Homework 2

Computer Vision 2022 Spring

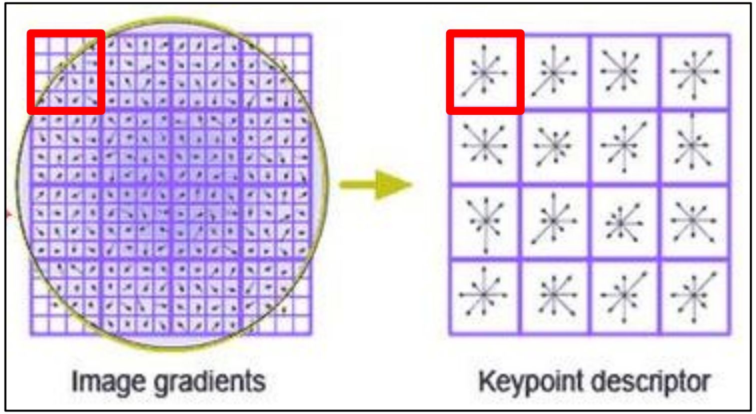
2022.4.14

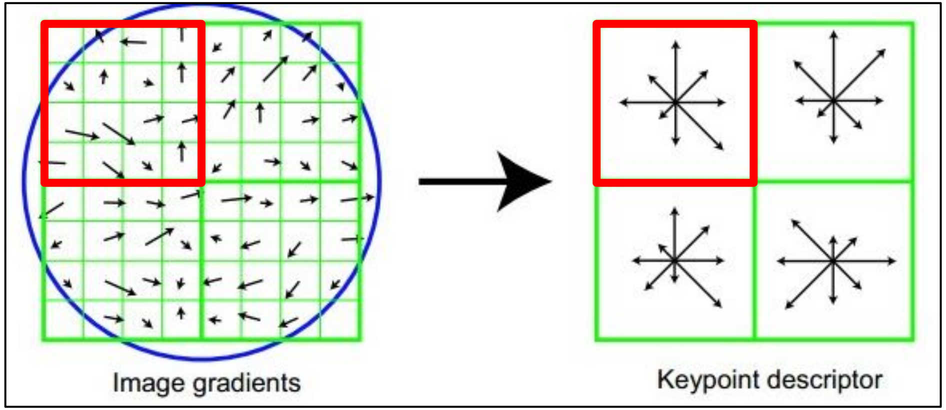
Image stitching

1. Detecting key point(feature) on the images
    - SIFT
2. Finding features correspondences (feature matching)
    - KNN
3. Computing homography matrix.
    - RANSAC
4. Stitching image (warp images into same coordinate system)
    - Homography

Feature Detection

- Finding features correspondences/compute homography matrix.
- SIFT – Scale Invariant Feature Detection
  - detect key points in the image and describe the points as 128-dimensional features (4 * 4 * 8).
- Check Ch.6、7 for more details of SIFT.





Install

- Python 3.6
- OpenCV : https://docs.opencv.org/4.5.5/
  - 4.5.5 (Recommend)
  - pip install opencv-python

1. SIFT in OpenCV

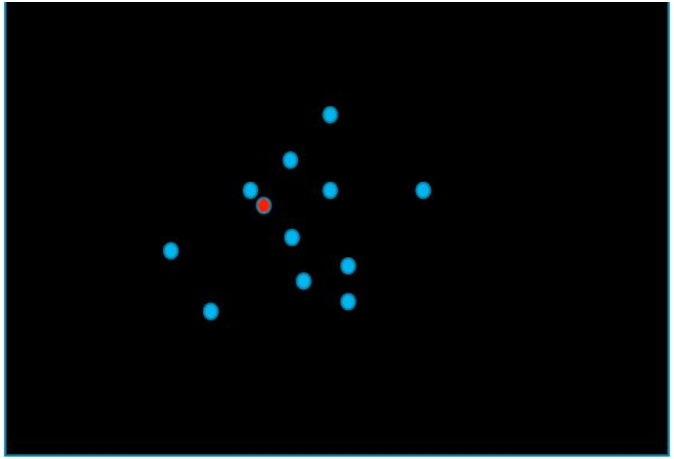
- Using OpenCV to detect SIFT key points of two images
- Input : gray scale image

