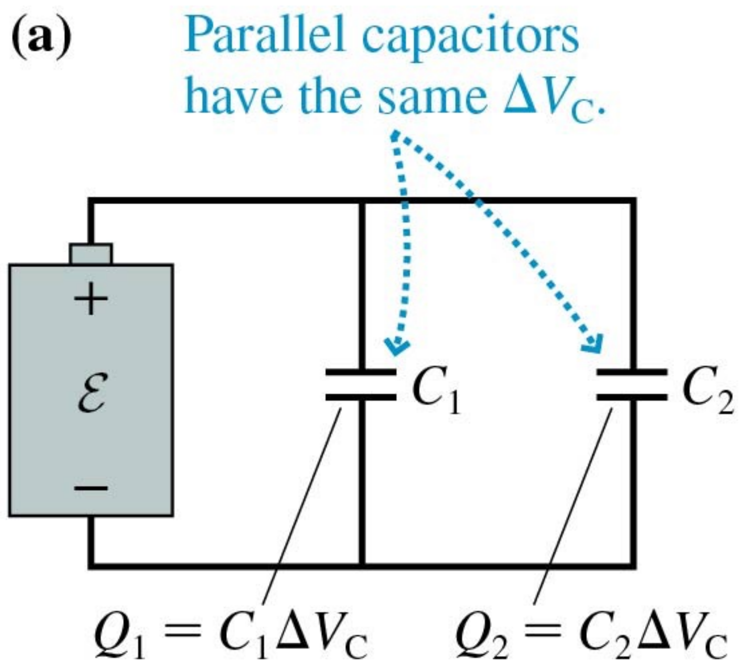
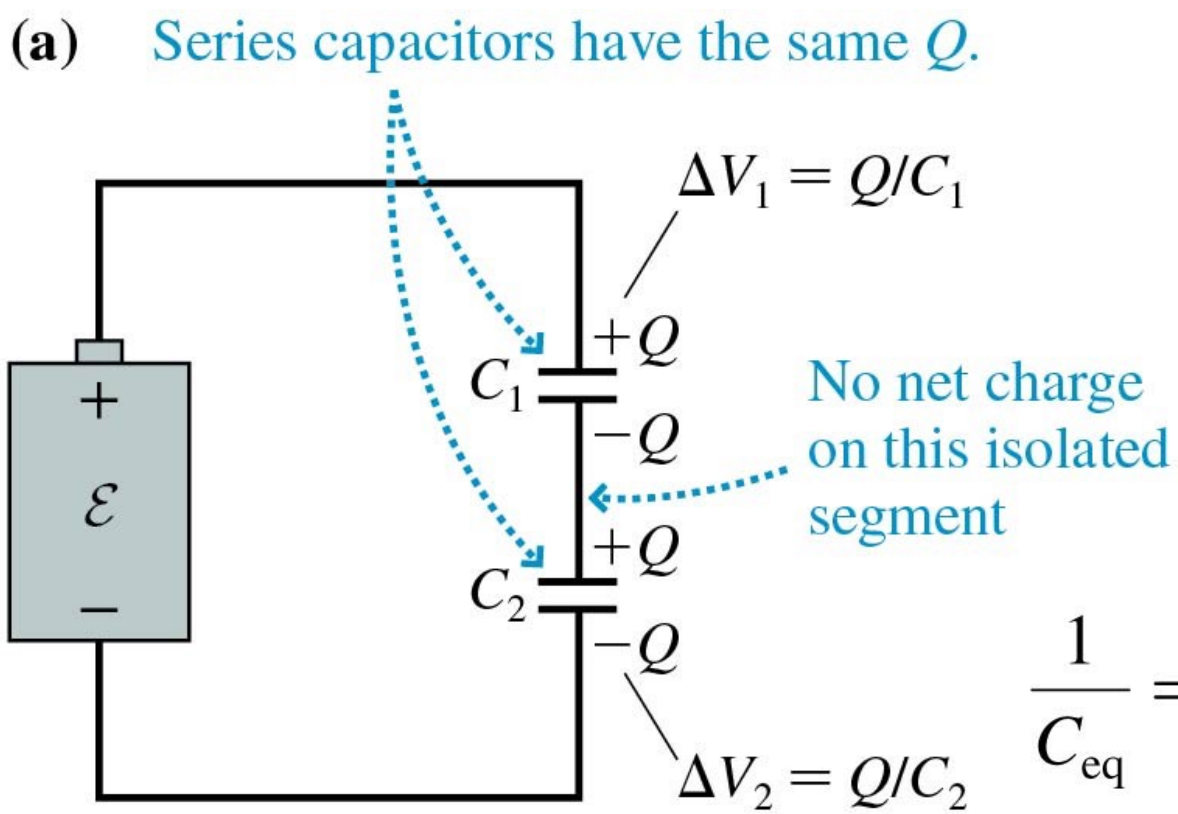
Capacitors in parallel



- Charge adds up:
  $Q = Q_1 + Q_2$
- Voltage is the same:
  $\Delta V_C = \Delta V_1 = \Delta V_2$

$$C_{eq} = \frac{Q}{\Delta V_C} = \frac{Q_1 + Q_2}{\Delta V_C} = \frac{Q_1}{\Delta V_C} + \frac{Q_2}{\Delta V_C} = C_1 + C_2$$

Capacitors in series



- Charge is the same:
  $Q = Q_1 = Q_2$
- Voltage adds up:
  $\Delta V_C = \Delta V_1 + \Delta V_2$

$$\frac{1}{C_{eq}} = \frac{\Delta V_C}{Q} = \frac{\Delta V_1 + \Delta V_2}{Q} = \frac{\Delta V_1}{Q} + \frac{\Delta V_2}{Q} = \frac{1}{C_1} + \frac{1}{C_2}$$

PHYS 2B: Resources available to help you succeed in this class

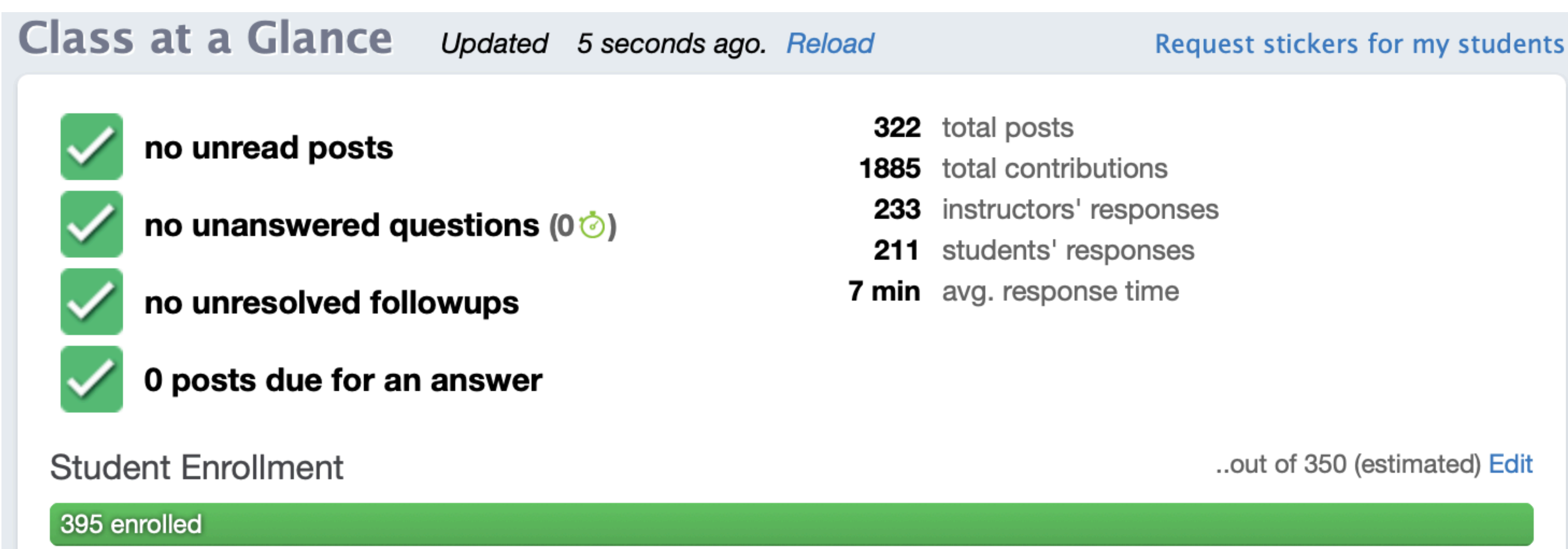
- We have many resources dedicated to this class:
  - Discussion sessions: Wed 1 hour (5 TAs!)
  - Office hours: 12+ hours
  - Tutorial center by your TAs: 10+ hours
  - Besides lectures and quiz: T, Th 2:00—3:20, F 5:00—5:50
  - **More than 24 hours per week of help <u>in person</u>!**

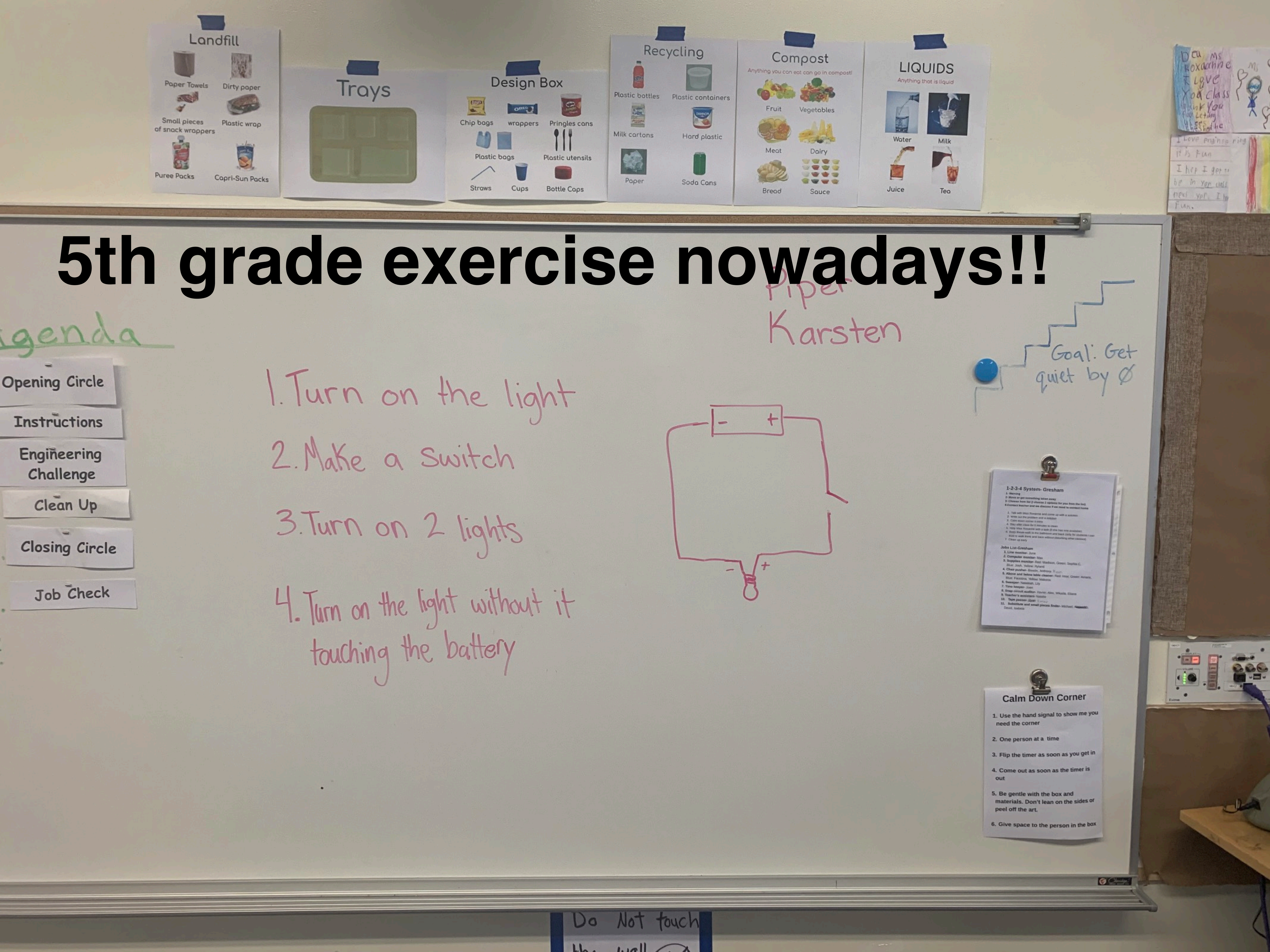
- piazza (24/7)
- many office hours (12+ hours, one for each section @271)
- discussion sessions (1 each, @254),
- tutorial center (10 additional hours by your discussion session TAs @271),
- OASIS,
- IDEA, engineering student center, PHYS 2B Wed 10-11:50
- Teaching + Learning Commons, Drop-in Geisel PHYS 2B
- Academic Achievement Hub (see image below)
- Library reserves (yes, now the physical book must be there)

