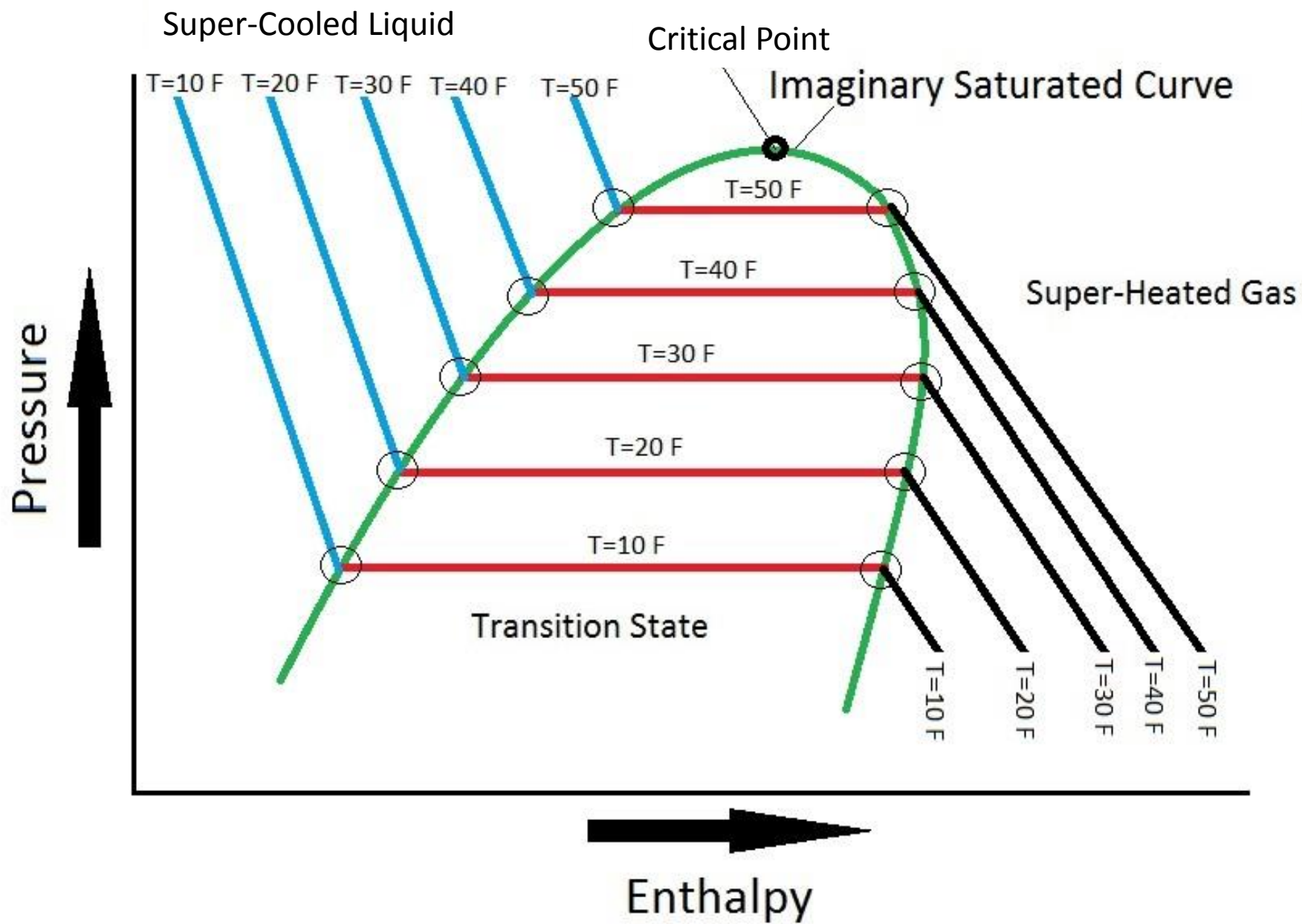


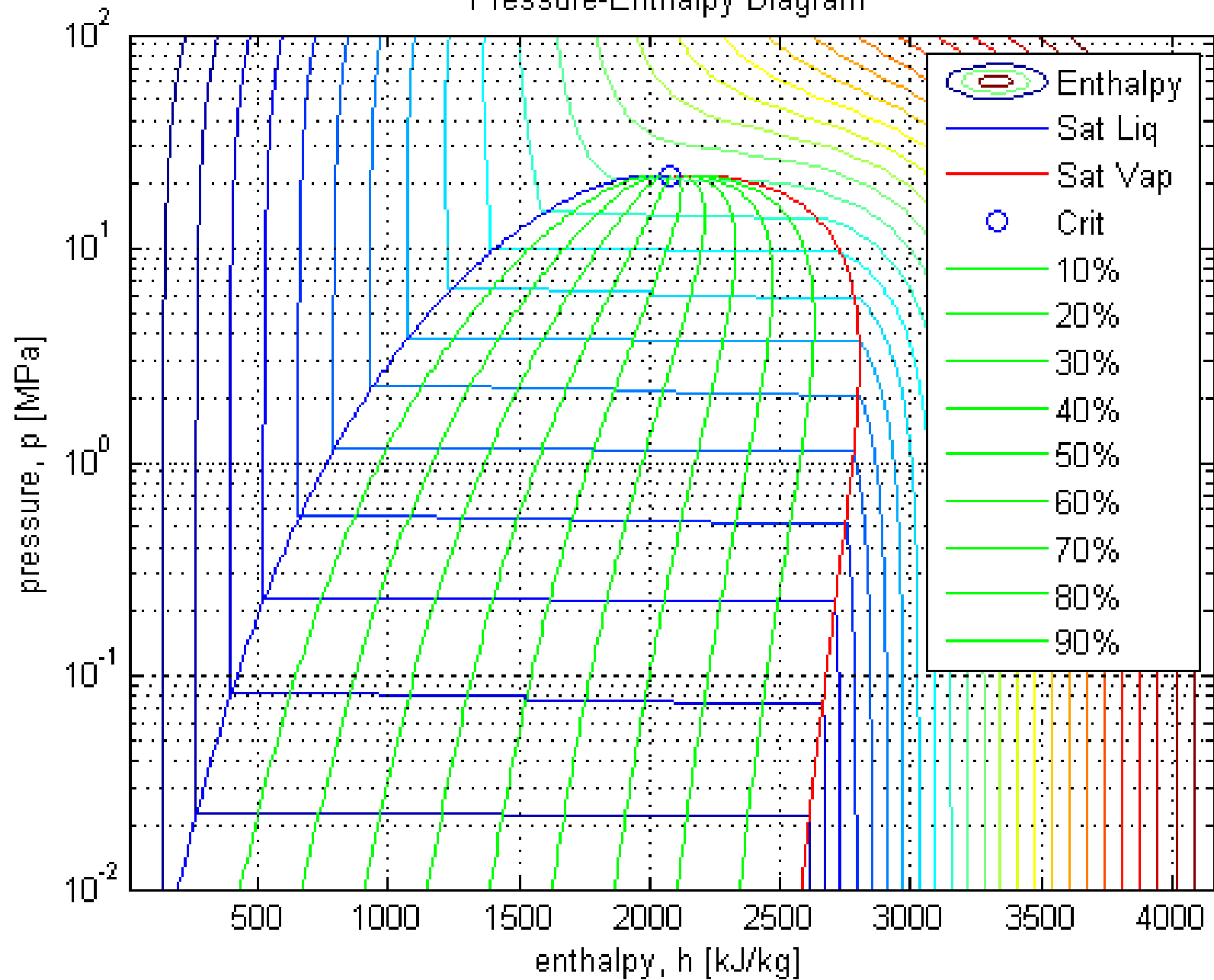
# ABE 201

# Biological Thermodynamics 1

Module 10:  
T P H and U Relationships



Pressure-Enthalpy Diagram



# Summary

- Thermodynamic properties (T, P, H, etc.) are related to one another by smooth functions with discontinuities at phase change boundaries
- This can be simplified as two types of thermal relationships: sensible and latent enthalpy (or internal energy)

$$\Delta H = \Delta H_{sensible} + \Delta H_{latent}$$

- Latent heat/enthalpy is the energy of phase change:

$$\Delta H_{latent} = \Delta H_{LV} \text{ or } \Delta H_{vaporization} \text{ or } \Delta H_{melting}$$

