N

Flux :

vector field $\vec{V}$
surface A

small flux
↓
small area
↓
$$d\Phi = \vec{V} \cdot d\vec{A}$$

$d\vec{A}$ points outward.

for surface $\vec{A}$, $\vec{E}$ in $\hat{x}$

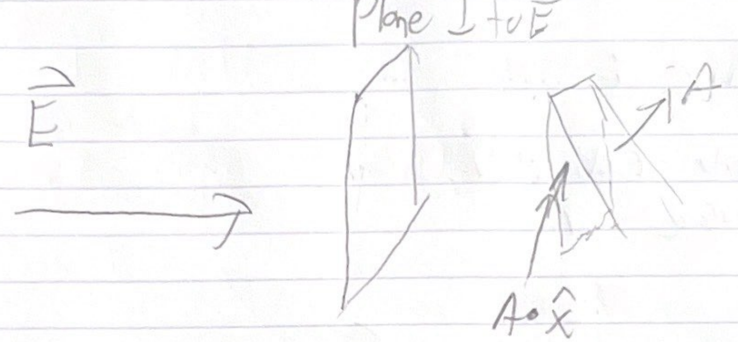
$\vec{E} \cdot \vec{A}$

$= E(\vec{A} \cdot \hat{x})$

↗

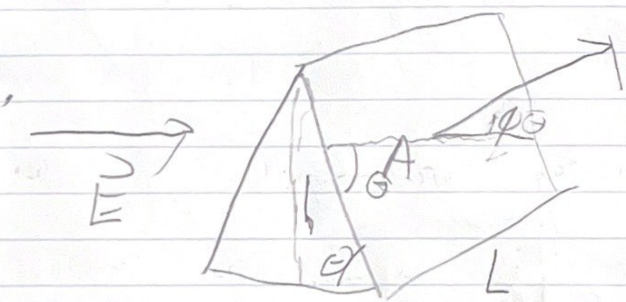
Projection of $\vec{A}$ onto plane perpendicular to $\vec{E}$.

i.e



The part of $\vec{A}$ that $\vec{E}$ "sees" for the flux is just that projection.

Q,



Flux through A?

A, $\vec{E} \cdot \vec{A} = E A_{projected} = \boxed{EhL}$

$\vec{E} \cdot \vec{A} = EA\cos\phi$

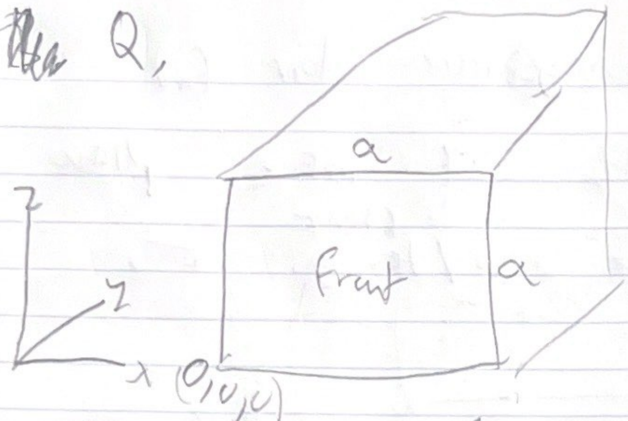
$= EA\cos(\frac{\pi}{2} - \theta) = EA\sin\theta$

$A = L\frac{h}{\sin\theta}$

$\Phi = \cancel{EA}\, E\frac{h\sin\theta}{\sin\theta}L = \boxed{EhL}$

Cube

Q,



side length a

$E = (Cx^2 + d)\,\hat{x}$

what is flux through each face?

A, Top, bottom, front, back are all 0 b/c $\vec{E}$ is $\perp$ to their surface.

Left: $E = (x^2+d) = d$
$x=0$
$\vec{E}\cdot\vec{A} = -d a^2$ because normal is to the Left, $\vec{E}$ to the right, Flux is going in

right:

$E = (Ca^2+d)$

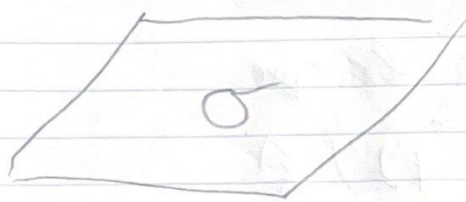
$\vec{E}\cdot\vec{A} = (Ca^2+d)a^2 = Ca^4 + da^2$

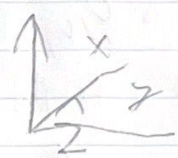
total flux is $Ca^4 + da^2 - da^2 = Ca^4$

not 0 if $C \neq 0$, so by Gausses law it must have charge inside.

Q, use gausses law find

$\vec{E}$ field of infinite plane of charge w/ charge density σ.



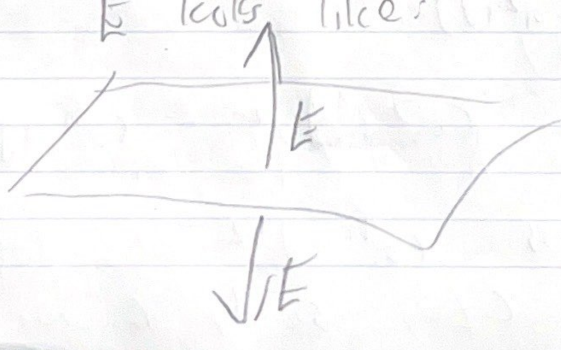


Notice symetry:

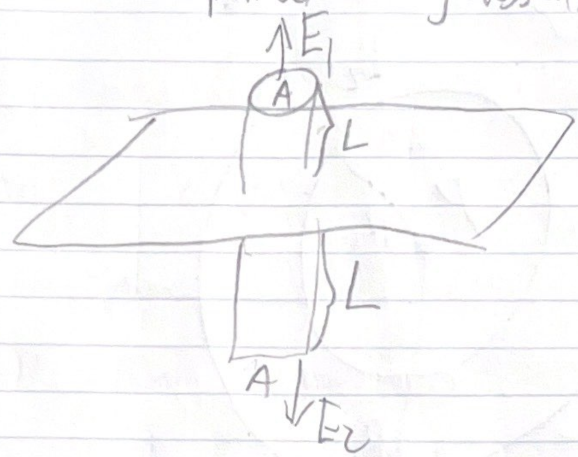
you can rotate it around $\vec{z}$, looks the same, so no component of $\vec{E}$ along plane (ie not x or y component)

can flip it over z=0, so top and bottom must have same magnitude opposite direction

so $\vec{E}$ looks like:



Use pillbox gaussian



by symmetry $|E_1| = |E_2|$

total flux is then

top bottom

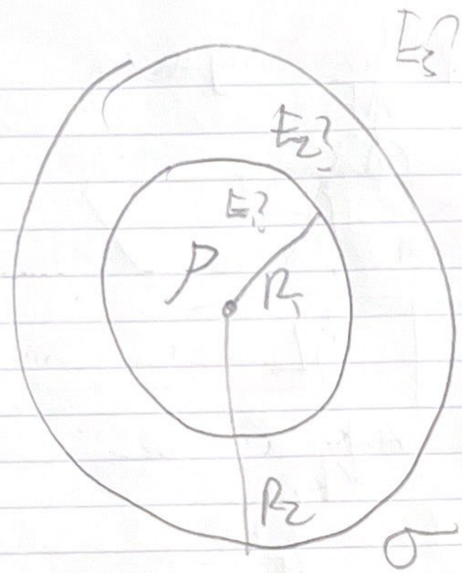
$EA + EA = 2EA$

$Q_{enc} = \sigma A$

so $\frac{\sigma A}{\varepsilon_0} = 2EA$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

Q.



solid sphere of uniform charge density ρ,
surrounded by uniform shell sphere charge density σ.

what is field in each region?

A. symmetry, gaussian surface is sphere

E_3?

$r > R_2$

$Q_{enc} = \rho \frac{4}{3} \pi R_1^3 + \sigma 4 \pi R_2^2$

~~E_3~~ Flux $= E_3 4 \pi r^2$

$\boxed{E_3 = \frac{Q_{enc}}{4 \varepsilon_0 \pi r^2}}$ looks ~~but~~ like point charge

E_2?

same thing, but

Q_{enc} is not just $\rho \frac{4}{3}\pi R_1^3$

So spherical shell does not contribute.

field inside spherical shell is 0.

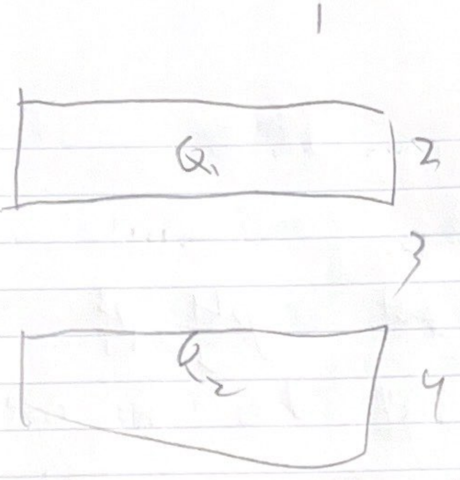
E_1?

depends on r

$Q_{enc} = \frac{4}{3}\pi r^3$

$E_1 = \frac{\frac{4}{3}\pi r^3 \rho}{4\pi r^2 \varepsilon_0} = \boxed{\frac{\rho r}{3\varepsilon_0}}$ linear in r

Q



conducting surfaces, 5
charged w/ Q_1 and Q_2.

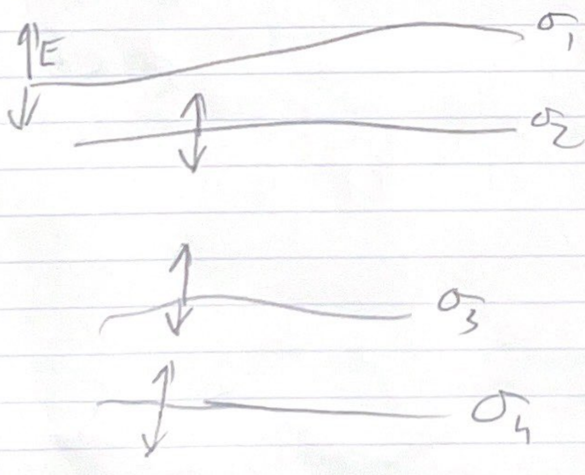
What is field in each region?

A. Conductors at static equilibrium have no field in interiors

so E in 1 and 4 $= 0$,

also have no net charge so problem reduces to

infinite planes!



in ~~the~~ region 1, all

fields from planes align

so $E =$

$$\frac{\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4}{2\varepsilon_0}$$

by charge conservation $Q_1 = (\sigma_1 + \sigma_2)A$

$Q_2 = (\sigma_3 + \sigma_4)A$

$$\Rightarrow \boxed{E = \frac{Q_1 + Q_2}{2A\varepsilon_0}}$$

symmetrically, in region 5

$$E = -\left(\frac{Q_1 + Q_2}{2A\varepsilon_0}\right)$$

in region 3 they subtract b/c opposite directions.

$$E = \frac{-(\sigma_1 + \sigma_2) + \sigma_3 + \sigma_4}{2\varepsilon_0} \Rightarrow$$

$$\boxed{\frac{-Q_1 + Q_2}{2A\varepsilon_0}}$$