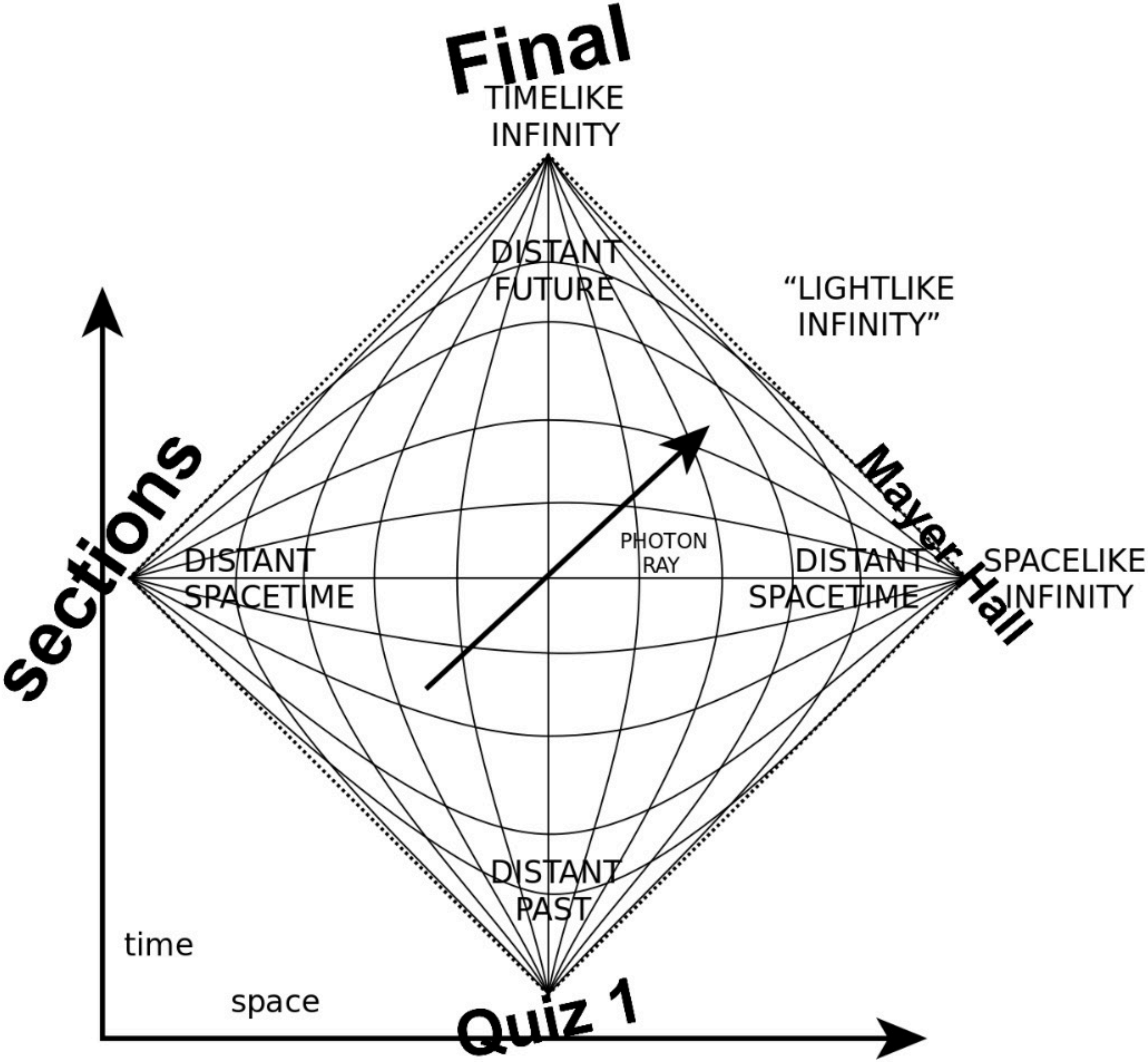
Minimum to pass this class:

know how to do <u>all</u> discussion problems.

Score < 7, attend sessions, use resources!




5th grade exercise nowadays!!
Landfill
Paper Towels
Dirty paper
Small pieces of snack wrappers
Plastic wrap
Puree Packs
Capri-Sun Packs
Trays
Design Box
Chip bags
wrappers
Pringles cans
Plastic bags
Plastic utensils
Straws
Cups
Bottle Caps
Recycling
Plastic bottles
Plastic containers
Milk cartons
Hard plastic
Paper
Soda Cans
Compost
Anything you can eat can go in compost!
Fruit
Vegetables
Meat
Dairy
Bread
Sauce
LIQUIDS
Anything that is liquid
Water
Milk
Juice
Tea
Karsten
Opening Circle
Instructions
Engineering Challenge
Clean Up
Closing Circle
Job Check
1. Turn on the light
2. Make a switch
3. Turn on 2 lights
4. Turn on the light without it touching the battery
Goal: Get quiet by Ø
1-2-3-4 System- Gresham
Calm Down Corner
1. Use the hand signal to show me you need the corner
2. One person at a time
3. Flip the timer as soon as you get in
4. Come out as soon as the timer is out
5. Be gentle with the box and materials. Don't lean on the sides or peel off the art.
6. Give space to the person in the box
Do Not touch



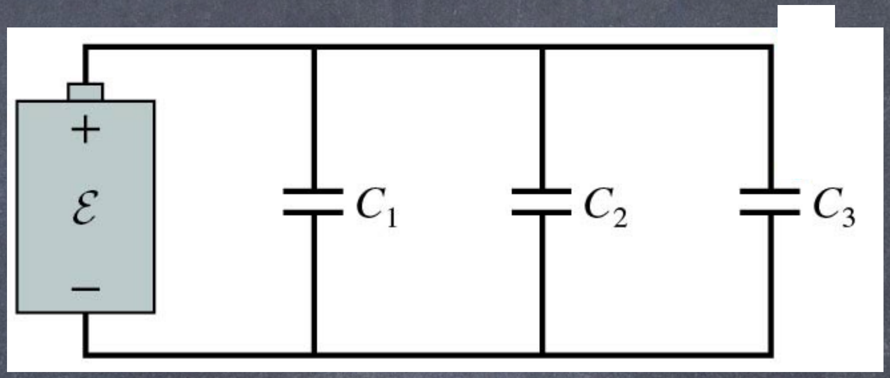
Final
TIMELIKE
INFINITY
DISTANT
FUTURE
"LIGHTLIKE
INFINITY"
sections
Mayer Hall
PHOTON
RAY
DISTANT
SPACETIME
DISTANT
SPACETIME
SPACELIKE
INFINITY
DISTANT
PAST
time
space
Quiz 1

Capacitors

Remember when you have multiple capacitors in a circuit that:

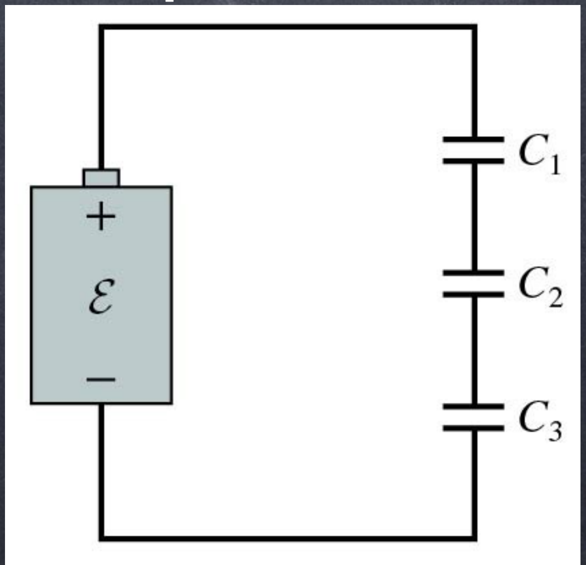
- Capacitors in **parallel** all have **same potential differences**
- The equivalent capacitance of the parallel capacitors also will have the same potential difference.

$$C_{\text{eq}} = C_1 + C_2 + C_3 + \cdots$$



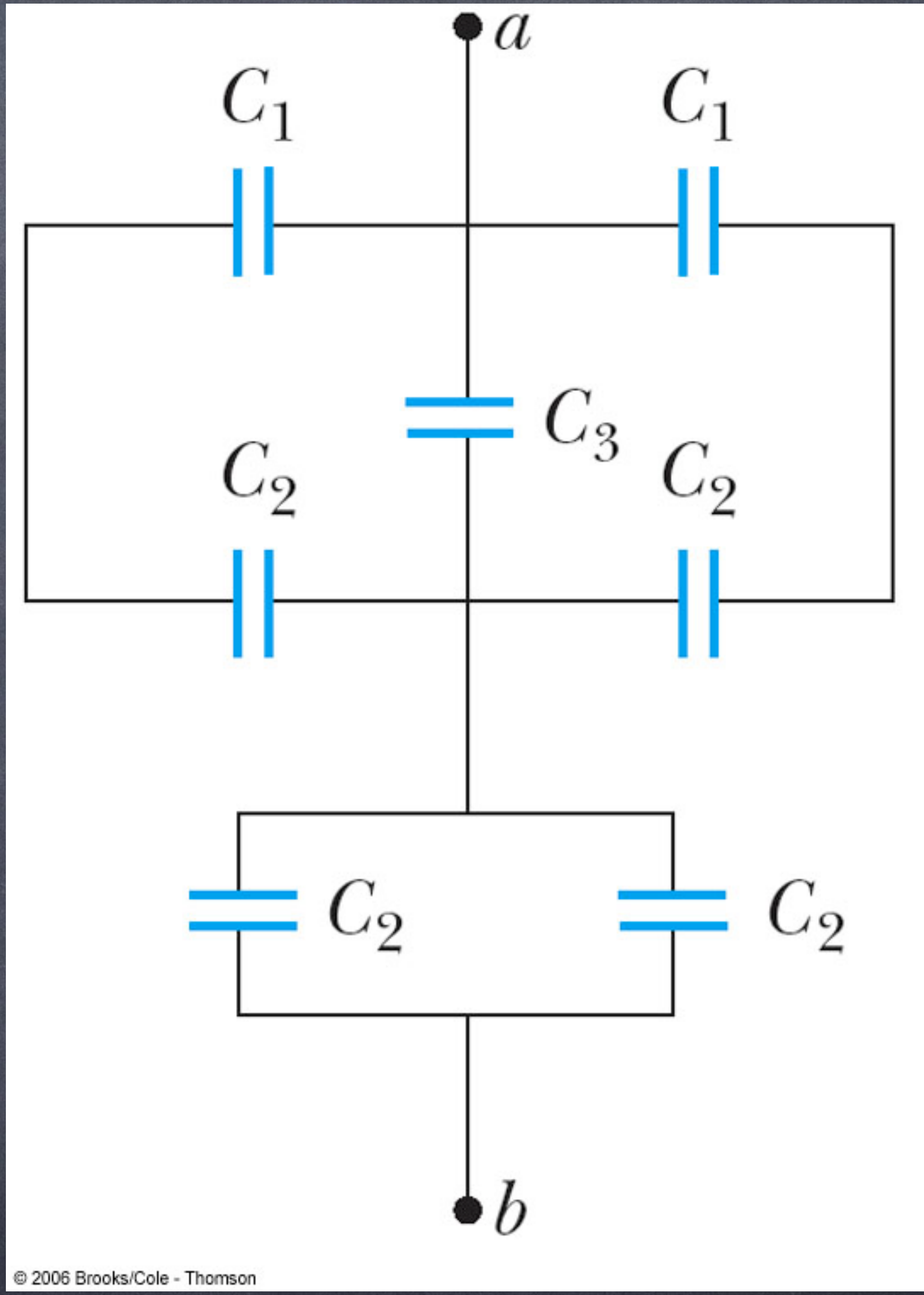
- Capacitors in **series** all have **same charge**
- The equivalent capacitor of the series capacitors also will have the same charge.

$$C_{\text{eq}} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots\right)^{-1}$$



Capacitor network, example

- Find the equivalent capacitance between points a and b in the group of capacitors connected in series as shown in the figure to the right (take $C_1 = 5.00\ \mu F$, $C_2 = 10.0\ \mu F$, and $C_3 = 2.00\ \mu F$). If the potential difference between points a and b is 60.0V, what is the charge stored on C_3?



- Answer
- Reduce the circuit by equivalent capacitance to find Q_3.
- Start with the C_1 and C_2 in series on the top.