- Sensible heat/enthalpy can be calculated using heat capacity ( $C_p$ ) as a function integrated over a temperature range.

$$\Delta H_{sensible} = \int_{T_1}^{T_2} m \cdot C_p dT$$

- Heat capacities for solids and most liquids are constants over large ranges of temperatures

$$\Delta H_{sensible} = \int m \cdot C_p dT = m \cdot C_p \int dT = m \cdot C_p \cdot \Delta T$$

- Heat capacities for biological materials can be measured, use tabulated values, or be estimated from the composition.

# Finding Heat Capacities

- Table B.2
  - What is  $C_p$  for Water (liquid) at 50 C?
  - What is  $C_p$  for Water (vapor) at 50 C?

From B.2 : Liquid H<sub>2</sub>O

$$C_p = 0.0754 \text{ kJ/mol-K for } T(0 - 100 \text{ C})$$
$$= 4.19 \text{ kJ/Kg-K}$$

For Vapor

$$C_p = .03346 + 6.88 \times 10^{-6} \times T + 7.604 \times 10^{-9} \times T^2 - 3.593 \times 10^{-12} \times T^3$$
$$= 0.0338 \text{ kJ/mol-K} = 1.88 \text{ kJ/kg-K}$$

# Example

- In a cooling process hydrogen ( $H_2$ ) is cooled from  $800^\circ\text{C}$  to  $200^\circ\text{C}$ . If  $10\text{ kmol/min}$  is delivered to the cooler, calculate the heat removal rate in kW using the heat capacity formulas (Table B.2)

**Table B.2** Heat Capacities<sup>a</sup>

$$\text{Form 1: } C_p[\text{kJ}/(\text{mol}\cdot^\circ\text{C})] \text{ or } [\text{kJ}/(\text{mol}\cdot\text{K})] = a + bT + cT^2 + dT^3$$

$$\text{Form 2: } C_p[\text{kJ}/(\text{mol}\cdot^\circ\text{C})] \text{ or } [\text{kJ}/(\text{mol}\cdot\text{K})] = a + bT + cT^{-2}$$

*Example:*  $(C_p)_{\text{acetone(g)}} = 0.07196 + (20.10 \times 10^{-5})T - (12.78 \times 10^{-8})T^2 + (34.76 \times 10^{-12})T^3$ , where  $T$  is in  $^\circ\text{C}$ .

*Note:* The formulas for gases are strictly applicable at pressures low enough for the ideal gas equation of state to apply.

