ABE 201 Biological Thermodynamics 1

Module 9:
1st Law of Thermodynamics
Energy Balances and
Enthalpy

Thermodynamics

- You can't win
- You can't break even
- You can't even get out of the game

Isaac Asimov

Summary

- Energy balances follow continuity equation
- Convention splits energy into "mass associated" and "pure energy" bins
- Mass-associated energy is split into "types" of energy
 - Kinetic energy
 - Potential energy
 - Everything else
- Enthalpy is a term of convenience (combines internal energy and flow work)

Intensive vs Extensive Properties

 Intensive properties do not depend upon amount of something

 Extensive properties <u>depend</u> upon amount of something.

- Intensive: Temperature
- Extensive: Mass, Volume

Converting <u>Extensive</u> Properties into <u>Intensive</u> Properties

- Ratios of extensive properties become intensive
- Mass = extensive, Volume = extensive
- Density (Mass/Volume) = <u>intensive!</u>

- Dividing an extensive property by mass is often called a <u>specific</u> property.
- Vol/mass = specific volume

$$Acc = In - Out$$

$$0 = (E_{mass} + E_{no_mass})_{in} - (E_{mass} + E_{no_mass})_{Out}$$
$$(E_{mass})_{out} - (E_{mass})_{in} = (E_{no_mass})_{in} - (E_{no_mass})_{Out}$$

$$\Delta \dot{H} + \Delta \dot{E}_k + \Delta \dot{E}_p = \dot{Q} - \dot{W}_s$$

$$\left[\sum \left(\hat{H}_{i,out} \dot{m}_{i,out} \right) - \sum \left(\hat{H}_{i,in} \dot{m}_{i,in} \right) \right]$$

$$+ \left[\sum \left(\frac{1}{2} \dot{m}_{i,out} v_{i,out}^{2} \right) - \sum \left(\frac{1}{2} \dot{m}_{i,in} v_{i,in}^{2} \right) \right]$$

$$+ \left[\sum \left(\dot{m}_{i,out} z_{i,out} g \right) - \sum \left(\dot{m}_{i,in} z_{i,in} g \right) \right] = \left[\sum Q_{in} - \sum Q_{out} \right]$$

$$-\left[\Sigma W_{s,out} - \Sigma W_{s,in}\right]$$

Closed System Energy Balance

$$\Delta U + \Delta E_k + \Delta E_p = Q - W$$

Open System Energy Balance

$$\Delta \dot{H} + \Delta \dot{E}_{k} + \Delta \dot{E}_{p} = \dot{Q} - \dot{W}_{s}$$

$$\Delta \dot{H} = \Delta \dot{U} + \Delta \left(\dot{P} \dot{V} \right)$$

Tabulated Enthalpy data

- Ĥ, and Û are tabulated only as ΔĤ or ΔÛ based on a reference state.
 - Common symbols h_f, h_g, h_{fg}, u_f, u_g
- Reference state must be the same in each table, if subtracting values from multiple tables.
- However, ΔĤ values should be the same in any table. i.e. difference between two Ĥ values.

Steam Tables

 Pure water as liquid and vapor...No air or other component present.

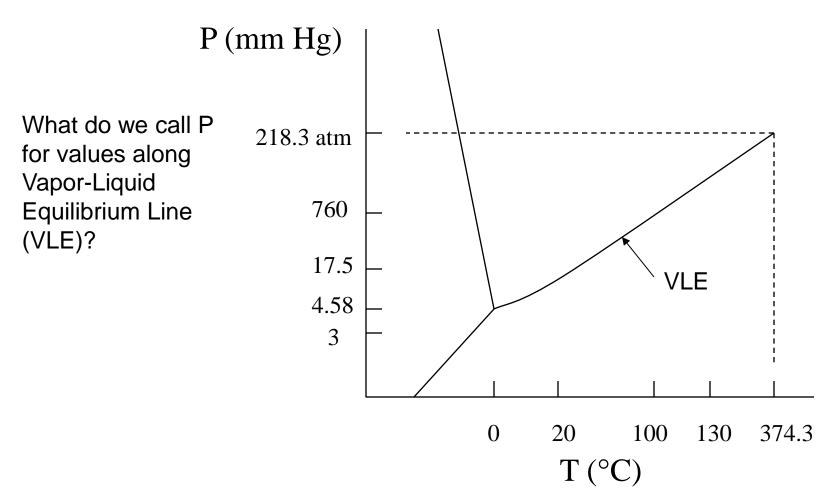


Table 1. Saturation (Temperature)