Capacitor network, example

- Answer
- Combine the two top capacitors that are in series (left and right are the same) to form an equivalent C_S:

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{5\mu\text{F}} + \frac{1}{10\mu\text{F}}$$

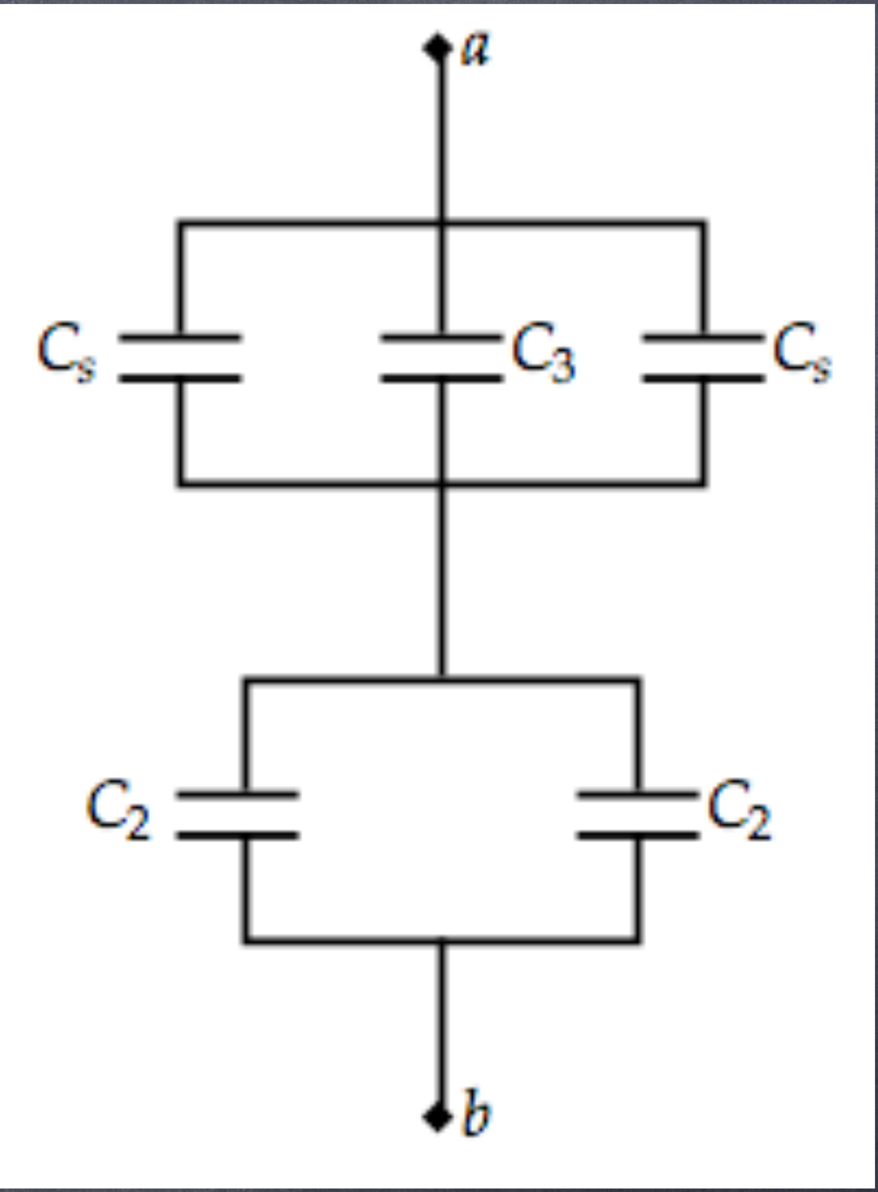
$$\frac{1}{C_S} = \frac{2}{10\mu\text{F}} + \frac{1}{10\mu\text{F}} = \frac{3}{10\mu\text{F}}$$

$$C_S = \frac{10\mu\text{F}}{3} = 3.33\mu\text{F}$$

- Next, combine the top three that are in parallel:

$$C_{\text{P1}} = C_S + C_3 + C_S$$

$$C_{\text{P1}} = 3.33\mu\text{F} + 2.00\mu\text{F} + 3.33\mu\text{F} = 8.66\mu\text{F}$$



Capacitor network, example

- Answer
- Next, combine the bottom two that are in parallel:

$$C_{P2} = C_2 + C_2$$

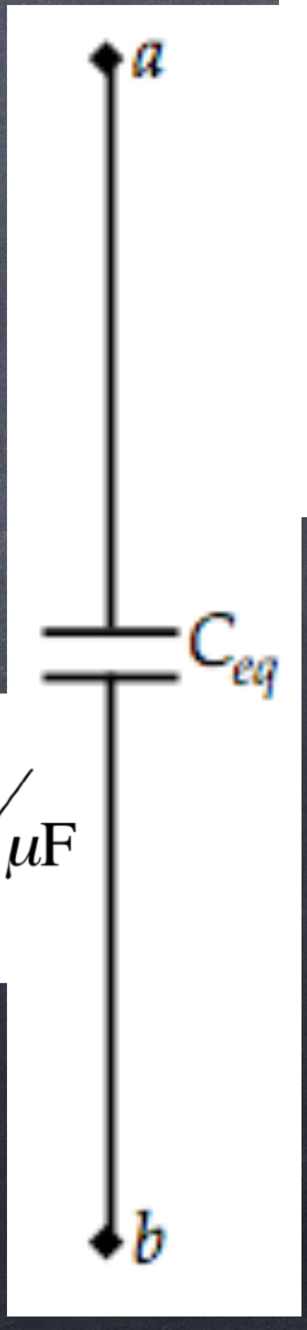
$$C_{P2} = 10.0\,\mu\text{F} + 10.0\,\mu\text{F} = 20.0\,\mu\text{F}$$

- Finally, combine the remaining two capacitors that are in series:

$$\frac{1}{C_{eq}} = \frac{1}{C_{P1}} + \frac{1}{C_{P2}} = \frac{1}{8.66\,\mu\text{F}} + \frac{1}{20\,\mu\text{F}}$$

$$\frac{1}{C_{eq}} = (0.1155)\,{}^{1}\!/_{\mu\text{F}} + (0.0500)\,{}^{1}\!/_{\mu\text{F}} = (0.1655)\,{}^{1}\!/_{\mu\text{F}}$$

$$C_{eq} = 6.04\,\mu\text{F}$$



Capacitor network, example

- Answer

- Next, we need to find the total charge stored on the equivalent capacitor:

$$Q_{eq} = C_{eq}(\Delta V_{ab})$$

$$Q_{eq} = 6.04\,\mu\text{F}(60\text{V}) = 362\,\mu\text{C}$$

- Looking back at the two equivalent capacitors in series (C_{p1} and C_{p2}), we see that they must have the same amount of charge:

$$Q_{eq} = Q_{P1} = Q_{P2} = 362\,\mu\text{C}$$

- This means that they potential difference across C_{p1} is:

$$\Delta V_{P1} = \frac{Q_{P1}}{C_{P1}}$$

$$\Delta V_{P1} = \frac{362\,\mu\text{C}}{8.66\,\mu\text{F}} = 41.8\text{V}$$

Capacitor network, example

- Answer

- We turn back to the three top capacitors in parallel (C_S, C_3, and C_S).

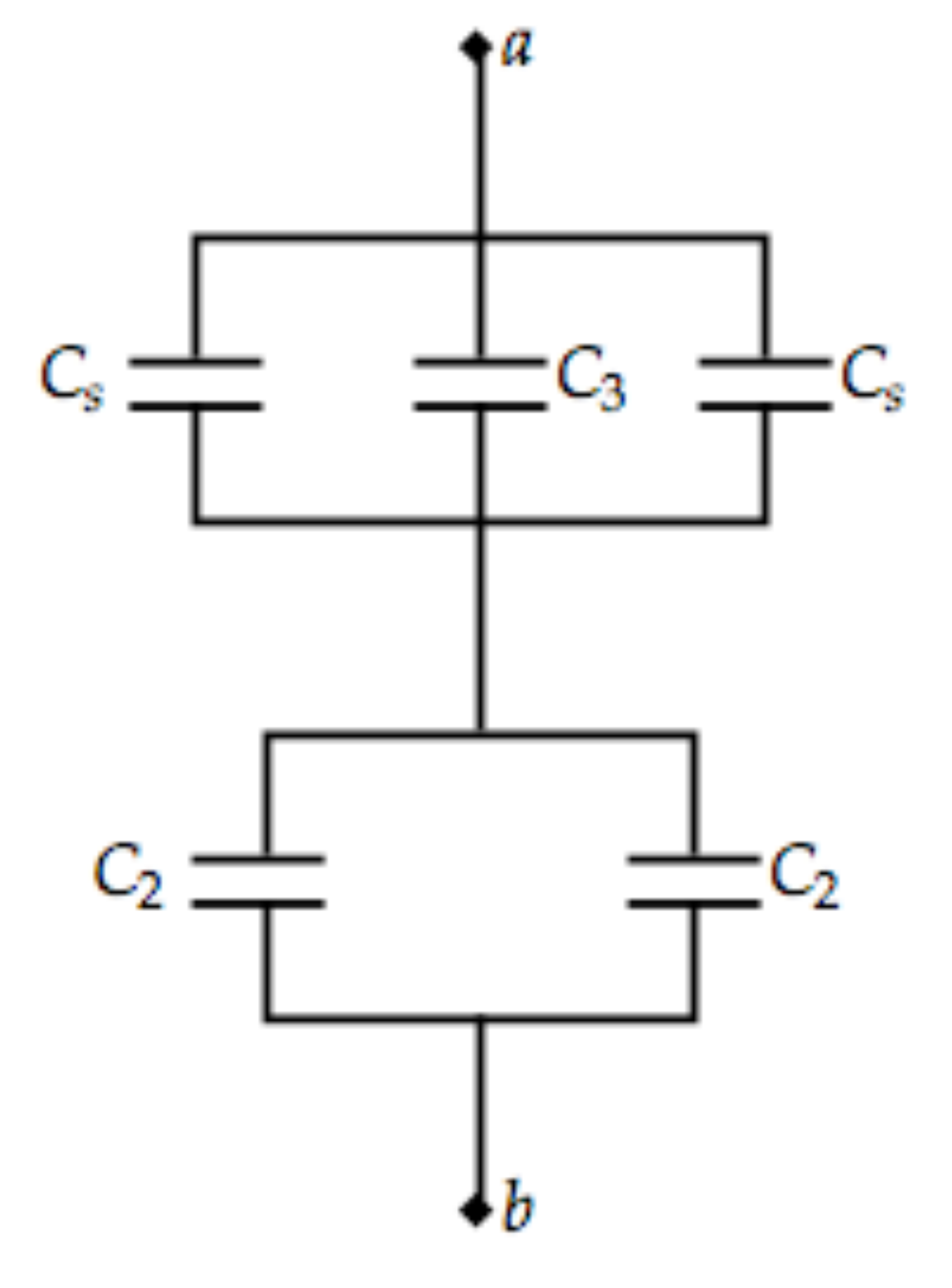
- We note that all three of these capacitors must have the same potential difference as each other (and C_{p1}).

$$\Delta V_{C3} = 41.8\text{V}$$

- This means that the charge on C_3 is:

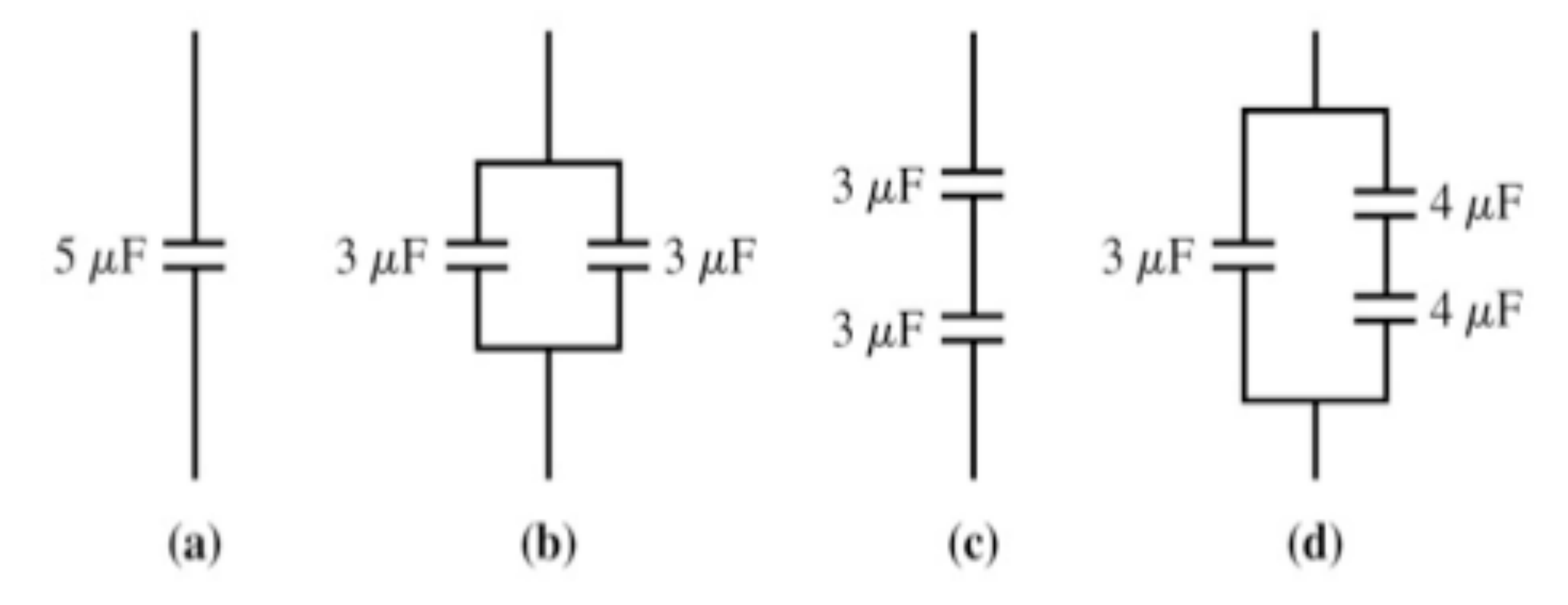
$$Q_3 = C_3\left(\Delta V_{C3}\right)$$

$$Q_3 = 2.00\mu\text{F}\left(41.8\text{V}\right) = 83.6\mu\text{C}$$



iClicker take home Question

- Rank in order, from largest to smallest the equivalent capacitance C_a to C_d of the circuits a to d.



- A) $C_d > C_b > C_a > C_c$.
- B) $C_d > C_b = C_c > C_a$.
- C) $C_a > C_b = C_c > C_d$.
- D) $C_b > C_a = C_d > C_c$.
- E) $C_c > C_a = C_d > C_b$.

Energy stored in a Capacitor

- Suppose you have a given capacitor charged to a potential difference ΔV with a charge ±q on either plate. How much energy would it take to transfer an small amount of charge dq?

- Start with the relationship between work and electric potential:

$$dW = (\Delta V)\,dq$$

$$dW = \left(\frac{q}{C}\right)dq$$

- If we wanted to calculate the entire work done to fully charge the capacitor in this manner (from q = 0 to q = Q) we have:

$$W = \int dW = \int_0^Q \left(\frac{q}{C}\right)dq$$

Energy of a Capacitor

- Since C merely depends on the geometry of the capacitor, we have:

$$W = \frac{1}{C}\int_0^Q (q)\,dq \qquad W = \frac{1}{C}\left(\frac{Q^2}{2}\right)$$

- Since the electric force is conservative, the energy stored (U_{elec}) in the capacitor is equivalent to the work put in:

$$\text{Energy stored} = \tfrac{1}{2}Q(\Delta V) = \tfrac{1}{2}C(\Delta V)^2 = \frac{Q^2}{2C}$$

- The main use of a capacitor is to store and then discharge energy.

