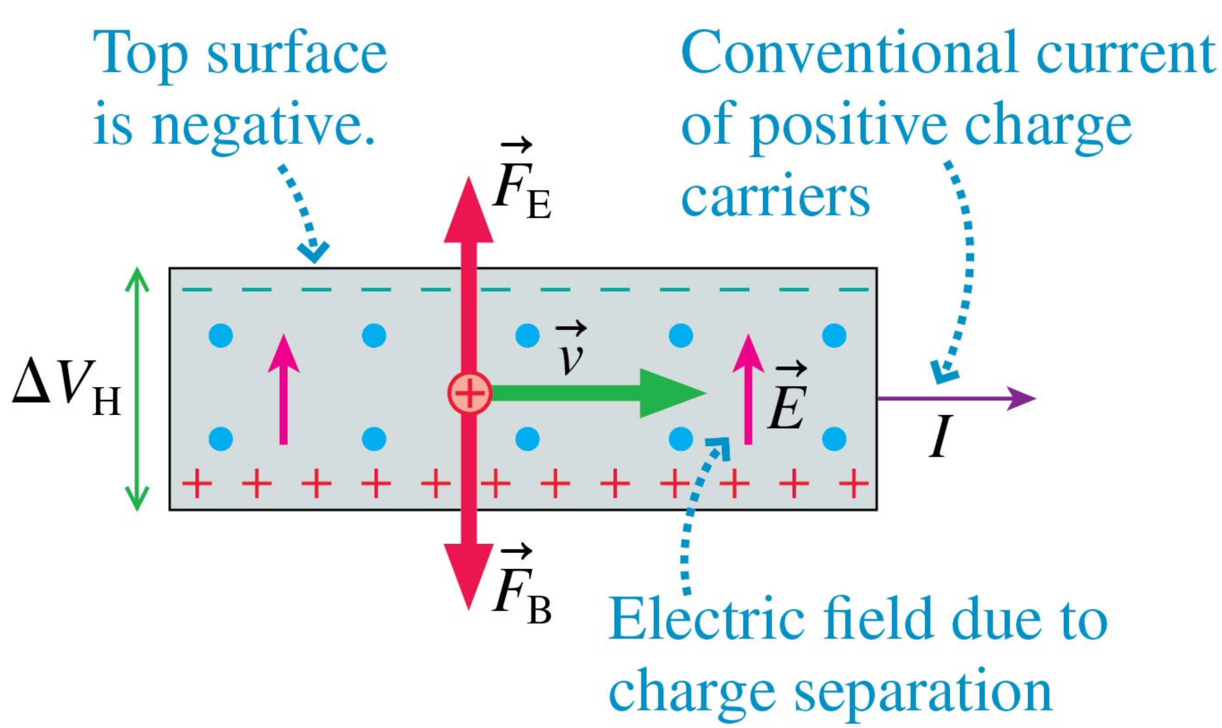
- This creates a measurable Hall voltage ΔV_{H} higher on the *top* surface.



The Hall Effect: electrons are the charge carriers

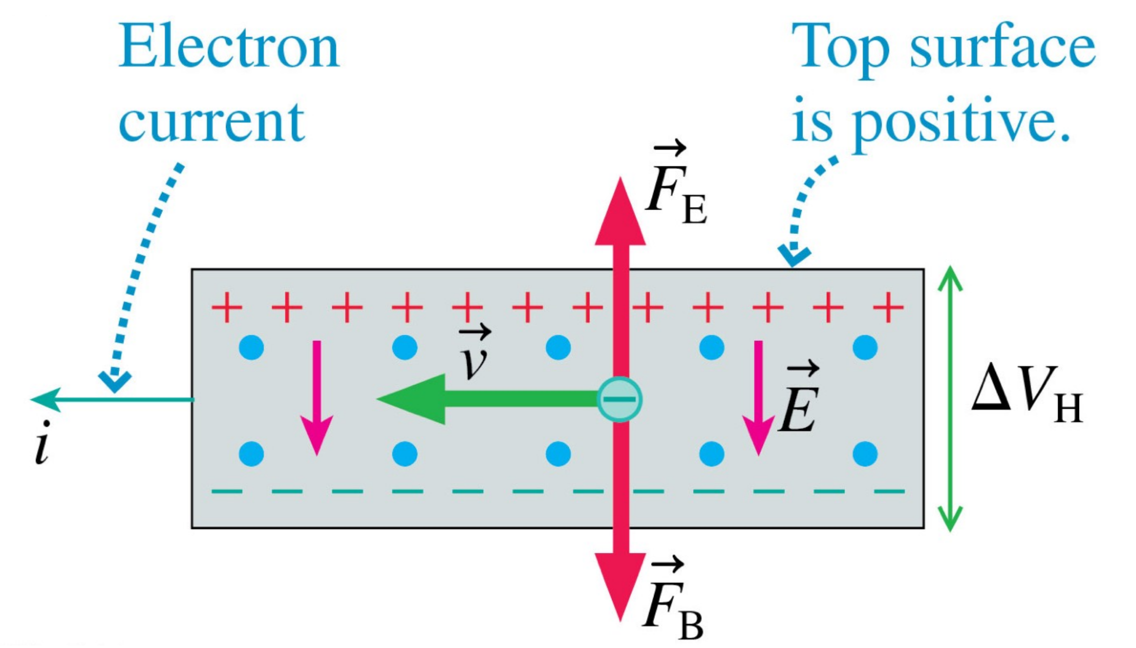
if the charge carriers are ***positive,***

the Hall voltage is higher at the **bottom**



if the charge carriers are ***negative,***

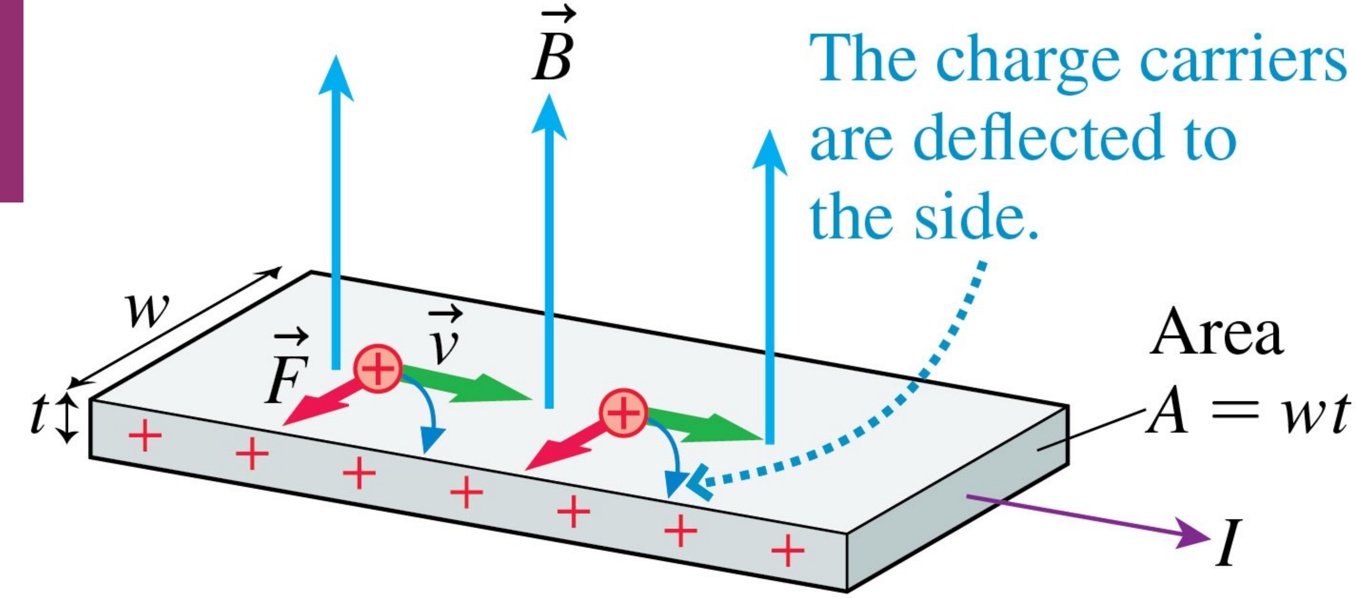
the Hall voltage is higher at the **top**



I, the current in both cases goes to the right

The Hall Effect

$$\vec{F}_{\text{on } q} = q\vec{v} \times \vec{B}$$



1. Charge carriers moving at drift speed v_d experience a magnetic force $\boldsymbol{F}_B = e\, v_d\, B$ perpendicular to both ***B*** and the current I.

2. Charge separation leads to an electric field $E = \Delta V_H / w$ inside conductor.

3. The steady-state condition is when the electric force balances the magnetic force: $\boldsymbol{F}_B = F_E$

$$e\, v_d\, B = eE = e\, \Delta V_H / w$$

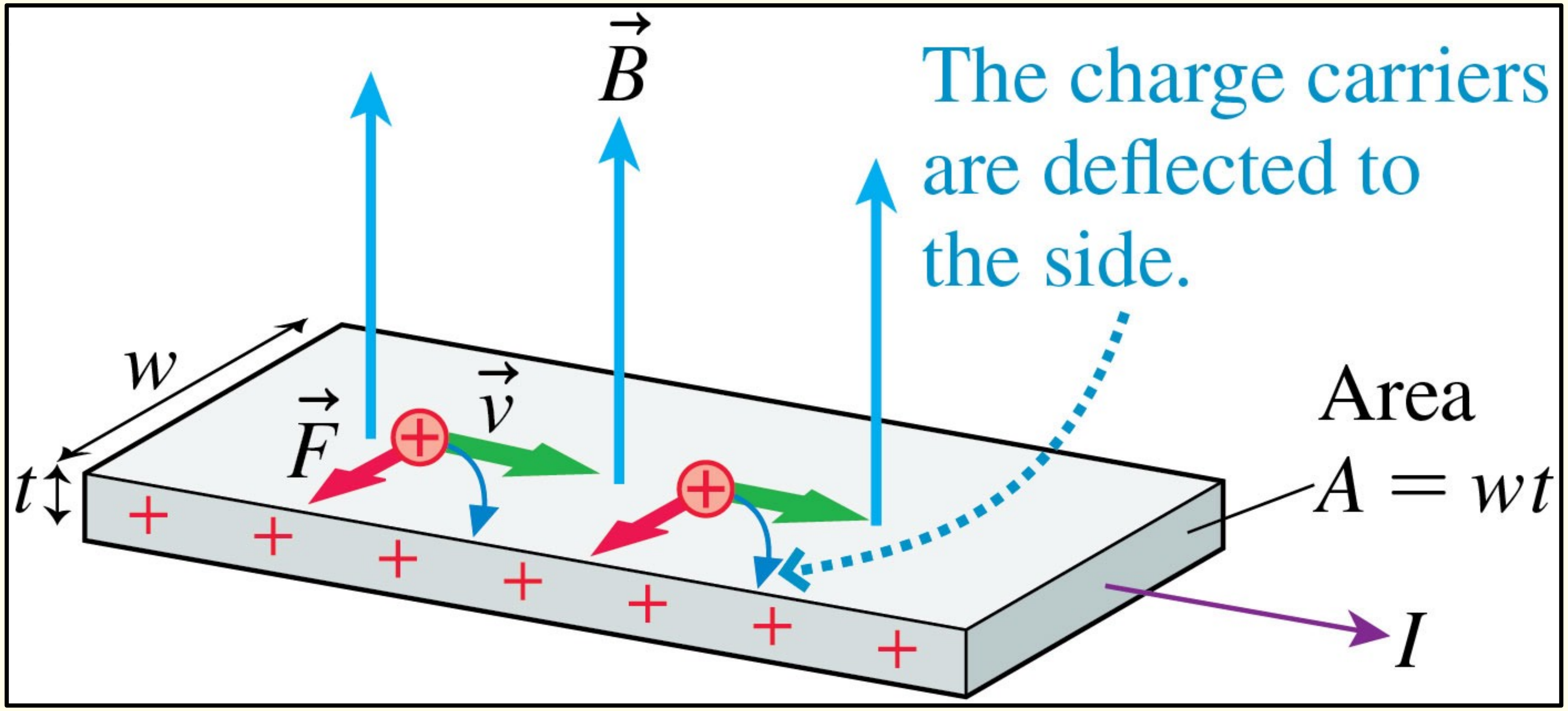
where v_d is the drift speed, which is $v_d = I/(wtne)$.

- From this we can find the Hall voltage: $\Delta V_H = \dfrac{IB}{tne}$

where n is the charge-carrier density (charge carriers per m^3).