		Densi	ty, kg/m³	Enthalpy, kJ/kg			Entro	py, kJ/(kg	Volume, cm ³ /g		
t, °C	p, MPa	$ ho_{ t L}$	$ ho_{ extsf{V}}$	$h_{ m L}$	$h_{ m V}$	Δh	$s_{ m L}$	SV	Δs	$v_{ m L}$	$v_{ m V}$
0.01	0.000 611 7	999.79	0.004 855	0.00	2500.9	2500.9	0.000 00	9.1555	9.1555	1.000 21	205 991.
1	0.000 657 1	999.85	0.005 196	4.18	2502.7	2498.6	0.015 26	9.1291	9.1138	1.000 15	192 439.
2	0.000 706 0	999.89	0.005 563	8.39	2504.6	2496.2	0.030 61	9.1027	9.0720	1.000 11	179 758.
3	0.000 758 1	999.92	0.005 952	12.60	2506.4	2493.8	0.045 89	9.0765	9.0306	1.000 08	168 008.
4	0.000 813 5	999.93	0.006 365	16.81	2508.2	2491.4	0.061 10	9.0505	8.9894	1.000 07	157 116.
5	0.000 872 6	999.92	0.006 802	21.02	2510.1	2489.0	0.076 25	9.0248	8.9486	1.000 08	147 011.
6	0.000 935 4	999.89	0.007 266	25.22	2511.9	2486.7	0.091 34	8.9993	8.9080	1.000 11	137 633.
7	0.001 002 1	999.86	0.007 757	29.43	2513.7	2484.3	0.106 37	8.9741	8.8677	1.000 14	128 923.
8	0.001 073 0	999.80	0.008 276	33.63	2515.6	2481.9	0.121 33	8.9491	8.8278	1.000 20	120 829.
9	0.001 148 3	999.74	0.008 826	37.82	2517.4	2479.6	0.136 24	8.9243	8.7881	1.000 26	113 304.
10	0.001 228 2	999.65	0.009 407	42.02	2519.2	2477.2	0.151 09	8.8998	8.7487	1.000 35	106 303.
11	0.001 313 0	999.56	0.010 021	46.22	2521.0	2474.8	0.165 87	8.8754	8.7096	1.000 44	99 787.
12	0.001 402 8	999.45	0.010 670	50.41	2522.9	2472.5	0.180 61	8.8513	8.6707	1.000 55	93 719.
13	0.001 498 1	999.33	0.011 355	54.60	2524.7	2470.1	0.195 28	8.8274	8.6321	1.000 67	88 064.
14	0.001 599 0	999.20	0.012 078	58.79	2526.5	2467.7	0.209 90	8.8037	8.5938	1.000 80	82 793.
15	0.001 705 8	999.06	0.012 841	62.98	2528.3	2465.4	0.224 46	8.7803	8.5558	1.000 94	77 875.
16	0.001 818 8	998.90	0.013 645	67.17	2530.2	2463.0	0.238 97	8.7570	8.5180	1.001 10	73 286.
17	0.001 938 4	998.73	0.014 493	71.36	2532.0	2460.6	0.253 43	8.7339	8.4805	1.001 27	69 001.
18	0.002 064 7	998.55	0.015 385	75.54	2533.8	2458.3	0.267 83	8.7111	8.4433	1.001 45	64 998.
19	0.002 198 3	998.36	0.016 325	79.73	2535.6	2455.9	0.282 18	8.6884	8.4063	1.001 64	61 256.
20	0.002 339 3	998.16	0.017 314	83.91	2537.4	2453.5	0.296 48	8.6660	8.3695	1.001 84	57 757.
21	0.002 488 2	997.95	0.018 354	88.10	2539.3	2451.2	0.310 73	8.6437	8.3330	1.002 05	54 483.
22	0.002 645 3	997.73	0.019 448	92.28	2541.1	2448.8	0.324 93	8.6217	8.2967	1.002 28	51 418.
23	0.002 811 1	997.50	0.020 598	96.46	2542.9	2446.4	0.339 08	8.5998	8.2607	1.002 51	48 548.
24	0.002 985 8	997.25	0.021 806	100.65	2544.7	2444.0	0.353 18	8.5781	8.2250	1.002 75	45 858.
25	0.003 169 9	997.00	0.023 075	104.83	2546.5	2441.7	0.367 22	8.5566	8.1894	1.003 01	43 337.
26	0.003 363 9	996.74	0.024 406	109.01	2548.3	2439.3	0.381 23	8.5353	8.1541	1.003 27	40 973.
27	0.003 568 1	996.47	0.025 804	113.19	2550.1	2436.9	0.395 18	8.5142	8.1191	1.003 54	38 754.
28	0.003 783 1	996.19	0.027 269	117.37	2551.9	2434.6	0.409 08	8.4933	8.0842	1.003 82	36 672.
20	0.004.000.2	005.00	0.000.005	101.55	2552.7	2422.2	0.422.04	0.4705	0.0406	1.004.11	24716