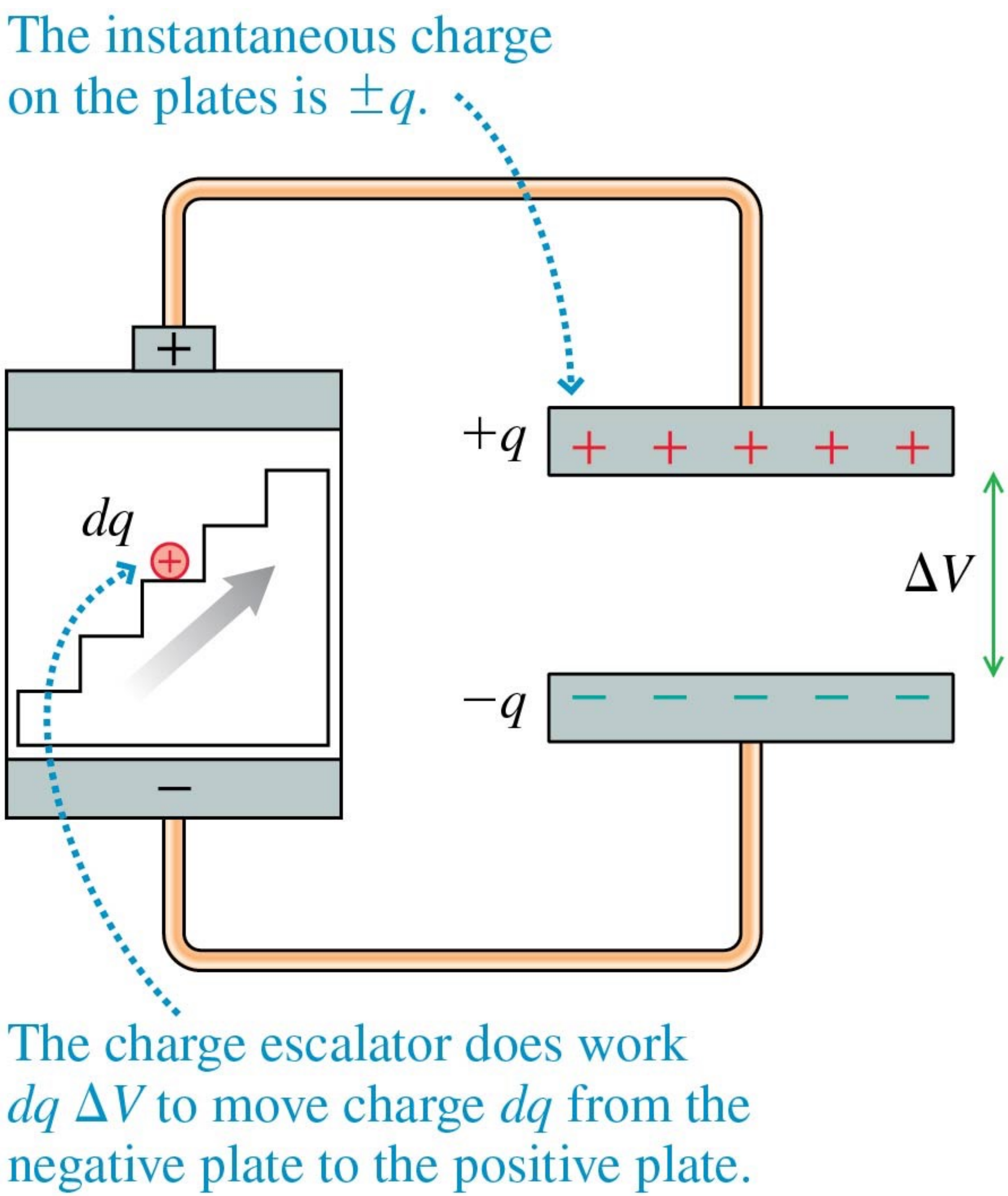
The Energy Stored in a Capacitor

- The figure shows a capacitor being charged.
- As a small charge *dq* is lifted to a higher potential, the potential energy of the capacitor increases by

$$dU = dq\,\Delta V = \frac{q\,dq}{C}$$

- The total energy transferred from the battery to the capacitor is

$$U_C = \frac{1}{C}\int_0^Q q\,dq = \frac{Q^2}{2C}$$



write eq on blackboard!

The Energy Stored in a Capacitor

- Capacitors are important elements in electric circuits because of their ability to store energy.
- The charge on the two plates is $\pm q$ and this charge separation establishes a potential difference $\Delta V = q/C$ between the two electrodes.
- In terms of the capacitor's potential difference, the potential energy stored in a capacitor is

$$U_{\mathrm{C}} = \frac{Q^2}{2C} = \frac{1}{2}C(\Delta V_{\mathrm{C}})^2$$

write eq on blackboard!

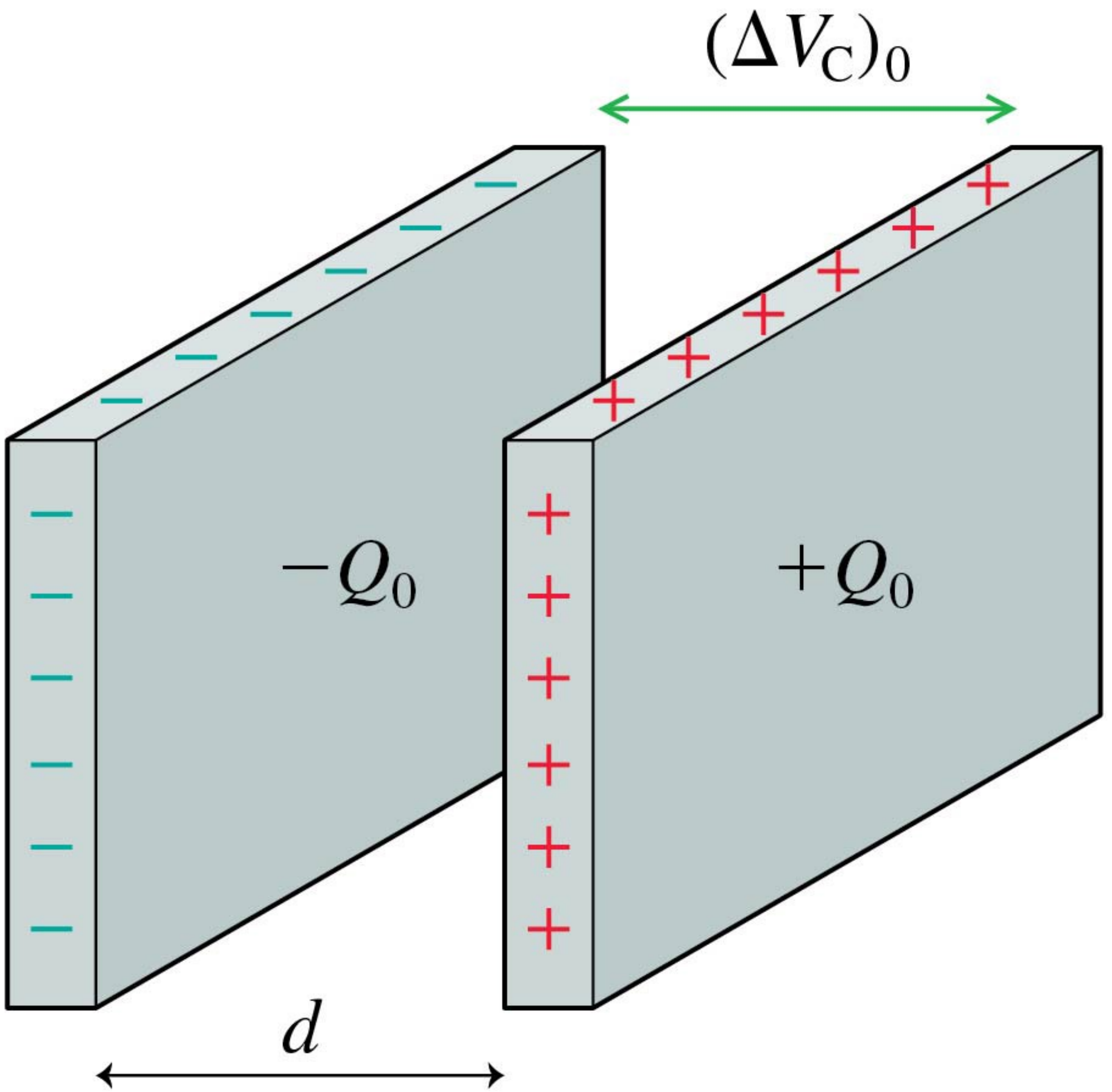
The Energy Stored in a Capacitor

- A capacitor can be charged slowly but then can release the energy very quickly.
- An important medical application of capacitors is the *defibrillator*.

- A heart attack or a serious injury can cause the heart to enter a state known as *fibrillation* in which the heart muscles twitch randomly and cannot pump blood.
- A strong electric shock through the chest completely stops the heart, giving the cells that control the heart's rhythm a chance to restore the proper heartbeat.

Dielectrics

- The figure shows a parallel-plate capacitor with the plates separated by a vacuum.
- When the capacitor is fully charged to voltage $(\Delta V_C)_0$, the charge on the plates will be $\pm Q_0$, where $Q_0 = C_0(\Delta V_C)_0$.
- In this section the subscript 0 refers to a vacuum-filled capacitor.
- How do we increase C?



Capacitance C_0 in vacuum

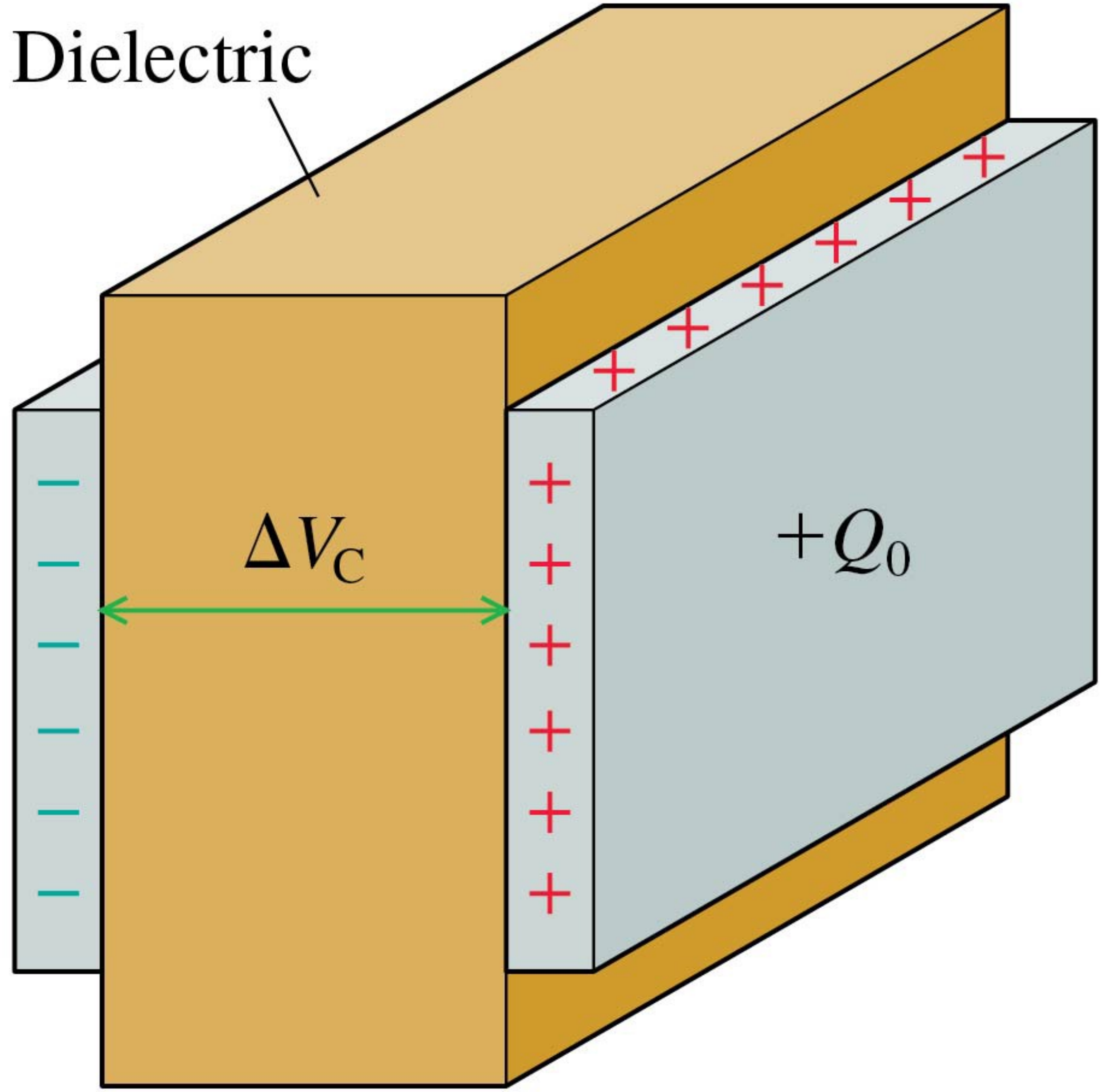
Dielectrics

- Now an insulating material is slipped between the capacitor plates.
- An insulator in an electric field is called a dielectric.
- The charge on the capacitor plates does not change

