

# ABE 201

# Biological Thermodynamics 1

Degree of Freedom Analysis  
and  
Multiple Unit Operations

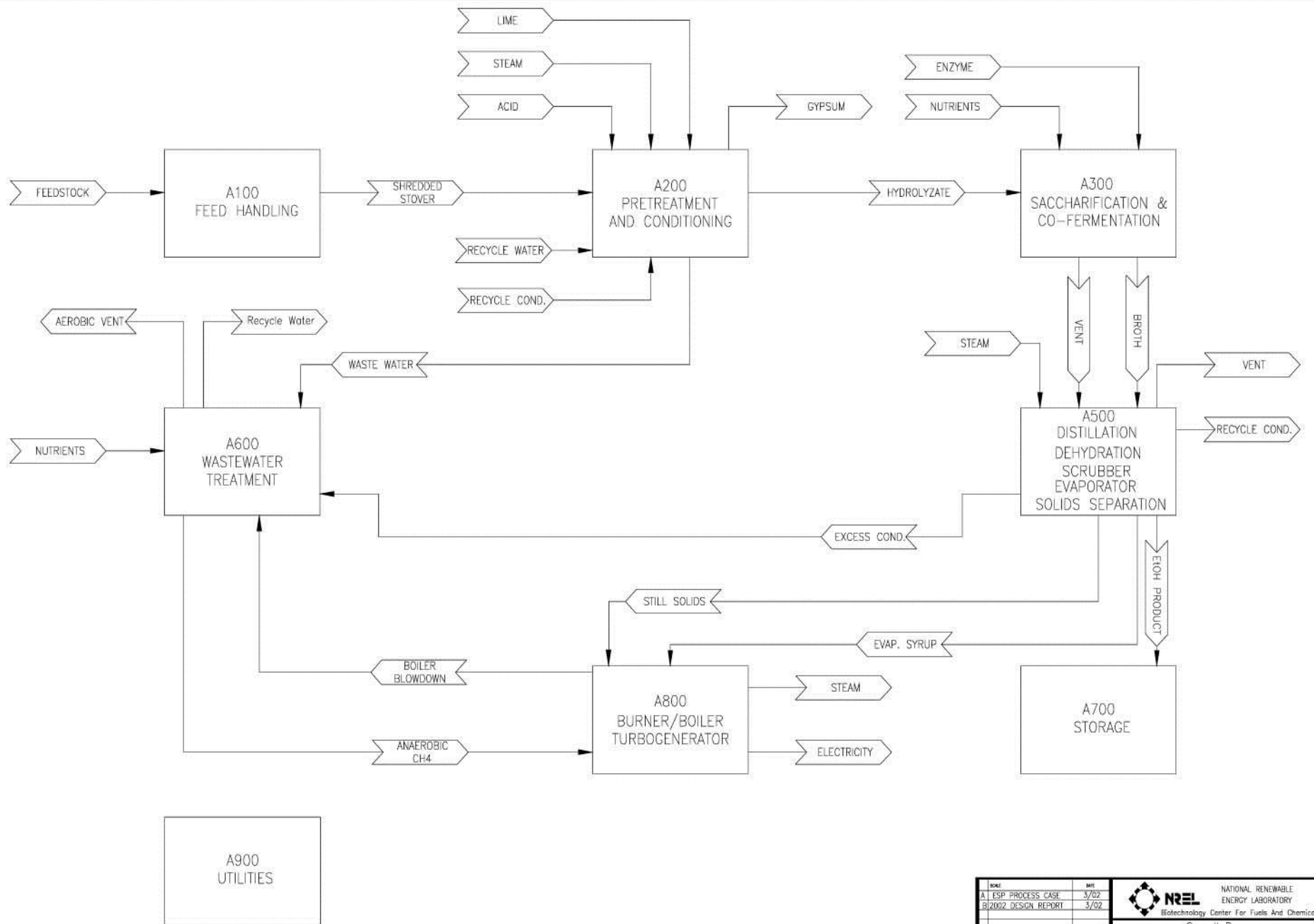
# Topics for Today

Analyzing complex biological processes

- Defining systems and subsystems
- Using Degree of Freedom Analysis

# Complex Biological Systems

- Biological processes have multiple unit operations
- Processes can be broken into multiple systems for analysis
- Determine which individual system (unit) to solve first by degree of freedom analysis
- Solve each of the systems, in order by DOF



