# ABE 201
# Biological Thermodynamics 1

Module 13: Integrated Mass and Energy Balances (1$^{st}$ and 2$^{nd}$ Law)

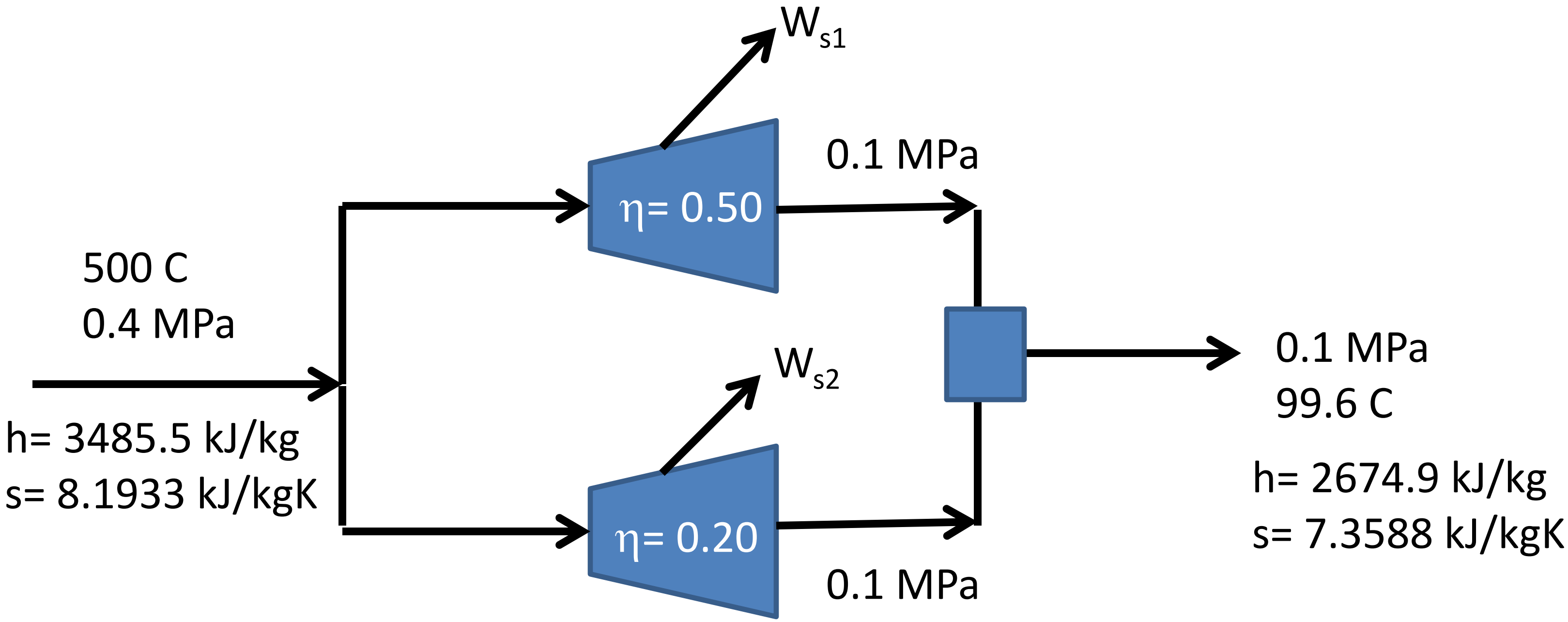
In Class Examples

# Example 1: Parallel Processing

Steam at 500C and 4 bar passes through two adiabatic turbines arranged in parallel. The steam leaves both turbines at 1 bar and are combined. After mixing, the steam is saturated vapor (100% quality) at 1 bar.

Turbine 1 has an efficiency of 50% and turbine 2 has an efficiency of 20%.

**Determine the fraction of the total flow that passes through each turbine.**




$$m_{in} = m_{50} + m_{20} \qquad 1 = m_{50} + m_{20}$$

$$\Delta\dot{H} + \cancel{\Delta\dot{E}_k} + \cancel{\Delta\dot{E}_p} = \cancel{\dot{Q}} - \dot{W}_s$$

$$\eta_{50} = 0.50 = \frac{W_{s,50}}{m_{50} * 3485.5}$$

$$\eta_{50} = 0.20 = \frac{W_{s,20}}{m_{20} * 3485.5} = \frac{W_{s,20}}{\left(1 - m_{50}\right) * 3485.5}$$

$$m_{in}(2674.9 - 3485.5) = -(W_{s,50} + W_{s,20})$$

$$(2674.9 - 3485.5) = -(0.50 * m_{50} * 3485.5 + 0.20 * \left(1 - m_{50}\right) * 3485.5)$$

$$m_{50} = 0.1085$$

$$m_{20} = 1 - m_{50} = 1 - 0.1085 = 0.8915$$

# Example 2:

Steam at 260ºC and 7.00 bar is expanded through a nozzle to 200ºC and 4.00 bar. Negligible heat is transferred from the nozzle to its surroundings. The approach velocity of the steam is negligible.