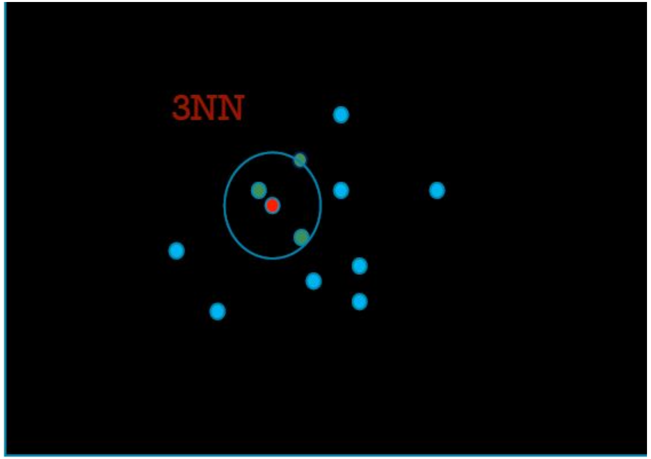
```
SIFT_Detector = cv2.SIFT_create()
kp, des = SIFT_Detector.detectAndCompute(img, None)
```

- output : keypoints (array), Descriptors (array)
- Keypoints store feature points
    - for a single keypoint you can use ".pt" to get the position of this key point on image [Ref]
- Descriptors store the 128-dimensional features
- The function name(detectAndCompute) of SIFT may be different with the version of OpenCV

2. Feature matching - KNN

- K-Nearest Neighbor
  - Finding the K closest neighbors to the target.
  - Brute-force : Comparing with the all 2-norm of SIFT feature (the 2-norm of descriptor)



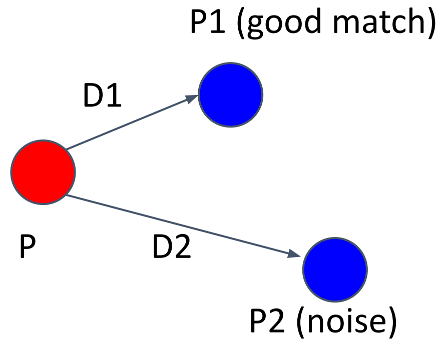


2. Feature matching - Lowe’s Ratio test

- Lowe’s Ratio test for eliminating bad match
  - A good match shold be able to be distinguished from noise

1. For every key point P in image1 using 2NN
   to get 2 matched key points P1 & P2 in image2
2. Computing the 2-norm of P1 & P2 between P named D1 , D2
3. If D1 < threshold * D2 then P1 is a good match

(threshold is a programmer defined ration between 0 to 1 ,
 the suggestion of OpenCV tutorial is 0.7~0.8)



3.Homography

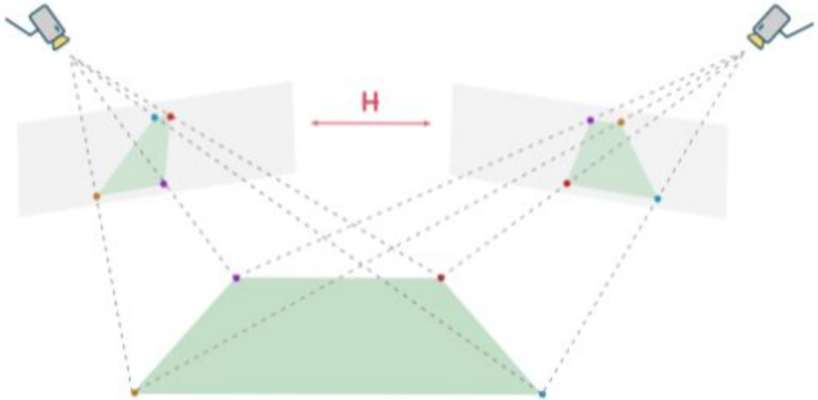
- Construct a linear system as: P2=HP1, P2 = (x2,y2,1) , P1 = (x1,y1,1) where P2 and P1 are correspondence points , H is homography matrix.