  $(Q = Q_0)$.
- However, the voltage has *decreased*:

  $$\Delta V_C < (\Delta V_C)_0$$

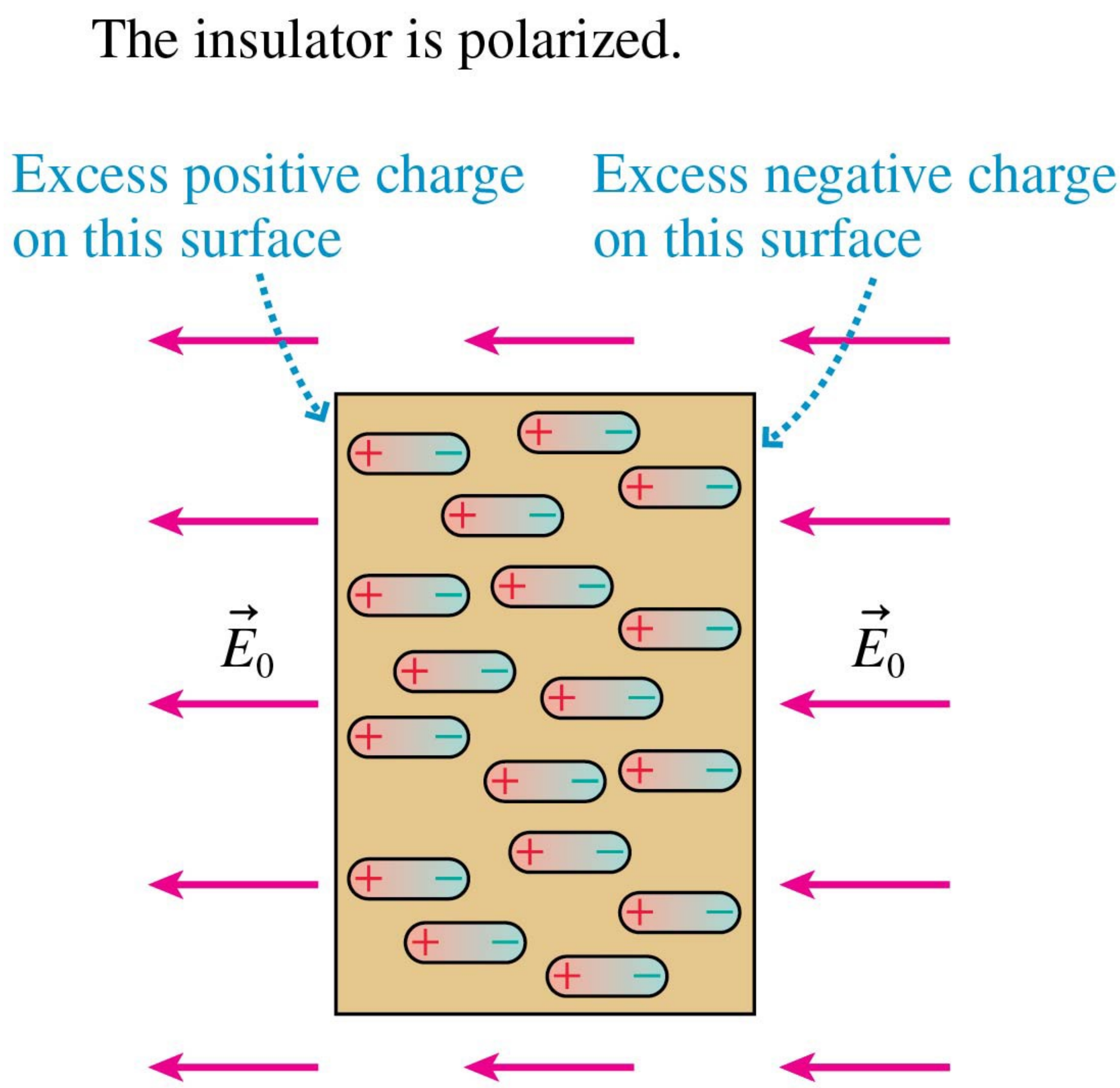
  WHY?



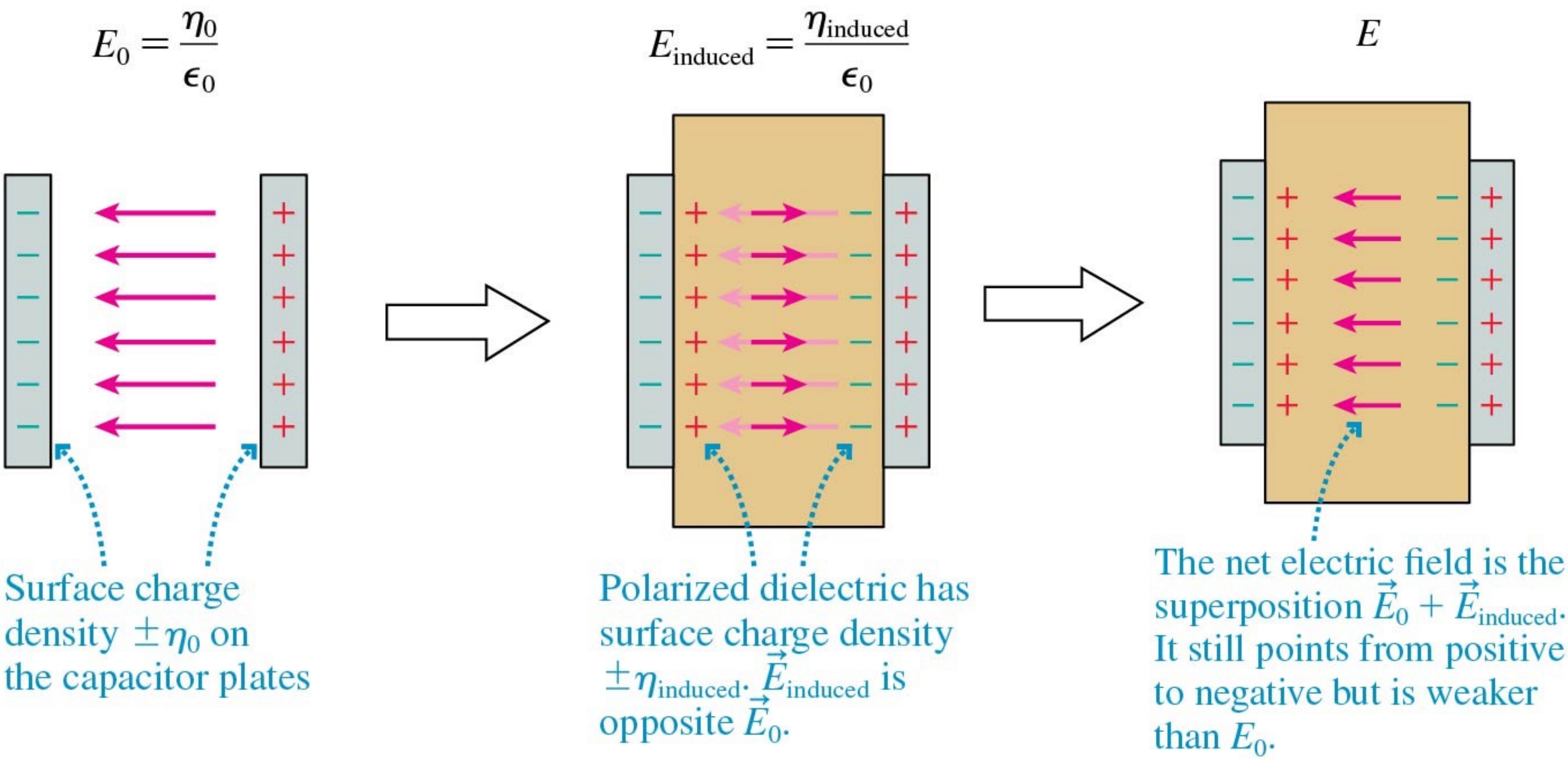
Capacitance $C > C_0$

Dielectrics

- The figure shows how an insulating material becomes polarized in an external electric field.
- The insulator as a whole is still neutral, but the external electric field separates positive and negative charge.

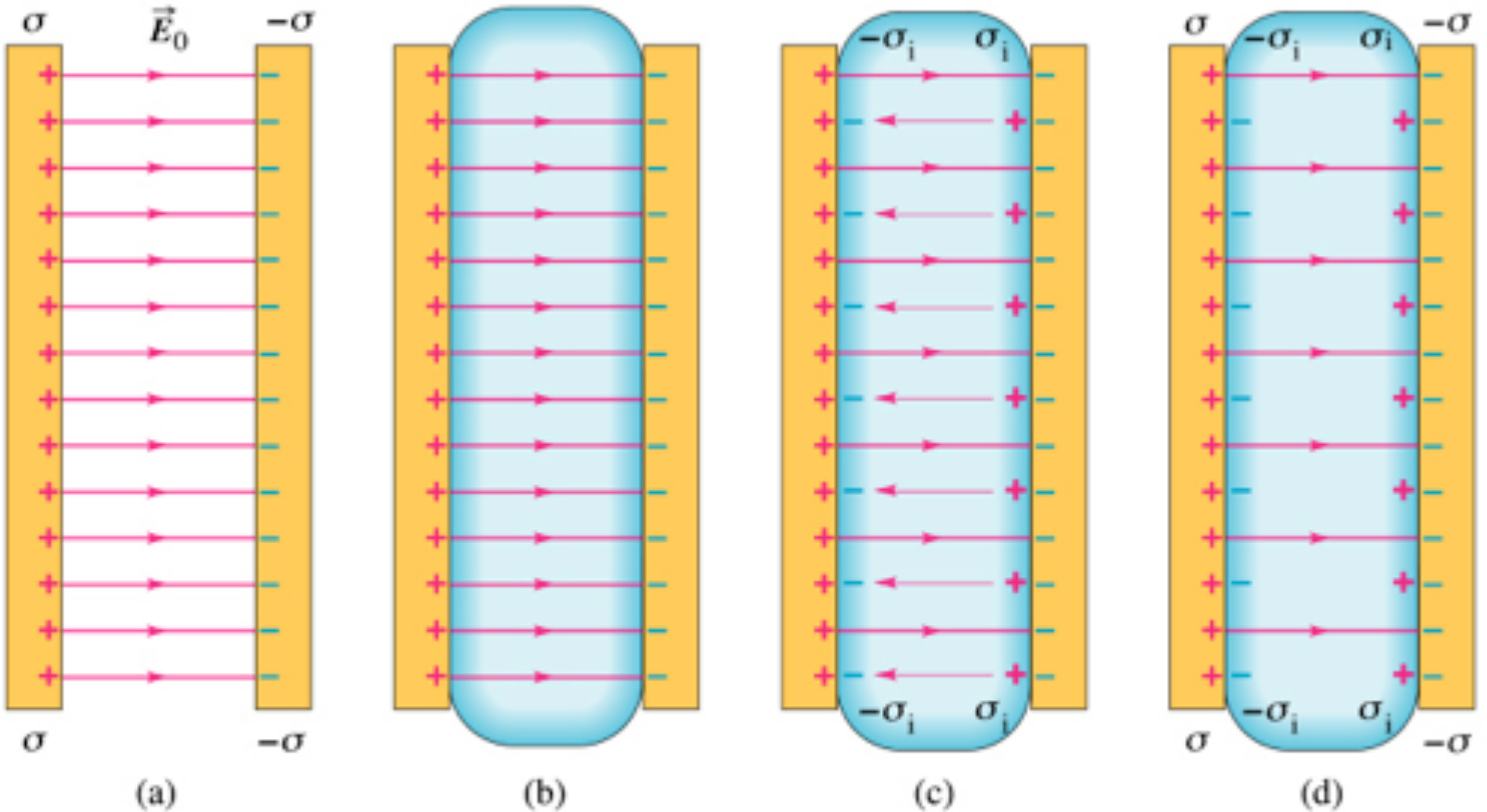


Dielectrics



Dielectrics

- Easily polarized materials have larger dielectric constants than materials not easily polarized.