Compound	Formula	Mol. Wt.	State	Form	Temp. Unit	$a \times 10^3$	$b \times 10^5$	$c \times 10^8$	$d \times 10^{12}$	Range (Units of $T$ )
Acetone	$\text{CH}_3\text{COCH}_3$	58.08	l	1	$^\circ\text{C}$	123.0	18.6			-30-60
			g	1	$^\circ\text{C}$	71.96	20.10	-12.78	34.76	0-1200
Acetylene	$\text{C}_2\text{H}_2$	26.04	g	1	$^\circ\text{C}$	42.43	6.053	-5.033	18.20	0-1200
Air		29.0	g	1	$^\circ\text{C}$	28.94	0.4147	0.3191	-1.965	0-1500
			g	1	K	28.09	0.1965	0.4799	-1.965	273-1800
Ammonia	$\text{NH}_3$	17.03	g	1	$^\circ\text{C}$	35.15	2.954	0.4421	-6.686	0-1200
Ammonium sulfate	$(\text{NH}_4)_2\text{SO}_4$	132.15	c	1	K	215.9				275-328
Benzene	$\text{C}_6\text{H}_6$	78.11	l	1	$^\circ\text{C}$	126.5	23.4			6-67
			g	1	$^\circ\text{C}$	74.06	32.95	-25.20	77.57	0-1200
Isobutane	$\text{C}_4\text{H}_{10}$	58.12	g	1	$^\circ\text{C}$	89.46	30.13	-18.91	49.87	0-1200
<i>n</i> -Butane	$\text{C}_4\text{H}_{10}$	58.12	g	1	$^\circ\text{C}$	92.30	27.88	-15.47	34.98	0-1200
Isobutene	$\text{C}_4\text{H}_8$	56.10	g	1	$^\circ\text{C}$	82.88	25.64	-17.27	50.50	0-1200
Calcium carbide	$\text{CaC}_2$	64.10	c	2	K	68.62	1.19	$-8.66 \times 10^{10}$	—	298-720
Calcium carbonate	$\text{CaCO}_3$	100.09	c	2	K	82.34	4.975	$-12.87 \times 10^{10}$	—	273-1033
Calcium hydroxide	$\text{Ca}(\text{OH})_2$	74.10	c	1	K	89.5				276-373
Calcium oxide	$\text{CaO}$	56.08	c	2	K	41.84	2.03	$-4.52 \times 10^{10}$		273-1173
Carbon	C	12.01	c	2	K	11.18	1.095	$-4.891 \times 10^{10}$		273-1373
Carbon dioxide	$\text{CO}_2$	44.01	g	1	$^\circ\text{C}$	36.11	4.233	-2.887	7.464	0-1500
Carbon monoxide	CO	28.01	g	1	$^\circ\text{C}$	28.95	0.4110	0.3548	-2.220	0-1500
Carbon tetrachloride	$\text{CCl}_4$	153.84	l	1	K	93.39	12.98			273-343
Chlorine	$\text{Cl}_2$	70.91	g	1	$^\circ\text{C}$	33.60	1.367	-1.607	6.473	0-1200
Copper	Cu	63.54	c	1	K	22.76	0.6117			273-1357

<sup>a</sup>Adapted in part from D. M. Himmelblau, *Basic Principles and Calculations in Chemical Engineering*, 3rd Edition, © 1974, Table E.1. Adapted by permission of Prentice-Hall, Inc., Englewood Cliffs, NJ.



# Answer:

- Part a: (Use table B.2)

$$\dot{Q} = \dot{n}\Delta\hat{H}$$

$$\Delta\hat{H} = \int_{800}^{200} C_p dT = \int_{800}^{200} (a + b \cdot T + c \cdot T^2 + d \cdot T^3) dT$$

$$\Delta\hat{H} = \left[ a \cdot T + \frac{b \cdot T^2}{2} + \frac{c \cdot T^3}{3} + \frac{d \cdot T^4}{4} \right]_{800}^{200} = -17.29 \text{ kJ} / \text{mol}$$

$$a = 28.84 \times 10^{-3}, b = 0.00765 \times 10^{-5}, c = 0.3288 \times 10^{-8}, d = -0.8698 \times 10^{-12}$$

$$\Delta H = \frac{-17.29 \text{ kJ}}{\text{mol}} \frac{10 \text{ kmol}}{\text{min}} \frac{1000 \text{ mol}}{1 \text{ kmol}} \frac{1 \text{ min}}{60 \text{ sec}} = -2882 \text{ kW}$$

# Using Tabulated Values

- How much heat must be removed to cool 2.5 kg green beans (85% moisture content) from 20 C to -20 C?

**Tabulated Heat Capacities for Selected Biological Materials**  
 from Transport Processes and Unit Operations 3<sup>rd</sup> Ed. by Geankoplis

Material	C <sub>p</sub> (kJ/kg-K)	Material	C <sub>p</sub> (kJ/kg-K)
Oak	2.39	Flour	1.80-1.88
Pine, yellow	2.81	Ice Cream (liquid)	3.27
Wool	1.36	Ice Cream (frozen)	1.88
Starch	1.218	Milk, whole	3.85
Glycerol	1.382	Olive Oil	2.01
Lactose	1.202	Green Beans (fresh)	3.81
Apples	3.73-4.02	Green Beans (frozen)	1.97
Bacon	3.43	Potatoes	3.52
Beef	3.43	Poultry (fresh)	3.31
Bread, white	2.72-2.85	Poultry (frozen)	1.55
Butter	2.30		
Eggs	3.18		
Corn (fresh)	3.32		
Corn (frozen)	1.77		

$$\Delta H + \Delta E_k + \Delta E_p = Q - W_s$$

$$\Delta H = Q$$

Break up into 3 parts

1: Cool from 20C to 0C

2: Freezing Process (phase change)