Application: The Hall Probe, magnetic field sensor

Hall voltage is proportional to the magnetic field $$\Delta V_H = \frac{IB}{tne}$$

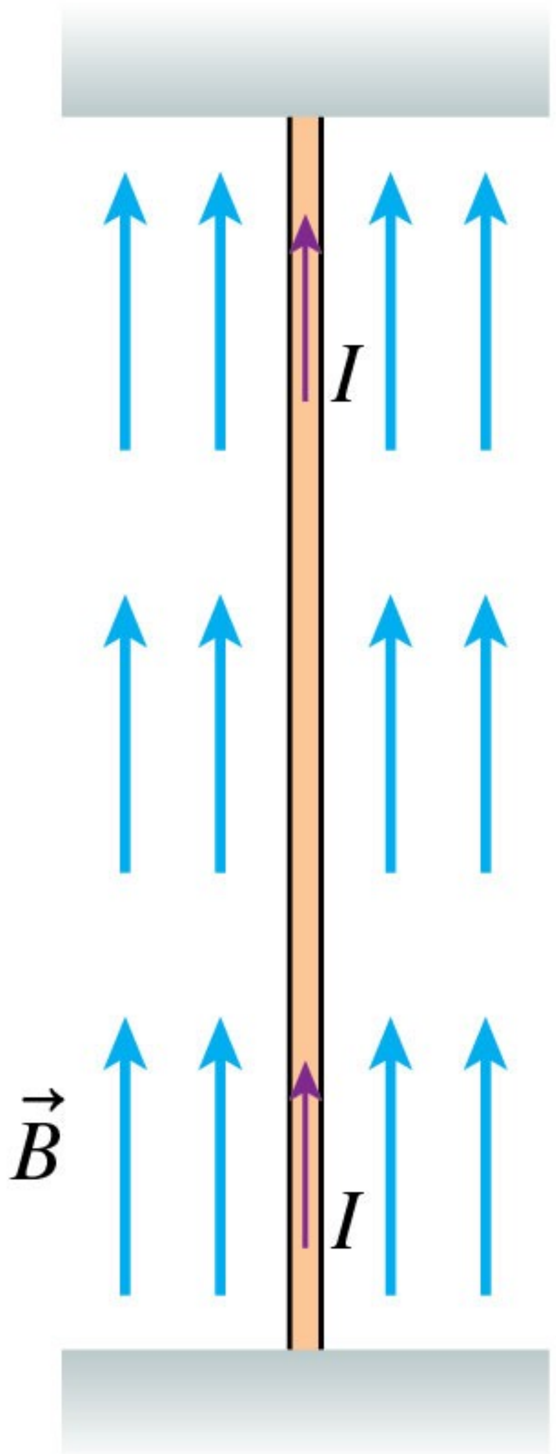


Example: A Hall probe consists of a strip of the metal bismuth that is 0.15 mm thick and 5.0 mm wide. Bismuth is a poor conductor with charge-carrier density $1.35 \times 10^{25}\ \mathrm{m}^{-3}$. The Hall voltage on the probe is 2.5 mV when the current through it is 1.5 A. What is the strength of the magnetic field, and what is the electric field strength inside the bismuth?

$$\boxed{\Delta V_H = \frac{IB}{tne}}$$

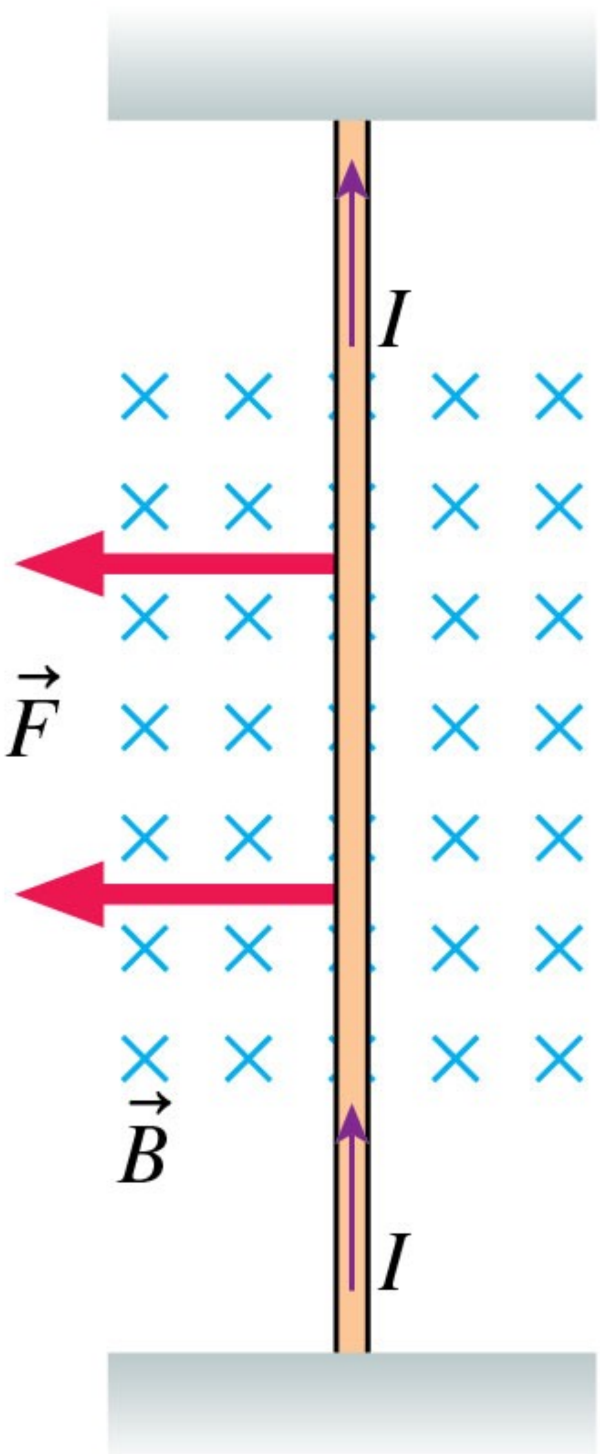
$$E = \Delta V_H / w$$

Magnetic Forces on Current-Carrying Wires



- There's *no* force on a current-carrying wire parallel to a magnetic field.

Magnetic Forces on Current-Carrying Wires



- A current perpendicular to the field experiences a force in the direction of the right-hand rule.
- If a wire of length l contains a current $I = q/\Delta t$, it means a charge q must move along its length in a time $\Delta t = l/v$.
- Thus we have $Il = qv$.
- Since $\vec{F} = q\vec{v} \times \vec{B}$, the magnetic force on a current-carrying wire is

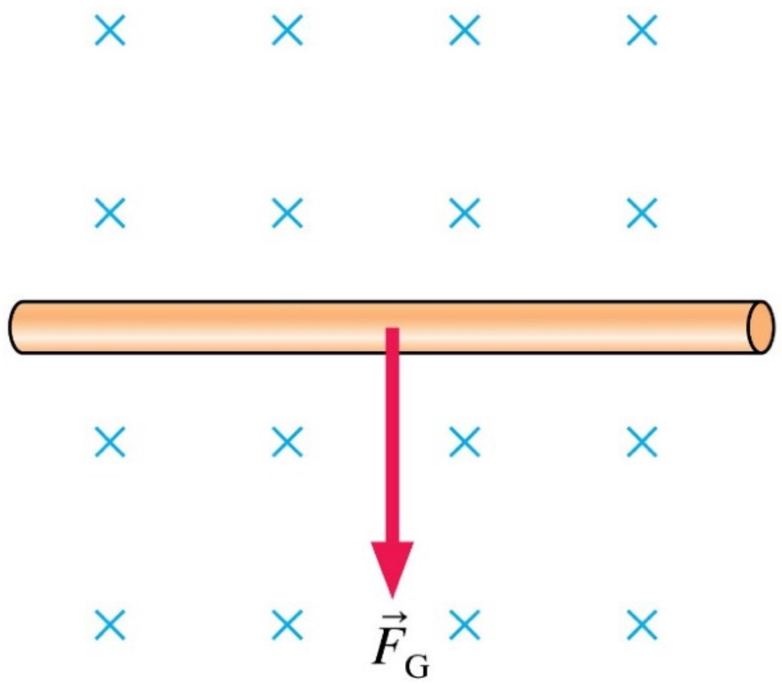
$$\vec{F}_{\text{wire}} = I\vec{l} \times \vec{B} = (IlB \sin\alpha, \text{ direction of right-hand rule})$$

iClicker question #15-3

$$\vec{F}_{\text{wire}} = I\vec{l} \times \vec{B}$$

The horizontal wire can be levitated—held up against the force of gravity—if the current in the wire is

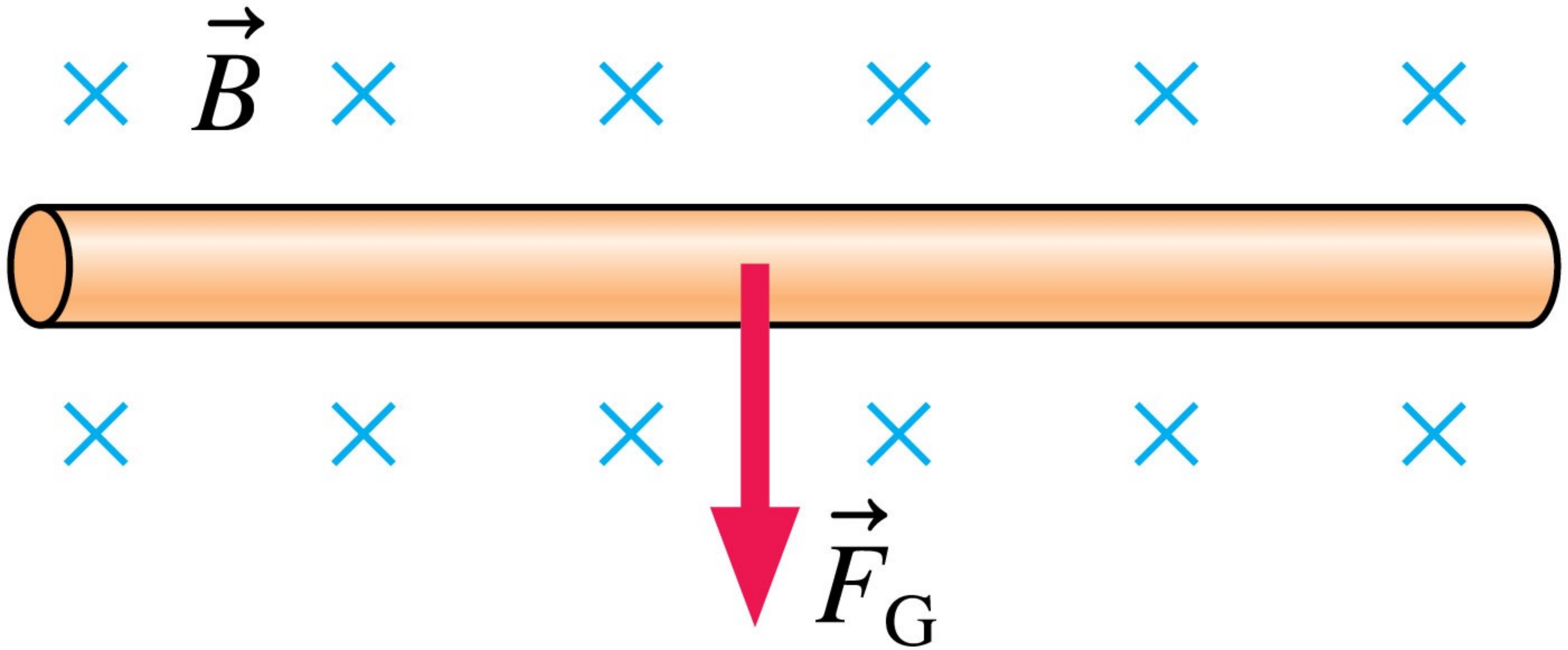
A. Right to left.
B. Left to right.
C. It can't be done with this magnetic field.



Example: Magnetic Levitation

$$\vec{F}_{\text{wire}} = I\vec{l} \times \vec{B}$$

The 0.10 T uniform magnetic field of **FIGURE 29.44** is horizontal, parallel to the floor. A straight segment of 1.0-mm-diameter copper wire, also parallel to the floor, is perpendicular to the magnetic field. What current through the wire, and in which direction, will allow the wire to "float" in the magnetic field?



MODEL The wire will float in the magnetic field if the magnetic force on the wire points upward and has magnitude mg, allowing it to balance the downward gravitational force.