Table 2. Saturation (Pressure)

		Densit	ty, kg/m ³	Enthalpy, kJ/kg			Entro	ppy, kJ/(kg	Volume, cm ³ /g		
p, MPa	t, °C	$ ho_\mathtt{L}$	$ ho_{ ext{V}}$	$h_{\mathtt{L}}$	$h_{ m V}$	Δh	$s_{\mathtt{L}}$	$s_{ m V}$	Δs	v_{L}	$v_{ m V}$
611.657 Pa	0.01	999.79	0.004 855	0.00	2500.9	2500.9	0.000 00	9.1555	9.1555	1.000 21	205 991.
0.0007	1.881	999.89	0.005 518	7.89	2504.3	2496.5	0.028 78	9.1058	9.0770	1.000 11	181 217.
0.0008	3.761	999.92	0.006 264	15.81	2507.8	2492.0	0.057 48	9.0567	8.9992	1.000 08	159 640.
0.0009	5.444	999.91	0.007 005	22.89	2510.9	2488.0	0.082 97	9.0135	8.9305	1.000 09	142 757.
0.0010	6.970	999.86	0.007 741	29.30	2513.7	2484.4	0.105 91	8.9749	8.8690	1.000 14	129 178.
0.0012	9.654	999.68	0.009 202	40.57	2518.6	2478.0	0.145 95	8.9082	8.7623	1.000 32	108 670.
0.0014	11.969	999.46	0.010 650	50.28	2522.8	2472.5	0.180 15	8.8521	8.6719	1.000 54	93 899.
0.0016	14.010	999.20	0.012 086	58.83	2526.5	2467.7	0.210 04	8.8035	8.5935	1.000 80	82 743.
0.0018	15.837	998.93	0.013 511	66.49	2529.9	2463.4	0.236 62	8.7608	8.5241	1.001 08	74 011.
0.0020	17.495	998.64	0.014 928	73.43	2532.9	2459.4	0.260 56	8.7226	8.4620	1.001 36	66 987.
0.0024	20.414	998.08	0.017 738	85.65	2538.2	2452.5	0.302 39	8.6567	8.3544	1.001 93	56 375.
0.0028	22.935	997.51	0.020 522	96.19	2542.8	2446.6	0.338 16	8.6012	8.2631	1.002 49	48 729.
0.0032	25.158	996.96	0.023 282	105.49	2546.8	2441.3	0.369 45	8.5533	8.1838	1.003 05	42 952.
0.0036	27.152	996.43	0.026 021	113.83	2550.4	2436.6	0.397 29	8.5110	8.1138	1.003 58	38 430.
0.0040	28.960	995.92	0.028 743	121.39	2553.7	2432.3	0.422 39	8.4734	8.0510	1.004 10	34 791.
0.0045	31.012	995.30	0.032 122	129.96	2557.4	2427.4	0.450 69	8.4313	7.9806	1.004 73	31 131.
0.0050	32.874	994.70	0.035 480	137.75	2560.7	2423.0	0.476 20	8.3938	7.9176	1.005 33	28 185.
0.0055	34.581	994.13	0.038 816	144.88	2563.8	2418.9	0.499 45	8.3599	7.8605	1.005 90	25 762.
0.0060	36.159	993.59	0.042 135	151.48	2566.6	2415.2	0.520 82	8.3290	7.8082	1.006 45	23 733.
0.0065	37.627	993.06	0.045 436	157.61	2569.3	2411.6	0.540 60	8.3007	7.7601	1.006 99	22 009.
0.0070	39.000	992.55	0.048 722	163.35	2571.7	2408.4	0.559 03	8.2745	7.7154	1.007 50	20 524.
0.0075	40.290	992.06	0.051 994	168.75	2574.0	2405.3	0.576 27	8.2501	7.6738	1.008 00	19 233.
0.0080	41.509	991.59	0.055 252	173.84	2576.2	2402.4	0.592 49	8.2273	7.6348	1.008 48	18 099.
0.0085	42.663	991.13	0.058 498	178.67	2578.3	2399.6	0.607 80	8.2060	7.5982	1.008 95	17 095.
0.0090	43.761	990.69	0.061 731	183.25	2580.2	2397.0	0.622 30	8.1858	7.5635	1.009 40	16 199.