$$\begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \Leftrightarrow \mathbf{x}_2 = H\mathbf{x}_1$$

In homogenous coordinates ($x_2' = x_2/z_2$ and $y_2' = y_2/z_2$)

$$x_2' = \frac{H_{11}x_1 + H_{12}y_1 + H_{13}z_1}{H_{31}x_1 + H_{32}y_1 + H_{33}z_1}$$
$$y_2' = \frac{H_{21}x_1 + H_{22}y_1 + H_{23}z_1}{H_{31}x_1 + H_{32}y_1 + H_{33}z_1}$$

$$x_2'(H_{31}x_1 + H_{32}y_1 + H_{33}) = H_{11}x_1 + H_{12}y_1 + H_{13}$$
$$y_2'(H_{31}x_1 + H_{32}y_1 + H_{33}) = H_{21}x_1 + H_{22}y_1 + H_{23}$$



3.Homography

- If we restrict h33 = 1

$$\begin{aligned} x'_2(H_{31}x_1 + H_{32}y_1 + 1) &= H_{11}x_1 + H_{12}y_1 + H_{13}z1 \\ y'_2(H_{31}x_1 + H_{32}y_1 + 1) &= H_{21}x_1 + H_{22}y_1 + H_{23}z1 \end{aligned}$$

$$\begin{aligned} x'_2 &= H_{11}x_1 + H_{12}y_1 + H_{13}z1 - H_{31}x_1x'_2 - H_{32}y_1x'_2 \\ y'_2 &= H_{21}x_1 + H_{22}y_1 + H_{23}z1 - H_{31}x_1y'_2 - H_{32}y_1y'_2 \end{aligned}$$

- For perspective transformation , you can use 4 pairs of match result to slove 8 unknow variable in homography matrix

$$\begin{bmatrix} \hat{x}_i z_a \\ \hat{y}_i z_a \\ z_a \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & \boxed{h_{33}} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1\hat{x}_1 & -y_1\hat{x}_1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 & -x_2\hat{x}_2 & -y_2\hat{x}_2 \\ x_3 & y_3 & 1 & 0 & 0 & 0 & -x_3\hat{x}_3 & -y_3\hat{x}_3 \\ x_4 & y_4 & 1 & 0 & 0 & 0 & -x_4\hat{x}_4 & -y_4\hat{x}_4 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1\hat{y}_1 & -y_1\hat{y}_1 \\ 0 & 0 & 0 & x_2 & y_2 & 1 & -x_2\hat{y}_2 & -y_2\hat{y}_2 \\ 0 & 0 & 0 & x_3 & y_3 & 1 & -x_3\hat{y}_3 & -y_3\hat{y}_3 \\ 0 & 0 & 0 & x_4 & y_4 & 1 & -x_4\hat{y}_4 & -y_4\hat{y}_4 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \end{bmatrix} = h_{33} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \\ \hat{x}_4 \\ \hat{y}_1 \\ \hat{y}_2 \\ \hat{y}_3 \\ \hat{y}_4 \end{bmatrix}$$

3.Homography

$$A = U\Sigma V^T$$

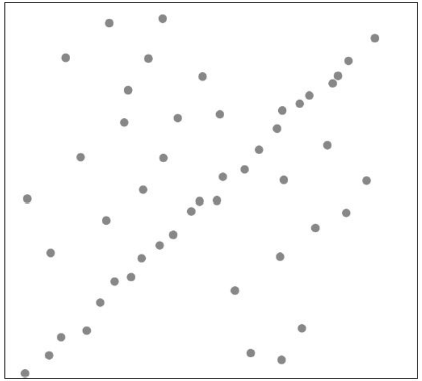
- Using SVD decomposition to find Least Squares error solution
- the solution = eigenvector of A^TA associated with the smallest eigenvalue (V stores the eigenvector of A^TA , Σ stores the singular value (root of eigen value))
  find the smallest number in Σ and H = corresponding vector in V^T
- Remember to normalize h33 to 1