Example: Magnetic Levitation

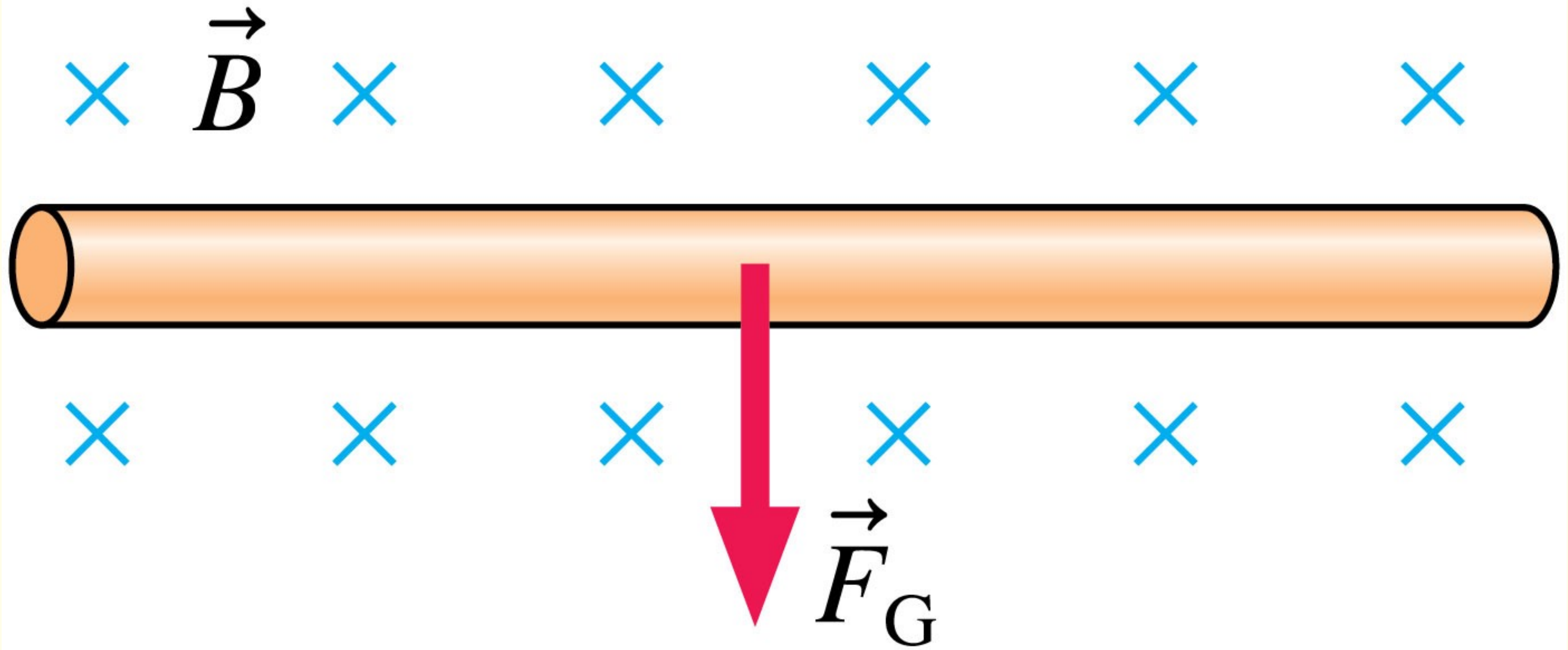
$$\vec{F}_{\text{wire}} = I\vec{l} \times \vec{B}$$

$$F = IlB = mg = \rho(\pi r^2 l)g$$

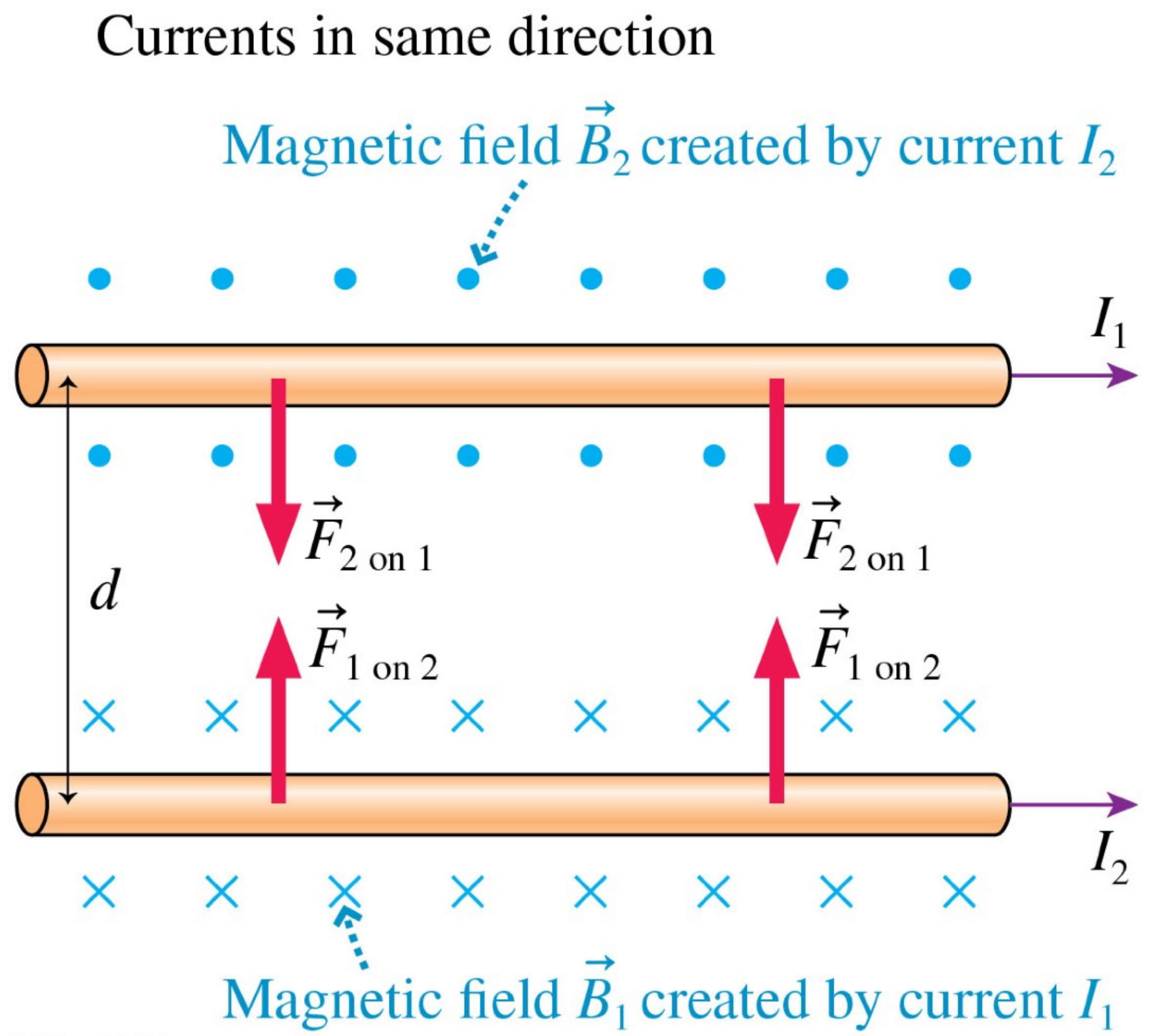
where $\rho = 8920\ \text{kg/m}^3$ is the density of copper. The length of the wire cancels, leading to

$$I = \frac{\rho \pi r^2 g}{B} = \frac{(8920\ \text{kg/m}^3)\pi(0.00050\ \text{m})^2(9.80\ \text{m/s}^2)}{0.10\ \text{T}}$$
$$= 0.69\ \text{A}$$

A 0.69 A current from left to right will levitate the wire in the magnetic field.

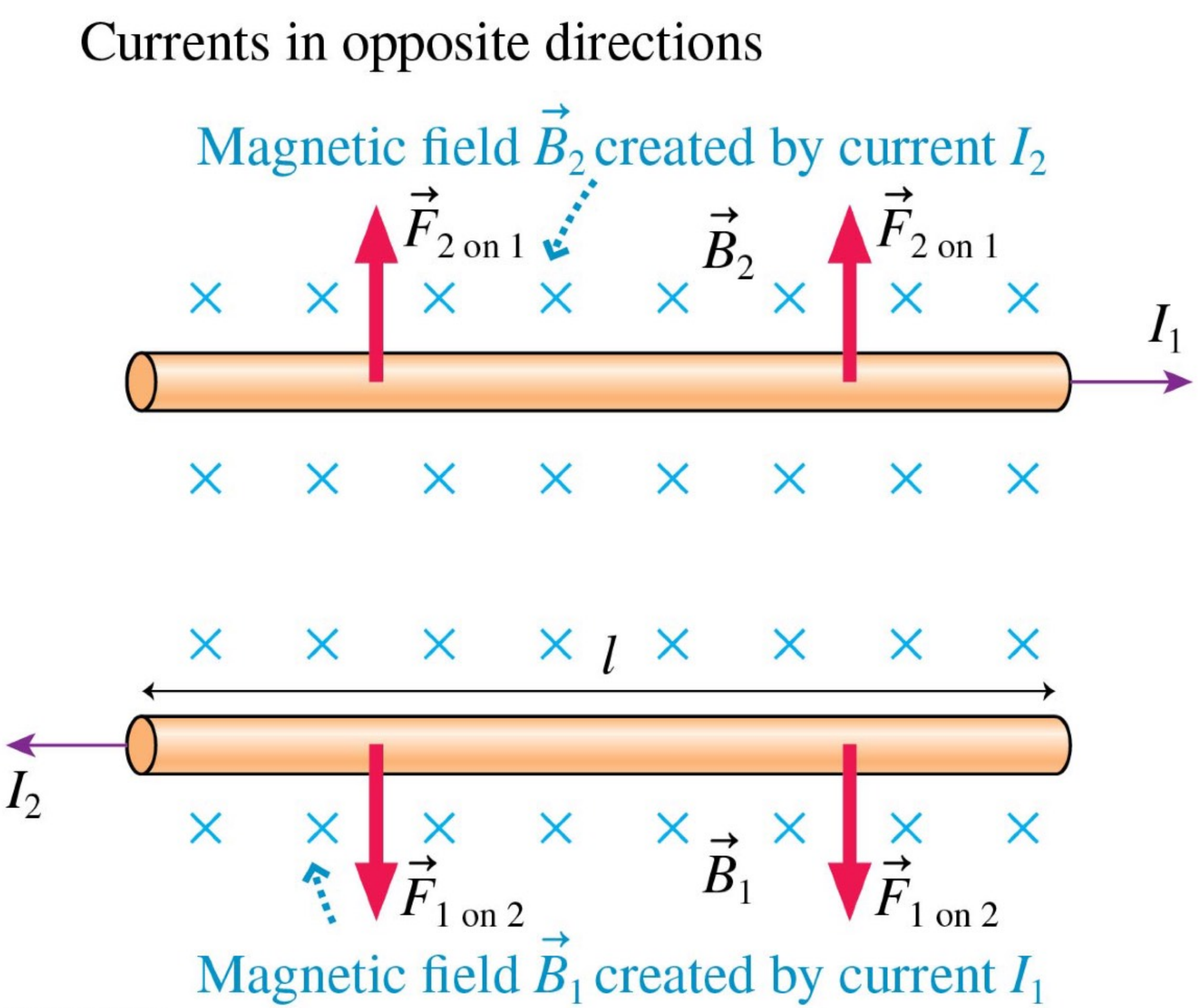


Magnetic Forces Between Parallel Current-Carrying Wires: Current in Same Direction



$$F_{\text{parallel wires}} = I_1 l B_2 = I_1 l \frac{\mu_0 I_2}{2\pi d} = \frac{\mu_0 l I_1 I_2}{2\pi d}$$