# Process Flow Diagrams/Sheets

## “PFD” or “PFS”

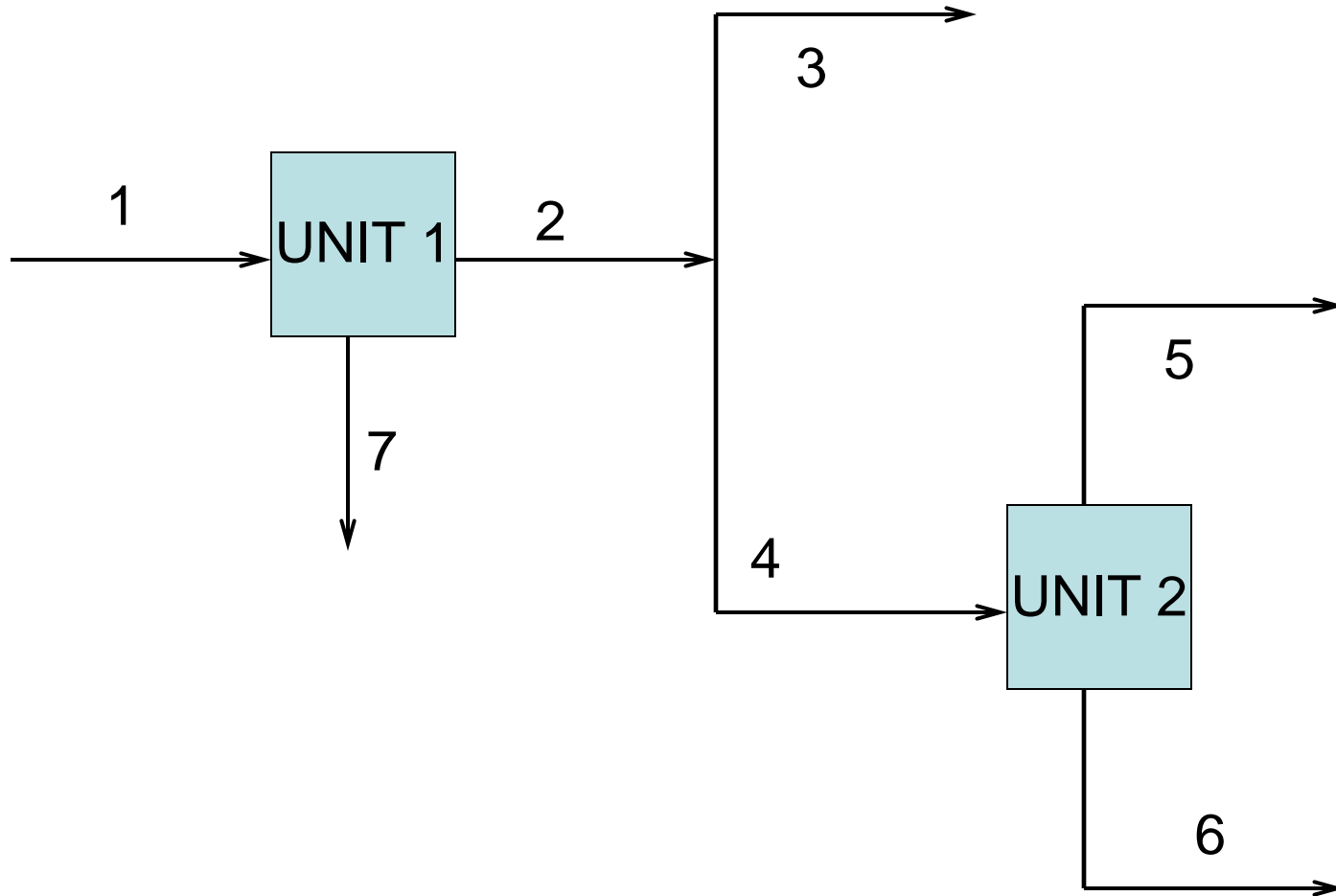
- Made of Unit Operations
  - Filtration, Fermentation, Distillation, Centrifugation, Drying
- Example from NREL
  - Processing Facility to Convert Corn Stalks to Biofuel (ethanol) and Biopower (electricity)
  - 25 PFDs with all stream compositions
  - 28 “compounds” tracked and balanced
  - Thermal energy tracked and balanced

