

Triangle inequality

Proposition: Triangle inequality. Let $v, w \in \mathbb{R}^n$

Then, $\|v + w\| \leq \|v\| + \|w\|$

Proof:

$$\begin{aligned}\|v + w\|^2 &= \langle v + w, v + w \rangle \\ &= \langle v, v + w \rangle + \langle w, v + w \rangle \\ &= \langle v, v \rangle + \langle v, w \rangle + \langle w, v \rangle + \langle w, w \rangle\end{aligned}$$

$\leq \|v\|^2 + 2|\langle v, w \rangle| + \|w\|^2$, where with the [Cauchy Schwarz Inequality](#), we can rewrite this as:

$$\begin{aligned}&\leq \|v\|^2 + 2\|v\|\|w\| + \|w\|^2 \\ &= (\|v\| + \|w\|)^2 \\ \implies &\|v + w\|^2 \leq (\|v\| + \|w\|)^2 \\ \implies &\|v + w\| \leq (\|v\| + \|w\|)\end{aligned}$$