Electric potential

Potential energy:

$$\Delta U = -\int \vec{F} \cdot d\vec{s}$$

$$\vec{F} = q\vec{E}$$

$$\Delta U = q\left(-\int \vec{E} \cdot d\vec{s}\right)$$

define this as electric potential, or voltage.

Sometimes called emf

inverting equation:

$$-\nabla V = E$$

in cartesian:

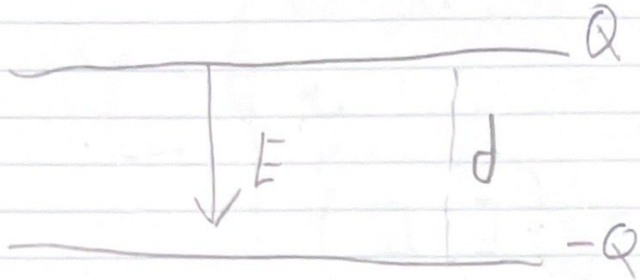
$$\frac{-\partial V}{\partial x_i} = \vec{E} \cdot \hat{x}_i \qquad x_i = x, y, z$$

For pairs of point charges

$$U = \frac{k q_1 q_2}{r}$$

~~Capacitor~~

Capacitor



$$E = \frac{Q}{A2\varepsilon_0} - \left(\frac{-Q}{A2\varepsilon_0}\right) = \frac{Q}{A\varepsilon_0}$$

voltage:

ds is opposite to $\vec{E}$, so $\vec{E}\cdot d\vec{s} = -E\,ds$

$$V_{top} - V_{bottom} = -\int_{bottom}^{top} \vec{E}\cdot d\vec{s} =$$

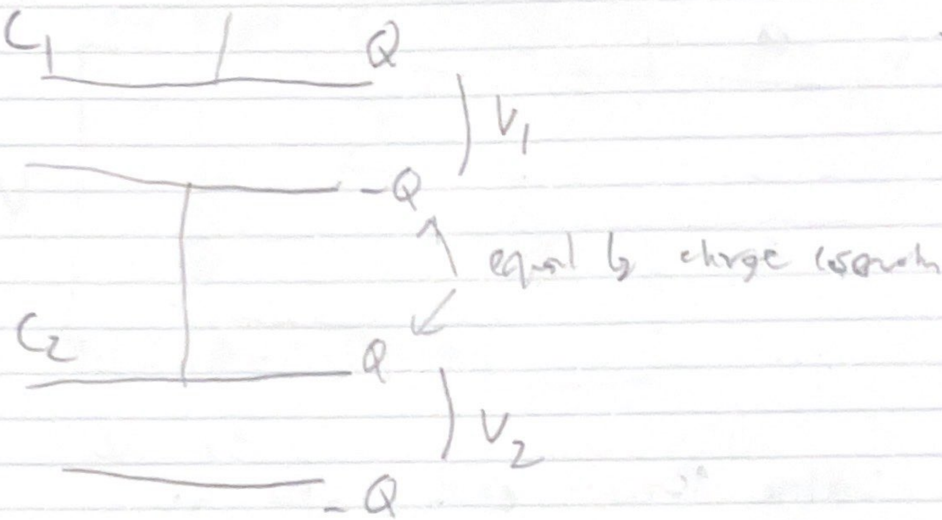
$$\int_0^d E\,ds = \frac{Q}{A\varepsilon_0} d = \Delta V$$

$$\frac{Q}{\Delta V} = \frac{A}{\varepsilon_0 d} = C$$ capacitance,

only depends on geometry (A, d)

or dielectric placed in between $(\varepsilon_0 \to \varepsilon)$

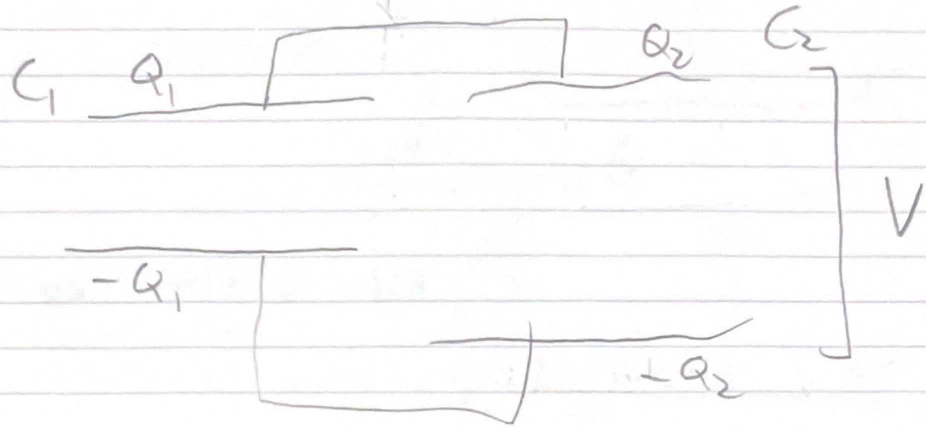
Series:



$$V_{total} = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$\Rightarrow \frac{V_{total}}{Q} = \boxed{\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2}}$$

parallel:



Voltage across each is the same

$$Q_{total} = Q_1 + Q_2 = C_1 V + C_2 V$$

$$\Rightarrow \frac{Q_{total}}{V} = \boxed{C_{total} = C_1 + C_2}$$

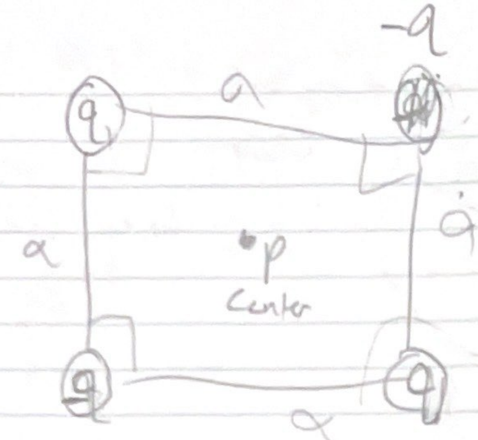
Energy:

to bring to equal charge;

$$\int dw = \int_0 V\,dq = \int_0^q \frac{q}{c}\,dq$$

$$= \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Q



total potential energy?