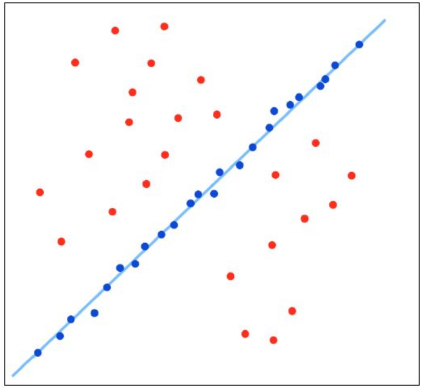
$$\begin{bmatrix} \hat{x}_i z_a \\ \hat{y}_i z_a \\ z_a \end{bmatrix} = \underbrace{\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}}_{\mathbf{H}} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1\hat{x}_1 & -y_1\hat{x}_1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 & -x_2\hat{x}_2 & -y_2\hat{x}_2 \\ x_3 & y_3 & 1 & 0 & 0 & 0 & -x_3\hat{x}_3 & -y_3\hat{x}_3 \\ x_4 & y_4 & 1 & 0 & 0 & 0 & -x_4\hat{x}_4 & -y_4\hat{x}_4 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1\hat{y}_1 & -y_1\hat{y}_1 \\ 0 & 0 & 0 & x_2 & y_2 & 1 & -x_2\hat{y}_2 & -y_2\hat{y}_2 \\ 0 & 0 & 0 & x_3 & y_3 & 1 & -x_3\hat{y}_3 & -y_3\hat{y}_3 \\ 0 & 0 & 0 & x_4 & y_4 & 1 & -x_4\hat{y}_4 & -y_4\hat{y}_4 \end{bmatrix}}_{\mathbf{A}} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \end{bmatrix} = h_{33} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \\ \hat{x}_4 \\ \hat{y}_1 \\ \hat{y}_2 \\ \hat{y}_3 \\ \hat{y}_4 \end{bmatrix}$$

Reference :
SVD : https://web.mit.edu/be.400/www/SVD/Singular_Value_Decomposition.htm
Homography : https://cseweb.ucsd.edu/classes/wi07/cse252a/homography_estimation/homography_estimation.pdf

3.RANSAC





Random Sample Consensus

Input : M match;

1. Randomly select 4 data points as inliers S.Find a homography matrix H to S.
2. Test all match(p1 , p2) against H, estimate p2’ = p1 * H
   - if the distance between p2’ and p2 is small , add the match to S, which is called a consensus set.
3. If $|S|$ is larger than ever, mark H as the best estimated H^*.
4. If some stopping criterion is satisfied, end
5. Else go to step 1.

Note that you can re-estimate the models with the consensus sets.

