Discrete Mathematics
(Assignment-4)

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Q.1(a)

Ans. (a) Vertex : A point in a graph where edges meet.

Example : In graph $G(V, E)$, $V = \{A, B, C\}$, vertices are A, B, C

(b) Edge : A line connecting two vertices.

Example : $E = \{(A, B), (B, C)\}$

(c) degree : Number of edges incident to a vertex

Example : $deg(A) = 1$, $deg(B) = 2$ in $E = \{(A, B), (B, C)\}$

(d) Path : A sequence of distinct vertices connected by edges.

Example : Path : $A \to B \to C$

(e) Cycle : A closed path that starts and ends at the same vertex.

Example : $A \to B \to C \to A$

Q.1(b)

Ans. • Planar graph : A graph that can be drawn on a plane without any edges crossing.

Euler's formula for planar graphs :

$V - E + F = 2$

if $V = 7$, $E = 12$

$\to 7 - 12 + F = 2 \Rightarrow F = 7$

$12 \leq 3(7) - 6 = 15 \Rightarrow$ Condition holds

Hence, such a graph can be planar.

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Q.1(c)

Ans.

Feature	Euler Path/Circuit	Hamiltonian Path/Circuit
- Definition	Traverse every edge exactly once.	Traverses every vertex exactly once.
- Existence condition	Path exists if 0 or 2 vertices have odd degree; circuit if all even degree.	Exists if graph contains a cycle visiting each vertex once.
- Focus	Edges	Vertices
- Example	Graph of Königsberg bridges (Euler circuit possible when even degrees)	In C_5 (Pentagon), Hamiltonian circuit = $A \to B \to C \to D \to E \to A$.

Q.1(d)

Ans. • Chromatic number ($\chi(G)$): Minimum number of colours required to colour vertices so that no two adjacent vertices share the same colour.

For cycle graphs: (C_n)

- If n is even $\to \chi = 2$
- If n is odd $\to \chi = 3$

Since C_5 has 5 vertices (odd),

$$\chi(C_5) = 3$$

Q.1(e)

Ans. (a) BFS (Breath First Search): Traverses level-wise using a queue.

Steps (Starting from A):

- Visit A → enqueue neighbours B, C
- Visit B → enqueue D
- Visit C → enqueue E
- Visit D, E → no new nodes

• BFS order: A, B, C, D, E

(b) DFS (Depth First Search): Traverses depth-wise using a stack.

Steps (start A):

- A → B → D (no new neighbours, backtrack)
- From A → C → E

• DFS order: A, B, D, C, E

Q.2 (a)

Ans. Euler's Formula:

For any connected planar graph,

$V - E + F = 2$

Proof (by induction on edges):

1. For a tree: V

- **Proof:** by induction on number of edges in graph
- **Base:** If $e=0$, graph consists of single node with a single face surrounding it. So, $1-0+1=2$ which is right.
- **Induction:** Suppose formula works for all graphs with no more than n edges. Let G_1 be a graph with $n+1$ edges.
- **Case 1:** G_1 doesn't contain a cycle. So, G_1 is a free tree and we already know the formula works for free trees.
- **Case 2:** G_1 contains at least one cycle. Pick an edge p that's on a cycle. Remove p to create a new graph G_1''.

Since the cycle separates plane into two faces, faces to either side of p must be distinct. When we remove edge p, merge these two faces. So, G_1'' has one fewer faces than G_1.

Since G_1'' has n edges, formula works for G_1'' by induction hypothesis. That is, $V'-e'+f'=2$. But $V'=V$, $e'=e-1$ & $f'=f-1$. Substituting,

$$V-(e-1)+(f-1)=2$$

So,

$$\boxed{V-e+F=2}$$

Q.2(b)

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Ans. (i) Kruskal's Algorithm (for MST):

1. Sort all edges in increasing weight order.
2. Pick the smallest edge that doesn't form a cycle (using union-find).
3. Repeat until all vertices are connected (V-1 edges).

Eg- Edges = {AB(2), AC(3), BC(4)}

→ Pick AB(2), (AC)(3)

MST edges = {AB, AC}, total weight = 5

(ii) Prim's Algorithm

1. Start with any vertex.
2. Add smallest edge connecting a vertex inside the MST to a vertex outside it.
3. Repeat until all vertices are included.

Eg- Start from A → choose AB(2), then AC(3).

MST same as Kruskal's

(iii) MST using Kruskals

1. Using all edges with weights.
2. Sort them ascendingly.
3. Select smallest edge if it doesn't make a cycle.
4. Continue till you get (V-1) edges.

Output: Minimum spanning tree with minimal total cost