3: Cool from 0C to -20C

$$Q = \Delta H_1 + \Delta H_2 + \Delta H_3$$

1:

$$\Delta H_1 = m * c_{p(\text{fresh})} * (T_2 - T_1) = 2.5 \text{ kg} (3.81 \text{ kJ/kg-C}) (0 - 20) = -190.5 \text{ kJ}$$

Heat of melting (table B.1 -> water  $\Delta H_m = 6.0095 \text{ kJ/mol}$ )

$$\begin{aligned} 2: \Delta H_2 &= m * x_{\text{water}} * \Delta H_{\text{freezing}} \\ &= 2.5(0.85) * (6.0095 \text{ kJ/mol}) (1 \text{ mol}/18\text{g}) (1000\text{g}/1\text{kg}) = -709.5 \text{ kJ} \end{aligned}$$

$$3: \Delta H_3 = m * c_{p(\text{frozen})} * (T_2 - T_1) = 2.5 * (1.97) (-20 - 0) = -98.5 \text{ kJ}$$

$$Q = \Delta H_1 + \Delta H_2 + \Delta H_3 = -190.5 - 709.5 - 98.5 = -998.5 \text{ kJ}$$

# Green Beans

- Using the Seibels Equation, what are:
  - The heat capacity of fresh green beans (85% MC)?
  - The heat capacity of frozen green beans (85% MC)?

Fresh Green Beans:

$$C_p = 0.837 + 3.348*(0.85) = 3.68 \text{ kJ/kg-K}$$

Compare to tabulated value:  $C_p = 3.81 \text{ kJ/kg-K}$

Frozen Green Beans:

$$C_p = 0.837 + 1.256*(0.85) = 1.91 \text{ kJ/kg-K}$$

Compare to tabulated value:  $C_p = 1.97 \text{ kJ/kg-K}$

# Using Choi and Okos Equation

- Calculate the heat capacity of a bacterial cell slurry that contains a bioproduct

Slurry Composition

75.0% water

17.5% carbohydrates

5.0% protein

2.0% lipids (fat)