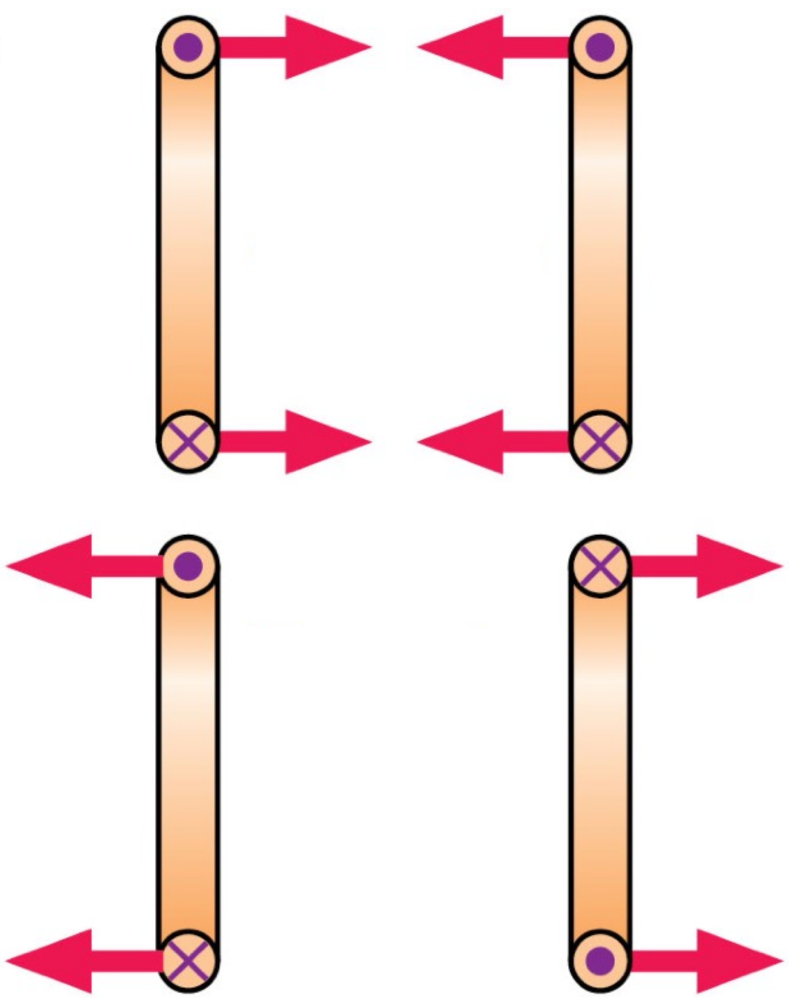
Magnetic Forces Between Parallel Current-Carrying Wires: Current in Opposite Directions

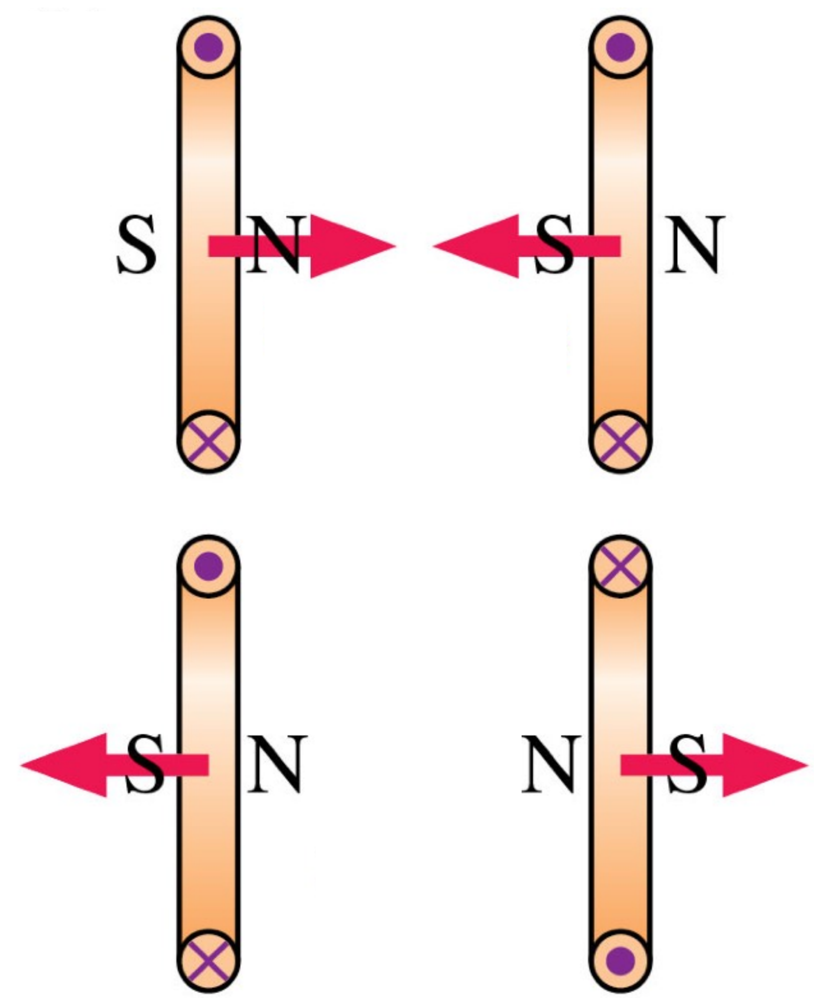


$$F_{\text{parallel wires}} = I_1 l B_2 = I_1 l \frac{\mu_0 I_2}{2\pi d} = \frac{\mu_0 l I_1 I_2}{2\pi d}$$

Forces on Current Loops

- The two figures show alternative but equivalent ways to view magnetic forces between two current loops.

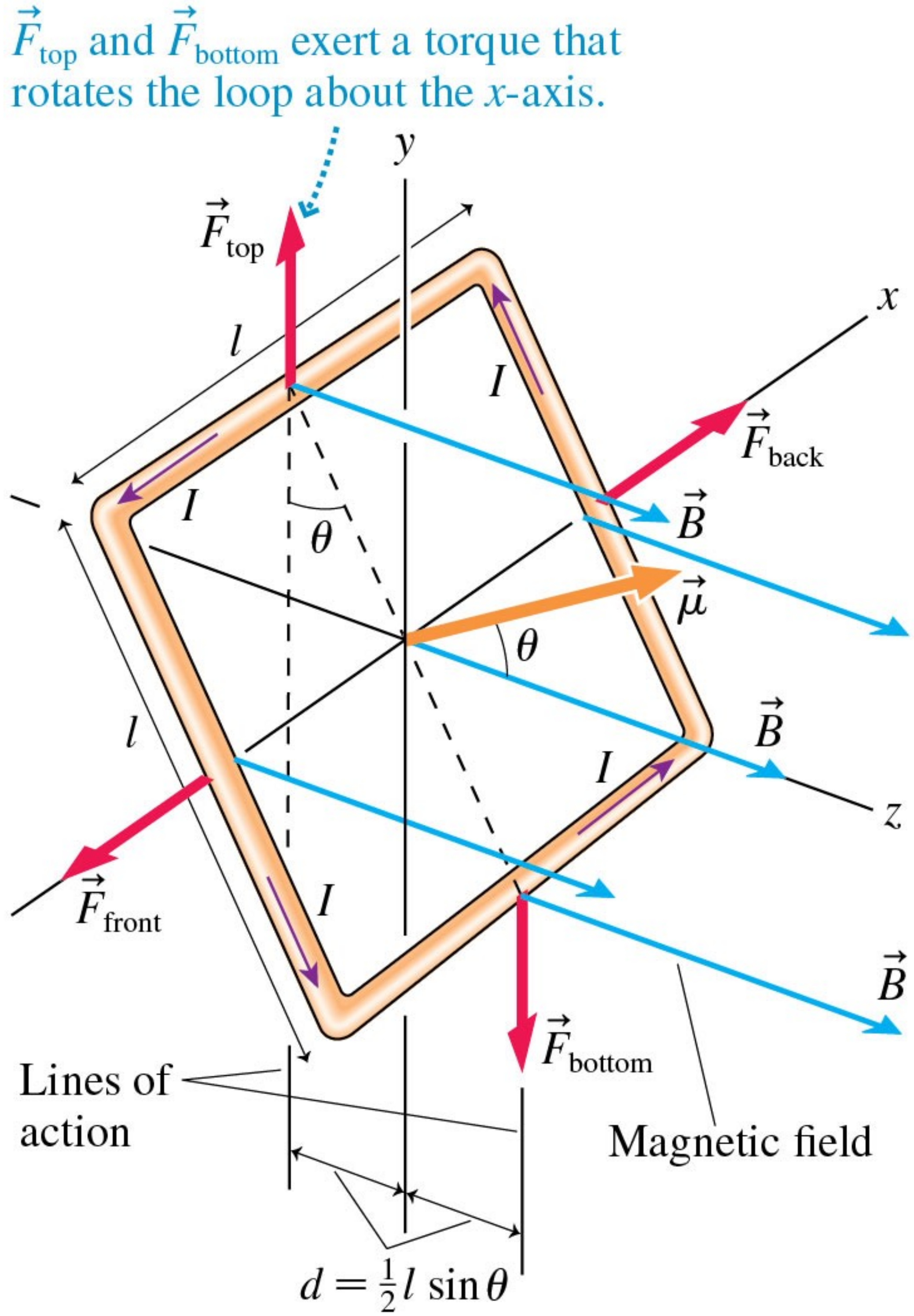




- Parallel currents attract, opposite currents repel.
- Opposite poles attract, like poles repel.

A Uniform Magnetic Field Exerts a Torque on a Square Current Loop

- $\vec{F}_{\text{front}}$ and $\vec{F}_{\text{back}}$ are opposite to each other and cancel.
- Both $\vec{F}_{\text{top}}$ and $\vec{F}_{\text{bottom}}$ exert a force of magnitude $F = I\,l\,B$ around a moment arm $d = ½\,l\,\sin\theta$.



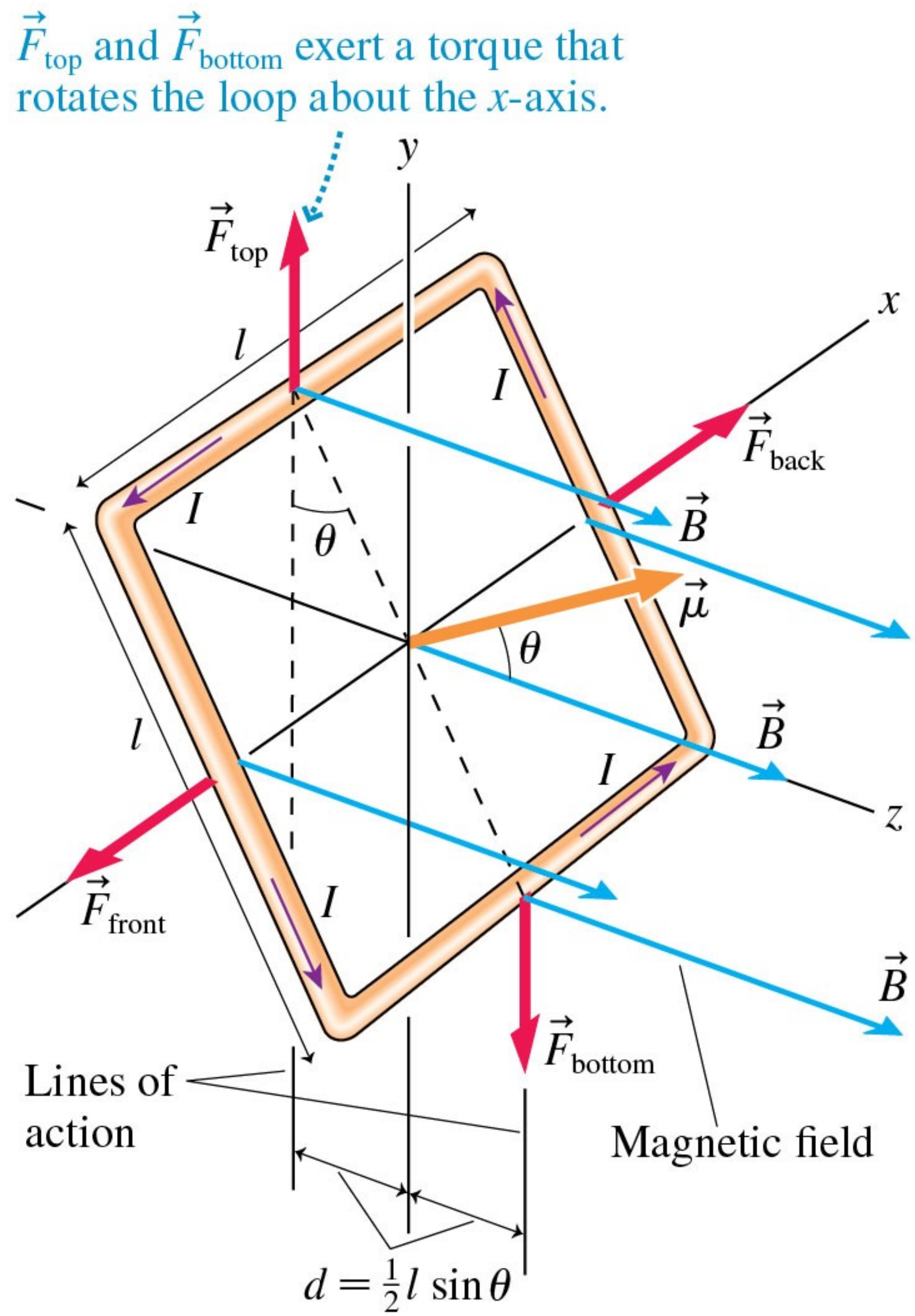
A Uniform Magnetic Field Exerts a Torque on a Square Current Loop