# 28 “Compounds” in Mass Balance

Individual Compounds	Soluble Solids (SS)	Insoluble Solids (IS)
Water	Glucose	Cellulose
Ethanol	Xylose	Xylan
Acetic Acid	Arabinose	Arabinan
Sulfuric Acid	Other Sugars	Other Sugar Polymers
Furfural	Cellobiose	Lignin
HMF	Glucose Oligomers	Gypsum
Carbon Dioxide	Xylose Oligomers	Ca(OH) <sub>2</sub>
Methane	Other Oligomers	
Oxygen	"Corn Steep Liquor"	
Nitrogen	Other SS	
Others		

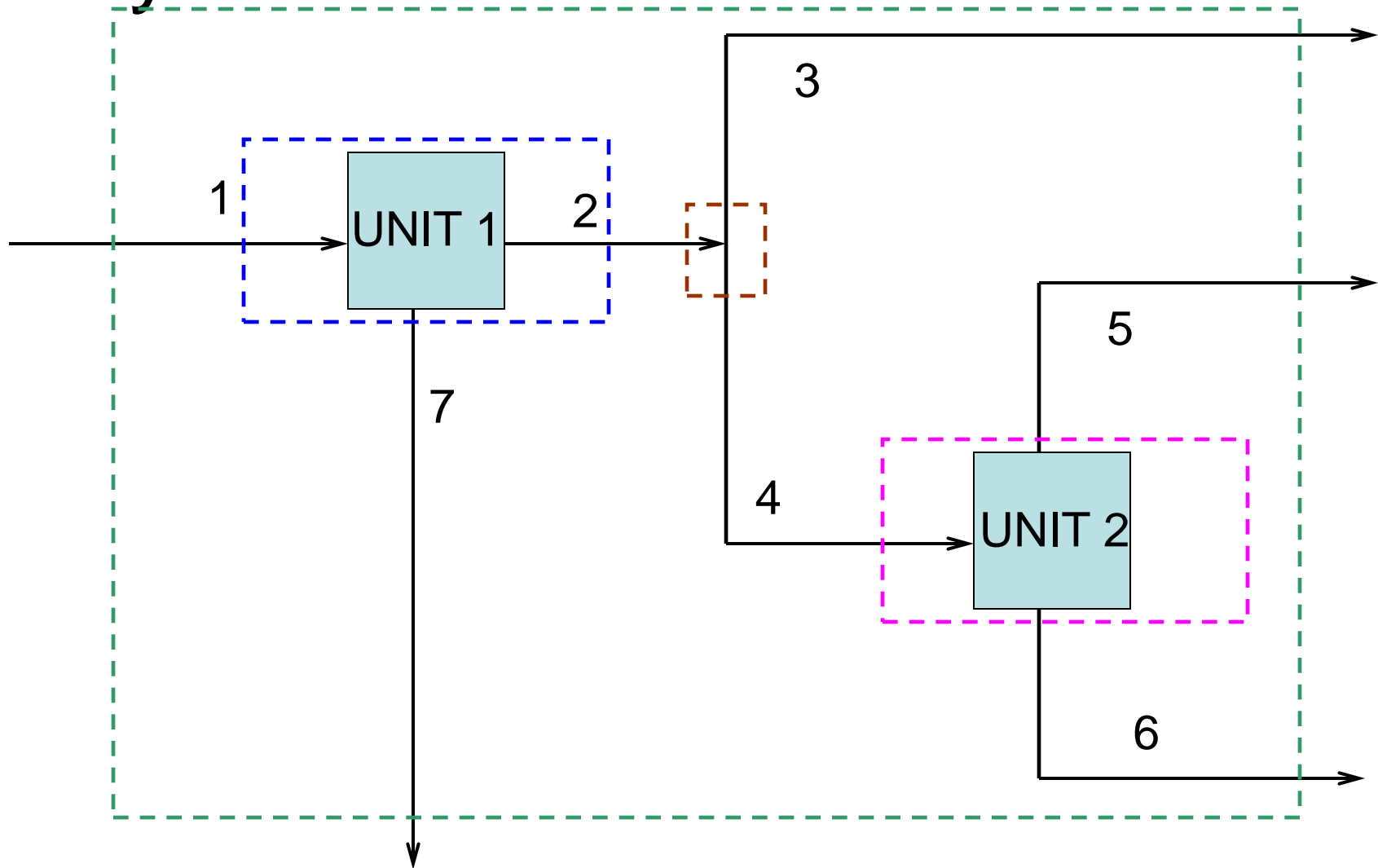


# Example:

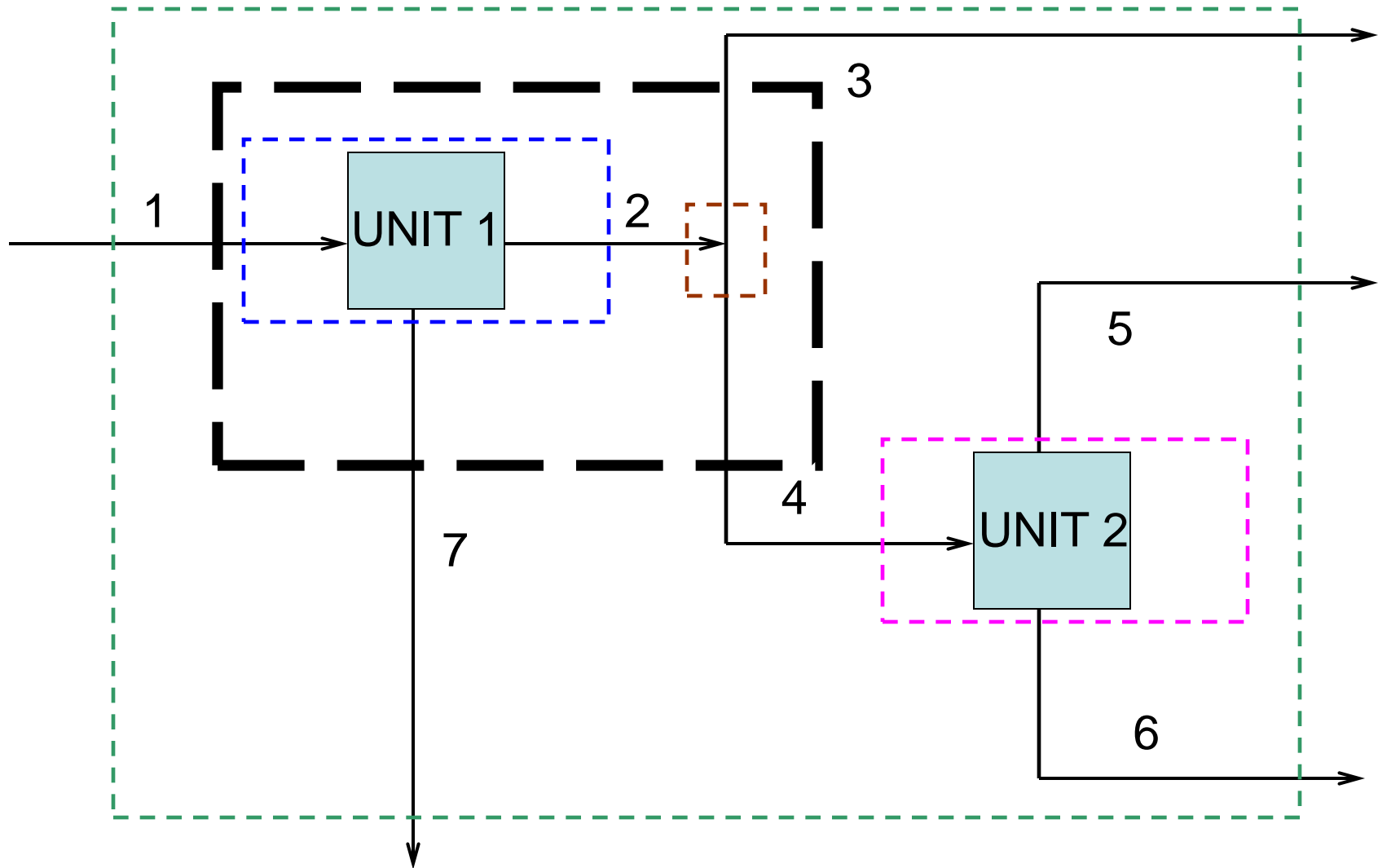




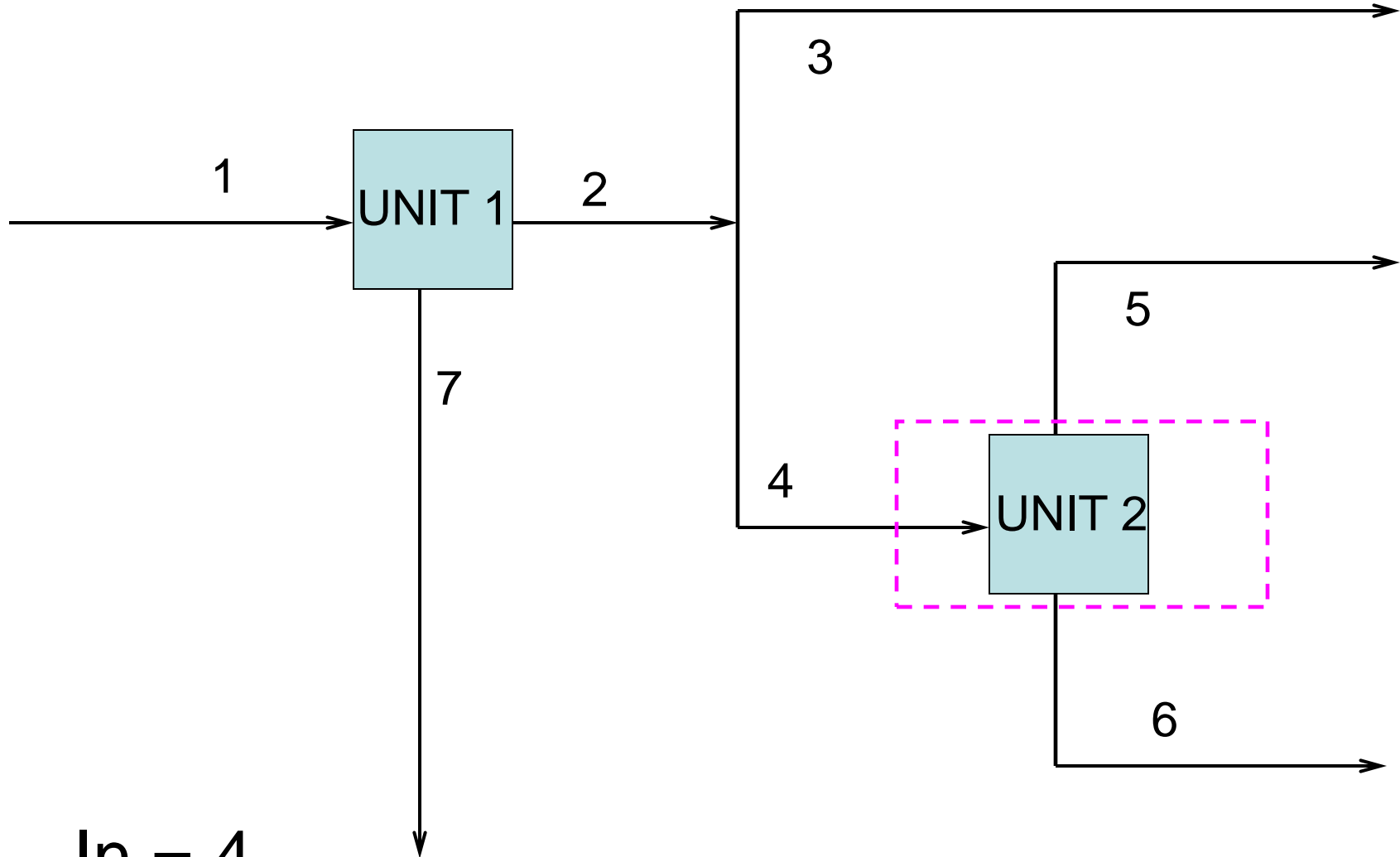
# Systems:



# Have We Drawn All Systems...



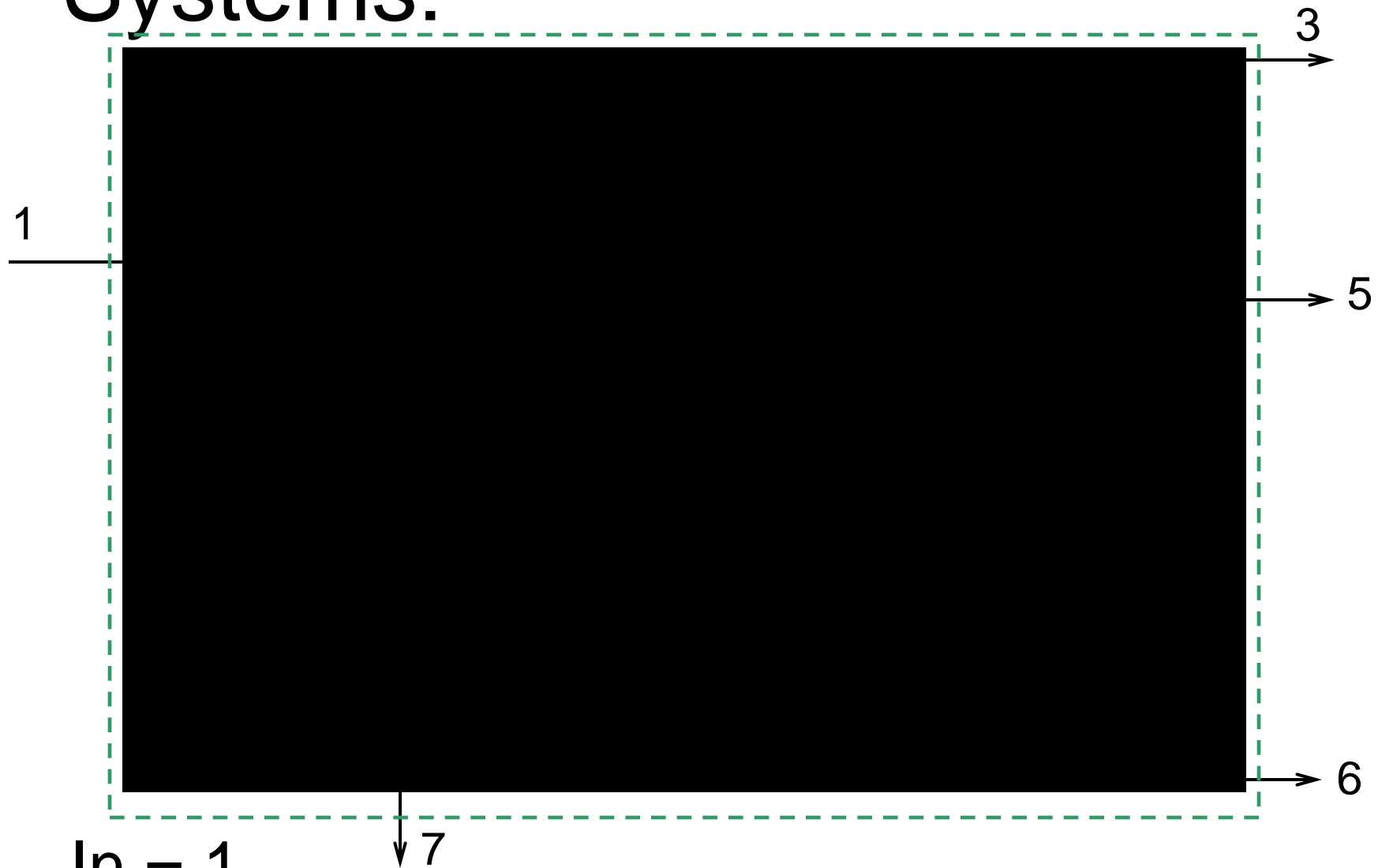
# Systems:



In = 4

Out = 5 + 6

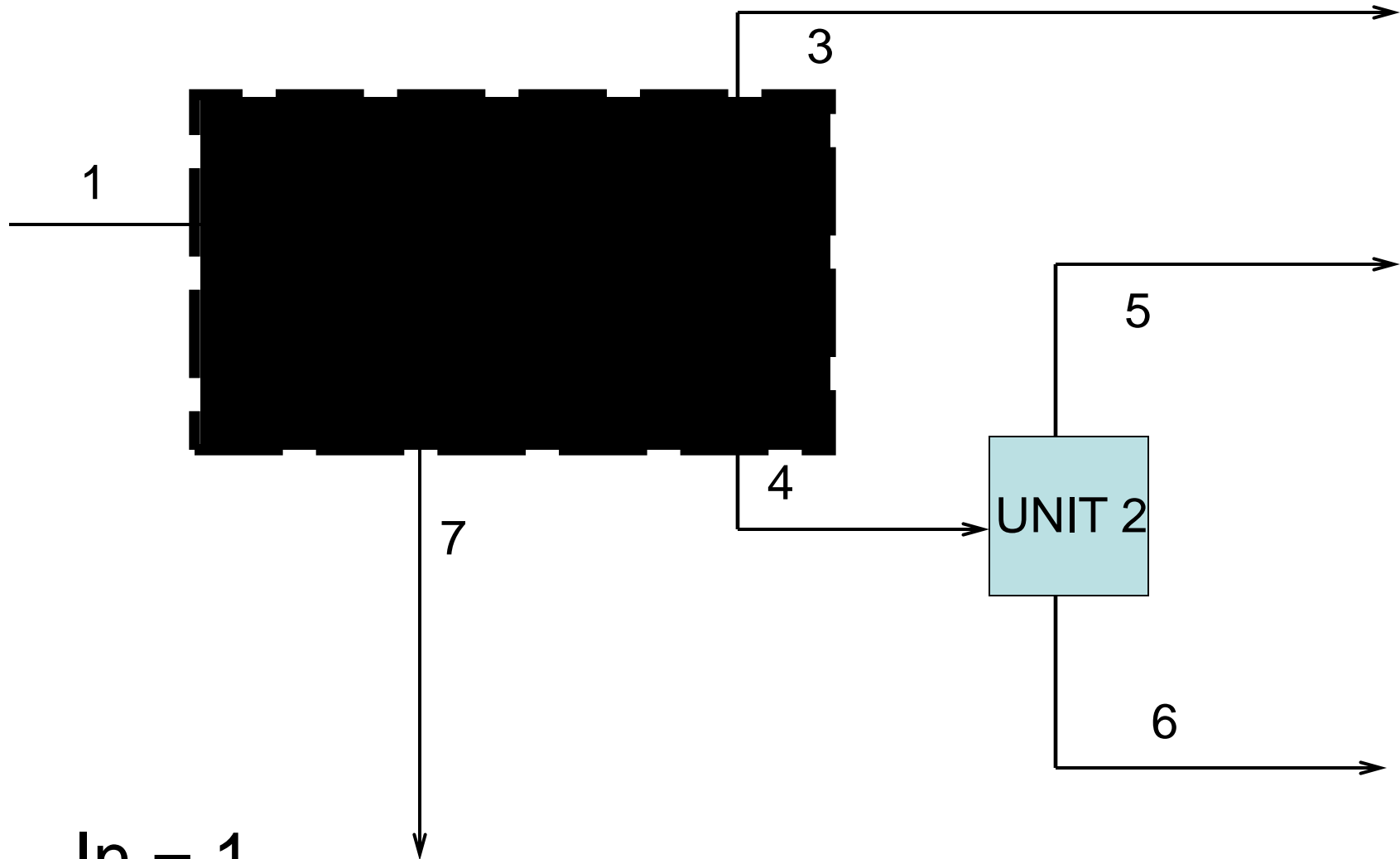
# Systems:



In = 1

Out = 3 + 5 + 6 + 7

# Systems:



$$\text{In} = 1$$

$$\text{Out} = 3 + 4 + 7$$

# Solving Complex Processes

- Some systems may have unique solutions
- Some systems may not have unique solutions
- How do we go about deciding how to proceed?

# Degree of Freedom Analysis

Variables	Equations	Unique Solution
X	$2X = 16$	$X = 8$
X	$2X + Y = 7$	$X = 2$
Y	$X + 4Y = 14$	$Y = 3$
X	$2X + Y + Z = 7$	$X = 1$
Y	$X + 4Y - 3Z = 0$	$Y = 2$
Z	$3X - Y + 5Z = 16$	$Z = 3$

# Degrees of Freedom Cont'd.

- Once your system is defined and knowns & unknowns identified:
  - Count unknown variables ( $V$ )
  - Count independent eqns.
    - **Material balances (B)** on individual components or the total system, note: maximum number of eqns. Is equal to the number of chemical species for a non reactive system.
    - **Physical property eqns. (P):** (ie. relations of mass, volume, density, specific gravity etc..)
    - **Physical constraints (C):** fractions of moles or mass must add to one because they are fractions.