a) Calculate the exit velocity
b) Determine the entropy generation

**7.22**

| Inlet | Outlet |
|---|---|
| $\dot{m}$ (kg/s) | $\dot{m}$ (kg/s) |
| 260°C, 7 bars | 200°C, 4 bars |
| $\hat{H}$ = 2974 kJ/kg | $\hat{H}$ = 2860 kJ/kg |
| $u_0 = 0$ | $u$ (m/s) |

$$\Delta\dot{H} + \Delta\dot{E}_k + \Delta\dot{E}_p = \dot{Q} - \dot{W}_s$$

$$\Big\Downarrow \quad \Delta\dot{E}_p = \dot{Q} = \dot{W}_s = 0$$

$$\Delta\dot{E}_k = -\Delta\dot{H} \Rightarrow \frac{\dot{m}u^2}{2} = -\dot{m}\left(\hat{H}_{\text{out}} - \hat{H}_{\text{in}}\right)$$

$$\Big\Downarrow$$

$$u^2 = 2\left(\hat{H}_{\text{in}} - \hat{H}_{\text{out}}\right) = \frac{(2)(2974-2860)\text{kJ}}{\text{kg}}\left|\frac{10^3\ \text{N}\cdot\text{m}}{1\ \text{kJ}}\right|\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}} = 2.28\times10^5\ \frac{\text{m}^2}{\text{s}^2} \Rightarrow \underline{\underline{u = 477\ \text{m/s}}}$$

$$\sum m_{in}\hat{s} - \sum m_{out}\hat{s} + S_{gen} = dS_{sys}$$

$$S_{gen} = (\text{S}_{out} - S_{in}) = (7.1723 - 7.1472) = 0.0251 kJ/kg - K$$

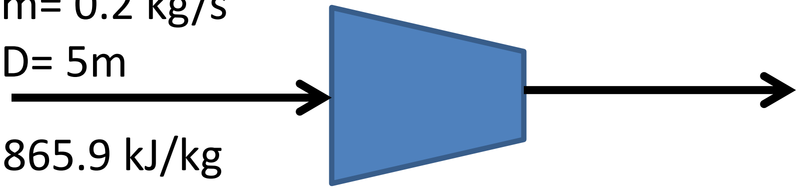
# Homework 12-1

Steam passes through an adiabatic nozzle at a rate of 0.2 kg/s. The nozzle has a circular inlet that has a diameter of 5 m and a circular outlet that has a diameter of 2 m. The steam enters at 0.3 MPa and 200°C exits into a chamber 0.1 MPa pressure.

a) What is the temperature and velocity of the exiting steam?

b) How much entropy is generated by the nozzle?

200 C
0.3 Mpa
m= 0.2 kg/s
D= 5m



0.1 MPa
T = ?
m= 0.2 kg/s
D= 2m

h= 2865.9 kJ/kg
V= 716.42 $m^3/kg$
s= 7.3131 kJ/kgK

h= 2855.7 kJ/kg
V= 2125.5 $m^3/kg$
s= 7.7934 kJ/kgK

$$\Delta \mathrm{H} + \Delta E_k + \cancel{\Delta E_p} = \cancel{Q} - \cancel{W_s}$$

$$\mathrm{m} * \mathrm{H_{out}} - \mathrm{m} * \mathrm{Hin} + \frac{m}{2} * (v^2{}_{out} \text{ - } v^2{}_{in}) = 0$$

$$\mathrm{m} * \mathrm{H_{out}} - \mathrm{m} * \mathrm{Hin} + \frac{m}{2} * \left(\left[\frac{m*V}{\frac{\pi}{4}D^2}\right]^2{}_{out} \text{ - } \left[\frac{m*V}{\frac{\pi}{4}D^2}\right]^2{}_{in}\right)*10^{-3}\ \mathrm{kJ/J} = 0$$