- The total torque is

  $$\tau = 2Fd = (Il^2)B\sin\theta = \mu B\sin\theta$$

  where $\mu = Il^2 = IA$ is the loop's magnetic dipole moment.
- Although derived for a square loop, the result is valid for a loop of any shape:

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$



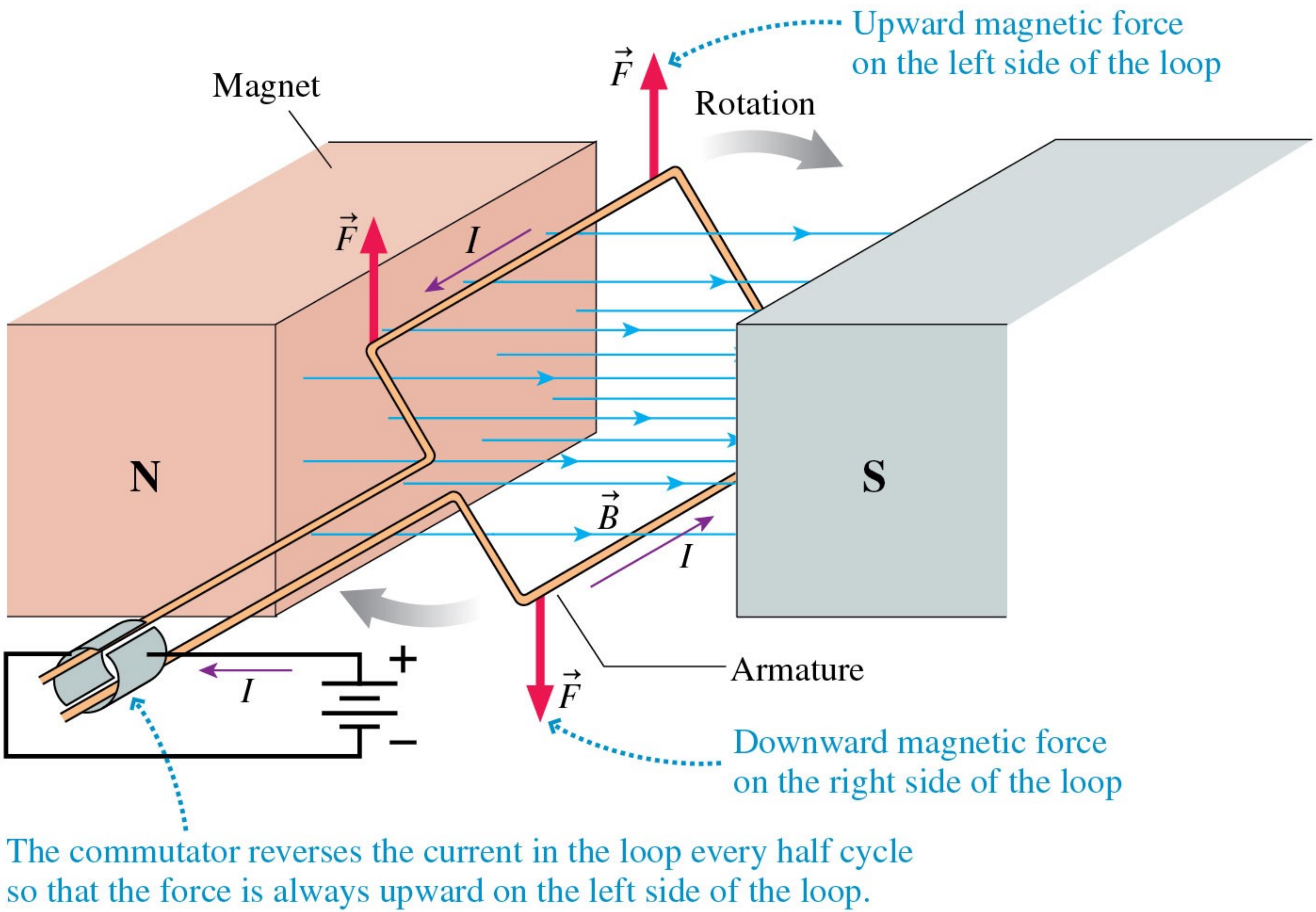
iClicker question #15-4

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

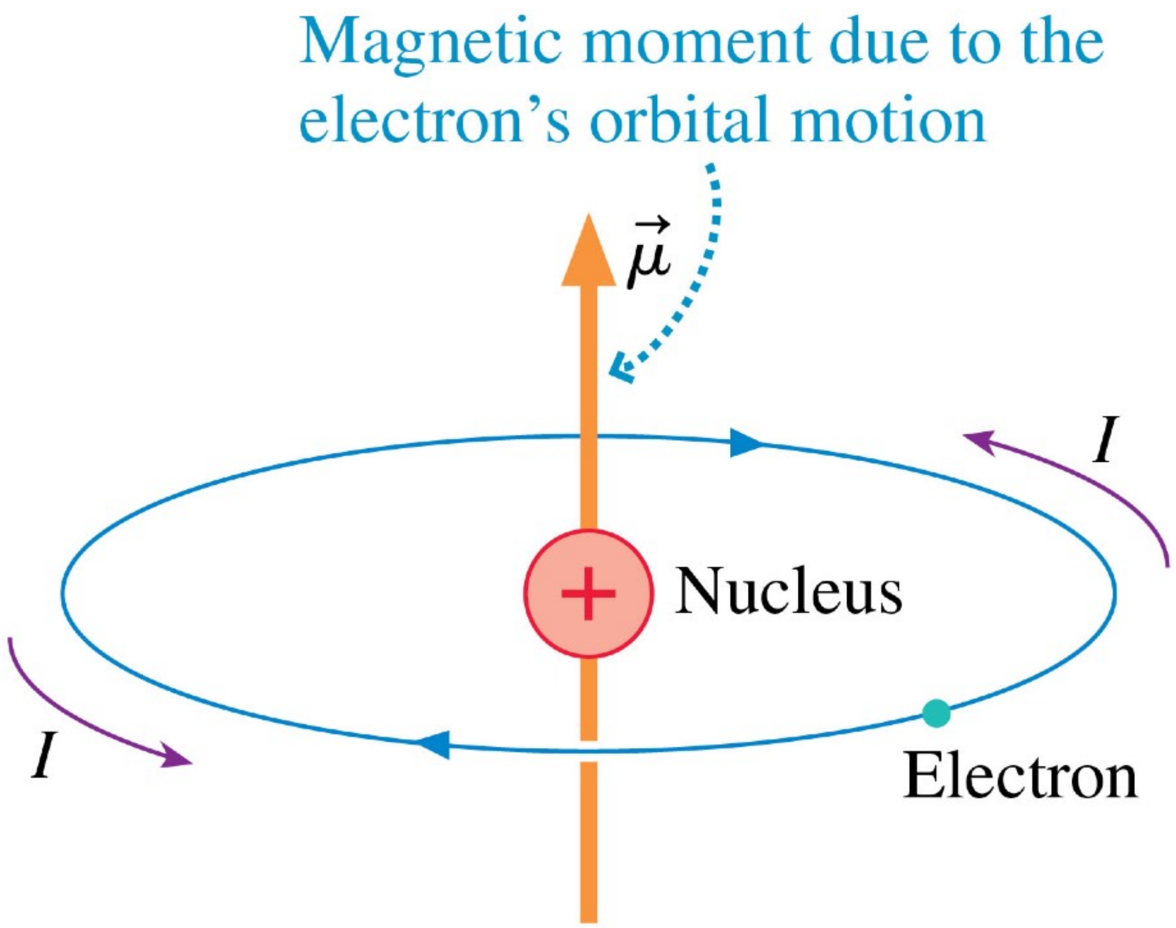
If released from rest, the current loop will

A. Move upward.
B. Move downward.
C. Rotate clockwise.
D. Rotate counterclockwise.
E. Do something not listed here.

A Simple Electric Motor

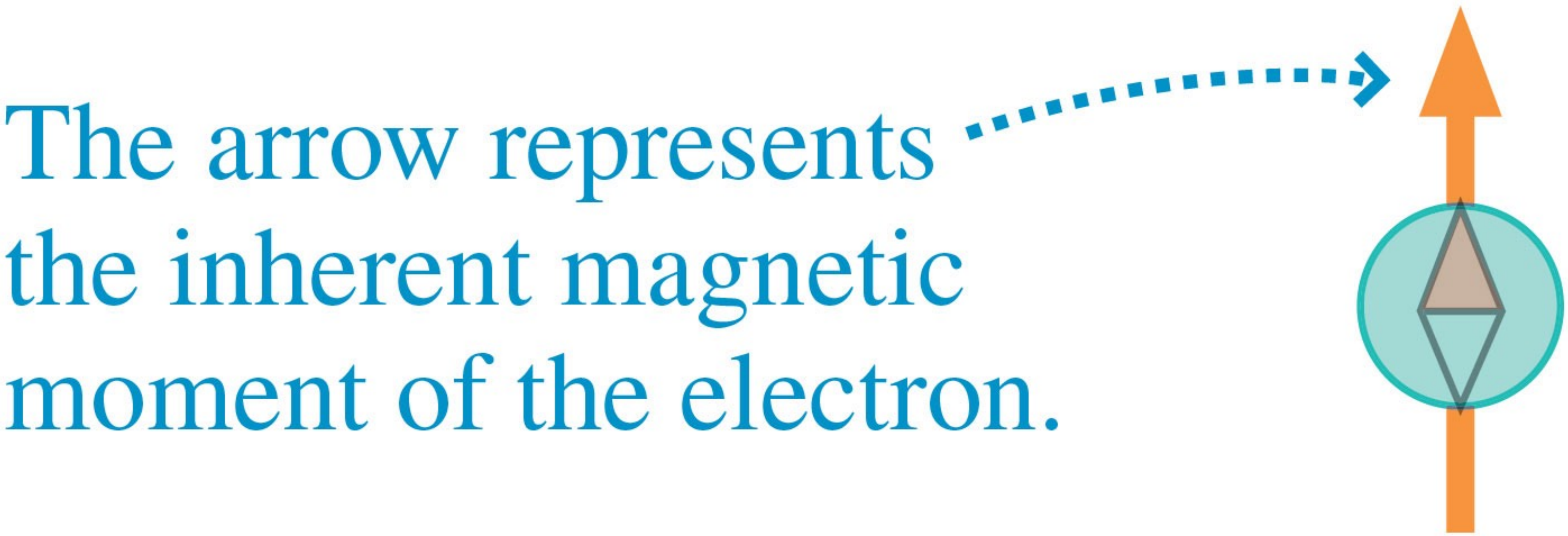


Atomic Magnets



- A plausible explanation for the magnetic properties of materials is the orbital motion of the atomic electrons.
- The figure shows a simple, classical model of an atom in which a negative electron orbits a positive nucleus.
- In this picture of the atom, the electron's motion is that of a current loop!
- An orbiting electron acts as a tiny magnetic dipole, with a north pole and a south pole.