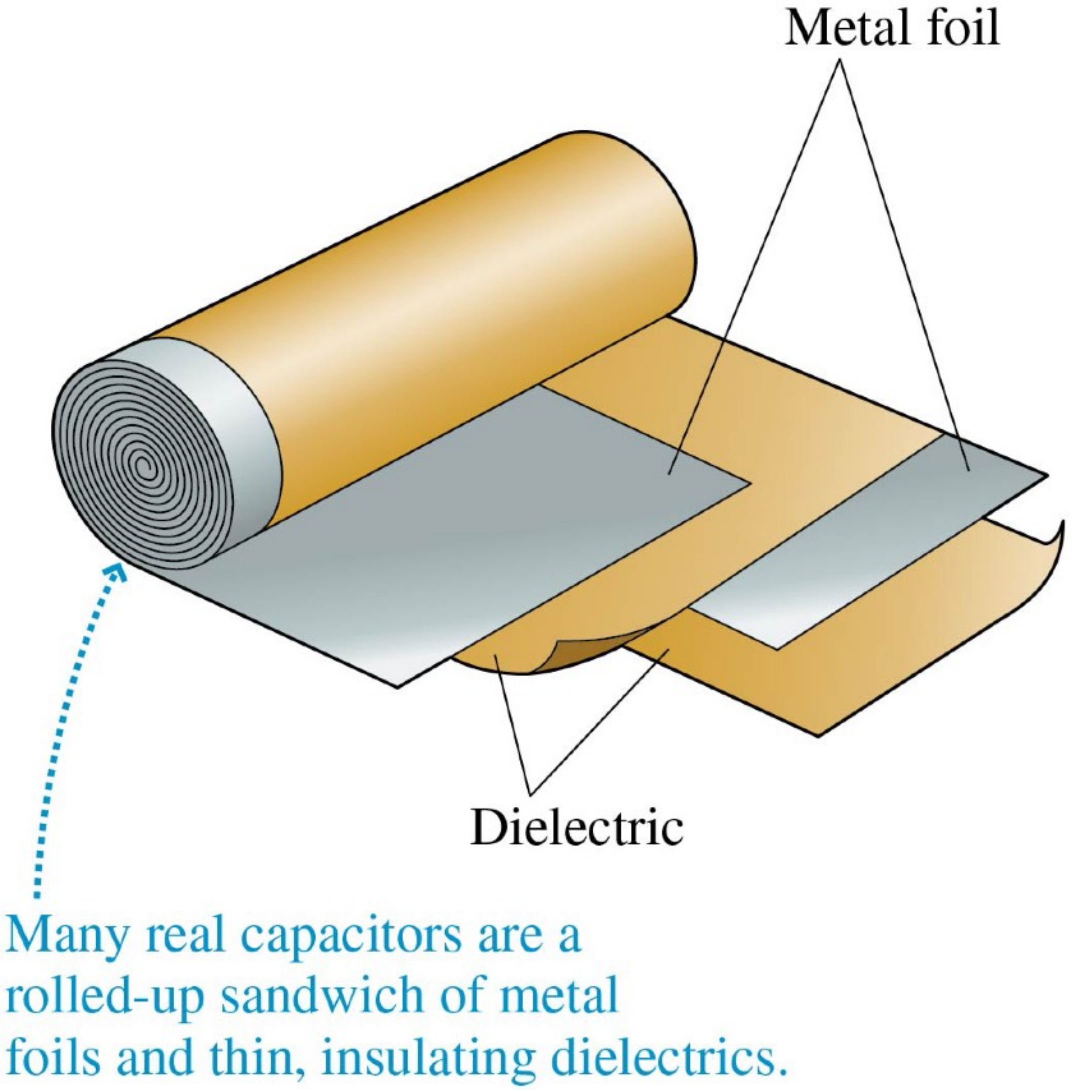
Dielectrics

- We define the **dielectric constant**: $$\kappa \equiv \frac{E_0}{E}$$
- The dielectric constant, like density or specific heat, is a property of a material.
- Easily polarized materials have larger dielectric constants than materials not easily polarized.
- Vacuum has $\kappa = 1$ exactly.
- **Filling a capacitor with a dielectric increases the capacitance by a factor equal to the dielectric constant:**

$$C = \frac{Q}{\Delta V_C} = \frac{Q_0}{(\Delta V_C)_0/\kappa} = \kappa \frac{Q_0}{(\Delta V_C)_0} = \kappa C_0$$

Dielectrics

- The production of a practical capacitor, as shown, almost always involves the use of a solid or liquid dielectric.
- All materials have a maximum electric field they can sustain without breakdown—the production of a spark.
- The breakdown electric field of air is about 3×10^6 V/m.
- A material's maximum sustainable electric field is called its **dielectric strength**.



Dielectrics

- Filling a capacitor with a dielectric increases the capacitance by a factor equal to the dielectric constant.
- Remember: the capacitance for a parallel-plate capacitor changes to:

$$C = \kappa \varepsilon_o \frac{A}{d}$$

- Note that the dielectric constant is a unitless variable.

Table 25-1

Some Properties of Dielectrics[a]

Material	Dielectric Constant κ	Dielectric Strength (kV/mm) (10^6 V/m)
Air (1 atm)	1.00054	3
Polystyrene	2.6	24
Paper	3.5	16
Transformer oil	4.5	
Pyrex	4.7	14
Ruby mica	5.4	
Porcelain	6.5	
Silicon	12	
Germanium	16	
Ethanol	25	
Water (20°C)	80.4	
Water (25°C)	78.5	
Titania ceramic	130	
Strontium titanate	310	8

For a vacuum, κ = unity.

Dielectric

- In general, you can say that in a region completely filled by a dielectric material of dielectric constant κ, that all equations containing ε_o are replaced with $\kappa\varepsilon_o$.
- If you have a partially filled region, the common thing to do is turn to Gauss' Law.
- For any given plate separation, there is a maximum electric field that can be produced in the dielectric before it breaks down and begins to conduct.
- This maximum electric field is called the **dielectric strength** (measured in N/C).

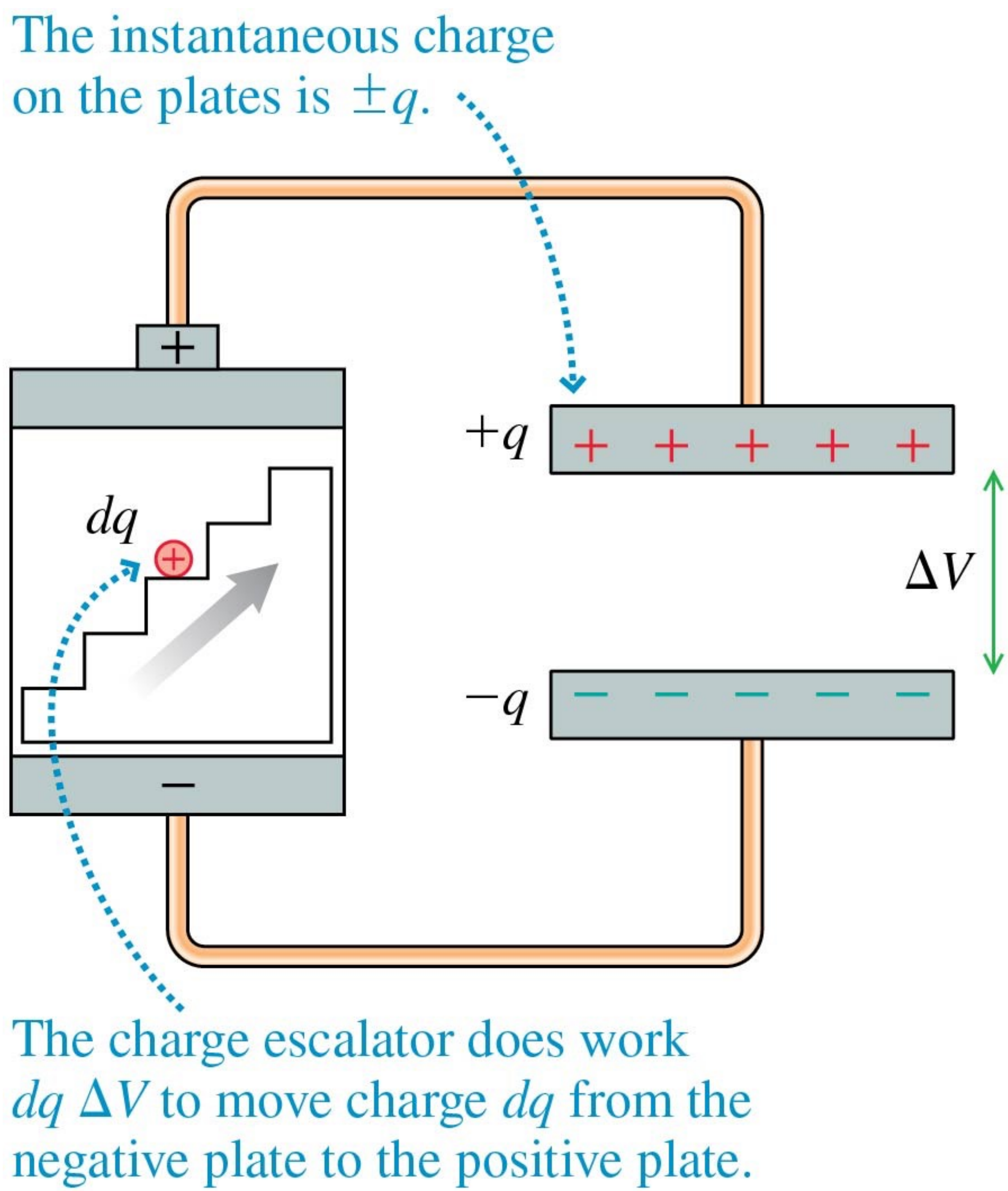
The Energy Stored in a Capacitor

- The figure shows a capacitor being charged.
- As a small charge *dq* is lifted to a higher potential, the potential energy of the capacitor increases by

$$dU = dq\,\Delta V = \frac{q\,dq}{C}$$

- The total energy transferred from the battery to the capacitor is

$$U_C = \frac{1}{C}\int_0^Q q\,dq = \frac{Q^2}{2C}$$



write eq on blackboard!

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- The charge on the two plates is $\pm q$ and this charge separation establishes a potential difference $\Delta V = q/C$ between the two electrodes.
- In terms of the capacitor's potential difference, the potential energy stored in a capacitor is

$$U_C = \frac{Q^2}{2C} = \frac{1}{2}C(\Delta V_C)^2$$

write eq on blackboard!

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- A strong electric shock through the chest completely stops the heart, giving the cells that control the heart's rhythm a chance to restore the proper heartbeat.

iClicker question 9-1

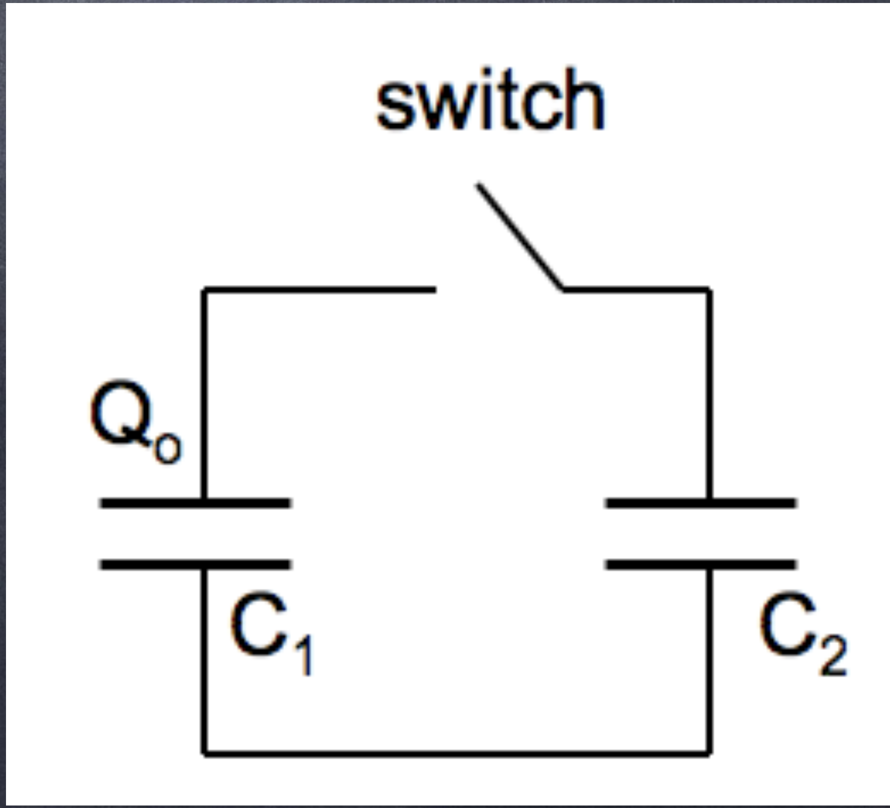
A capacitor charged to $1.5\ \mathrm{V}$ stores $2.0\ \mathrm{mJ}$ of energy. If the capacitor is charged to $3.0\ \mathrm{V}$, it will store

A. $1.0\ \mathrm{mJ}$
B. $2.0\ \mathrm{mJ}$
C. $4.0\ \mathrm{mJ}$
D. $6.0\ \mathrm{mJ}$
E. $8.0\ \mathrm{mJ}$

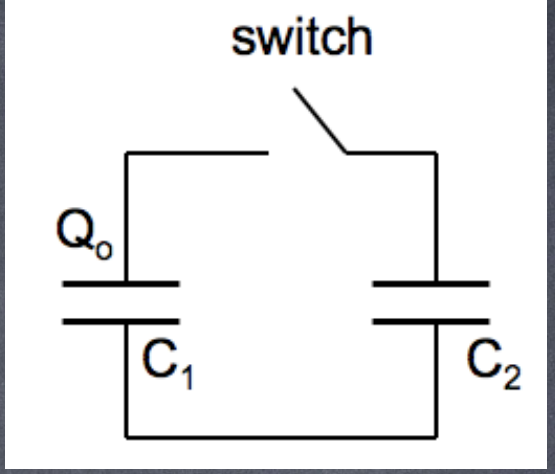
Capacitance, example

- <u>Example</u>
- A 3.55 µF capacitor (C_1) is charged to a potential difference $\Delta V_o = 6.30$ V, using a battery. The charging battery is then removed, and the capacitor is connected to an uncharged 8.95 µF capacitor (C_2). After the switch is closed, charge flows from C_1 to C_2 until an equilibrium is established with both capacitors at the same potential difference, ΔV_f. What energy is stored in the two capacitors after the switch has been closed?

- Answer
- Usually no coordinate system needs to be defined for a capacitor (start with the original charge amount Q_o.)