4. Stitching image

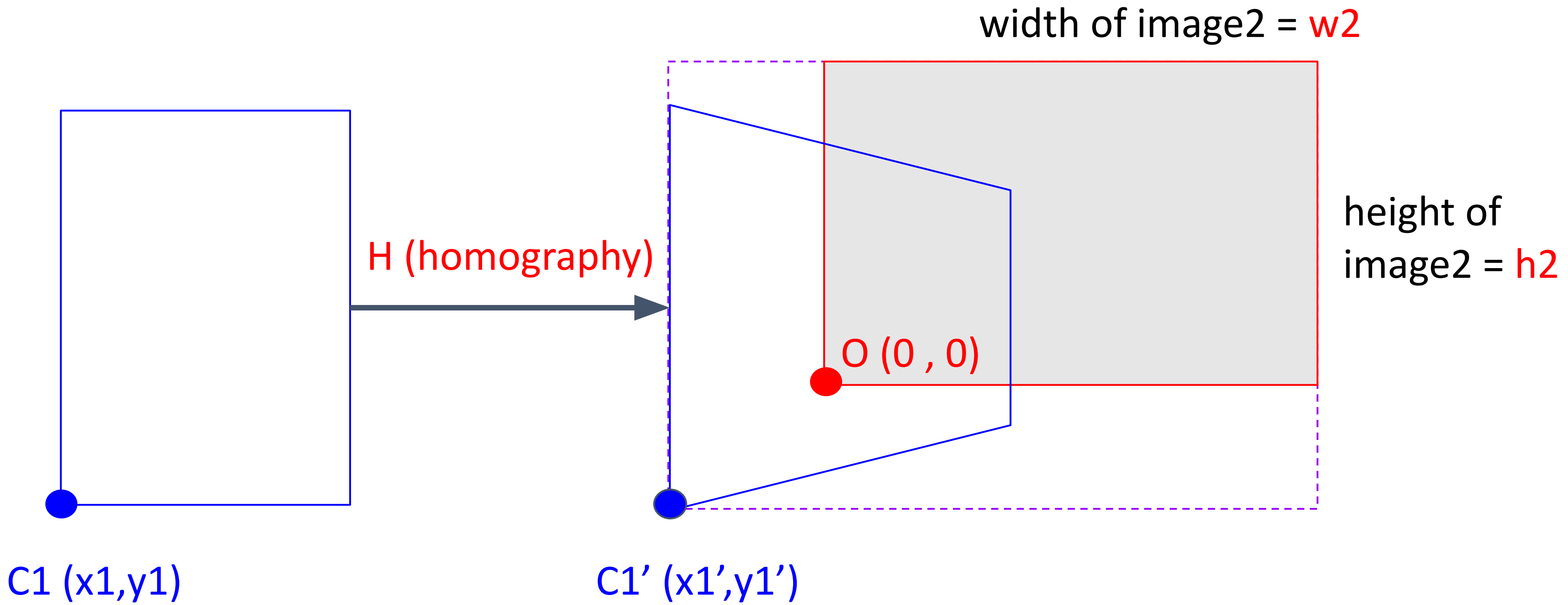
1. Using homography matrix H to calculate the position of 4 corners of image1 in the perspective of image2
2. Using image1 after perspective transformation to analyze the size which we need to combine two image together of
3. Using cv2.warpPerspective(src, M, dsize, ...) to warp the whole image1
   - src is source image1 , M is homography matrix H , dsize is output image size
   - `warped_l = cv2.warpPerspective(src=img1, M=H, dsize=size)`
4. Concating two images (for better results you can use blending or some ways to improve the quality of overlap part)

For stitching images you can use any function of OpenCV

4. Stitching image - more detail

corners' = corners * H
x1' = min(min(corners'_x),0)
y1' = min(min(corners'_y),0)