T = 190 C
Vout = 135 m/s

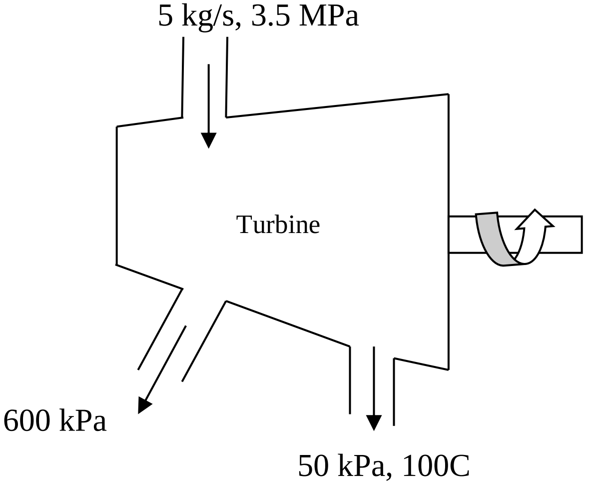
| T | H | V | dH (kJ) | dEk (kJ) | Equation |
|---|---|---|---|---|---|
| 175 | 2826.1 | 2054.9 | -7.96 | 1.71 | -6.25 |
| 190 | 2855.7 | 2125.5 | -2.04 | 1.83 | -0.21 |
| 195 | 2865.6 | 2149 | -0.06 | 1.87 | 1.81 |

$$\sum m_{in}\hat{s} - \sum m_{out}\hat{s} + S_{gen} = \frac{dS_{sys}}{dt}$$

$$S_{gen} = (\mathrm{S}_{out} - S_{in})\,\mathrm{m} = (7.7934 - 7.3131)(0.2) = 0.0961 kJ / K$$

# Homework 12-2

An isentropic steam turbine processes 5 kg/s of steam at 3.5 MPa, which is exhausted at 50 kPa and 100 C. Five percent of this flow is diverted for feedwater heating at 600 kPa. Determine the inlet steam temperature and power produced by this turbine, in kW.



1: steam inlet

2: outlet for feedwater heating

3: outlet exhaust

Energy Balance

$$\Delta\dot{H} + \Delta\dot{E}_k + \Delta\dot{E}_p = Q - W$$

$$\Delta\dot{E}_k, \Delta\dot{E}_p = 0$$

$$Isentropic : reversible, adiabatic \quad \therefore Q = 0,\ s_1 = s_2 = s_3$$

$$\Delta\dot{H} = -W$$

$$(\dot{m}_2 h_2 + \dot{m}_3 h_3) - (\dot{m}_1 h_1) = -W$$

3. $At\ P_3 = 50kPa, T_3 = 100C,$ steam exists as a superheated vapor.