Capacitance, example



- Answer
- The original charge, Q_o, shared by the two capacitors is:

$$Q_o = Q_1 + Q_2$$

- Next, turn to the definition of capacitance:

$$C = \frac{Q}{\Delta V}$$

- Substituting back into our charge equation gives us:

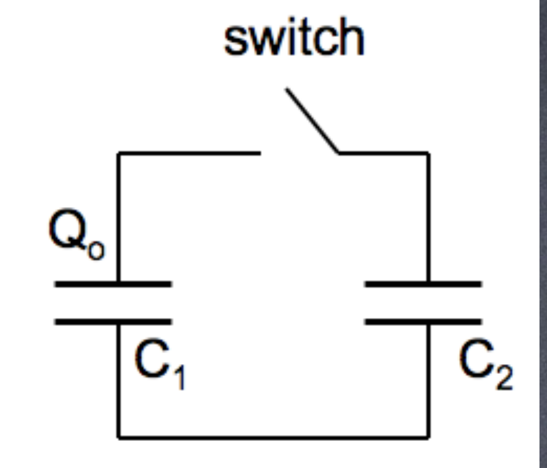
$$C_1(\Delta V_o) = C_1(\Delta V_1) + C_2(\Delta V_2)$$

- But we know the final potential difference is the same between the two capacitors:

$$\Delta V_1 = \Delta V_2 = \Delta V_f$$

$$C_1(\Delta V_o) = C_1(\Delta V_f) + C_2(\Delta V_f) = (C_1 + C_2)\Delta V_f$$

Capacitance



- Answer
- Solving for $\mathscr{D}V_f$ gives us:

$$\Delta V_f = \frac{C_1(\Delta V_o)}{(C_1 + C_2)}$$

$$\Delta V_f = \frac{3.55\mu\text{F}(6.30\text{V})}{(3.55\mu\text{F} + 8.95\mu\text{F})} = 1.79\text{V}$$

- Putting this common potential difference into the energy equation gives us:

$E_0 = 3.55\mu\text{F}(6.3\text{V})^2/2 = 70.5\mu\text{J}$

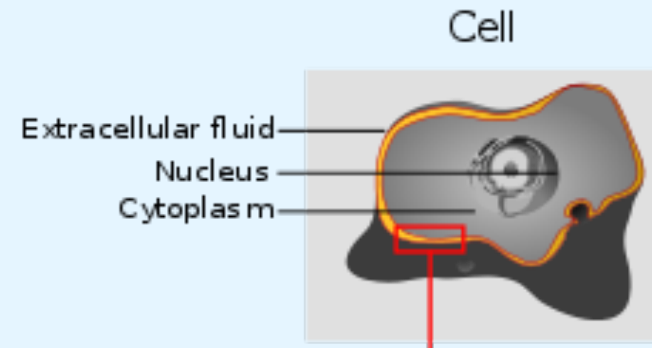
$$E_{Tot} = E_1 + E_2$$

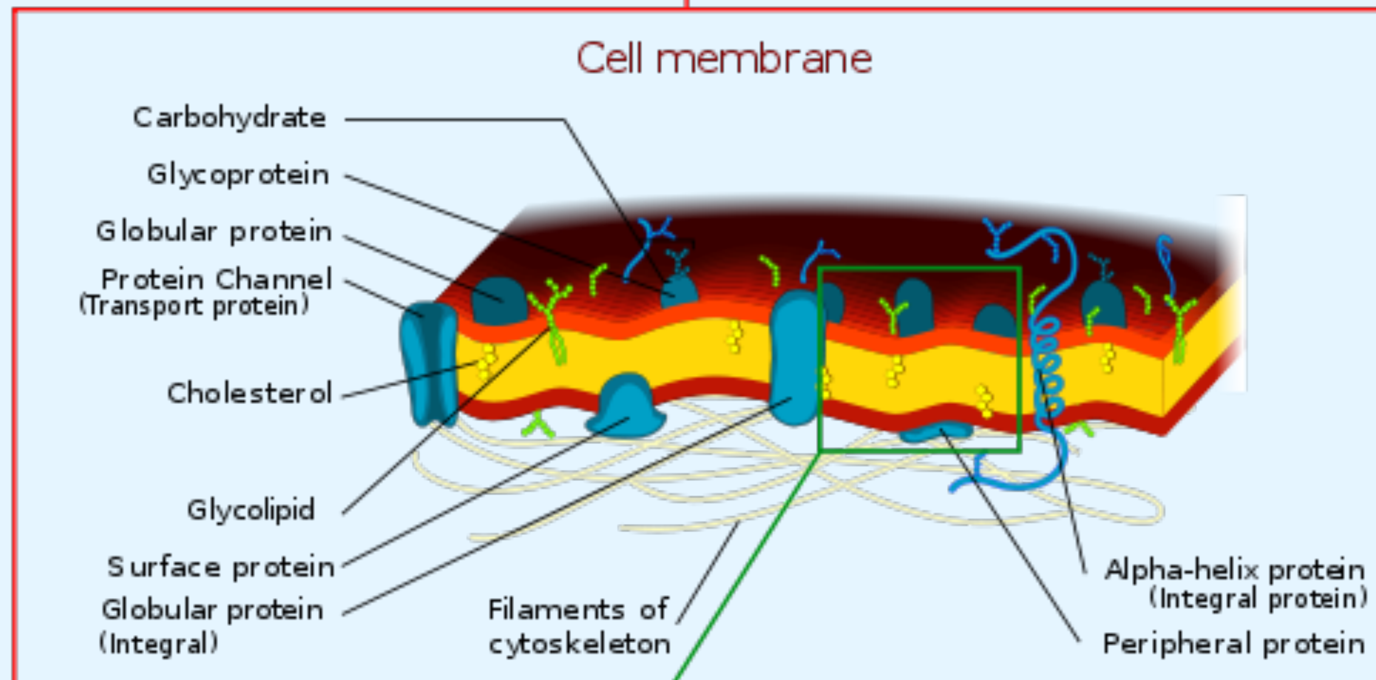
$$E_{Tot} = \tfrac{1}{2}C_1(\Delta V_f)^2 + \tfrac{1}{2}C_2(\Delta V_f)^2$$

$$E_{Tot} = \tfrac{1}{2}(C_1 + C_2)(\Delta V_f)^2$$

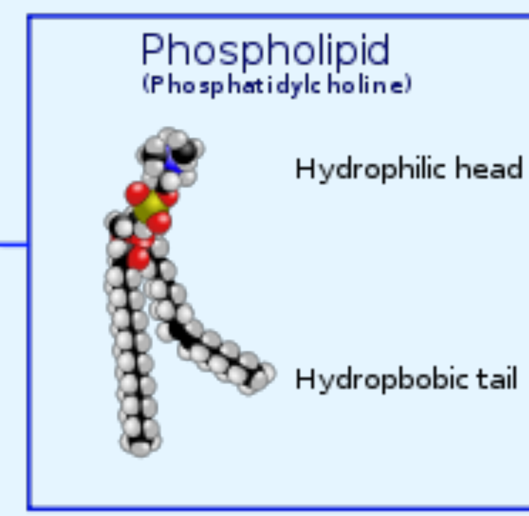
$$E_{Tot} = \tfrac{1}{2}(3.55\mu\text{F} + 8.95\mu\text{F})(1.79\text{V})^2 = 20.0\mu\text{J}$$

Cellular technology

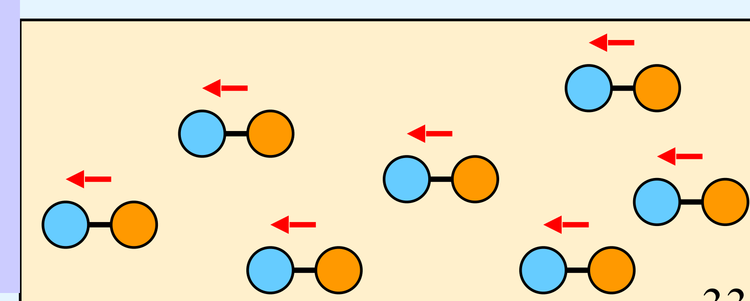




"... local dielectric properties can play crucial roles in membrane functions."
-- Local Dielectric Properties Around Polar Region of Lipid Bilayer Membranes, *J. Membrane Biol.* 85,225-231 (1985)

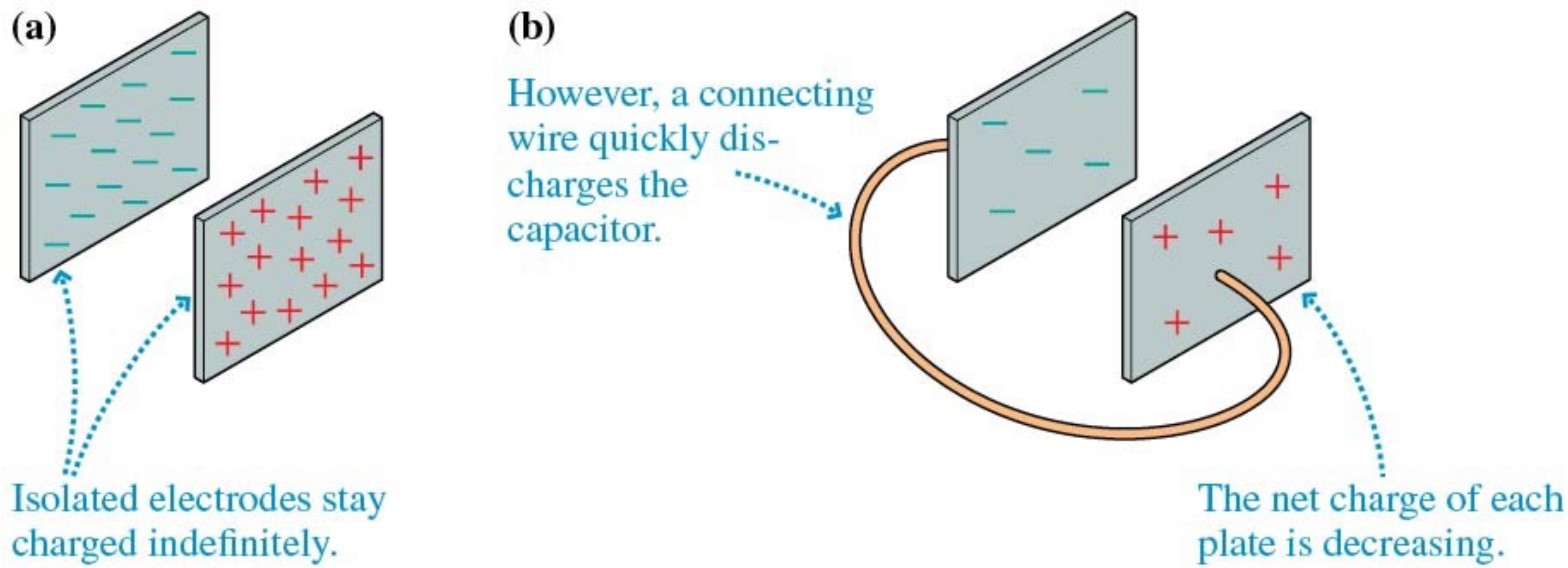


"An important physical effect of cholesterol is to increase the membrane's internal electrical dipole potential, which is one of the major mechanisms by which it modulates ion permeability. The dipole potential ... arises because of the alignment of dipolar residues ... in the .. interior of the membrane. ... its magnitude can vary from ~100 to >400 mV.... Recent investigations have suggested that it affects numerous different biological membrane processes."
-- Cholesterol Effect on the Dipole Potential of Lipid Membranes, *Biophys J.* 2006 June 1; 90(11).



- How does a capacitor get discharged?

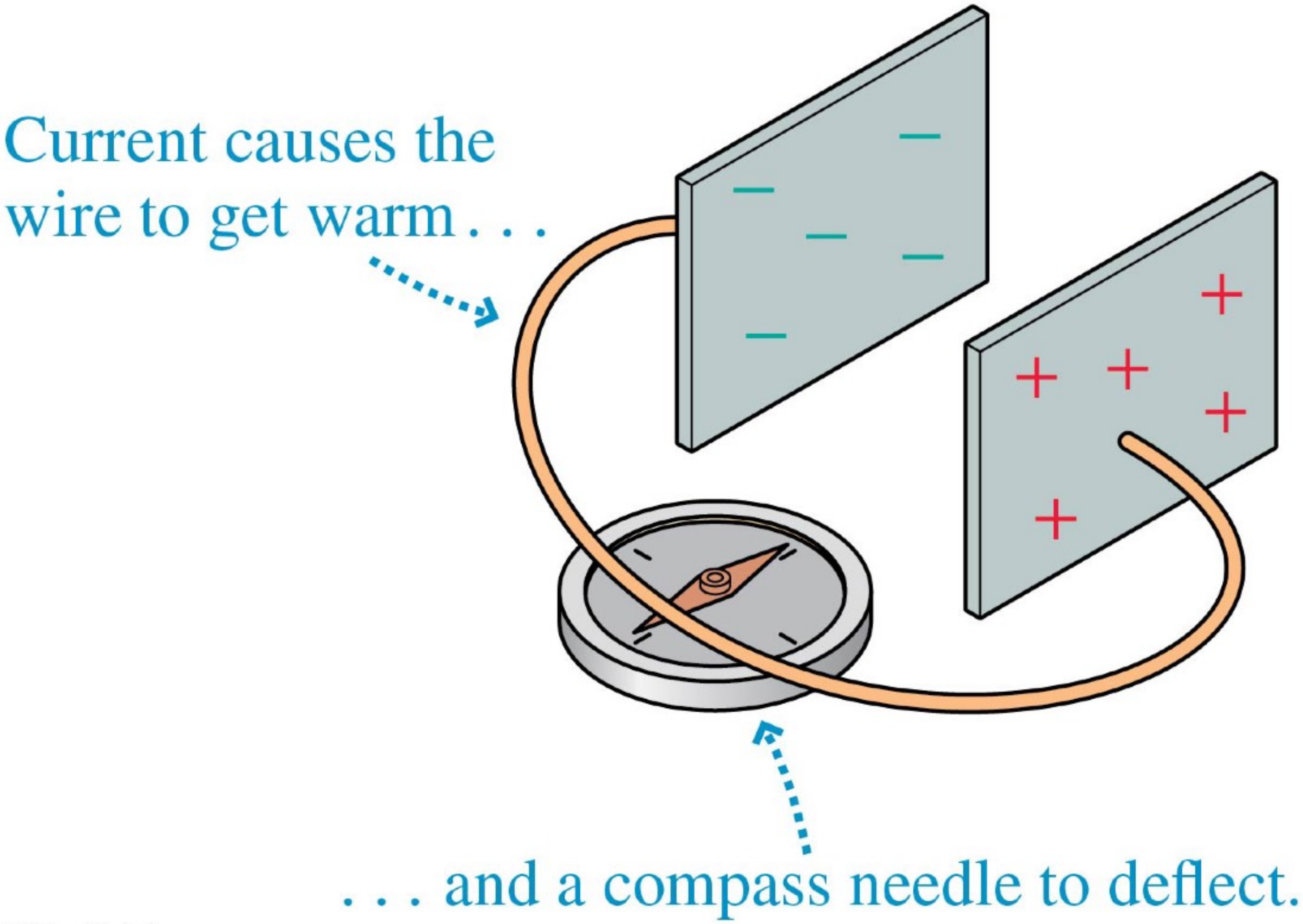
- Figure (a) shows a charged capacitor in equilibrium.
- Figure (b) shows a wire discharging the capacitor.