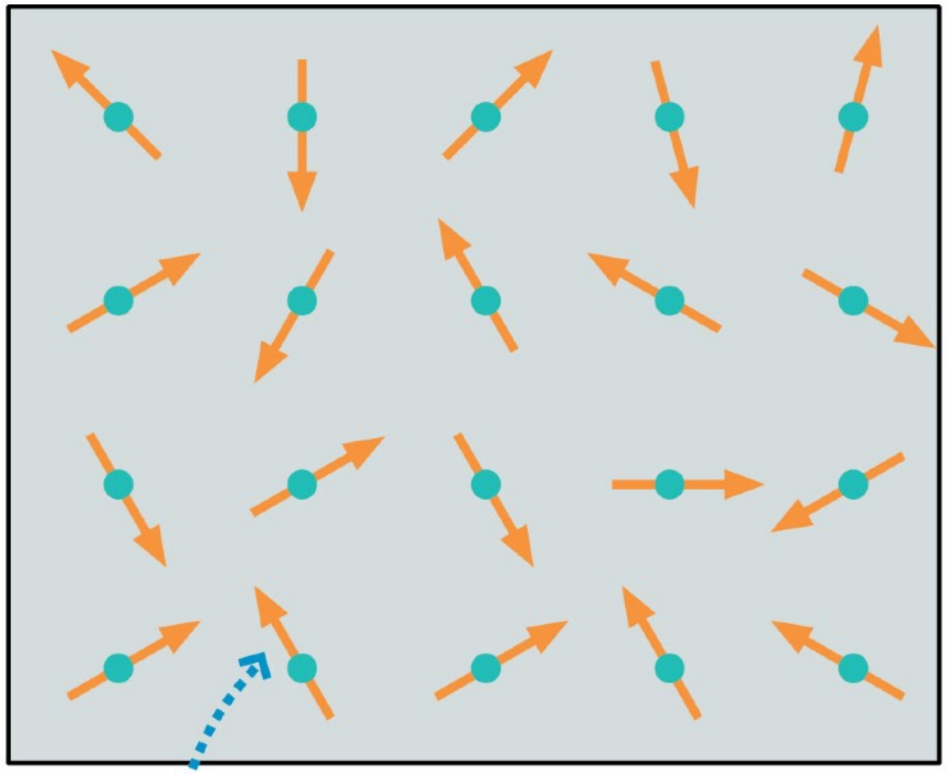
The Electron Spin



- An electron's inherent magnetic moment is often called the electron *spin* because, in a classical picture, a spinning ball of charge would have a magnetic moment.
- While it may not be spinning in a literal sense, an electron really is a microscopic magnet.

Magnetic Properties of Matter

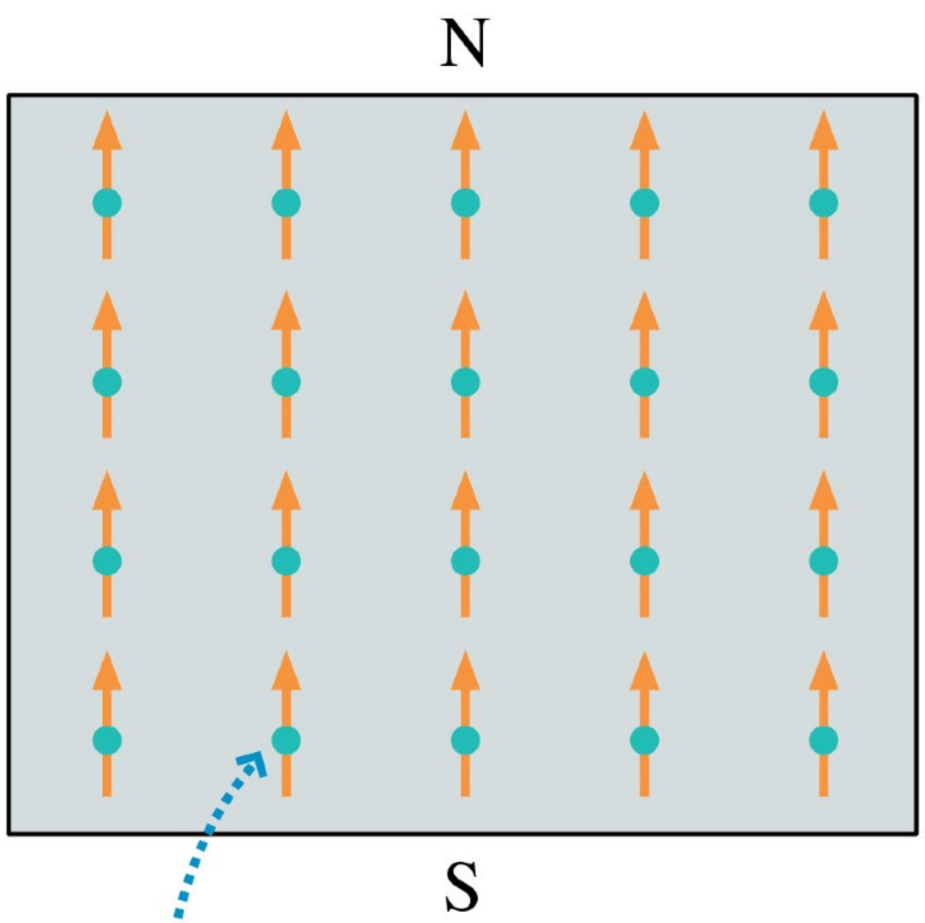
- For most elements, the magnetic moments of the atoms are randomly arranged when the atoms join together to form a solid.
- As the figure shows, this random arrangement produces a solid whose net magnetic moment is very close to zero.



The atomic magnetic moments due to unpaired electrons point in random directions. The sample has no net magnetic moment.

Ferromagnetism

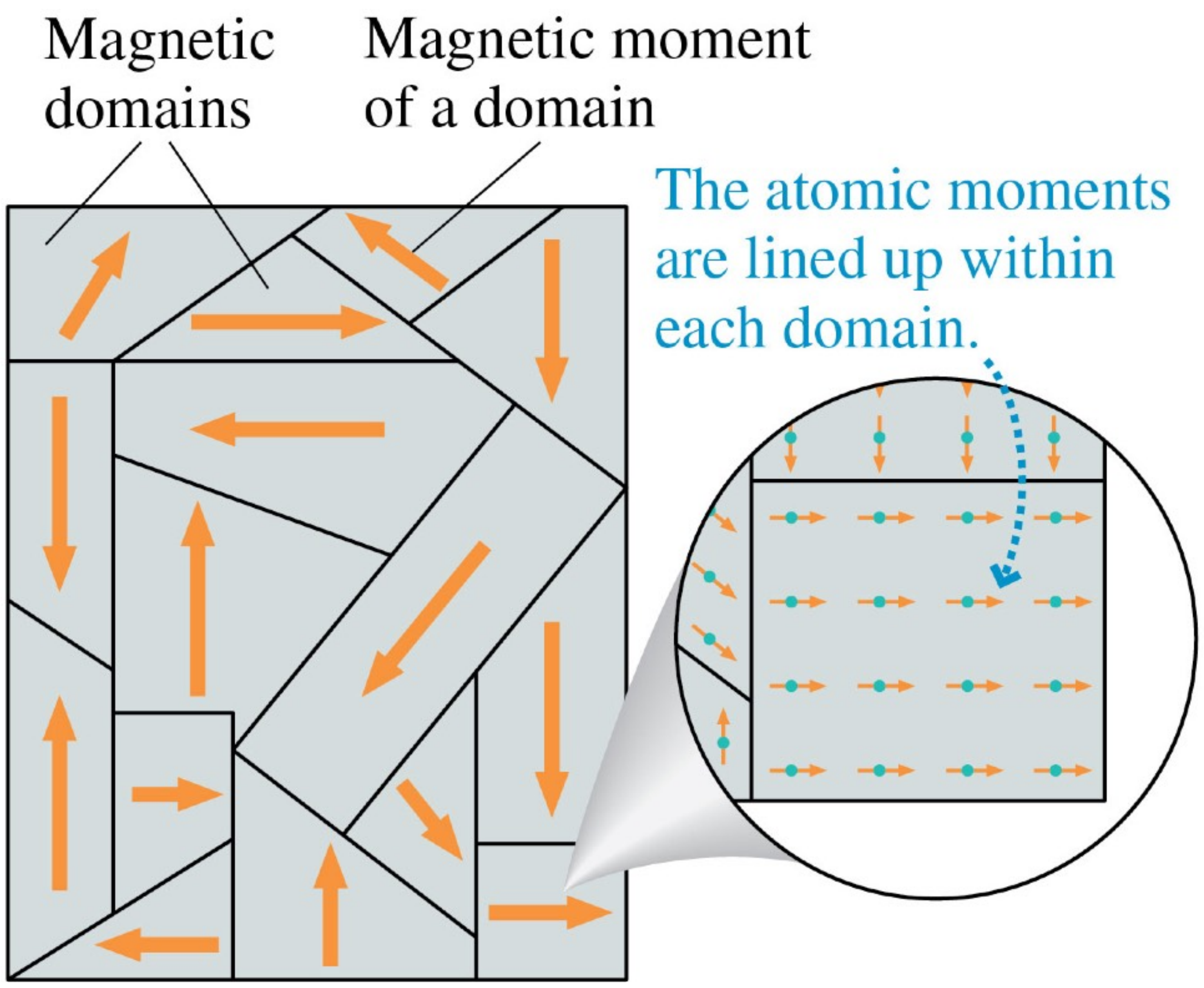
- In iron, and a few other substances, the atomic magnetic moments tend to all line up in the *same* direction, as shown
in the figure.
- Materials that behave in this fashion are called **ferromagnetic**, with the prefix *ferro* meaning "iron-like."



The atomic magnetic moments are aligned.
The sample has north and south magnetic poles.

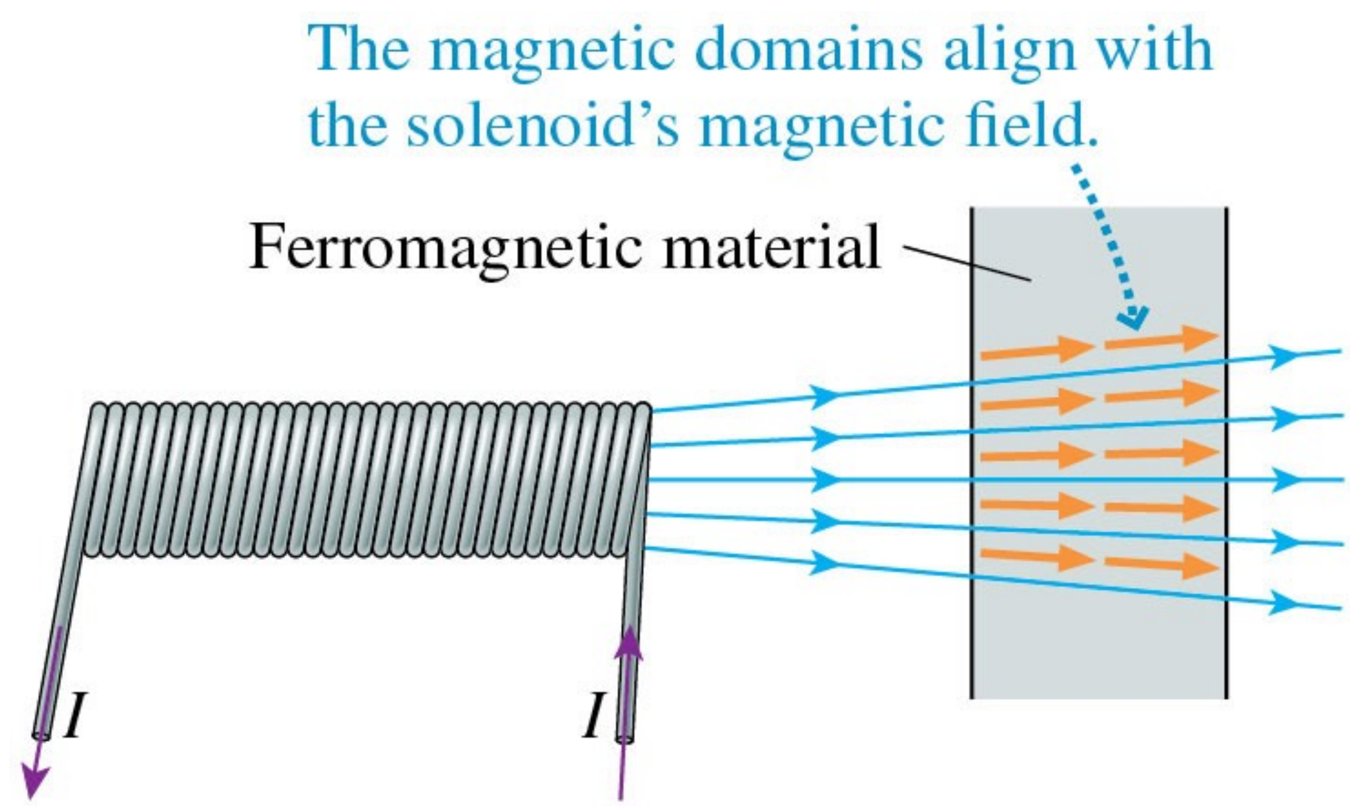
Ferromagnetism

- A typical piece of iron is divided into small regions, typically less than $100\ \mu m$ in size, called **magnetic domains**.
- The magnetic moments of all the iron atoms within each domain are perfectly aligned, so each individual domain is a strong magnet.
- However, the various magnetic domains that form a larger solid are randomly arranged.