0.5% ash

$$c_p = 4.180 * X_w + 1.711 * X_p + 1.928 * X_f \\ + 1.547 * X_c + 0.908 * X_a$$

$$= 4.18(0.75) + 1.711(0.05) + 1.928(0.02) \\ + 1.547(0.175) + 0.908(0.005)$$

$$= 3.53 \text{ kJ/kg-K}$$

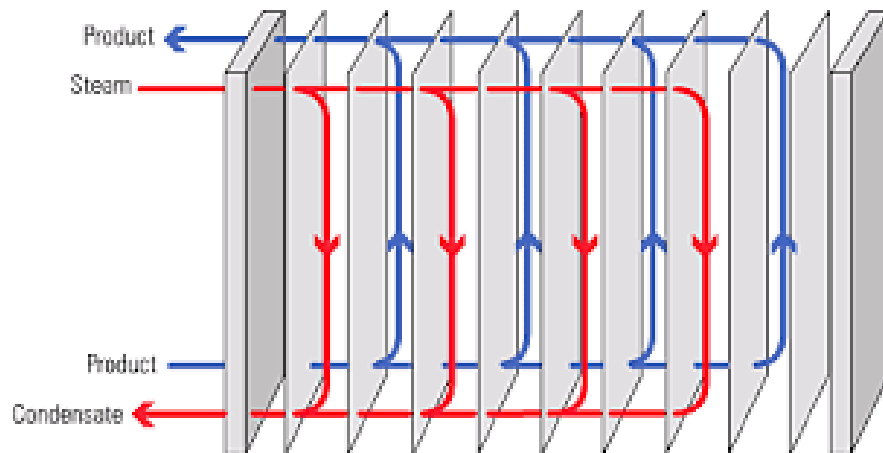
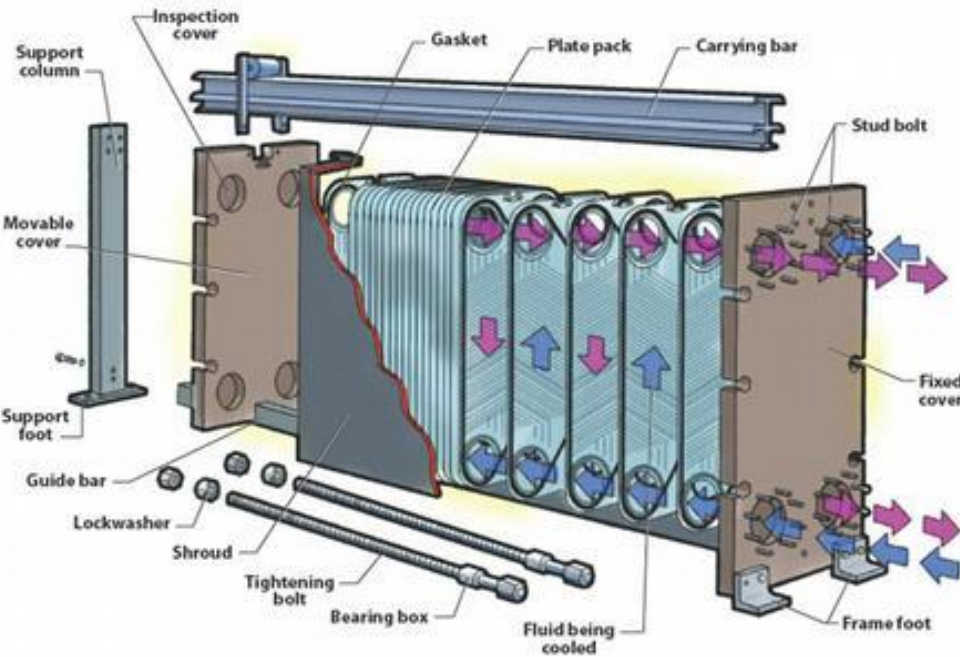


# Mass and Energy Balance

Whole milk is pasteurized at a rate of 250 kg/hr by heating from 4 C to 72C in an adiabatic plate-and-frame heat exchanger using superheated steam (2 bar, 140 C) at a rate of 12.5 kg/hr.

If the steam leaves the heat exchanger at 2 bar, what is the final conditions of the steam?

# Plate and Frame Heat Exchangers



**Tabulated Heat Capacities for Selected Biological Materials**  
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Eggs	3.18		
Corn (fresh)	3.32		
Corn (frozen)	1.77		

$$\Delta H + \Delta E_k + \Delta E_p = Q - W_s$$

$$\Delta H = 0$$

$$\Delta H_{\text{milk}} = \Delta H_{\text{steam}}$$

$$\begin{aligned}\Delta H_{\text{milk}} &= m \cdot c_p \cdot (T_2 - T_1) = 250 \text{ kg/hr} (3.85 \text{ kJ/kg-C}) (72 - 4) \\ &= 65450 \text{ kJ/hr}\end{aligned}$$

$$\begin{aligned}\Delta H_{\text{steam}} &= m_{\text{steam}} \cdot (H_{\text{initial, 2bar, 140C}} - H_{\text{final}}) \\ &= 12.5(2748.3 \text{ kJ/kg} - H_{\text{final}}) = 34353.75 - 12.5 \cdot H_{\text{final}}\end{aligned}$$

$$H_{\text{final}} = 2487.7 \text{ kJ/kg}$$

$$H_{\text{sat(2bar,v)}} = 2706.2 \text{ kJ/kg}$$

$$H_{\text{sat(2bar,l)}} = 504.70 \text{ kJ/kg}$$

$$H_{\text{final}} = x H_{\text{sat(2bar,v)}} + (1-x) H_{\text{sat(2bar,l)}}$$

Where x = quality

$$2487.7 = x(2706.2) + (1-x) 504.70$$

$$x = 0.90$$

**Outlet steam is saturated steam (120.2 C, 2 bar) at 90% quality.**