# Degrees of Freedom Cont'd.

$DF = \# \text{ Unknowns} - \# \text{ Independent Eqns.}$

$DF = V - B - P - C$

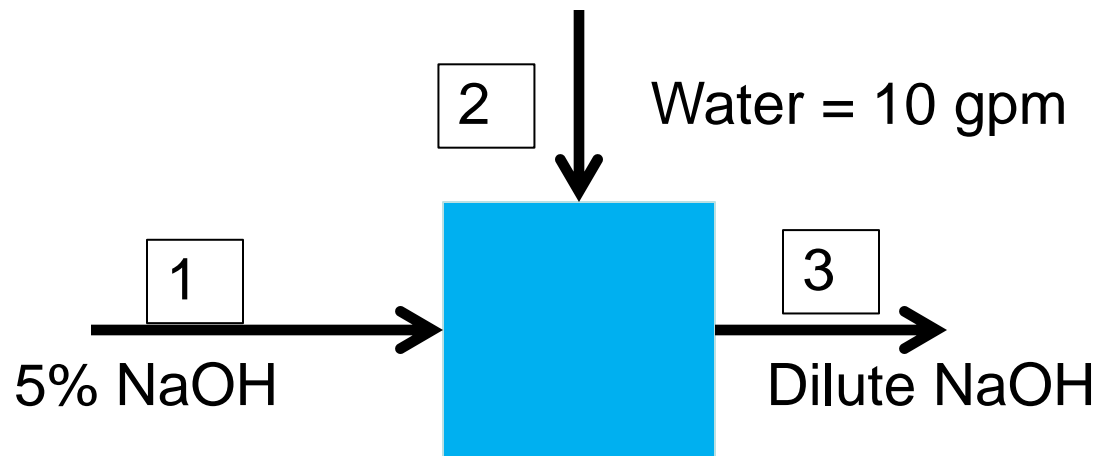
$DF > 0$  - underspecified problem, infinite solns.

$DF = 0$  - problem can be solved exactly

$DF < 0$  - overspecified, contains redundancy,  
and possibly an erroneous relationship if  
indeed all of your equations are  
“independent”.

# Sodium Hydroxide Example

- A 5.00% wt solution of sodium hydroxide is being diluted by a 10.0 gpm stream of pure water.
- What is the weight percent of sodium hydroxide in the stream exiting the system at  $2.00 \times 10^2$  lbm/min?
- Assume steady-state conditions.



Composition	1	2	3
NaOH	0.05	0	?
Water	x1w	1	?

Mass	1	2	3
NaOH	?	0	?
Water	?	?	?
Total	?	?	200 lbm

Mass Balance	1	2	3
NaOH	$0.05 * m1$	0	$X3n * 200$
Water	$0.95 * m1$	83.4 lbm	$X3w * 200$
Total	$m1$	83.4 lbm	200 lbm

Variables (V) =  $m1$   $X3n$   $X3w$  = 3

Balances (B) = NaOH Water = 2

Physical Properties (P) =  $\rho_{\text{water}} = 8.34 \text{ lbm/gal}$  = 0

Constraints (C) =  $X3n + X3w = 1$  = 1

$$DF = V - B - P - C = 3 - 2 - 0 - 1 = 0$$

System is solvable

# Tips for DOF Analysis

- Solve what you can without complex algebra!
- If you have a **physical property** that allows you to calculate mass or energy, use it to eliminate a variable!
- Remember that fractions must sum to 1 (**physical constraint**). If you can solve missing fractions based on what you're given, do so! Eliminate a variable!

# Summary

- Defining Systems in Complex Processes
  - Boundaries must be contiguous
  - Boundaries define what's in and what's out!
  - Careful selection of system boundaries can be used to solve complex problems
- Degree of Freedom Analysis
  - Determine if a unique solution is possible for the system as you have defined it.
  - The number of equations must equal or exceed the number of unknown variables