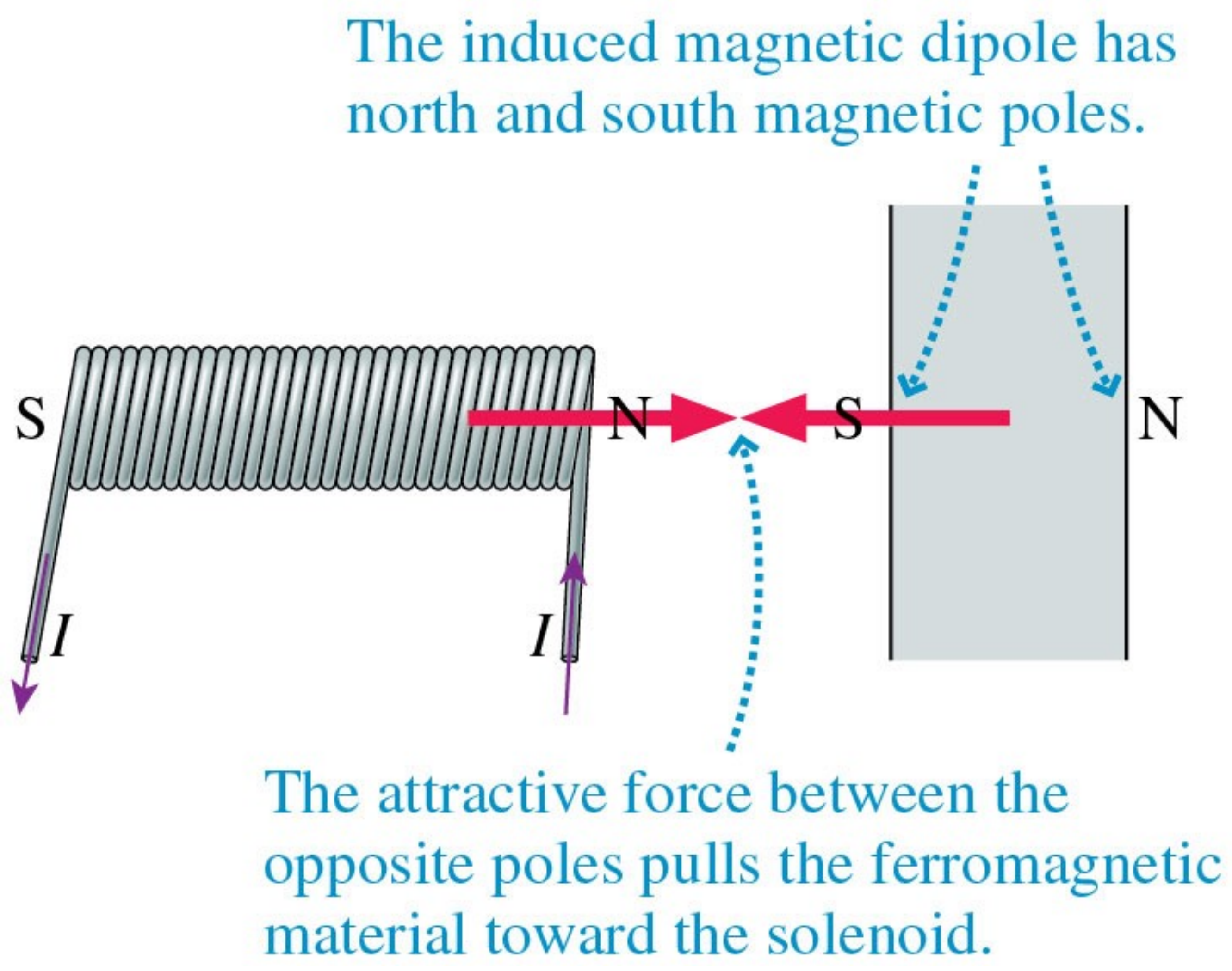
Induced Magnetic Dipoles

- If a ferromagnetic substance is subjected to an *external* magnetic field, the external field exerts a torque on the magnetic dipole of each domain.
- The torque causes many of the domains to rotate and become aligned with the external field.

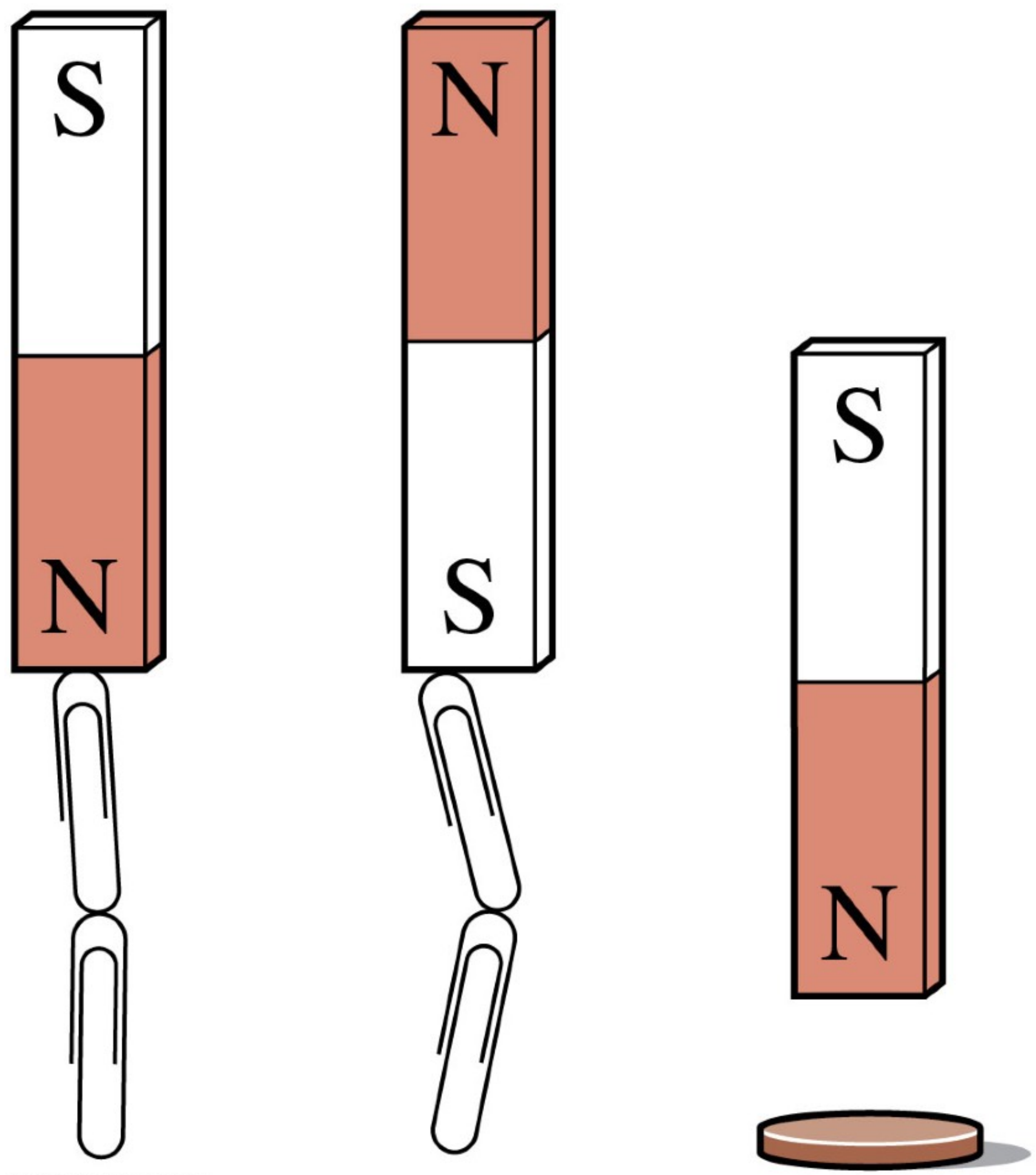


Induced Magnetic Dipoles

- The induced magnetic dipole always has an *opposite* pole facing the solenoid.
- Consequently the magnetic force between the poles *pulls* the ferromagnetic object to the electromagnet.



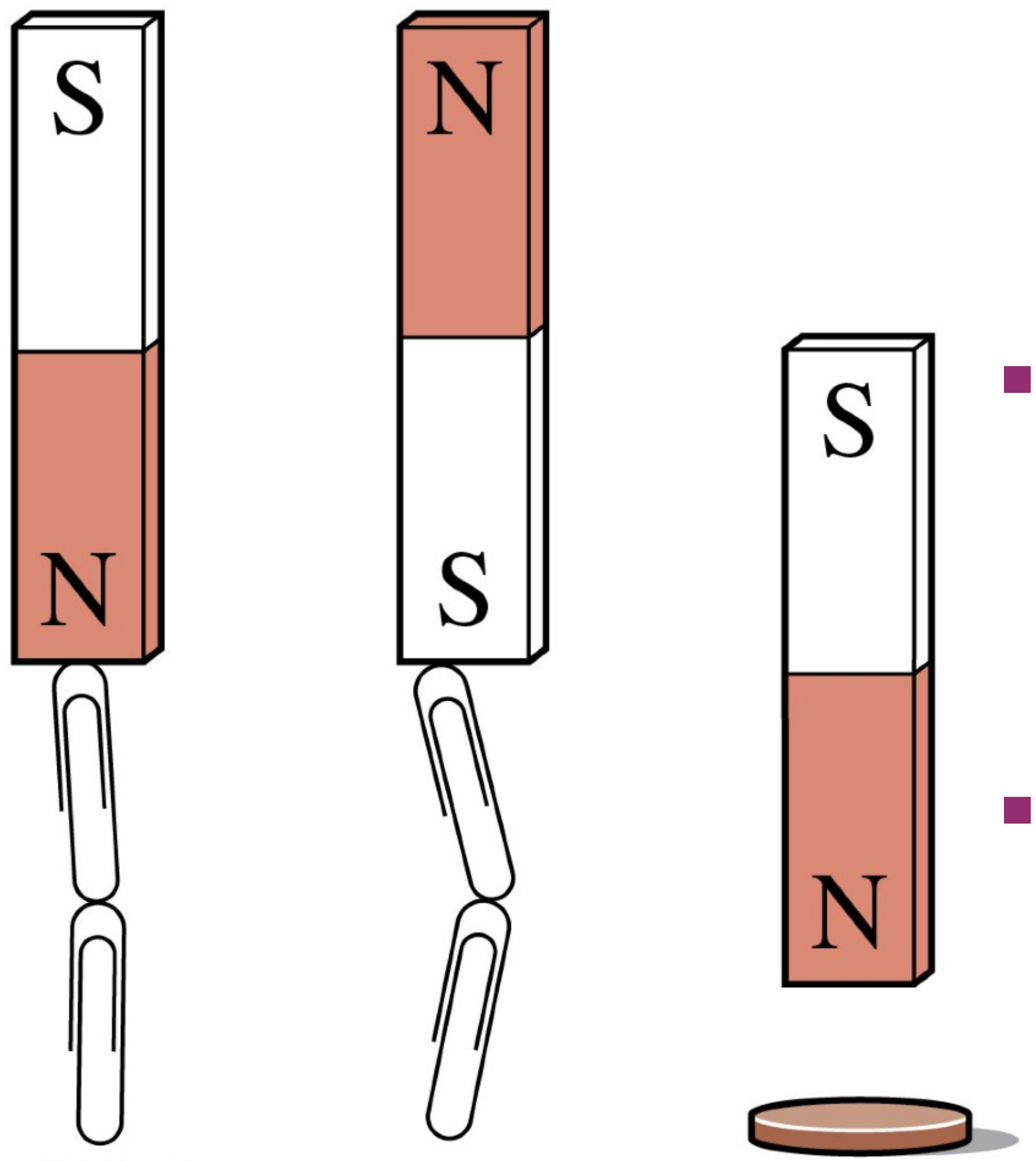
Induced Magnetism



- Now we can explain how a magnet attracts and picks up ferromagnetic objects:

1. Electrons are microscopic magnets due to their spin.
2. A ferromagnetic material in which the spins are aligned is organized into magnetic domains.
3. The individual domains align with an external magnetic field to produce an induced magnetic dipole moment for the entire object.