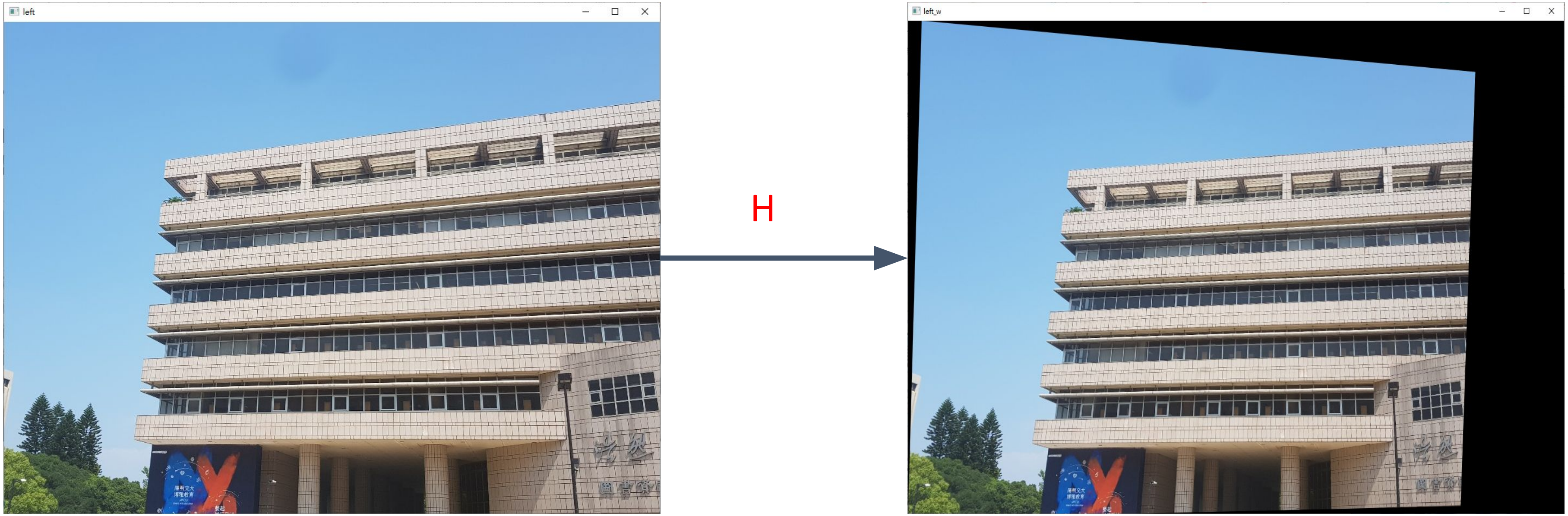
- Assume image1 is on left hand side and image2 is on right hand side
- Size we need = (w2 + abs(x1') , h2 + abs(y1'))



4. Stitching image - more detail

- Example for image1 applys perspective transformation

```
warped_l = cv2.warpPerspective(src=img1, M=H, dsize=size)
```



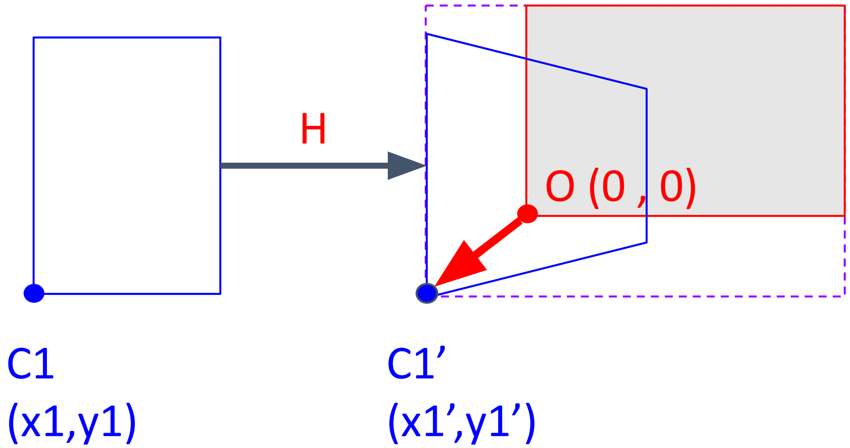
4. Stitching image - more detail

- For image2 using affine translation to move the image2 origin O to C1' .
  Let two image in smae image size and it's eayier to combine they.

- Your translation matrix need multiply H because we need translate it in the perspective of image2

Affine translation matrix (A) = $\begin{bmatrix} 1 & 0 & \Delta x \\ 0 & 1 & \Delta y \\ 0 & 0 & 1 \end{bmatrix}$ * H