- *As* the capacitor is discharging, there is a *current* in the wire.

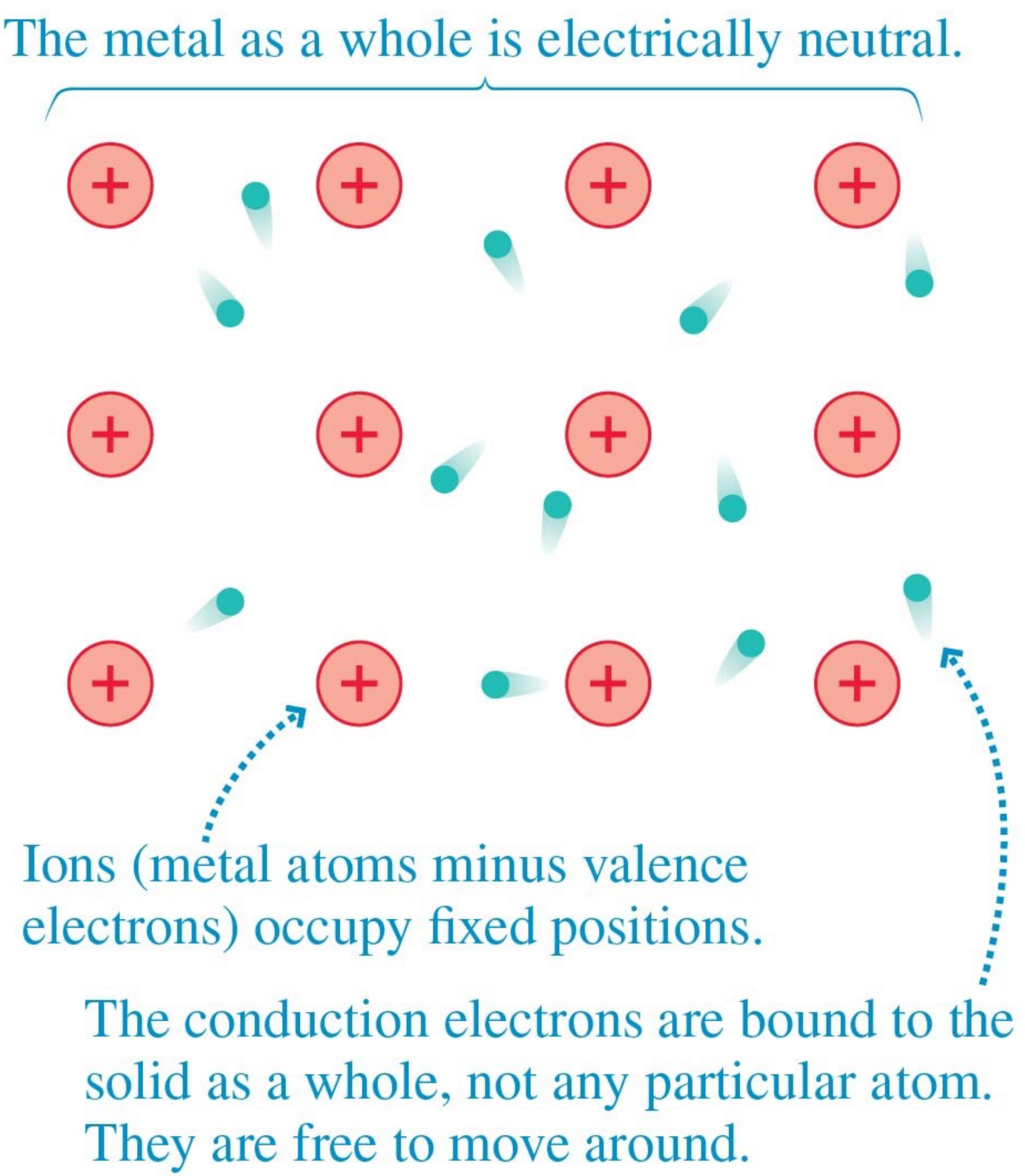
Electric Current

- When a current is flowing, the conductors are *not* in electrostatic equilibrium.
- Though you cannot see current directly, there are certain *indicators* that current is present in a wire.



Charge Carriers

- The outer electrons of metal atoms are only weakly bound to the nuclei.
- In a metal, the outer electrons become detached from their parent nuclei to form a fluid-like *sea of electrons* that can move through the solid.
- **Electrons are the charge carriers in metals.**

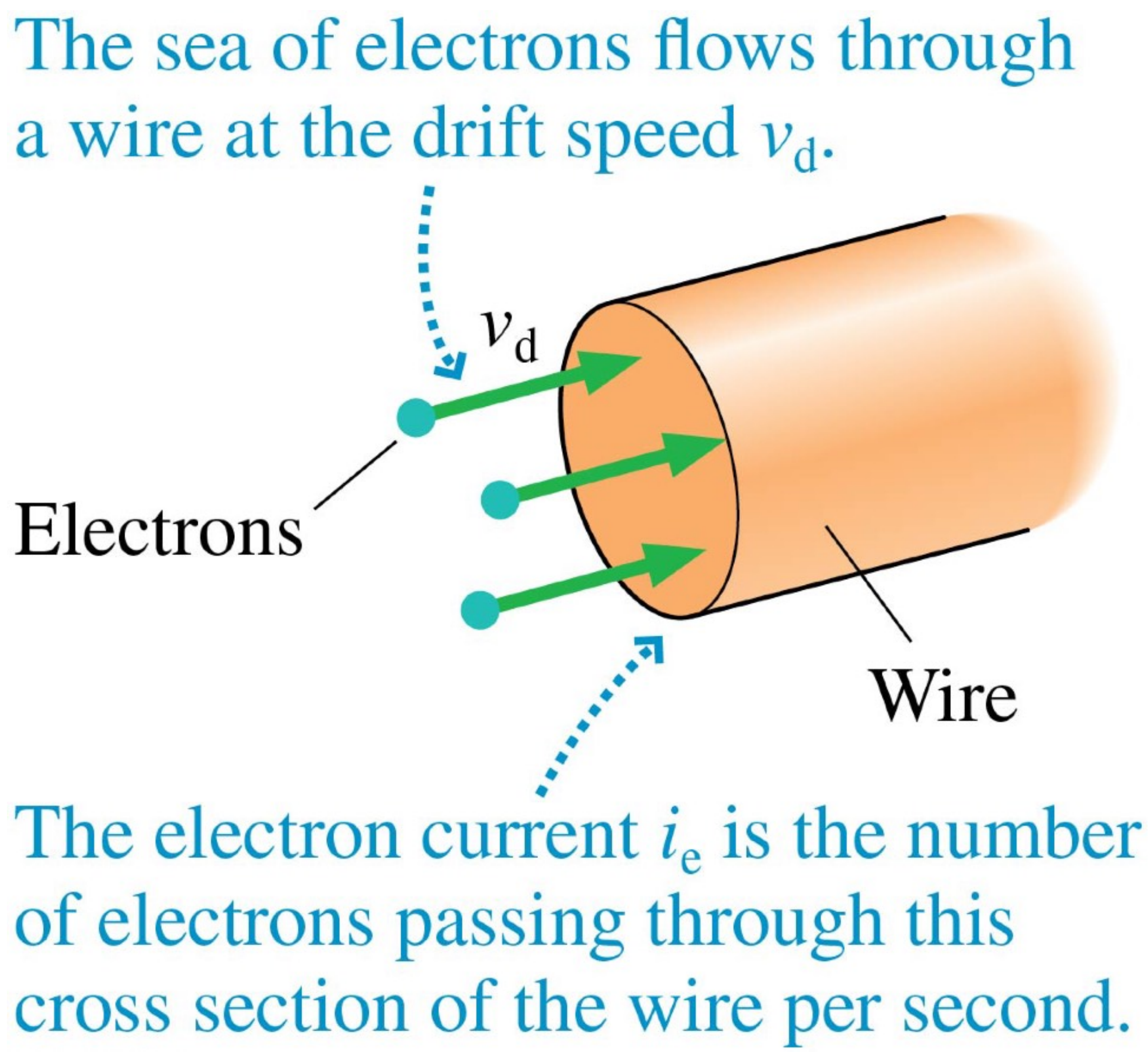


The Electron Current

- We define the **electron current** i_e to be the number of electrons per second that pass through a cross section of the conductor.

- The number N_e of electrons that pass through the cross section during the time interval Δt is

$$N_e = i_e \Delta t$$



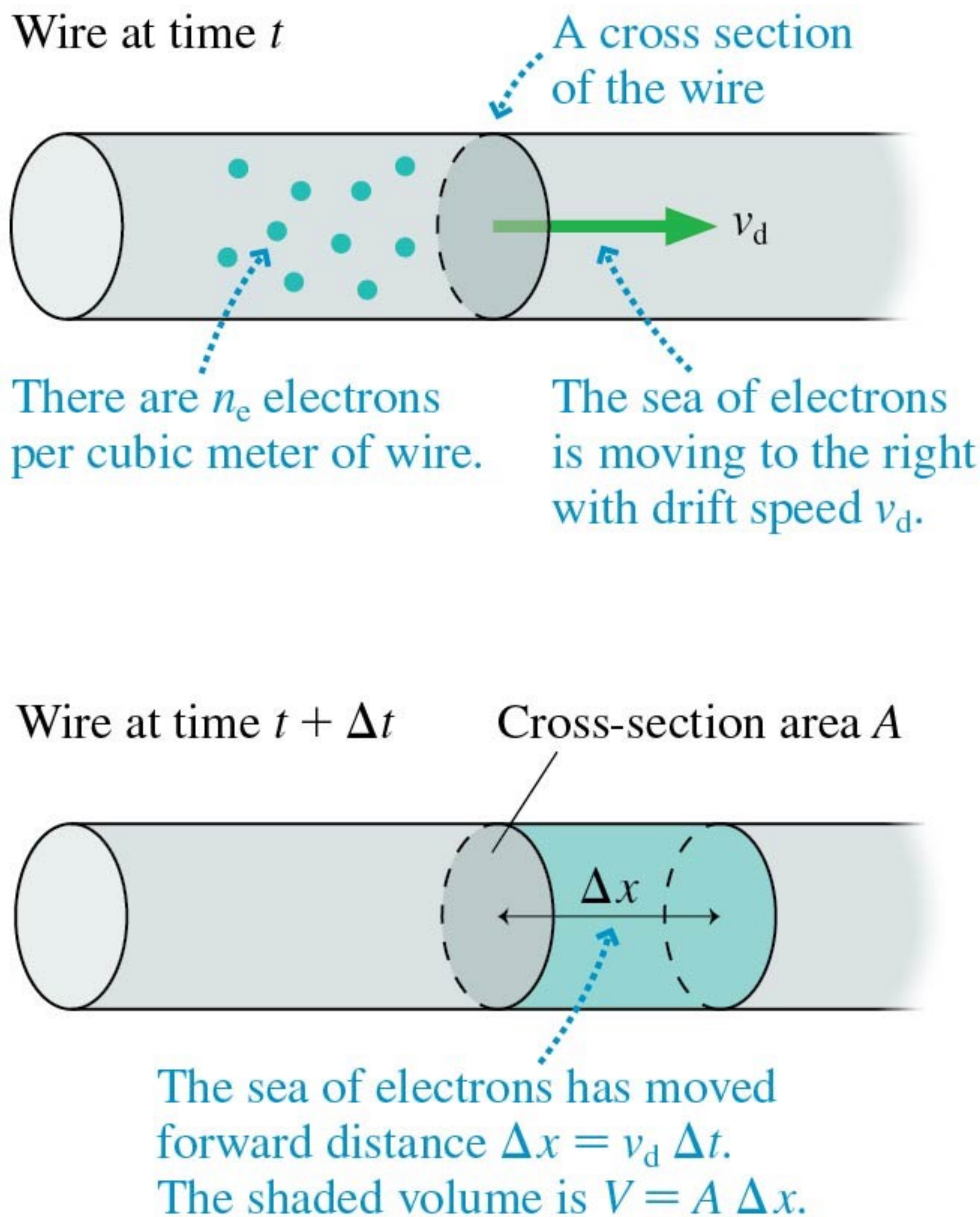
The Electron Current

- If the number density of conduction electrons is n_e, then the total number of electrons in the shaded cylinder is

$$N_e = n_e V$$
$$= n_e A \Delta x$$
$$= n_e A v_d \Delta t$$

- So the electron current is

$$\boxed{i_e = n_e A v_d}$$



iClicker question 9-2

A wire carries a current. If both the wire diameter and the electron drift speed are doubled, the electron current increases by a factor of

A. 2.
B. 4.
C. 6.
D. 8.
E. Some other value.