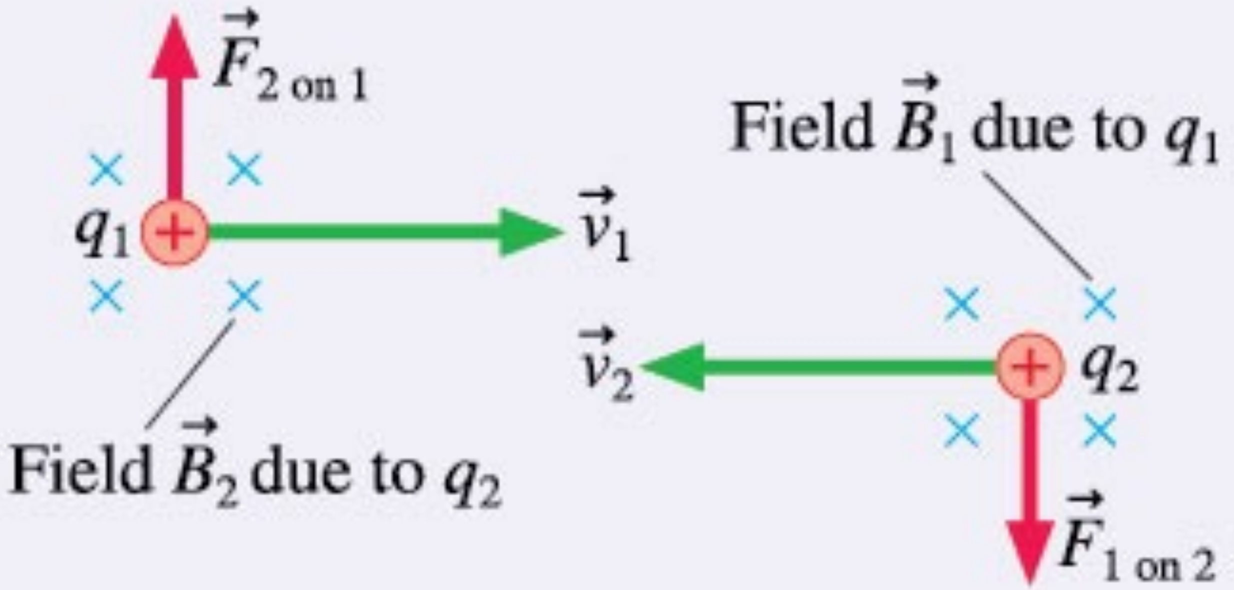
Induced Magnetism



- An object's magnetic dipole may not return to zero when the external field is removed because some domains remain "frozen" in the alignment they had in the external field.
- Thus a ferromagnetic object that has been in an external field may be left with a net magnetic dipole moment after the field is removed.
- In other words, the object has become a **permanent magnet**.

General Principles

At its most fundamental level, **magnetism** is an interaction between moving charges. The magnetic field of one moving charge exerts a force on another moving charge.

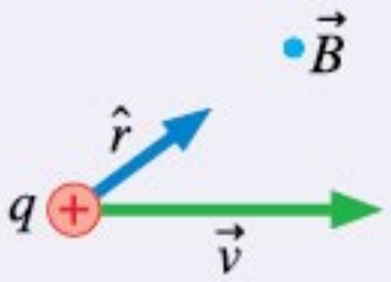


General Principles

Magnetic Fields

The **Biot-Savart law** for a moving point charge

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$$



Magnetic field of a current

MODEL Model wires as simple shapes.

VISUALIZE Divide the wire into short segments.

SOLVE Use superposition:

- Find the field of each segment Δs.
- Find $\vec{B}$ by summing the fields of all Δs, usually as an integral.

An alternative method for fields with a high degree of symmetry is **Ampère's law**:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{through}}$$

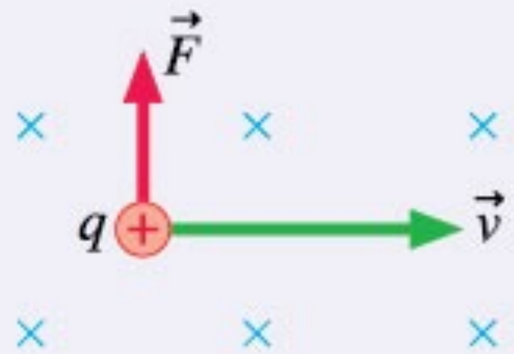
where I_{through} is the current through the area bounded by the integration path.

Magnetic Forces

The magnetic force on a moving charge is

$$\vec{F} = q\vec{v} \times \vec{B}$$

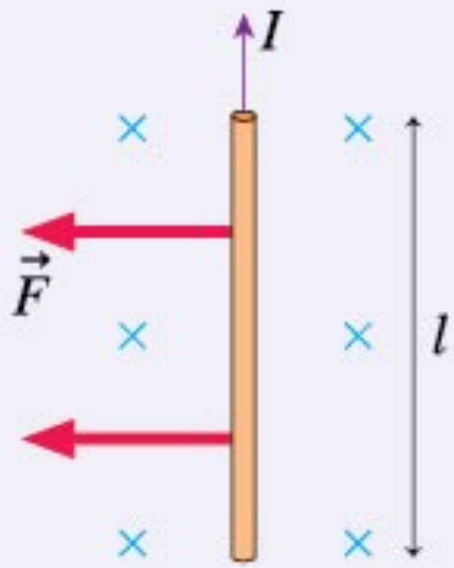
The force is perpendicular to $\vec{v}$ and $\vec{B}$.



The magnetic force on a current-carrying wire is

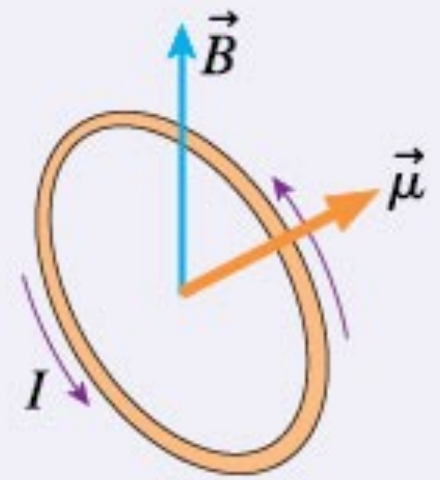
$$\vec{F} = I\vec{l} \times \vec{B}$$

$\vec{F} = \vec{0}$ for a charge or current moving parallel to $\vec{B}$.



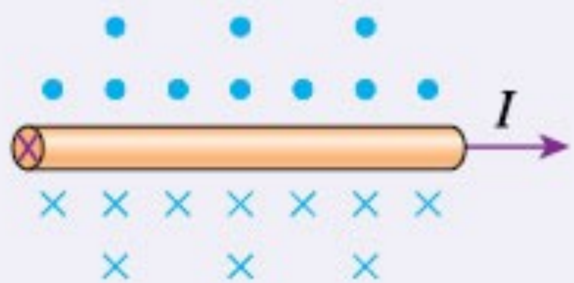
The magnetic torque on a magnetic dipole is

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$



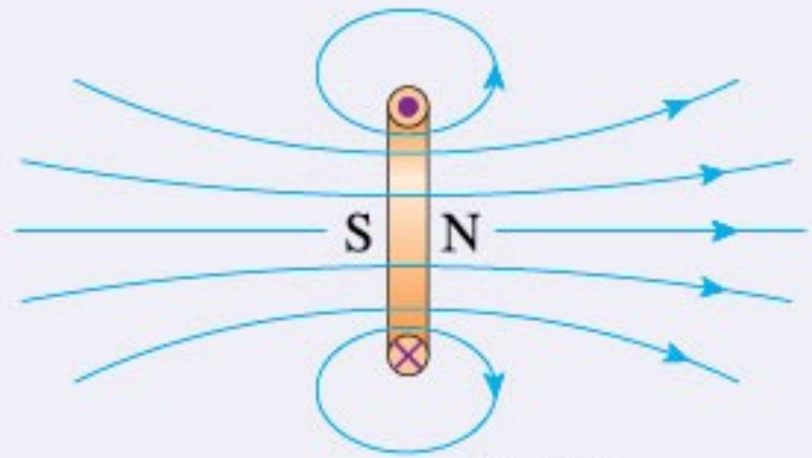
Applications

Wire



$$B = \frac{\mu_0}{2\pi}\frac{I}{r}$$

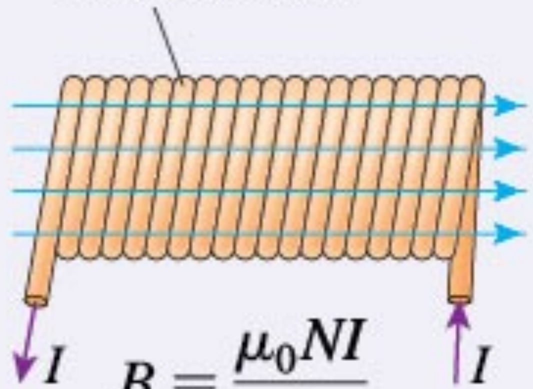
Loop



$$B_{\text{center}} = \frac{\mu_0}{2}\frac{NI}{R}$$

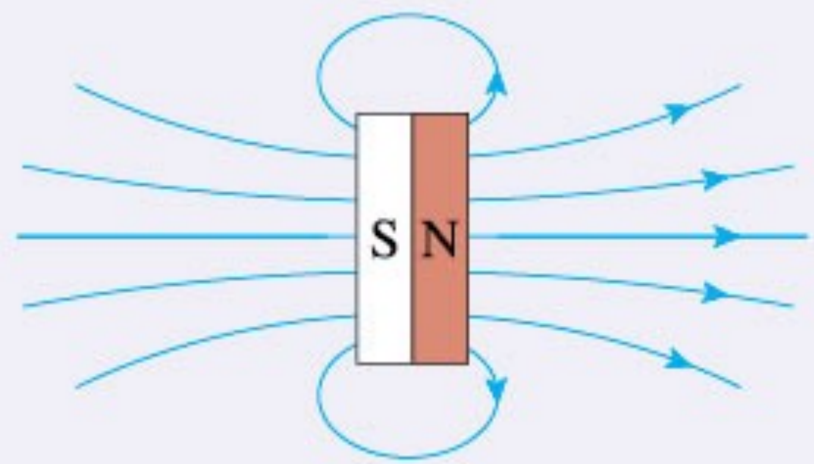
Solenoid

Uniform field



$$B = \frac{\mu_0 NI}{l}$$

Flat magnet



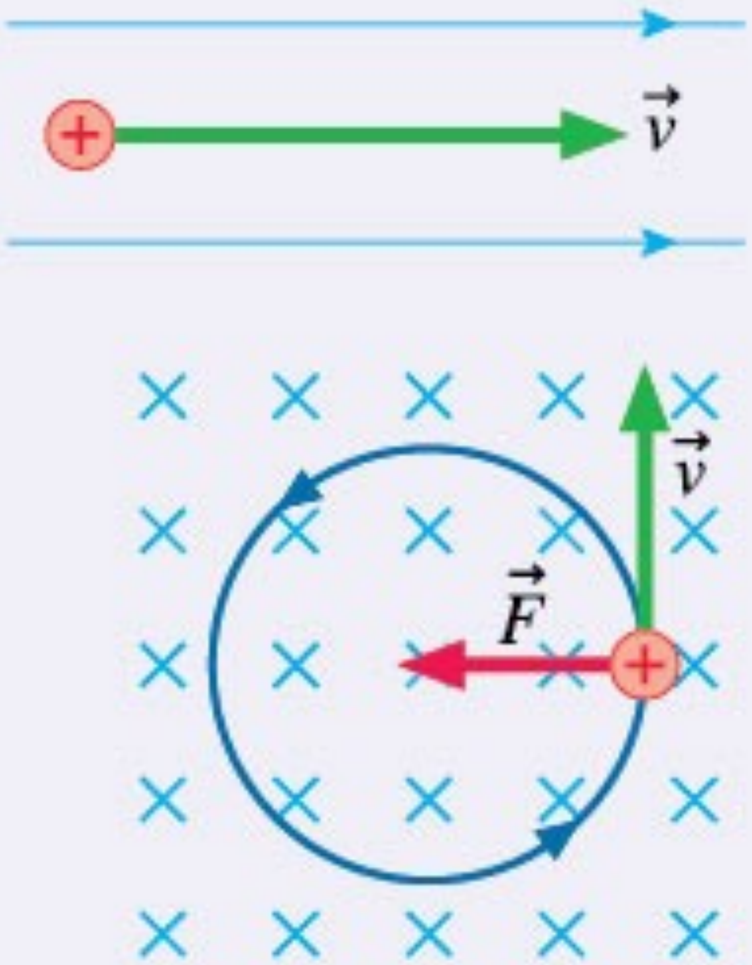
Right-hand rule

Point your right thumb in the direction of I. Your fingers curl in the direction of $\vec{B}$. For a dipole, $\vec{B}$ emerges from the side that is the north pole.

Charged-particle motion

No force if $\vec{v}$ is parallel to $\vec{B}$

Circular motion at the cyclotron frequency $f_{\text{cyc}} = qB/2\pi m$ if $\vec{v}$ is perpendicular to $\vec{B}$



Applications