Cross product

Definition: Cross product on $\mathbb{R}^3$

Take some $v, w \in \mathbb{R}^3$. Cross product of the vector will be based on the determinant of the components of v, w in the following way:

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \times \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} \det \begin{bmatrix} v_2 & w_2 \\ v_3 & w_3 \end{bmatrix} \\ \det \begin{bmatrix} v_3 & w_3 \\ v_1 & w_1 \end{bmatrix} \\ \det \begin{bmatrix} v_1 & w_1 \\ v_2 & w_2 \end{bmatrix} \end{bmatrix}$$

- The cross product will be a vector also in $\mathbb{R}^3$ that is orthogonal to both of the vectors v, w
  - We can see this by taking the plane formed by v and w, and creating a vector that is orthogonal to that plane

> Proof: Cross product orthogonality
>
> For our first case, we have to work to show that $v \ \ (v \times w) = 0 \in \mathbb{R}$.
>
> $$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \ \begin{bmatrix} v_2 w_3 - v_3 w_2 \\ v_3 w_1 - v_1 w_3 \\ v_1 w_2 - v_2 w_1 \end{bmatrix}$$
>
> $$= v_1(v_2 w_2 - v_3 w_2) + v_2(v_3 w_1 - v_1 w_3) + v_3(v_1 w_2 - v_2 w_1)$$
>
> When expanded, this simplifies to zero.
>
> For our second case, we have to show that $w \ \ (v \times w) = 0 \in \mathbb{R}$
>
> $$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} \ \begin{bmatrix} v_2 w_3 - v_3 w_2 \\ v_3 w_1 - v_1 w_3 \\ v_1 w_2 - v_2 w_1 \end{bmatrix}$$
>
> $$= w_1(v_2 w_2 - v_3 w_2) + w_2(v_3 w_1 - v_1 w_3) + w_3(v_1 w_2 - v_2 w_1)$$
>
> Which when expanded, also simplifies to zero.

Proof: Say that $v, w, u \in \mathbb{R}^3$. Then, $\det\begin{bmatrix} v & w & u \end{bmatrix} = w \ (w \times u)$
TBA