Table 3. Compressed Water and Superheated Steam

0.01 MPa $(t_s = 45.806 ^{\circ}\text{C})$					$0.02 \text{ MPa } (t_s = 60.058 ^{\circ}\text{C})$					$0.03 \text{ MPa } (t_s = 69.095 ^{\circ}\text{C})$			
v	ρ	h	S	t, °C	v	ρ	h	S	t, °C	v	ρ	h	S
1.010 27	989.83	191.81	0.649 20	$t_s(L)$	1.017 16	983.13	251.42	0.832 02	$t_{s}(L)$	1.022 24	978.25	289.27	0.944 07
14 670.	0.068 166	2583.9	8.1488	$t_{\rm s}({ m V})$	7648.0	0.130 75	2608.9	7.9072	$t_{\rm s}({ m V})$	5228.4	0.191 26	2624.5	7.7675
*1.000 20	999.80	-0.03	-0.000 15	0	1.000 20	999.80	-0.02	-0.000 15	0	1.000 19	999.81	-0.01	-0.000 15
1.000 08	999.92	21.03	0.076 25	5	1.000 07	999.93	21.04	0.076 25	5	1.000 07	999.93	21.05	0.076 25
1.000 34	999.66	42.03	0.151 09	10	1.000 34	999.66	42.04	0.151 08	10	1.000 33	999.67	42.05	0.151 08
1.000 94	999.06	62.99	0.224 46	15	1.000 94	999.06	63.00	0.224 46	15	1.000 93	999.07	63.01	0.224 46
1.001 84	998.17	83.92	0.296 48	20	1.001 83	998.17	83.93	0.296 48	20	1.001 83	998.17	83.94	0.296 48
1.003 00	997.01	104.84	0.367 22	25	1.003 00	997.01	104.84	0.367 22	25	1.002 99	997.02	104.85	0.367 22
1.004 41	995.61	125.74	0.436 75	30	1.004 41	995.61	125.75	0.436 75	30	1.004 40	995.62	125.76	0.436 75
1.006 04	993.99	146.64	0.505 13	35	1.006 04	994.00	146.65	0.505 13	35	1.006 03	994.00	146.66	0.505 12
1.007 89	992.18	167.54	0.572 40	40	1.007 88	992.18	167.54	0.572 40	40	1.007 88	992.19	167.55	0.572 39
1.009 92	990.17	188.44	0.638 61	45	1.009 92	990.18	188.44	0.638 61	45	1.009 92	990.18	188.45	0.638 61
14 867.	0.067 263	2592.0	8.1741	50	1.012 15	988.00	209.35	0.703 81	50	1.012 14	988.00	209.36	0.703 80
15 101.	0.066 220	2601.6	8.2036	55	1.014 55	985.66	230.26	0.768 02	55	1.014 55	985.66	230.27	0.768 02
15 335.	0.065 211	2611.2	8.2326	60	1.017 13	983.16	251.18	0.831 29	60	1.017 12	983.16	251.19	0.831 29
15 568.	0.064 233	2620.7	8.2611	65	7764.8	0.128 79	2618.6	7.9360	65	1.019 87	980.52	272.12	0.893 65
15 801.	0.063 285	2630.3	8.2891	70	7882.6	0.126 86	2628.3	7.9646	70	5242.8	0.190 74	2626.3	7.7727
16 034.	0.062 366	2639.8	8.3167	75	8000.2	0.125 00	2638.0	7.9927	75	5322.0	0.187 90	2636.2	7.8013
16 267.	0.061 474	2649.3	8.3439	80	8117.6	0.123 19	2647.7	8.0202	80	5401.0	0.185 15	2646.0	7.8292
16 500.	0.060 607	2658.9	8.3707	85	8234.8	0.121 44	2657.4	8.0474	85	5479.7	0.182 49	2655.8	7.8567
16 732.	0.059 766	2668.4	8.3971	90	8351.8	0.119 73	2667.0	8.0741	90	5558.3	0.179 91	2665.5	7.8837
16 964.	0.058 947	2677.9	8.4232	95	8468.7	0.118 08	2676.6	8.1004	95	5636.8	0.177 41	2675.3	7.9103
	0.058 152	2687.5	8.4489	100	8585.5	0.116 48	2686.2	8.1263	100	5715.1	0.174 97	2685.0	7.9365
	0.057 378	2697.0	8.4742	105	8702.2	0.114 91	2695.8	8.1519	105	5793.3	0.172 61	2694.7	7.9623
	0.056 624		8.4993	110	8818.7	0.113 40	2705.4	8.1771	110	5871.4	$0.170\ 32$	2704.3	7.9877
	0.055 890		8.5240	115	8935.2	0.111 92	2715.0	8.2020	115	5949.5	0.168 08	2714.0	8.0128
18 124.	0.055 176	2725.6	8.5484	120	9051.6	0.110 48	2724.6	8.2266	120	6027.4	0.165 91	2723.7	8.0375