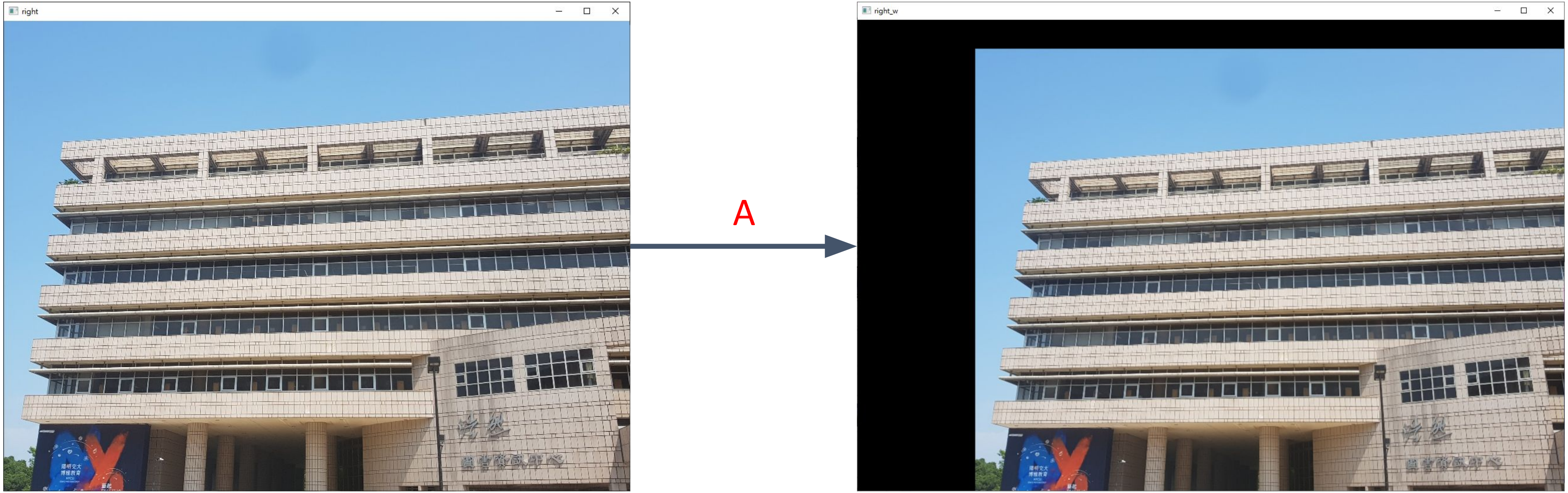
In this case $\Delta x = -x1^{'}, \Delta y = -y1^{'}$
Because origin moves to down left,
image2 needs move to top right relatively



4. Stitching image - more detail

- Example for using affine translation to move the image2 origin O to C1’

```
warped_r = cv2.warpPerspective(src=img2, M=A, dsize=size)
```



Requirements

- You are only allowed to use the function of OpenCV mentioned in previous slides. Please implement all (key point matching ,RANSAC , Homography ...) by youself
  - For submission you can use :
    - SIFT
  - For debugging only:
    - KNN match : BFMatcher()
    - Homography : findHomography()
- But there is no limitation of “image stitching” only (You can use any function provided by OpenCV)

Other tips

- Using Blending when you concate the two image
- Preprocessing for more easily concating multiple image : Cylindrical projection

Grading

50% Stitching 2 images together

SIFT (10%)

KNN (10%)

RANSAC (15%)

Homography (15%)

30% Report (Don't just paste the code with comment)

1. explain your implementation
2. show the result of stitching 2 images
3. try to stitch more images as you can and compare with them

10% stitching at least 4 images clearly

10% stitching at least 4 images seamlessly with blending (bonus)

Deadline

- Deadline : 2022/05/2 (Mon.) 11:59 pm
- Please zip the all files and name it as {studentID}_HW2.zip : ex 310553013_HW2.zip (wrong file format may get -5% panelty)
    - Zip file format:
        - 1. {studentID}_report.pdf
        - 2. your code
- Penalty of 10% of the value of the assignment per late week
    - late a week : your_score * 0.9
    - late two week : your_score * 0.8 …
- E3 forum : https://e3.nycu.edu.tw/mod/forum/view.php?id=278296

Result

blending

Cylindrical projection

Result

Sample of concating 8 image together