V at p?

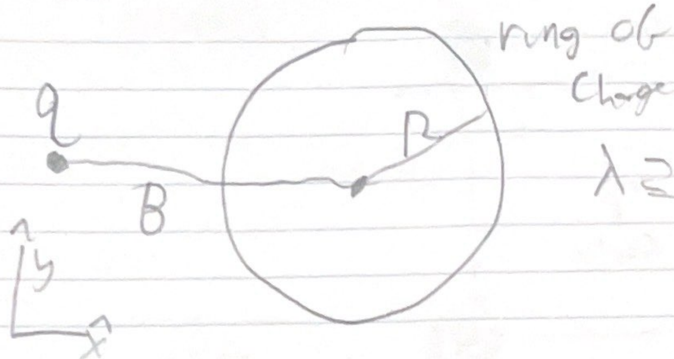
A: sum over all pairs:

$$\boxed{\frac{kq^2}{\sqrt{2}a} + \frac{kq^2}{\sqrt{2}a} - 4\frac{kq^2}{a}}$$

$$V(p) = \boxed{0}$$

$$\frac{2k}{\sqrt{2}a}(2q - 2q) = 0$$

Q



$\lambda =$ linear charge density

what is $\vec{E}$ at p? what is V?

A. $\boxed{\vec{E} = \frac{kq}{B^2}\hat{x}}$ ring cancels

$$V = \frac{kq}{B} + \int_0^{2\pi R} \frac{k\lambda\, dL}{R} =$$

$$\boxed{\frac{kq}{B} + k\pi 2\lambda}$$

Q: $V = ax^2 + cy + d$

What is $\vec{E}$?

A. $\frac{\partial V}{\partial x} = 2ax$

$\frac{\partial V}{\partial y} = c$

$\frac{\partial V}{\partial z} = 0$

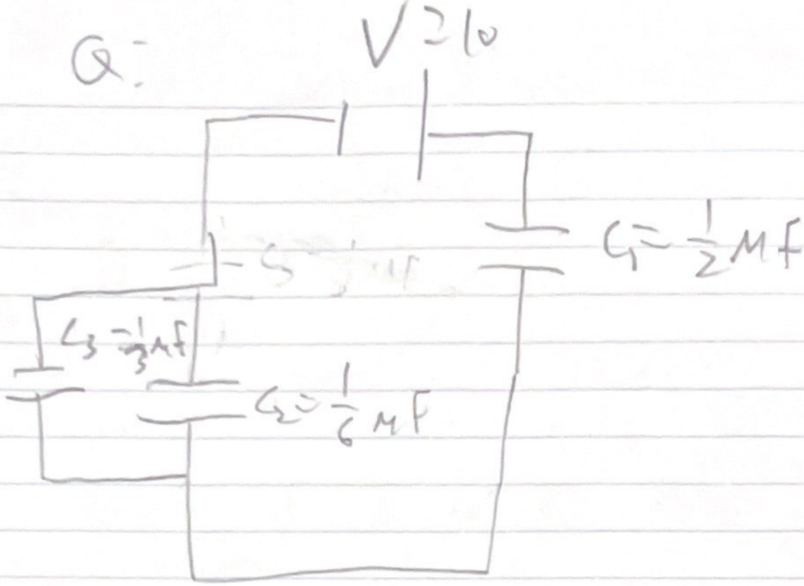
$\vec{E} = -2ax\,\hat{x} + -c\,\hat{y}$

Q: $E = x^2 \hat{x}$ x in metrs.

ΔV betwen $x=0$, $x=3$

$V(3) - V(0) = -\int_0^3 x^2 dx = \boxed{-9}$

Q:



Total C? Total U?

individual U?

A. Total $C = \left(\frac{1}{C_1} + \frac{1}{C_3+C_2}\right)^{-1}$

$= (2+2)^{-1} = \boxed{\frac{1}{4}\ \mu F}$

B. $U = \frac{1}{2}CV^2 = \frac{100}{8} = \boxed{12.5\ \mu J}$

$Q_{tot} = C_{tot} V = 2.5\ \mu C$

tup of total capacitor is just tup of C_1

$U(C_1) = \frac{1}{2}\frac{Q_1^2}{C_1} = \frac{1}{2}\frac{Q_{tot}^2}{C_1} = \boxed{6.25\ \mu J}$

$V_1 = \frac{Q_1}{C_1} = 5V$

so across other 2 capacitors is $10 - 5 = 5V$

$U(C_2) = \frac{1}{2} C_2 V_2^2 = \frac{25}{6} = \boxed{4.17\ \mu J}$

$U(C_3) = \frac{1}{2} C_3 V_3^2 = \frac{25}{12} = \boxed{2.08\ \mu J}$

~~yes~~ $U_1 + U_2 + U_3 = 12.5\ \mu J$

same total