Steam Tables

- NIST Tables
 - What is the Reference State?
 - What does p(MPa) in 2nd column represent?
 - Compare the saturation vapor pressure of water at 4 C to the pressure given in the steam table for 4 C
- Can vapor exist at 4C and 0.000 9 MPa?
- Can vapor exist at 4C and 0.000 7 bar?

- The reference state is liquid water at the triple point
- The pressures are the <u>saturation vapor</u> <u>pressures</u> for water at that temperature
- 1 MPa = 10 bar
- No at 4 C and P=0.009 the water is liquid!
 Not on the VLE line!
- Yes, at 4 C and P = 0.007 the water is superheated steam! Below VLE line.

Steam Tables

- Look at V water (far right)
 - As T increases → V increases for liquid water (but very little)
- Look at V steam
 - As T increases → V decreases for steam.
 Why?
 - Can we assume this steam is ideal gas?
 - Does PV/RT = 1.0 at T = 10 C?
 - How about T = 100 C?

At 10C, PV/RT = 0.998 At 100C, PV/RT = 0.983

Example use of Tables

- 1. Find $\hat{H}(H_2O(liquid)) = 0$
- 2. Find $\hat{H}(H_2O(vapor))$ 24C & 1.0 MPa) =
- 3. Find $\hat{H}(H_2O(vapor))$ sat'd & 1.0 MPa) =
- 4. What is specific volume at 640 C & 2.0 MPa?
- 5. Find m for 1000 m³/h of sat'd steam at 1.0 MPa.
- 6. m for 1000 m³/hr of steam at 1.0 MPa and 700C.
- 7. What is Kinetic energy of H₂O(vapor) 250 kg/min at 0.5 MPa, 310 C through a 0.5 meter dia pipe?

- 1. We have to assume H is independent of P for liquid. H = 100.65 kJ/kg
- 2. Doesn't exist because sat pressure of water vapor at 24C is 0.00298 MPa!
- 3. H = 2777.1 kJ/kg
- 4. Superheated steam table -> H = 3781.0 kJ/kg
- 5. 1000 m³/h of sat'd steam at 1.0 mPa = 1000 m3/h / (194.36 cm3/g * $1000g/kg * 1m3/100^3cm3$) = 5145 kg/h
- 6. v = 0.44783 m3/kg mdot = 1000 m3/hr / 0.44783 kg/m3 = 2233 kg/h
- 7. V =0.53216 m3/kg. U(velocity) = 250 kg/min x 1min/60s x 0.53216 m3/kg / (pi*0.25^2) = 11.3 m/s