$h_3 = 2682.4kJ/kg, s_3 = 7.6953kJ/kg \cdot K$

1. $P_1 = 3.5MPa, s_1 = s_3 = 7.6953kJ/kg \cdot K \quad (isentropic)$

$T_1 = 704C, h_1 = 3919kJ/kg$

2. $P_2 = 600kPa, s_2 = s_3 = 7.6953kJ/kg \cdot K \quad (isentropic)$

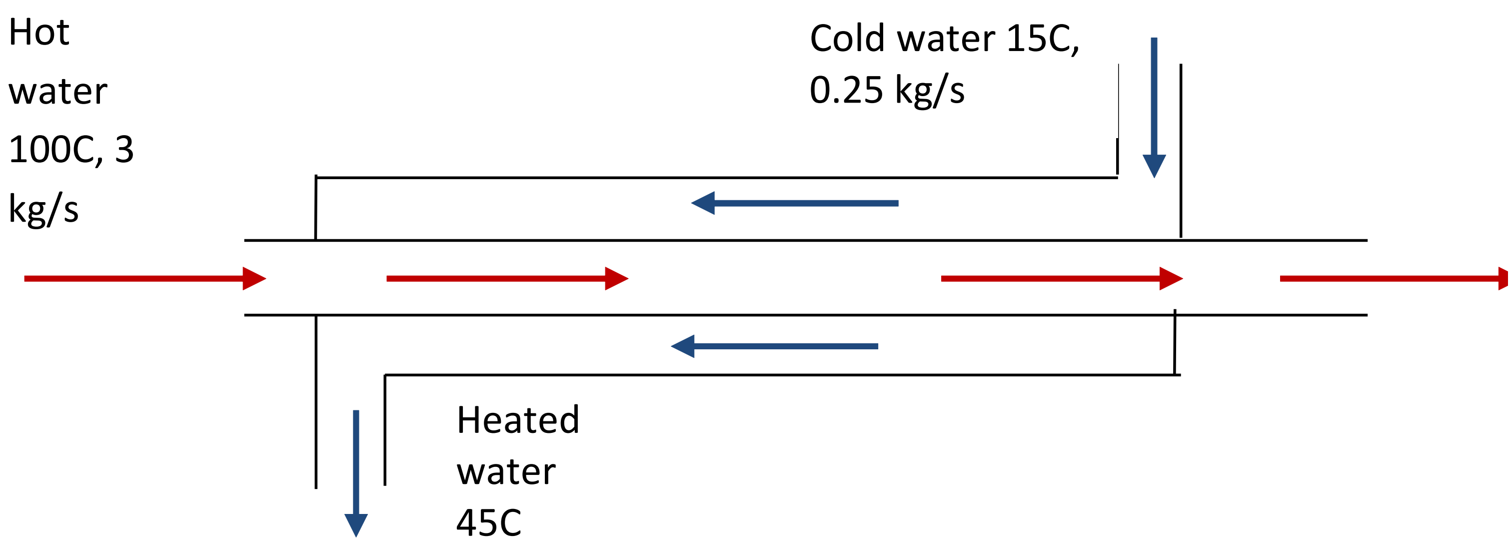
$h_2 = 3261.5kJ/kg$

$$\therefore W = (\dot{m}_1 h_1) - (\dot{m}_2 h_2 + \dot{m}_3 h_3) = \frac{5kg}{s} \cdot \frac{3919kJ}{kg} - \left(\frac{5kg}{s} \cdot 0.05 \cdot \frac{3261.5kJ}{kg} + \frac{5kg}{s} \cdot 0.95 \cdot \frac{2682.4kJ}{kg}\right)$$

$$= 6038.2kJ/s = 6038.2kW$$

# Homework 12-3

Cold water at 15 $^{o}$C enters a thin-walled, double-pipe, counter-flow heat exchanger at a rate of 0.25 kg/s and heated to 45 $^{o}$C by hot water coming from a water heater at 100 $^{o}$C and a rate of 3 kg/s. The heat exchanger is insulated well so the heat transfer between the exchanger and surroundings is negligible. Determine (a) the rate of heat transfer and (b) the rate of entropy generation in the heat exchanger. Assume that the heat capacity of the water is 4.18 kJ/kg-K.



Energy Balance for an open system

for cold water

$\Delta\dot{\mathrm{H}}=\dot{\mathrm{Q}}$

$$\dot{m}h_2-\dot{m}h_1=Q=mc_p(T_2-T_1)=0.25kg/s\times 4.18kJ/kg\cdot K\times(318.15-288.15)K=31.35kJ/s$$

for hot water

$\Delta\dot{\mathrm{H}}=-\dot{\mathrm{Q}}$

$$\dot{m}h_2-\dot{m}h_1=-Q=-mc_p(T_2-T_1)=-3kg/s\times 4.19kJ/kg\cdot K\times(T_2-373.15)K=31.35kJ/s$$

$$\therefore T_2=97.5C$$

Entropy Balance

$$\dot{\mathrm{S}}_{in}-\dot{S}_{out}+\dot{S}_{gen}=\Delta\dot{S}_{system}$$

$$\Delta\dot{S}_{system}=0(\text{steady flow})$$

$$\dot{m}_{cold}S_{cold\_in}+\dot{m}_{hot}S_{hot\_in}-\dot{m}_{cold}S_{cold\_out}-\dot{m}_{hot}S_{hot\_out}+\dot{S}_{gen}=0$$

$$\dot{S}_{gen}=\dot{m}_{cold}(S_{cold\_out}-S_{cold\_in})+\dot{m}_{hot}(S_{hot\_out}-S_{hot\_in})$$

$$=\dot{m}_{cold}c_{p,cold}\ln\left(\frac{T_{cold\_out}}{T_{cold\_in}}\right)+\dot{m}_{hot}c_{p,hot}\ln\left(\frac{T_{hot\_out}}{T_{hot\_in}}\right)$$

$$=0.25\cdot 4.18\cdot\ln\left(\frac{273.15+45}{273.15+15}\right)+3\cdot 4.19\cdot\ln\left(\frac{273.15+97.5}{273.15+100}\right)=0.019kJ/s=0.019kW$$