 $Ek = \frac{1}{2} \times m \times u^2 = \frac{1}{2} \times (250 \text{ kg/min} \times 1/60) \times 11.3^2 \times 1N/1 \text{kg m/s}^2$ = 266 W

Steam Calculators

- Many available, some free
 - Windows, Mac, Linux
 - Android and iOS apps

- National Institute of Standards and Technology (NIST)
 - http://webbook.nist.gov/chemistry/fluid/

Practical Steam

- In processing conditions, steam is usually expressed in terms of <u>gauge</u> pressure.
- In the U.S. this usually means psi.
- In common engineering parlance, this is expressed as "XX pound steam"
- Example: What is the temperature and specific enthalpy of saturated 150 pound steam?

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(150 + 14.7) psi * 0.101325 MPa / 14.696 psi
= 1.136 MPa
T = 185 C = 366 F
H = 2781 kJ/kg = 1196 BTU/lb
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Steam Quality

- In some situations, it is possible to have "wet" steam, i.e. steam with small droplets (mist) of liquid water.
- The <u>quality</u> of wet steam is the percent/fraction of the mixture that is vapor
- The mixture is at <u>saturation</u> conditions by definition.

Example of Steam Quality

 10 bar steam is 75% quality. What is the specific enthalpy of the mixture?

 How much heat must be removed to completely condense this steam? 10 bar = 1.0 MPa

HI = 762.52 kJ/kg, Hv = 2777.1 kJ/kg

H = (0.25)(762.52) + (0.75)(2777.1)= **2274.2** kJ/kg

Q = (0.75)Hfv = 0.75(2777.1 - 762.52) = 1510.9 kJ/kg

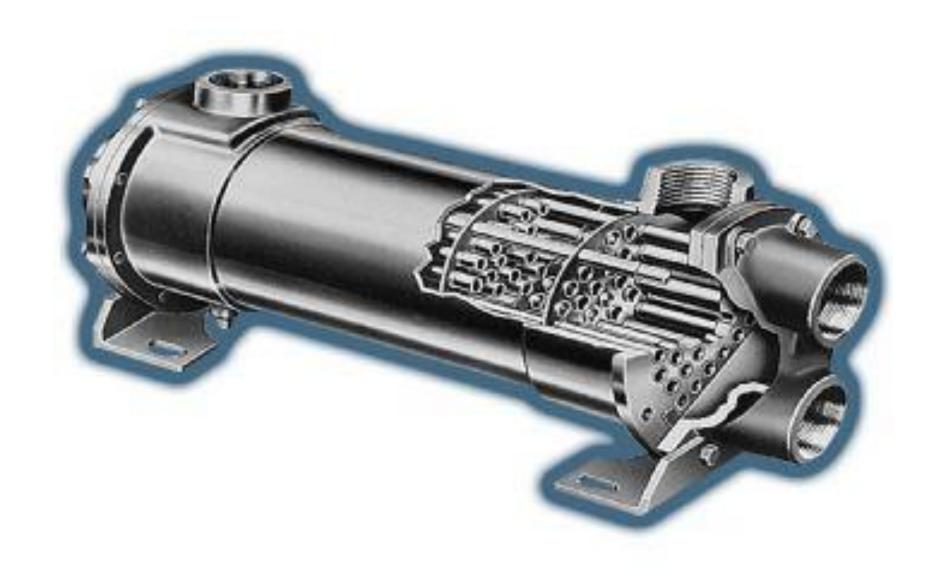
Energy Balance Steps

- 1. Attempt to solve material balances
- 2. Write energy balance form (open or closed) and simplify
- Choose reference state
- Construct a table for enthalpies or internal energies
- Solve for enthalpies or internal energies, and find changes
- 6. Solve for potential and kinetic energy changes
- Solve for the variable of interest usually Q and/or W_S

Pasteurization of Milk

High temperature, short time (HTST) pasteurization involves heating milk to 72C and holding for 15 seconds before re-cooling. Milk (100 L/sec) enters an adiabatic shell-and-tube heat exchanger at 4 C and leaves at 72 C. Steam (10 bar, saturated) is used to heat the milk. The exiting steam is at 10 bar and 94 C. There are no significant height or velocity changes for either fluid.

Assuming the thermodynamic properties of milk are that of water, how much steam is required by this process?



Shell and Tube Heat Exchanger







Shell and Tube Heat Exchangers

Pasteurization of Milk



	In	Out			
Tube	T = 4 C	T = 72 C			
	H = 16.8 kJ/kg	H = 301.4 kJ/kg			
	V = 0.0010	V = 0.0010			
Shell	P = 10 bar	T = 94 C			
	T = 179.9	H = 393.8 kJ/kg			
	H= 2776.2 kJ/kg				

$$\Delta \dot{H} + \Delta \dot{E}_k + \Delta \dot{E}_p = \dot{Q} - \dot{W}_s$$

$$\Delta \dot{H} = 0$$

$$\left(\hat{H}_{\textit{milk},\textit{out}} m_{\textit{milk}} + \hat{H}_{\textit{steam},\textit{out}} m_{\textit{steam}}\right) - \left(\hat{H}_{\textit{milk},\textit{in}} m_{\textit{milk}} + \hat{H}_{\textit{steam},\textit{in}} m_{\textit{steam}}\right) = 0$$

$$\left(\hat{H}_{milk,out} - \hat{H}_{milk,in}\right) m_{milk} + \left(\hat{H}_{steam,out} m_{steam} - \hat{H}_{steam,in} m_{steam}\right) = 0$$

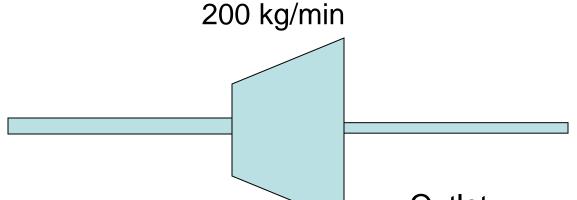
$$(301.4 - 16.8)$$
 * $[100 \text{ L/s}$ * $(1 \text{ m/}1000 \text{ L})$ / $(0.0010 \text{ m3/kg})]$ + $(393.8 - 2776.2)$ * $m_{\text{steam}} = 0$

$$(284.6 \text{ kJ/kg}) * [100 \text{ kg/s}] + (-2382.4 \text{ kJ/kg}) * m_{\text{steam}} = 0$$

$$m_{steam} = (28460 \text{ kJ/s}) / (2382.4 \text{ kJ/kg}) = 11.9 \text{ kg/s}$$

Steam Turbine

- Two hundred kg/min of steam enters a steam turbine at 350C and 40 bar through a 7.5-cm diameter line and exits at 75C and 5 bar through a 5 cm line after expanding adiabatically. The exiting stream may be vapor, liquid, or "wet" steam.
- If the exiting stream is "wet," what is its temperature?
- How much energy is transferred to the turbine?
 (Neglect ΔE_p but not ΔE_k)



ID = 7.5 cm

Inlet

T = 350 C

P = 40 bar = 4.0 MPa

Superheated

H = 3093.3 kJ/kg

 $V = 0.0665 \text{ m}^3/\text{kg}$

Outlet

ID = 5 cm

T = 75 C

P = 5 bar = 0.5 MPa

 T_{sat} = 151.8 C, all liquid

H = 314.03 kJ/kg

 $V = 1.03 \times 10^{-3} \text{ m}^3/\text{kg}$

$$\Delta \dot{H} + \Delta \dot{E}_k + \Delta \dot{E}_p = \dot{Q} - \dot{W}_s$$

$$\Delta \dot{H} + \Delta \dot{E}_k = -\dot{W}_s$$

$$(\hat{H}_{out} - \hat{H}_{in})m + \frac{1}{2}m(v_{out}^2 - v_{in}^2) = -\dot{W}_{s}$$

(314.03 - 3093.3)(3.33kg / s)

$$+\frac{1}{2}3.33kg / s \left[\frac{3.33*1.03x10^{-3}}{3.14*0.025^{2}} \right]^{2} - \left[\frac{3.33*0.0665}{3.14*0.0375^{2}} \right]^{2} \right] = -\dot{W}_{s}$$

$$\Delta v = (1.74m / s - 50.2m / s) = 48.5m / s$$

$$-9255 - 4.183 = -W_s$$

$$W_s = 9259kW$$